[12]L. A. Malov and V. G. Solov'ev, Yad. Fiz. **5**, 566 (1967) [Sov. J. Nucl. Phys. **5**, 403 (1967)]; A. L. Komov, L. A. Malov, and V. G. Solov'ev, Soobshchenie (Communication), JINR R4-5693 (1971); S. P. Ivanova, A. L. Komov, L. A. Malov, and V. G. Solov'ev, Izv. Akad. Nauk SSSR, Ser. Fiz. **37**, 911 (1973); S. P. Ivanova, A. L. Komov, L. A. Malov, and V. G. Solov'ev, Preprint JINR R4-8582 (1975); Izv. Akad. Nauk SSSR, Ser. Fiz. **39**, 1612 (1975).
[13]F. A. Gareev, S. P. Ivanova, L. A. Malov, and V. G. Soloviev, Nucl. Phys. A **171**, 134 (1971).
[14]R. V. Jolos, V. G. Soloviev, and K. M. Zheleznova, Phys. Lett. B **25**, 393 (1967); R. V. Jolos, U. M. Finer, V. G. Soloviev, and K. M. Zheleznova, Phys. Lett. B **27**, 614 (1968).
[15]G. Kyrchev, V. G. Solov'ev, and Ch. Stoyanov, Izv. Akad. Nauk SSSR, Ser. Fiz. (1975); Preprint JINR R4-8611 (1975).
[16]S. P. Ivanova, A. L. Komov, G. Kyrchev, V. G. Solov'ev, and Ch. Stoyanov, Tezisy XXVI Soveshchaniya po Yadernoĭ Spektroskopii i Strukture Yadra (Proc. 26th Conference on Nuclear Spectroscopy and Nuclear Structure), Baku (1976).
[17]S. P. Ivanova, A. L. Komov, G. Kyrchev, V. G. Soloviev, and Ch. Stojanov, Preprint JINR E4-9010 (1975).
[18]V. G. Soloviev, Nucl. Phys. **69**, 1 (1965).
[19]A. Bohr, Nuclear Structure, Dubna Symposium 1968, IAEA, Vienna (1968), p. 179; J. O. Rasmussen, Nuclear Structure, Dubna Symposium 1968, IAEA, Vienna (1968), p. 169; R. A. Sorensen, Nuclear Structure, Dubna Symposium 1968, IAEA, Vienna (1969), p. 27; S. T. Belyaev, Nuclear Structure, Dubna Symposium 1968, IAEA, Vienna (1968), p. 155; N. I. Pyatov, Arkiv. Fys. **36**, 667 (1967).
[20]B. N. Kalinkin, Ya. Grabovski, and F. A. Gareev, Acta. Phys. Pol. **30**, 999 (1966); F. A. Gareev, S. P. Ivanova, and B. N. Kalinkin, Acta Phys. Pol. **30**, 461 (1966); F. A. Gareev, S. P. Ivanova, and B. N. Kalinkin, Izv. Akad. Nauk SSSR, Ser. Fiz. **33**, 1690 (1968).
[21]F. A. Gareev, S. P. Ivanova, V. G. Solov'ev, and S. I. Fedotov, Fiz. Élem. Chastits At. Yadra. **4**, 357 (1973) [Sov. J. Part. Nucl. **4**, 148 (1973)].
[22]F. A. Gareev, S. P. Ivanova, and V. V. Pashkevich, Yad. Fiz. **11**, 1200 (1970) [Sov. J. Nucl. Phys. **11**, 667 (1970)]; Preprint JINR E4-4704 (1969); S. P. Ivanova, A. L. Komov, and N. Yu. Shirikova, Soobshchenie (Communication), JINR R4-8406 (1974).
[23]C. E. Bemis *et al.*, Phys. Rev. C **8**, 1466 (1973).
[24]J. L. C. Ford *et al.*, Phys. Rev. Lett. **27**, 1232 (1971); F. K. McGowan *et al.*, Phys. Rev. Lett. **27**, 1741 (1971).
[25]V. M. Strutinskiĭ, Yad. Fiz. **3**, 614 (1966) [Sov. J. Nucl. Phys. **3**, 449 (1966)]; Nucl. Phys. A **95**, 420 (1967).
[26]N. P. Newly, Phys. Rev. **125**, 2063 (1962); N. I. Pyatov, Izv. Akad. Nauk SSSR, Ser. Fiz. **27**, 1436 (1963); N. I. Pyatov and A. S. Chernyshev, Izv. Akad. Nauk SSSR, Ser. Fiz. **28**, 1173 (1964).
[27]E. Arbman *et al.*, Nucl. Phys. **21**, 406 (1960); M. Arnoux *et al.*, Compt. Rend. **169**, 317 (1969); M. Herment *et al.*, Compt. Rend. **273**, 801, 1053 (1971); W. Kurcewicz *et al.*, J. Phys. (Paris) **34**, 159 (1973); J. Dalmasso *et al.*, C.R. Acad. Sci. Ser. B **278**, 97 (1974); J. Dalmasso *et al.*, Compt. Rend. **273**, 509, 568 (1971).
[28]V. M. Gorbachev, Yu. S. Zamyatin, and A. A. Lbov, Osnovnye Kharakteristiki Izotopov Tyazhelykh Élementov (Basic Quantum Numbers of Isotopes of Heavy Elements), Second Edition, Atomizdat, Moscow (1975).
[29]Th. W. Else and J. R. Huizenga, Nucl. Phys. A **187**, 545 (1972).
[30]F. K. McGowan *et al.*, Phys. Rev. C **10**, 1146 (1974).
[31]J. V. Maher *et al.*, Phys. Rev. C **5**, 1380 (1972).
[32]R. F. Casten *et al.*, Phys. Lett. B **40**, 333 (1972).
[33]B. B. Back *et al.*, Nucl. Phys. A **217**, 116 (1973).
[34]W. R. McMurray *et al.*, Z. Phys. **253**, 289 (1972).
[35]F. S. Stephens, F. Asaro, and I. Perlman, Phys. Rev. **100**, 1543 (1955).
[36]T. Yamazaki, Nucl. Data A **1**, 453 (1966).
[37]F. S. Stephens *et al.*, Phys. Rev. Lett. **15**, 420 (1965).
[38]S. Bjørnholm *et al.*, Nucl. Phys. A **118**, 261 (1968).
[39]K. Neergard and P. Vogel, Nucl. Phys. **149**, 217 (1970).
[40]S. Bjørnholm *et al.*, Nucl. Phys. **42**, 469 (1963).
[41]S. A. Baranov *et al.*, Zh Éksp. Teor. Fiz. **41**, 1475 (1961) [Sov. Phys. JETP **14**, 1053 (1962)].
[42]R. E. Bell *et al.*, Kgl. Dan. Vid. Selsk. Mat.-Fys. Medd. **32**, No. 12 (1960).
[43]L. Varnell, Nucl. Phys. A **144**, 429 (1970).
[44]C. M. Lederer, Thesis, Univ. California (1963); UCRL-11028 (1963).
[45]S. A. Baranov *et al.*, Yad. Fiz. **5**, 241 (1967) [Sov. J. Nucl. Phys. **5**, 169 (1967)].
[46]A. Bjørnholm and O. B. Nielsen, Nucl. Phys. **42**, 642 (1963).
[47]A. H. Wapstra, Nucl. Phys. A **97**, 641 (1967).
[48]A. H. Wapstra, Physica **37**, 261 (1967).
[49]G. Ardisson and C. Ardisson, Compt. Rend. Ser. B **280**, 377 (1975).
[50]P. W. De Lange *et al.*, Nuovo Cimento **14**, 681 (1959).
[51]Ong Ping Hok, Physica **22**, 465 (1956).
[52]P. G. Hansen *et al.*, Phys. Lett. B **24**, 95 (1967).
[53]C.-F. Leang, Compt. Rend. **255**, 3155 (1962).
[54]S. A. Baranov *et al.*, Yad. Fiz. **7**, 727 (1968) [Sov. J. Nucl. Phys. **7**, 442 (1968)].
[55]E. K. Hyde *et al.*, The Nuclear Properties of the Heavy Elements, Vol. II, Prentice-Hall, Englewood Cliffs, New Jersey (1964).
[56]S. Bjørnholm *et al.*, Phys. Rev. **130**, 2000 (1963).
[57]S. Bjørnholm *et al.*, Nucl. Phys. A **118**, 241 (1968).
[58]J. S. Boyno *et al.*, Nucl. Phys. A **209**, 125 (1973).
[59]N. Trautmann *et al.*, Proc. 3rd Intern. Protactinium Conf. Schloss Elman, Germany (1969).
[60]N. Trautmann *et al.*, Z. Nat. a **23**, 2127 (1968).
[61]C. M. Lederer *et al.*, Nucl. Phys. A **135**, 36 (1969).
[62]S. A. Baranov *et al.*, Zh. Éksp. Teor. Fiz. **43**, 1135 (1962) [Sov. Phys. JETP **16**, 801 (1963)].
[63]A. Backlin *et al.*, Lund. Proc. Intern. Symp. Neutron Capture Gamma-Ray Spectr., Studsvik, IAEA (1969), p. 141.
[64]W. R. Kane *et al.*, Phys. Rev. Lett. **25**, 953 (1970).
[65]J. S. Boyno *et al.*, Bull. Amer. Phys. Soc. **17**, 463 (1972).
[66]K. Katori *et al.*, Phys. Rev. C **8**, 2336 (1973).
[67]S. A. Baranov *et al.*, Yad. Fiz. **1**, 557 (1965) [Sov. J. Nucl. Phys. **1**, 397 (1965)].
[68]F. S. Stephens *et al.*, in: Coulomb Excitation (eds. K. Alder and W. Winter), Academic Press, New York (1966), p. 208.
[69]R. M. Diamond and F. S. Stephens, Arkiv. Fys. **36**, 221 (1967).
[70]A. B. Smith, Nucl. Phys. **47**, 633 (1963).
[71]E. Barnard *et al.*, Nucl. Phys. 80, 46 (1966).
[72]A. T. G. Ferguson, Izv. Akad. Nauk SSSR, Ser. Fiz. **31**, 74 (1967).
[73]A. M. Fridman *et al.*, Phys. Rev. C **9**, 760 (1974).
[74]J. Borggreen *et al.*, Nucl. Phys. **29**, 515 (1962).
[75]R. A. Glass *et al.*, J. Inorg. Nuclear Chem. **13**, 181 (1960).
[76]B. S. Dzhelepov *et al.*, Zh. Éksp. Teor. Fiz. **45**, 1360 (1963) [Sov. Phys. JETP **18**, 937 (1964)].
[77]S. A. Baranov *et al.*, Yad. Fiz. **4**, 1108 (1966) [Sov. J. Nucl. Phys. **4**, 798 (1967)].
[78]S. A. Baranov *et al.*, Yad. Fiz. **10**, 1110 (1969) [Sov. J. Nucl. Phys. **10**, 632 (1970)].
[79]M. R. Schmorak *et al.*, Phys. Rev. Lett. **24**, 1507 (1970).
[80]L. P. Bilibin *et al.*, Izv. Akad. Nauk SSSR, Ser. Fiz. **30**, 217 (1966).
[81]M. R. Schmorak *et al.*, Phys. Rev. Lett. **24**, 1507 (1970).
[82]S. W. Yates *et al.*, Bull. Amer. Phys. Soc. **20**, 97 (1975).
[83]R. A. Meyer *et al.*, Bull. Amer. Phys. Soc. **17**, 464 (1972).
[84]P. R. Fields *et al.*, Nucl. Phys. A **208**, 269 (1973).
[85]G. N. Flerov *et al.*, At. Énerg. **17**, 310 (1964) [Sov. Atomic Energy].

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# Preequilibrium decay in nuclear reactions

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Models of preequilibrium emission of particles from excited nuclei are discussed. The agreement of the predictions of these models with experiment is analyzed for various characteristics and reaction channels. Possible new applications of models of preequilibrium decay are noted. The relation of the preequilibrium approach to general methods of nuclear-reaction theory is discussed.

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## INTRODUCTION

In nuclear physics a comparatively complete study has been made of two extreme types of nuclear reactions described by the mechanisms of direct and equilibrium processes: The first mechanism assumes participation in the reaction of only the simplest nuclear configurations, for example, quasiparticles; the second mechanism, on the other hand, assumes that the only important role is played by the many-particle configuration typical of the compound nucleus. Between these mechanisms lies a broad and still little studied region—the so-called reactions of preequilibrium decay of nuclei, to which the present article is devoted.

The equilibrium statistical model[1] has made an important contribution to the understanding of the physics of nuclear decay. However, it is clear that in the transition to higher excitation energies the assumption that equilibrium is established in the nuclear system becomes less and less well justified. Development of the theory of direct nuclear reactions and particularly progress in the distorted-wave and coupled-channel methods has permitted explanation of the appearance of a high-energy component and asymmetry of the angular distribution of reaction products at high energies. It is interesting also that there is a great deal of experimental data which show a systematic deviation from the two extreme approaches, for example, precision measurements of the spectra of particles in proton-nuclear reactions carried out by Bertrand and Peelle[2] for a wide range of target nuclei. Both intuition and an analysis of the experimental data suggest that in this case it is necessary to consider the very evolution of the nuclear system toward the equilibrium state.

The first model of this type was proposed by Serber for description of the interaction of a high-energy nucleon with a nucleus.[3] This model assumes that the energy introduced by the primary particle is systematically redistributed among the nuclear components as the result of a series of direct nucleon-nucleon collisions (the intranuclear cascade). Sufficiently energetic particles can leave the nucleus. Here it is possible to consider the motion of the particles on the basis of a quasiclassical approach, introducing the concept of a particle trajectory in the nucleus. The physical picture of the process is similar to relaxation in a gas due to collisions between molecules.

A departure from the equilibrium scheme in the region of moderate excitations was made by Izumo,[4] who developed the model of partial equilibrium. However, substantial progress is due to the work of Griffin[5] and Blann,[6] who were responsible for the origin of preequilibrium emission models. A somewhat different approach—the Fermi-gas relaxation model—was proposed by Harp, Miller, and Berne.[7,8] The general ideas of preequilibrium emission were further developed in subsequent papers and became the subject of discussion of many conferences and special seminars.[9–12]

There is no question but that statistical equilibrium in a nucleus at excitation energies of tens of MeV is established as the result of a very complicated process, a combination of different types of excitation—quasiparticle, cluster, and collective. However, even simple versions of exciton models, which are a rather crude approximation of the real process, contain interesting physics and permit a new look at some phenomena which have already become familiar.

We shall set forth here the physical bases of models of preequilibrium decay (MPD), we shall discuss the general predictions of the preequilibrium approach and compare the most popular versions of the exciton

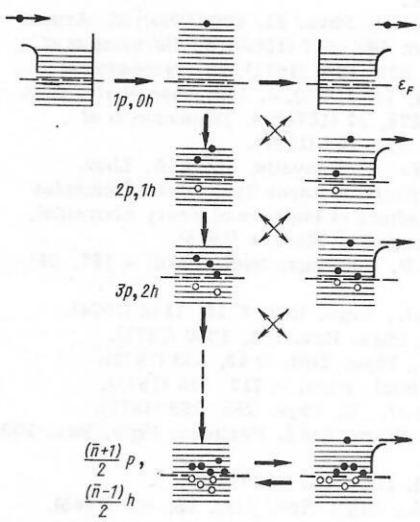


FIG. 1. Diagram of a nucleon-nuclear reaction in terms of the exciton model.

model, and we shall consider the Fermi-gas relaxation model. We shall not discuss the intranuclear-cascade model, since these questions have been sufficiently covered in the book by Barashenkov and Toneev.[13] We shall analyze the experimental data by means of MPD and we shall discuss some questions of the description of preequilibrium emission in terms of nonequilibrium statistical physics and the general methods of nuclear-reaction theory.

## 1. MODELS OF PREEQUILIBRIUM DECAY

### The exciton model. Qualitative discussion.

All modifications of the exciton model are based on the following assumptions: The nucleus is considered as a system of fermions with a weak two-particle residual interaction; the excited states are classified according to the number of excitons $n$, by which we mean the total number of excited particles $p$ and holes $h$, $n=p+h$; all states with identical numbers of excitons are considered *a priori* to be identical. The reaction occurs in accordance with the simple scheme arbitrarily illustrated in Fig. 1. Having entered the region of nuclear potential, the nucleon as a result of the first interaction with a component of the nucleus forms a three-quasiparticle doorway state of the $2p1h$ type, all configurations of this state being assumed equally probable. The two-particle nature of the residual interactions imposes definite selection rules on the permissible transitions: $\Delta n=0$ or $\pm 2$. Since in the initial stage of the reaction a transition to larger $n$ values involves a substantial extension of the configuration space, transitions with $\Delta n=+2$ turn out to be most probable (in Fig. 1 the thickness of the arrows corresponds to the strength of the transitions). Consequently, the excited system will preferentially develop towards states of increasing complexity, successively passing through states with $n=5$, 7, 9, and so forth. With the approach to equilibrium the transitions with $\Delta n=0$ and $-2$ acquire greater significance. In the state of dynamic equilibrium characterized by an average number of particles $\bar{n}$, all three permissible types of transitions are equally probable and in this way the two stages of the process are described in a unified manner in terms of the MPD.

In the course of the evolution of the system, among the possible configurations with a fixed number of excitons there are those in which at least one nucleon is in a single-particle state in the region of the continuum (see the right-hand part of Fig. 1), i.e., the particle can be emitted by the nucleus. It is easy to see that the simplest nuclear states possess the highest probability of emitting a particle with a high kinetic energy $\varepsilon$. It is just this possibility of describing the emission of particles in the stage of establishment of statistical equilibrium in the excited nuclear system which is one of the most attractive features of the MPD.

### Master equations.

Cline and Blann[14] and subsequently other authors[15–17] have developed the quantitative formulation of the nuclear relaxation process described above in terms of the kinetic or master equation

$$\frac{\partial P(n,t)}{\partial t}=\lambda_+(n-2,E)P(n-2,t)+\lambda_-(n+2,E)P(n+2,t) - \Big[\lambda_+(n,E)+\lambda_-(n,E)+\sum_i\Gamma_i(n,E)\Big]P(n,t). \tag{1}$$

This equation relates the change in the probability $P(n,t)$ of population of an $n$-exciton state with the probabilities of the direct processes $\lambda_{\Delta n}(n,E)$ to the neighboring $n-2$ and $n+2$ states and with the probability of emission of a particle into the continuum

$$\Gamma_i(n,E)=\int^{E-B_i} W_i(n,\epsilon)\,d\epsilon. \tag{2}$$

Here $W_i(n,\epsilon)$ determines the energy spectrum of particles of the $i$th form emitted from a state with excitation energy $E$ and number of excitons $n$; $B_i$ is the binding energy of this particle, and the summation is carried out over all open emission channels.

The system of equations (1) written for each state in the preequilibrium state is supplemented by the initial condition

$$P(n,t=0)=\delta(n-n_0). \tag{3}$$

The energy of the doorway state is $E=\epsilon_0+B_0$, where $\epsilon_0$ is the kinetic energy of the incident particle; $B_0$ is its binding energy.

The probability of population $P(n,t)$ obtained from solution of the system of Eq. (1) permits us also to write down such physical quantities as the instantaneous energy spectrum of emitted particles

$$I_i(\epsilon,t)\,d\epsilon\,dt=\sum_{\substack{n=n_0\\ \Delta n=2}} P(n,t)\,W_i(n,\epsilon)\,d\epsilon\,dt \tag{4}$$

and the complete preequilibrium spectrum

$$N_i(\epsilon)\,d\epsilon=\int_0^{t_{equ}} I_i(\epsilon,t)\,d\epsilon\,dt=\sum_{\substack{n=n_0\\ \Delta n=2}} W_i(n,\epsilon)\,t_n\,d\epsilon, \tag{5}$$

where the lifetime of an $n$-quasiparticle state is

$$t_n=\int_0^{t_{equ}} P(n,t)\,dt. \tag{6}$$

The summation in Eqs. (4) and (5) is carried out over all states of the preequilibrium stage with allowance for selection rules in $\Delta n$. The quantity $t_{equ}$ is fixed by the moment of reaching the value $n=\bar{n}$.

In a state of dynamic equilibrium the probability population ceases to depend on time and is determined only by the density $w(n,E)$ of $n$-exciton states, i.e. $P(n,t_{equ})=w(n,E)/\sum_n w(n,E)$.

It is possible to generalize the master equation to allow for the possibility of successive emission of several particles, transitions with $\Delta n=0$, and so forth. However, even these complicated systems are solved comparatively simply as the result of coupling of the kinetic equations with random Markov processes.[17]

The time evolution described above is actually an example of a discontinuous Markov process[18]: The time variable varies continuously, and at some random moment a discontinuous change occurs in the state of the system, the behavior of the system in the future being completely determined by its state at the present moment. From this point of view Eq. (1) is a Kolmogorov–Chapman equation[18] for a specified random process.

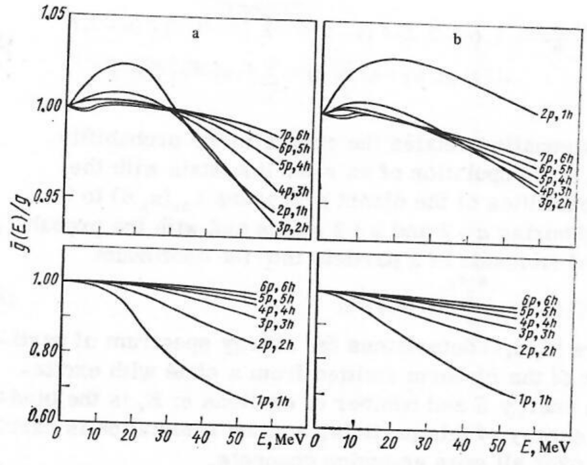


FIG. 2. Comparison of density of states with a given number of quasiparticles for the case of single-particle levels: a—rectangular potential well with Fermi energy $\varepsilon_F = 20$ MeV; b—cut-off oscillator potential with $\varepsilon_F = 40$ MeV and $\omega(p, h, E)$ calculated in the equidistant approximation with $g = A/13.3$ MeV$^{-1}$.[24] For convenience we have expressed the results in terms of the effective single-particle density $\bar{g}$ defined according to Eq. (8) with the left side found numerically.

The interpretation of nuclear relaxation in terms of random processes suggests a simple means of solving the master equation: modeling of the random process by means of the Monte-Carlo method.[17] However, before discussing these solutions, it is necessary to determine the densities of excited nuclear states $w(n, E)$, the rates of intranuclear transitions $\lambda_{\Delta n}(n, E)$, and the rates of emission of particles $W_i(n, \epsilon)$.

## Density of excited states of the intermediate nucleus.

A particular feature of our problem is that it is necessary to know the density of excited states with a fixed number of excited particles and holes. To find $w(n, E)$ one can use the methods developed in statistical mechanics (see the reviews of Ericson[19] and Stavinskiĭ[20]), considering the number of excitons $n$ as an additional conserved additive quantum number. In the general case results can be obtained only numerically. However, if we neglect the effect of the Pauli principle, which assumes smallness of $n$, and consider the excitons as a gas of $n$ Boltzmann particles, then the expression for the distribution function is written out at once:

$$Z(\beta) = \sum_k \exp(E_k \beta) = \frac{1}{n!}\left[\int g(\varepsilon)\exp(\varepsilon\beta)\, d\varepsilon\right]^n = \frac{1}{n!}(g/\beta)^n.$$

In deriving this relation the sum over single-particle states is replaced by an integral, introducing the density of single-particle states $g(\varepsilon)$, and then using the approximation of an equidistant spectrum, setting $g(\varepsilon) = g$. The factor $n!$ takes into account the indistinguishability of classical particles. Thus, we have

$$w(n, E) = \frac{2}{(2\pi i)}\int_{\sigma - i\infty}^{\sigma + i\infty} d\beta Z(\beta)\exp(-\beta E) = \frac{g(gE)^{n-1}}{(n-1)!\,n!}. \qquad (7)$$

The density of intermediate states was evaluated by Griffin by means of this formula in his version of the exciton model.[6] A more accurate relation is that suggested by V. M. Strutinskiĭ[21] and by Ericson,[19] where the particles and holes are treated as two different Boltzmann gases. Then we have $Z(\beta) = (g/\beta)^{p+h}(p!h!)$, which gives

$$w(p, h, E) = g(gE)^{p+h-1}/[p!h!(p+h-1)!]. \qquad (8)$$

It is interesting to investigate to what degree the simple relations such as (7) and (8) are valid. Comparison of these relations with the density of particle-hole excitations calculated in the independent-fermion model with the equidistant scheme of single-particle levels shows the high accuracy of Eq. (8), which improves with increase of $n$, while the deviation of Eq. (7) reaches several orders of magnitude.[22]

*Effect of the Pauli principle* on the density of intermediate states in the equidistant model has been investigated by Williams,[23] who showed that this effect can be expressed approximately by the analytical dependence

$$w(p, h, E) = g(gE - \mathcal{A})^{p+h-1}/[p!\,h!\,(p+h-1)!], \qquad (9)$$

i.e., exclusion of multiple population of single-particle levels in this approximation leads to a shift of the effective excitation energy by an amount $\mathcal{A}/g$, where

$$\mathcal{A} = (p^2 + h^2 + p - 3h)/4. \qquad (10)$$

*The accuracy of the equidistant approximation* has been analyzed[24] by comparison of $w(p, h, E)$ for three types of distributions of single-particle levels: equidistant distribution, for a Fermi-gas potential with $g(\varepsilon) \sim \varepsilon^{1/2}$, and for a cut-off oscillator potential. An example of this analysis is given in Fig. 2. The closeness of the ratio $\bar{g}(E)/g$ to unit indicates the rather high accuracy of Eq. (8).

*In real nuclei* the nonuniform distribution of levels is a reflection of nuclear-structure effects. Specific calculations have been carried out by several authors[25–28] and are given in Fig. 3. The results of investigation of the *effect of nuclear shell structure* can be characterized as follows. The equidistant model is a good approximation for the density of states at comparatively high energies. As we should expect, in the region of low excitations Eqs. (7) and (8) do not portray the actual behavior of the density of quasiparticle states. For

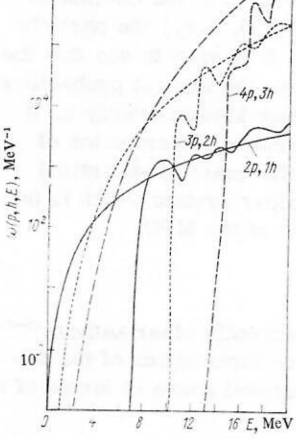


FIG. 3. Density of quasiparticle states calculated with the Nilsson scheme for the filled shell $Z = 50$ ($A = 115$) and according to Eq. (8) (monotonic curves).[28]

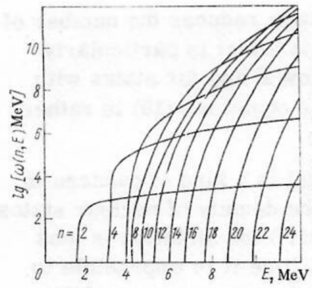


FIG. 4. Density of quasiparticle states of a heavy nucleus with inclusion of pairing in the superfluid nucleus model.[26] The single-particle density is $g = 18.3\ \mathrm{MeV}^{-1}$ and the ground-state correlation function is $\Delta_0 = 0.85$ MeV.

magic nuclei the deviations in this case can reach several orders of magnitude; however, for nuclei located between filled shelves the predictions of the equidistant model are quite satisfactory. The nature of the discrepancies near magic nuclei is such that results based on use of real single-particle schemes can be reproduced by the equidistant model with a shift of the ground-state energy.

A. V. Ignatyuk and Yu. V. Sokolov[26] have discussed the *effect of residual interactions of the correlation type* in terms of a general statistical approach. The results shown in Fig. 4 show that taking into account the pairing effect leads to a ground-state energy shift increasing with increasing number of excitons $n$ and to appearance of a characteristic step-like dependence of the thermodynamic quantities. The possibility of existence of such a structure was first pointed out by V. M. Strutinskiĭ.[21]

The appearance of structure can be traced qualitatively even in the Boltzmann-gas approximation if we take into account that for formation of a classical particle it is necessary to overcome pairing forces, expending some energy $\Delta$. Then, for example, for an even–even nucleus we obtain directly from Eq. (8)

$$w(n, E) = g^n (E - \Delta\cdot n)^{n-1}/\{[(n/2)!]^2 (n-1)!\}. \qquad (11)$$

Generalization of Eq. (11) to the case of nuclei with an arbitrary number of nucleons has been carried out by Dovbenko and Ignatyuk.[29]

The Strutinskiĭ–Ericson approximation (8) is widely used in MPD. The discrepancies noted serve as warning that Eq. (8) must be used with a certain degree of caution, particularly near magic nuclei. However, the real situation turns out to be more successful, since a statistical description involves, as a rule, not absolute but relative densities of excited states. The relative values depend much more weakly on the nuclear structure, as we can directly convince ourselves below.

### Probability of emission of particles from preequilibrium states.

Using the principle of detailed balance, the rate of decay $W_i(p,h,\epsilon)$ of an $n$-quasiparticle system with emission of a nucleon to the continuum can be expressed in terms of the cross section for the inverse reaction $\sigma_{\mathrm{inv}}(\epsilon)$:

$$W_i(p, h, \epsilon)\,d\epsilon = [(2s_i+1)/(\pi^2\hbar^3)]\,\mu_i \mathcal{R}_i(n) \times \epsilon\sigma_{\mathrm{inv}}(\epsilon)\, w(p-1, h, U)\,d\epsilon / w(p, h, E). \qquad (12)$$

Here $w(p, h, E)$ and $w(p-1, h, U)$ are the densities of states of the decaying nucleus and the residual nucleus with $U = E - B_i - \varepsilon$; $s_i$ and $\mu_i$ are the spin and reduced mass of the emitted particle. The factor $\mathcal{R}_i(n)$ assures that the selected exciton is a nucleon of type $i$.[14,30] Since Eq. (12) involves the ratio of densities of excited states, the result is not very sensitive to the specific choice of $w(p,h,E)$. Thus, the absolute values of the density of $2p1h$ states calculated with various assumptions as to the behavior of $g(\varepsilon)$, do not differ greatly from each other (see Fig. 2), but the probabilities of nucleon emission $W_i(2,1,\epsilon)$, as shown in Fig. 5, turn out to be nearly the same for all three models. In particular, the approximation of the equidistant model is rather good. Substitution of Eq. (8) into Eq. (12) gives ($\mathcal{R}_i(n) \approx 1$)

$$W_i(p, h, \epsilon)\,d\epsilon = [2\mu_i/(\pi^2\hbar^3)]\,\epsilon\sigma_{\mathrm{inv}}(\epsilon)\,[p(n-1)/(gE)]\,(U/E)^{n-2}\,d\epsilon. \qquad (13)$$

In the case of emission of a complex particle it is necessary to take into account all states not only of the residual nucleus but also of the emitted particle, the number of which is $w(p_i, 0, E-U)/g$. As before, $\mathcal{R}_i(n)$ takes into account the fact that the chosen combination of $p_i$ nucleons from the total number $p$ of excited particles has a proton-neutron composition corresponding to the $i$th particle. In terms of the independent-particle model the factor $\mathcal{R}_i(n)$ can be calculated by combinatorial methods.[15] However, the existence of the necessary isotopic composition in the selected grouping still does not guarantee formation of a single system. We introduce an additional factor $\gamma_i$ which gives the probability of "condensation" of this grouping into the complex particle $i$. Finally, we have[31]

$$W_i(p, h, \epsilon)\,d\epsilon = \frac{2s_i+1}{\pi^2\hbar^3}\,\mu_i\epsilon\sigma_{\mathrm{inv}}(\epsilon)\,\gamma_i \times \mathcal{R}_i(n)\,\frac{w(p_i, 0, E-U)}{g}\cdot\frac{w(p-p_i, h, U)}{w(p, h, E)}\,d\epsilon, \qquad (14)$$

where the parameter $\gamma_i$ is found from comparison with experiment.

An alternative approach has been suggested by Milazzo–Colli[32,33] with application to $\alpha$-particle emission. These authors assumed the existence in the nucleus of $\alpha$-particle excited states. Such an $\alpha$ cluster (or $\alpha$ hole) was considered as one exciton with a single-particle density $g_\alpha = g/4$. If we now introduce the probability of "preparation" of the $\alpha$ cluster $\varphi$, then by analogy with Eq. (12) we can write for the probability of $\alpha$-particle emission

$$W_\alpha(p, h, \epsilon)\,d\epsilon = \frac{(2s_i+1)}{\pi^2\hbar^3}\,\mu_\alpha\epsilon\sigma_{\mathrm{inv}}(\epsilon) \times \frac{\varphi K^\alpha_{p,h}\, w(p-p_\alpha, h, U)}{[\varphi K^\alpha_{p,h} + (1-\varphi)K^\nu_{p,h}]\, w(p, h, E)}\,d\epsilon, \qquad (15)$$

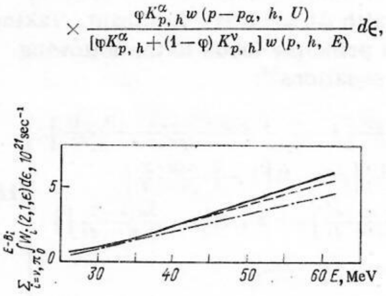


FIG. 5. Rate of emission of nucleons from $2p1h$ states of the excited system $p + {}^{89}Y$.[24] The curves were calculated with various assumptions regarding $g(\varepsilon)$: Solid curve—equidistant distribution, dashed curve—for rectangular potential well, dash-dot curve—for cut-off oscillator potential.

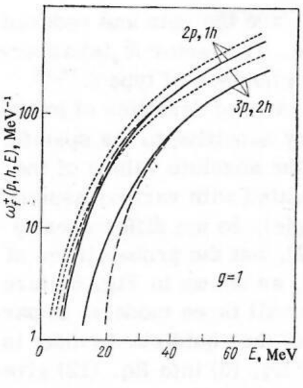


FIG. 6. Density of available $2p1h$ and $3p2h$ states: solid curves —exact numerical calculation[34]; dashed curve—approximate calculation with inclusion of Pauli principle [see Eq. (18)]; dotted curve—without inclusion of Pauli principle [see Eq. (17)].

where the index $\nu$ refers to the nucleons; the factor $K^i_{p,h}$, which take into account charge conservation and the difference in the single-particle density of nucleons and $\alpha$ particles, is easily calculated.[32] The probability of preparation of $\alpha$ particles remains a free parameter.

### Probabilities of intranuclear transitions.

In view of the assumption of smallness of the residual two-particle interactions in determination of the transition probability $\lambda_{\Delta n}$ per unit time, we can use first-order time-dependent perturbation theory,[5,6,30]

$$\lambda_{\Delta n}(n, E) = (2\pi/\hbar)\langle |M|^2\rangle w_f^{\Delta n}(n, E), \tag{16}$$

where $\langle |M|^2\rangle$ is the averaged matrix element of an intranculear transition with change of the number of excitons by $\Delta n$ and conservation of the energy $E$; $w_f^{\Delta n}$ is the density of final states actually achieved in this transition. The difference of the quantity $w_f^{\Delta n}$ from the density of intermediate states lies in the fact that, as the result of the selection rule $\Delta n = 0$ and $\pm 2$, a part of the $(n+\Delta n)$-quasiparticles states can be populated only by multiple two-particle interactions.[30] In the approximation of the equidistant single-particle model, Eq. (8), we have

$$\left.\begin{aligned} w_f^{+}(p, h, E) &= \frac{\lambda_+(p, h, E)\hbar^2}{2\pi\langle |M|^2\rangle} = \frac{g}{2}\cdot\frac{(gE)^2}{(p+h+1)};\\ w_f^{0}(p, h, E) &= \frac{\lambda_0(p, h, E)\hbar^2}{2\pi\langle |M|^2\rangle} = \frac{g^2E}{2(p+h)}[p(p-1)\\ &\qquad + 4ph + h(h-1)];\\ w_f^{-}(p, h, E) &= \frac{\lambda_-(p, h, E)\hbar^2}{2\pi\langle |M|^2\rangle} = \frac{g}{2}[ph(p+h-2)]. \end{aligned}\right\} \tag{17}$$

These equations were obtained for the first time by Williams[30] and have been given more accurately by Oblozinsky *et al.*[34] For small values of $n$ we have $w_f^+ \gg w_f^-$ and transitions with $\Delta n = +2$ are dominant. Taking into account the Pauli principle leads to the following approximation of the equations[34]:

$$\left.\begin{aligned} w_f^{+}(p, h, E) &= \frac{g(gE-\mathcal{A})^2}{2(n+1)}\left[1-\frac{n+1}{n}\frac{p(p-1)+ph+h(h-1)}{gE-\mathcal{A}}\right];\\ w_f^{0}(p, h, E) &= \frac{g(gE-\mathcal{A})}{2n}\left\{p(p-1)\left[1-\frac{3}{4}\cdot\frac{n(p-2)}{gE-\mathcal{A}}\right]\right.\\ &\qquad \left.+4ph\left[1-\frac{3n(n-2)}{8(gE-\mathcal{A})}\right]+h(h+1)\left[1-\frac{3n(h-2)}{4(gE-\mathcal{A})}\right]\right\};\\ w_f^{-}(p, h, E) &= \frac{g}{2}ph(n-2)\left[1-\frac{(n-1)}{(n+2)}\right.\\ &\qquad \left.\times\frac{(p-1)(p-2)+(h-1)(h-2)}{8(gE-\mathcal{A})}\right], \end{aligned}\right\} \tag{18}$$

where the shift $\mathcal{A}$ is determined by Eq. (10). The accuracy of Eq. (17) and (18) is shown in Fig. 6. Taking into account the Pauli principle reduces the number of available final states, and its effect is particularly noticeable in the region of low $E$ and for states with large $n$. The accuracy of the equations (18) is rather high.

While the equidistant model is a kind of panacea in the question of calculating the density of nuclear states, in regard to the quantity $\langle |M|^2\rangle$ the situation is less well defined. At the present time it is impossible to calculate with any accuracy the mean square of the matrix element. Therefore in analysis of experiments on the basis of MPD, $\langle |M|^2\rangle$ is either considered a free parameter or is evaluated from the probability of collision of nucleons in nuclear matter.

The collision probability of a particle with a nucleon of the nucleus per unit time is expressed in terms of the effective interaction cross section $\langle\sigma\rangle$ and the density of nucleons $\rho$ or in terms of the mean free path $\Lambda$ of the particle:

$$\lambda_{\text{coll}} = v\rho\langle\sigma\rangle \equiv v/\Lambda, \tag{19}$$

where $v$ is the averaged relative velocity of the colliding particles.

If we assume independence of the cross section for free scattering of nucleons, then[35]

$$\langle\sigma\rangle \sim (\varepsilon_0 + B_0)^2, \tag{20}$$

where $\varepsilon_0$ is the kinetic energy of the incident nucleon. In a realistic case the calculations can be carried out only numerically. For infinite nuclear matter such calculations have been carried out by Kikuchi and Kavai[36] by the Monte Carlo method, and their results have been approximated by Blann[37]:

$$\lambda_{\text{coll}} = 1.4\cdot 10^{21}(\varepsilon_0 + B_0) - 6\cdot 10^{18}(\varepsilon_0 + B_0)^2. \tag{21}$$

Here the energies are expressed in MeV and $\lambda_{\text{coll}}$ in sec$^{-1}$. Improvement of this relation to the case of a system with finite size has been carried out by Gadioli *et al.*[24] The results are shown in Fig. 7, together with a calculation in accordance with Eq. (21). It can be seen that particle-hole symmetry exists only near the Fermi energy $(\varepsilon - \varepsilon_F) \approx 0$. For excitations of the order of the depth of the potential well it is necessary to make a distinction between collisions of an excited particle and collisions of a hole.

The mean free path of a particle in the nucleus can be related to the imaginary part of the optical potential, $\Lambda = \hbar v/(2W_{\text{opt}})$, and consequently

$$\lambda_{\text{coll}} = 2W_{\text{opt}}/\hbar. \tag{22}$$

It is noteworthy that if the quantity $W_{\text{opt}}$ is extracted

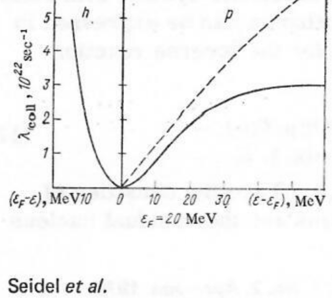


FIG. 7. Rate of collision of a particle (right portion) and hole (left portion) in a system of fermions enclosed in a rectangular potential well[24]: dashed curve—infinite nuclear matter [see Eq. (21)].

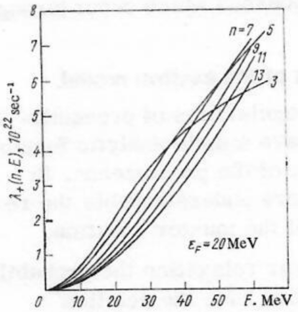


FIG. 8. Probability of transitions with $\Delta n = +2$ for $n$-quasiparticle states of a system of fermions[24] enclosed in a rectangular potential well [see Eqs. (25) and (26)].

from experiment (for example, from analysis of experiments on elastic scattering), there is no longer a need for any additional averaging procedure.

How is the *probability of exciton–exciton interaction* expressed by $\lambda_{\text{coll}}$? This question is frequently solved by identification of the collision probability with the probability $\lambda_+$ of a transition from a $1p0h$ state.[38]

$$\lambda_{\text{coll}}(\varepsilon_0) = \lambda_+(1, 0, \varepsilon_0 + B_0). \tag{23}$$

Assuming further the independence of the mean square of the matrix element of the number of excitons, $\langle |M|^2 \rangle$ is found from Eq. (16).

Gudima *et al.*[17] proceeded from a relation similar to Eq. (23) for transitions from each $n$-quasiparticle state, but here the collision probability was determined not by the total excitation energy, but by the mean relative energy of the colliding excitons $\langle \varepsilon_{\text{coll}} \rangle$. In this approximation the effective interaction cross section becomes a function of the number of excitons,

$$\langle \sigma \rangle = \eta_{\Pi}(\varepsilon/\varepsilon_F)\, \sigma(\langle \varepsilon_{\text{coll}} \rangle), \tag{24}$$

where the factor $n$ takes into account the effect of the Pauli principle.[35,36] The quantity $\langle \varepsilon_{\text{coll}} \rangle$ is made up of the average kinetic energy of the excited particle $\varepsilon_F + E/n$ found by averaging $\varepsilon$ over all possible configurations of the $n$-exciton state, and the kinetic energy of the "partner" $(3/5)\varepsilon_F$, averaged over the spectrum of a degenerate Fermi gas.[17] The probabilities $\lambda_0$ and $\lambda_-$ are determined from Eq. (17).

Gadioli *et al.*[24] established the relationship under discussion by means of the equation

$$\lambda_+(p, h, E) = p\langle \lambda_{\text{coll},p} \rangle + h\langle \lambda_{\text{coll},h} \rangle, \tag{25}$$

where the average value of the rate of collision of one excited particle $\langle \lambda_{\text{coll},p} \rangle$ or hole $\langle \lambda_{\text{coll},h} \rangle$ is taken over all $ph$ configurations:

$$\left.\begin{aligned} \langle \lambda_{\text{coll},p} \rangle &= \frac{1}{pw(p, h, E)} \int_0^E du\, \lambda_{\text{coll},p}(u) \\ &\quad \times w(p-1, h, E-u)\, w(1, 0, u); \\ \langle \lambda_{\text{coll},h} \rangle &= \frac{1}{hw(p, h, E)} \int_0^E du\, \lambda_{\text{coll},h}(u) \\ &\quad \times w(p, h-1, E-u)\, w(0, 1, u). \end{aligned}\right\} \tag{26}$$

The necessity of distinguishing collisions of particles and holes has already been discussed above. The transition probabilities calculated from Eqs. (25) and (26) for the case of a rectangular potential well are given in Fig. 8; use was made here of the values of $\lambda_{\text{coll}}$ shown in Fig. 7. The results show that independence of the matrix element of $n$, which follows from the inverse proportionality of $\lambda_+$ to the number of excitons [see Eq. (17)], is observed only at small excitations when the particle-hole symmetry is not violated. In regard to the energy dependence, at low excitation energies we have $\lambda_+ \sim E^2$ and consequently $\langle |M|^2 \rangle \approx \text{const}$. However, already at an energy of about 20 MeV $\lambda_+ \sim E$, which corresponds to $\langle |M|^2 \rangle \sim E^{-1}$. With further increase of the energy $E$, the probabilities of intranuclear transitions approach saturation, beginning with $\lambda_+$ at small $n$. Since the mass number $A$ does not enter explicitly anywhere in the probability of exciton–exciton interaction, according to Eq. (17) we have $\langle |M|^2 \rangle \sim A^{-3}$, since $g \sim A$. As follows from Ref. 24, these qualitative conclusions remain valid when potentials of arbitrary shape are used.

Approximations of a somewhat different type have led to the so-called hybrid model.[37] Here it is assumed that of all $n$-exciton states the main contribution to emission of particles with energy $\varepsilon$ is from those which have one particle in the continuum with energy $\varepsilon$. The transition to other states is accomplished as the result of collision of just this particle, other collisions not being considered. Then

$$\lambda_+(p, h, E) = \lambda_{\text{coll}}(\varepsilon + B_i). \tag{27}$$

It should be noted that the energy dependence of Eq. (7) and of the results shown in Fig. 8 are very similar, especially for $E \gtrsim 20$ MeV. We should therefore expect that the characteristics of a preequilibrium process in the approaches being considered also will not differ substantially.

## Dynamics of preequilibrium decay.

As already noted, in solution of the master equation (1) it is convenient to use an interpretation of the nuclear relaxation in terms of random Markov processes.[17] The method permits taking into account both all types of intranuclear transitions $\lambda_+$, $\lambda_-$ and $\lambda_0$, and also the

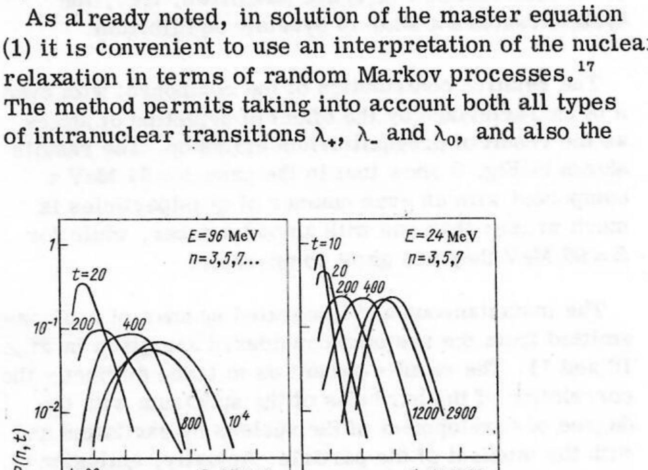


FIG. 9. Probability of population $P(n,t)$ of $n$-exciton states for a nuclear system with $g = 5.2\ \text{MeV}^{-1}$, $n_0 = 3$, and excitation energies $E = 24$ and 96 MeV.[17] The time scale is expressed in units of $1/\lambda_+$ ($n = 3$, $E = 24$ MeV). States with even values of $n$ are populated only as the result of nucleon emission.

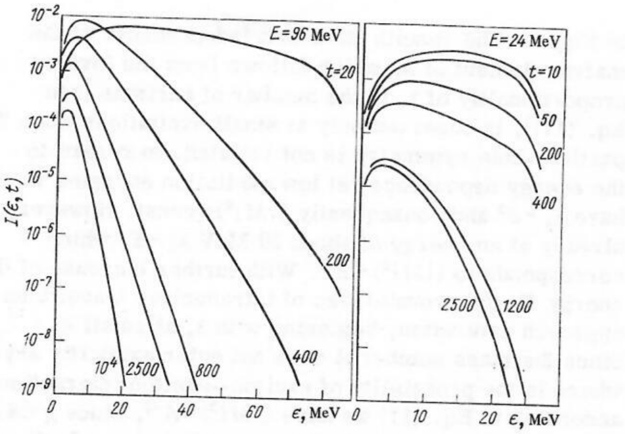


FIG. 10. Prompt-neutron spectra corresponding to the filling probabilities $P(n,t)$ shown in Fig. 9 [see Eq. (4)].

successive emission of several particles in the stage of establishment of equilibrium.

Solutions of the master equation for a system with mass number $A=64$ are given in Fig. 9 for two values of excitation energy. Here the density of single-particle states was calculated in terms of the equidistant model (8) and the quantity $\langle |M|^2 \rangle$ was extracted from the estimate[17] of the nucleon mean free path in nuclear matter [see Eqs. (19) and (24)]. As can be seen from the figure, the population function of the states $P(n, t)$, which at the initial moment had the form of a $\delta$ function, shifts with increasing $t$ toward larger values of $n$ and is more and more strongly smeared out. On reaching a moment of time $t \sim 3\times10^3$, which on the absolute scale corresponds to a quite reasonable relaxation time for the system of about $6\times10^{-19}$ sec, the location and shape of the distribution $P(n, t)$ are stabilized, i.e., the system reaches a state of dynamic equilibrium.

The relative contribution of the component with even $n$ is characterized by the effect of depletion of states as the result of preequilibrium emission. The results shown in Fig. 9 show that in the case $E=24$ MeV a component with an even number of quasiparticles is much weaker than one with an odd number, while for $E=96$ MeV they are quite comparable.

The instantaneous and integrated spectra of neutrons emitted from the systems considered are given in Figs. 10 and 11. The results permit us to trace distinctly the correlation of the hardness of the spectrum with the degree of envelopment of the nucleus by excitation and with the moment of the particle. Actually, emission of particles with maxima energy occurs in the earliest stage of relaxation from the simplest quasiparticle states; equilibrium emission makes its contribution primarily in the region of small values of $\varepsilon$.

It is interesting to note that the rate of emission of particles (see Fig. 11) decreases rapidly on approach to a state of dynamic equilibrium. This means that the compound system, having reached equilibrium, will "live" a comparatively long time with respect to particle emission. Thus, in MPD the nuclear processes are separated in a natural way in time into fast, preequilibrium, and slower processes which occur through the compound nucleus.

**Simplified analytic versions of the exciton model.** To bring out the general regularities of preequilibrium decay it is useful to have simple analytic formulas which reflect the essence of the phenomenon. In addition, this would make more understandable the results of numerical solution of the master equation.

In the initial stage of nuclear relaxation the probabilities of intranuclear transitions have the relation $\lambda_+ \gg \lambda_0 \gg \lambda_-$, as follows from Eq. (17). In particular, for the example considered above with $E=24$ MeV we have $\lambda_+/\lambda_- \sim 10^3$ and $10^2$ for $n=3$ and 5, respectively. Thus, in the first approximation we can assume that the system evolves with successive $\lambda_+$ transitions, beginning with $n=n_0$. This corresponds to the master equation (1), in which it is necessary to discard terms containing $\lambda_-$. The formal solution of the simplified equation is written as

$$P(n,t)=\begin{cases}\exp\{-[\lambda_+(n,E)+\Gamma(n,E)]t\}, & \text{if } n=n_0;\\ -\lambda_+(n-2,E)\exp\{-[\lambda_+(n,E)+\Gamma(n;E)]t\} & \\ \times\int_0^t d\tau P(n-2,\tau)\exp\{[\lambda_+(n,E)+\Gamma(n,E)]\tau\}, & \text{if } n>n_0,\end{cases} \tag{28}$$

where we have used the designation $\Gamma(n,E)=\sum_i \Gamma_i(n,E)$. Substituting Eq. (28) into Eq. (6), we find the mean lifetime of an $n$-quasiparticle state

$$t_n=t_{n-2}\lambda_+(n-2,E)/[\lambda_+(n,E)+\Gamma(n,E)].$$

When we take into account the initial condition

$$t_0=[\lambda_+(n_0,E)+\Gamma(n_0,E)]^{-1}$$

we have

$$t_n=\frac{1}{\lambda_+(n,E)+\Gamma(n,E)}\prod_{n'=n_0+2}^{n}\left[1-\frac{\Gamma(n'-2,E)}{\lambda_+(n'-2,E)+\Gamma(n'-2,E)}\right] \equiv \frac{\mathcal{D}(n)}{\lambda_+(n,E)+\Gamma(n,E)}, \tag{29}$$

where $\mathcal{D}(n)$ is the depletion factor.

The expression for the spectrum of emitted particles now takes the following form:

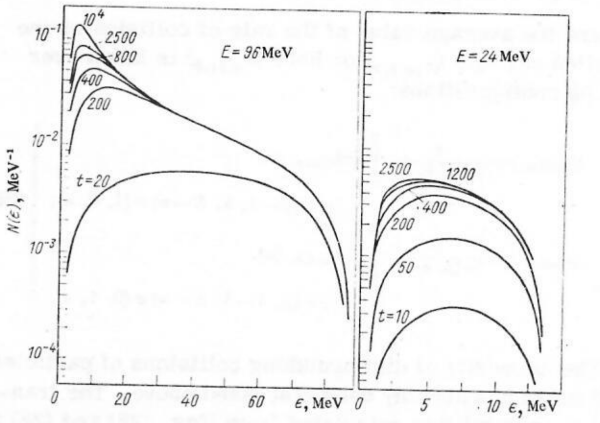


FIG. 11. Neutron spectra integrated over time for the nuclear system described in Fig. 9 [see Eq. (5)].

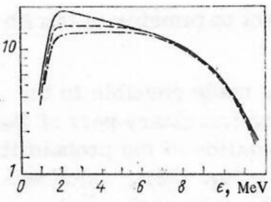


FIG. 12. Comparison of the shape of the preequilibrium neutron spectra obtained in various versions of the exciton model for a system with $n_0=3$, $g=4.3$ MeV$^{-1}$, and $E=24$ MeV: solid curve—solution of the master equation[30]; dashed curve—Williams' model [see Eq. (31)]; dot-dash curve—Blann's model [see Eq. (32)].

$$N_i(\epsilon)\,d\epsilon=\sum_{\substack{n=n_0\\ \Delta n=2}}^{\bar n} W_i(n,\epsilon)\frac{\mathscr{D}(n)\,d\epsilon}{\lambda_+(n,E)+\Gamma(n,E)}. \qquad (30)$$

In comparison with experiment, various modifications of Eq. (30), differing mainly in the choice of $\lambda_+(n,E)$, are often used. The most popular of these are the following.

*The Williams approximation.*[30] The rate of intranuclear transitions is evaluated according to Eq. (16) with use of Eq. (17) for $\omega_f^{\Delta n}$. Assuming independence of $\langle|M|^2\rangle$ of $n$, and $\lambda_+(n,E)\gg\Gamma(n,E)$, we obtain

$$N_i(\epsilon)\,d\epsilon=\frac{(2s_i+1)\,\mu_i\epsilon\sigma_{\rm inv}(\epsilon)}{\pi^3\hbar^2 g^4\langle|M|^2\rangle E^3}\sum_{\substack{n=n_0\\ \Delta n=2}}^{\bar n}(U/E)^{n-2}\,p\,(n^2-1)\,d\epsilon. \qquad (31)$$

*Blann's approximation.*[14] In contrast to the preceding model, the lifetime in any $n$-quasiparticle state is assumed identical, $t_n=t_0=\text{const}$. Then

$$N_i(\epsilon)\,d\epsilon=\frac{(2s_i+1)\,\mu_i\epsilon\sigma_{\rm inv}(\epsilon)}{\pi^2\hbar^3 gE}\sum_{\substack{n=n_0\\ \Delta n=2}}^{\bar n}\left(\frac{U}{E}\right)^{n-2}p\,(n-1)\,t_0\,d\epsilon. \qquad (32)$$

In both expressions the upper summation index $\bar n$ is equal to the average number of excitons in the equilibrium state. The value of $\bar n$ can be found, for example, from the condition

$$\lambda_+(\bar n,E)=\lambda_-(\bar n,E),$$

which gives

$$\bar n=\sqrt{2gE}. \qquad (33)$$

In the case of nucleon-nuclear reactions, Eqs. (31) and (32) are sometimes written in another form with replacement of $p$ by $(n+1)/2$.

*Griffin's approximation.*[5] This is the first model of preequilibrium decay. Griffin assumed a nuclear penetrability equal to unity, which led to loss of the energy factor $\epsilon^{1/2}\sigma_{\rm inv}(\epsilon)$ in Eq. (32).

In Fig. 12 the spectra calculated from Eqs. (31) and (32) are compared with the energy distribution found from solution of the master equation. All three curves have been reconciled in absolute value in the high-energy part. Although the analytic expressions somewhat underestimate the contribution of low-energy particles, the shapes of the distributions turn out to be very similar. Therefore we can analyze the general properties of MPD on the basis of Eqs. (31) and (32).

We must note first of all that the dependence on mass number enters into Eqs. (31) and (32) only through the single-particle density $g$ and, consequently, the predicted shape of the spectrum of particles emitted in the preequilibrium stage should not depend on $A$. In the case of equilibrium emission (compound nucleus) the energy distribution, on the other hand, is extremely sensitive to $g$, since this quantity is contained in the argument of the exponential.

We can expect that the initial number of excitons $n_0$ is determined by the nature of the bombarding particle, and not by the target nucleus. Then for identical initial excitation energy the preequilibrium spectra of particles should be similar also in absolute value.

In addition, the spectral distribution is very sensitive to $n_0$, and this is true especially for the fastest particles. In fact, for large values of $\epsilon$, i.e., $U/E\ll 1$, the dominant term in the sum is the term with the minimum value of the power-law exponent

$$N_i(\epsilon)\,d\epsilon\sim\epsilon\sigma_{\rm inv}(\epsilon)\,(U/E)^{n_0-2}\,d\epsilon. \qquad (34)$$

Thus, the hardness of the spectrum is greater, the smaller $n_0$.

No dependence of the spectra on the specific particle-hole composition of $n_0$ is evident from Eqs. (31) and (32). However, calculations with use of the master equation show that such a dependence exists.[14] The effect is due to the fact that for emission of a nucleon it is necessary to have an excited particle, and not an exciton excited as a whole. As a result the spectrum turns out to be more energetic for an initial configuration, for example, $2p1h$ than for a $1p2h$ state, although in both cases $n_0=3$. In addition, it is necessary to take into account the neutron-proton composition of the system. Its effect is understandable if we use for $W_i(\epsilon)$ the more general expression (14). It should be noted that the effects discussed appear more clearly in the relative yields of the particles.

In regard to the Pauli principle, its effect for the most part reduces to an increase of the lifetime of an $n$-quasiparticle state, the lifetime being greater, the higher the complexity of the state [see Eqs. (16), (18), and (30)]. However, this effect is partially compensated by the fact that when the Pauli principle is taken into account the equilibrium state is reached at smaller values of $n$.[1)] As a result the preequilibrium spectra calculated with inclusion of the Pauli principle and without it differ insignificantly, and this difference appears only in the low-energy part of the spectral distributions.

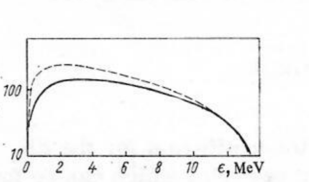


FIG. 13. Spectrum of protons emitted by an excited system with $n_0=3$, $A=100$, and $E=21$ MeV: solid curve—Blann's model [see Eq. (32)]; dashed curve—hybrid model [see Eq. (35)].

1)This fact follows from Eq. (33), where $gE$ must be replaced by $gE-\mathcal{A}$.

*The hybrid model approximation*[37,39] is based on consideration, out of all possible transitions, of only the most important with respect to nucleon emission, namely, transitions resulting from collisions of quasiparticles with a more complex state or transitions to the continuum through a virtual single-particle state with energy $\epsilon$. This means that the lifetime of the entire system is determined by the lifetime with respect to such single-particle states. By analogy with Eq. (30) we have for the emission of nucleons

$$N_i(\epsilon)\,d\epsilon=\sum_{\substack{n=n_0\\ \Delta n=2}}^{\bar{n}}\mathscr{P}_i(n)\frac{w(n-1,\,U)}{w(n,\,E)}\left[\frac{\lambda_i(\epsilon)/g}{\lambda_{\text{coll}}(\epsilon+B_i)+\lambda_i(\epsilon)/g}\right]\mathscr{D}(n), \quad (35)$$

where

$$\lambda_i(\epsilon)=(2s_i+1)\,\mu_i\epsilon\sigma_{\text{inv}}(\epsilon)/(\pi^2\hbar^3),$$

which can easily be seen by comparison with Eq. (12); $\lambda_{\text{coll}}$ is given by Eq. (21). The interpretation of Eq. (35) becomes obvious if we note that the ratio $w(n-1, U)/w(n,E)$ is the fraction of the exciton states which have in the continuum a nucleon with energy $\epsilon$, and that the term in square brackets determines the relative probability of emission of this nucleon.

Comparison of neutron spectra calculated with the hybrid and exciton models is given in Fig. 13. As expected, the hybrid model somewhat exaggerates the contribution to the region of small $\varepsilon$.

A development of the hybrid model is the approximation of *the geometry-dependent hybrid model*.[40,41] This model permits tracing the effect of the diffuseness of the nuclear density and potential on the characteristics of preequilibrium emission. The importance of such effects is quite evident. In the transition to the periphery of the nucleus, the mean free path increases and consequently the lifetime of the excited state decreases. The point is that the drop in density is accompanied by a decrease in the depth of the potential well, which is reflected in the number of hole states. Effectively it turns out that in peripheral collisions in nucleon-nuclear reactions the doorway state is more likely to be $2p0h$ than $2p1h$. The change of nuclear density should also affect the single-particle density parameter and the selection rules due to the Pauli principle.

In the first approximation these effects can be taken into account if we consider partial waves of the entrance channel and assume that for each partial wave the reaction develops in a spherical layer of thickness $\lambdabar$ and with a radius equal to the initial impact parameter. Then we will have for the cross section for emission of the $i$th particle with energy in the interval $d\epsilon$ at $\epsilon$[40,41]

$$\sigma_i(\epsilon)\,d\epsilon=\pi\lambdabar^2\sum_{l=0}^{\infty}(2l+1)\,T_lN_i(\epsilon)\,d\epsilon, \quad (36)$$

where $T_l$ is the transmission coefficient for the $l$th partial wave of the incident particle, which can be found, for example, in terms of the optical model; the function $N_i(\epsilon)$ is calculated according to Eq. (35), but for the averaged density of the shell $l\lambdabar < r < (l+1)$. In the general case, averaging of the density and potential of the nucleus must be carried out along the primary-particle trajectory,[40] but is frequently possible to average only over the variation of the impact parameter in the $l$th shell.[41]

A number of advantages are made possible in the developed scheme by use of the imaginary part of the optical potential $W_{\text{opt}}$ for calculation of the probabilities of intranuclear transitions [see Eq. (22)], which was demonstrated by Blann[41] for the case of the proton spectra in $(p, p'xn)$ reactions.

As a detailed analysis showed, taking into account the diffuseness of the distribution of nuclear matter has a substantial effect on the characteristics of particle emission from the doorway state; for states with $n > n_0$, averaging over the entire nuclear volume is a sufficiently good approximation.

### The Fermi-gas relaxation model.

Like the exciton model, this model describes the establishment of statistical equilibrium in an excited nuclear system. One of the main differences of this approach lies in the fact that a nuclear state is characterized not by the number of particles and holes, but by the filling numbers of the single-particle nuclear states $\nu_i(t)$. The physical assumptions on which the Fermi-gas relaxation model is based can be formulated as follows[7,8]: The nucleus is considered as a gas of independent nucleons enclosed in the nuclear volume $V$; specification of the filling numbers $\nu_i(t)$ completely determines the state of the system at any moment of time; change in the population of the states occurs as the result of scattering of two nucleons located in different single-particle states. The two-particle collision mechanism proposed also results in evolution of the system toward the equilibrium state.

To simplify the calculations it was assumed also that the probabilities of two-particle transitions with emission of nucleons depend only on the energy of the states involved in the transition and do not change greatly in the energy interval $\Delta\varepsilon$. The latter condition permits all single-particle states to be broken down into groups with a step $\Delta\varepsilon$. The number of states in the $i$th group is defined as

$$g_i=\int_{\varepsilon_i-\Delta\varepsilon/2}^{\varepsilon_i+\Delta\varepsilon/2}g(\epsilon)\,d\epsilon; \quad (37)$$

the number of nucleons in this group is $N_i(t)=g_i\nu_i(t)$. As the density of single-particle states $g(\varepsilon)$, the density of states for free motion is chosen,

$$g(\epsilon)=\sqrt{2}\,Vm^{3/2}\epsilon^{1/2}/(\pi^2\hbar^3), \quad (38)$$

where $m$ is the nucleon mass; the single-particle state energy $\varepsilon$ is measured from the bottom of the potential well. Similar relations define the densities of states for emission of nucleons, but their energy $\epsilon$ is measured from the level $\varepsilon_F+B$; $\varepsilon_F$ is the Fermi energy.

In the initial state all levels below $\varepsilon_F$ are occupied and the incident particle is in a single-particle state in the continuum with energy $\varepsilon_0=\varepsilon_F+B+\epsilon_0$. As a result of collisions, part of the nucleons are raised to states with energy $\varepsilon > \varepsilon_F$ and the energy introduced $\epsilon_0+B$ is redistributed among an ever increasing number of nucleons. A nucleon can be emitted if it has energy

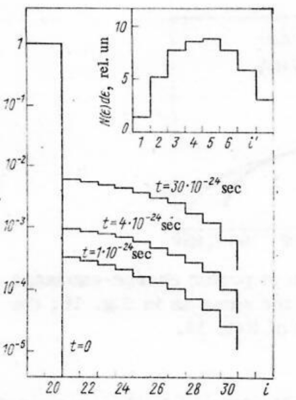


FIG. 14. Time dependence of probability of population of the $i$th energy interval in the Fermi-gas relaxation model for the system $n+{}^{52}Cr$ at $\varepsilon_0 = 14$ MeV ($\Delta\varepsilon = 1.8$ MeV). At time $t=0$ all states with $i \le 20$ are filled; in the figure we have plotted only values for $i \ge 18$. The change of $\nu_i(t)$ for $i \le 20$ coincides with $\nu_i(0)$ within the thickness of the line. In the insert we have shown the spectrum of neutrons emitted in the initial stage of relaxation.

$\varepsilon > \varepsilon_F + B$. The relaxation of a one-component Fermi gas is quantitatively described by the system of equations[7]:

$$\frac{d\nu_i(t)}{dt} = \sum_{j,k,l} \{\lambda_{kl\to ij} g_k g_l g_j \nu_k(t)\nu_l(t)[1-\nu_i(t)][1-\nu_j(t)] - \lambda_{ij\to kl} g_k g_l g_j \nu_i(t)\nu_j(t)[1-\nu_k(t)][1-\nu_l(t)]\}\,\delta(\epsilon_i+\epsilon_j-\epsilon_k-\epsilon_l) - \sum_{i'} \lambda_{i\to i'} g_{i'} \nu_i(t)\,\delta(\epsilon_{i'}-\epsilon_i+\epsilon_F+B). \tag{39}$$

Here we have for the number of nucleons emitted in the laboratory system with energy in the $i$th group $\epsilon_{i'}$

$$dN_{i'}(t)/dt = \sum_i g_i \nu_i(t)\lambda_{i\to i'}\delta(\epsilon_{i'}-\epsilon_i+\epsilon_F+B)$$

$$(i = 1, 2, 3, \ldots \varepsilon_F + B + \epsilon_0 \text{ и } i' = 1, 2, 3, \ldots, \epsilon_0).$$

The probabilities meaning of the individual terms in the right-hand side of Eq. (39) is easily discerned from comparison with the master equation (1). Factors of the type $(1-\nu_i(t))$ take into account the Pauli principle, and the δ function assures that energy is conserved. The rate of intranuclear transitions $\lambda_{kl\to ij}$ and of nucleon emission $\lambda_{i\to i'}$ are found from the quasiclassical relations [compare with Eq. (19)]:

$$\lambda_{kl\to ij} = \sigma(\epsilon_k+\epsilon_l)\,[2(\epsilon_k+\epsilon_l)/m]^{1/2}/[V \times \sum_{m,n}' g_m g_n \delta(\epsilon_k+\epsilon_l-\epsilon_m-\epsilon_m)];$$

$$\lambda_{i\to i'} = \sigma_{\text{inv}}(\epsilon_{i'})\,[2\epsilon_{i'}/m]^{1/2}/(g_i V'),$$

where $\sigma$ and $\sigma_{\text{inv}}$ are the cross sections for free nucleon—nucleon scattering and nucleon-nuclear absorption, respectively; Σ′ denotes summation over the transitions allowed by the Pauli principle. The whole scheme is easily generalized to the case of two-component composition of the nucleus.[8]

A solution of the system of ordinary differential equations (39) can be obtained by the Runge—Kutta method with accelerated convergence.[8,42] Since the lifetime of any configuration of the complete system is defined with respect to a variation of $\nu_i(t)$, the lifetime $t_i$ will be a function only of the energy of the single-particle state $\varepsilon_i$, and not of the introduced excitation $(\varepsilon_0 + B)$ as in the exciton model,[8] i.e.,

$$t_i \sim 1/[\lambda(\epsilon_i) + \lambda_{i\to i'}(\epsilon_i|\epsilon_{i'})]. \tag{40}$$

The quantity $\lambda(\varepsilon_i)$ is similar in meaning to $\lambda_{\text{coll}}$ introduced by Eq. (19). As shown above, the hybrid model of preequilibrium decay is based on the possibility of replacing the lifetime of an $n$-exciton state $t_n$ [Eq. (29)] by the lifetime (40).

An example of the time evolution of a two-component Fermi system is given in Fig. 14. We have also shown in that figure the preequilibrium spectrum of emitted neutrons, whose shape differs substantially from the equilibrium spectrum.

It should be emphasized that in the Fermi-gas relaxation model we consider directly all states of the system without use of the hypothesis that they are *a priori* equally probable. For this reason it is no longer necessary to average the transition probabilities over the available states, which results in substantial difficulties in the exciton model and is solved analytically only for the case of an equidistant spectrum of single-particle levels. In addition, in the Fermi-gas relaxation model all transition probabilities are expressed on an absolute scale and consequently there is no question of calcu-

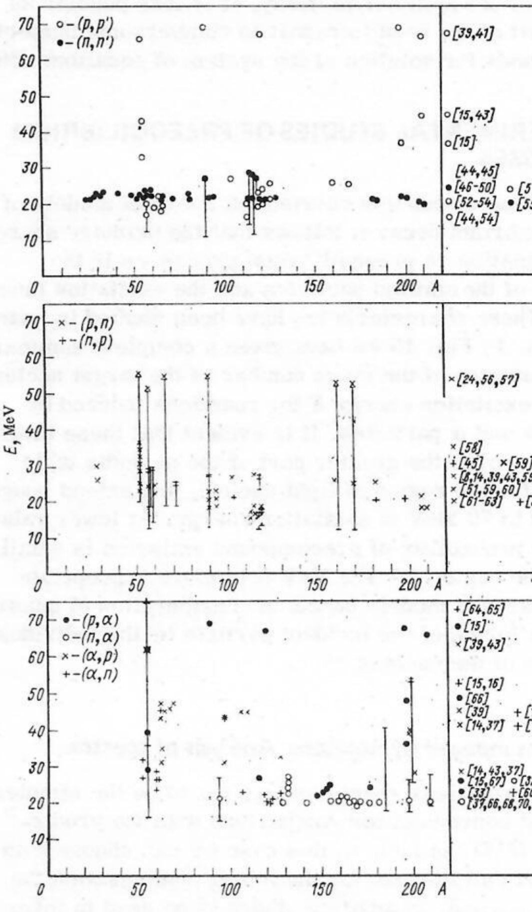


FIG. 15. Summary of experimental data analyzed by means of MPD: The lines denote the energy regions where the excitation functions have been studied.

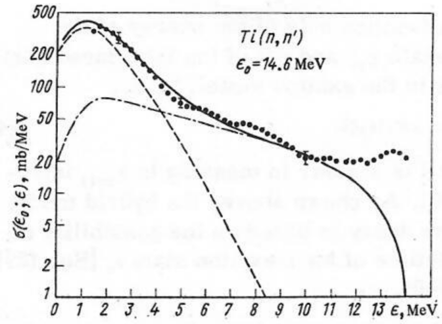


FIG. 16. Analysis of spectrum of inelastically scattered neutrons in the approximation of Eq. (41): dot-dash curve—preeqilibrium component [see Eq. (32)]; dashed curve—evaporative component [see Eq. (42)]; solid curve—combined distribution; the experimental points have been taken from Ref. 46.

lation of $\langle |M|^2\rangle$ and normalized of the results. However, in this approach, as in the exciton model, the question of the angular distribution of the emitted particles is not discussed, and the role of conservation of angular momentum remains unclear; still greater difficulties in comparison with the exciton variant of the MPD arise in the attempt to describe the emission of complex particles.

Although the Fermi-gas relaxation model is more systematic than the purely phenomenological exciton models of preequilibrium decay, it is less popular as the result of the need to resort to cumbersome numerical methods for solution of the system of equations (39).

## 2. EXPERIMENTAL STUDIES OF PREEQUILIBRIUM PROCESSES

From the discussion carried out above of models of preequilibrium decay it follows that the primary source of information on preequilibrium processes is the spectra of the emitted particles and the excitation functions. These characteristics have been studied by many workers. In Fig. 15 we have given a complete summary with indication of the mass number of the target nucleus and the excitation energy $E$ for reactions induced by nucleons and $\alpha$ particles. It is evident that these investigations cover the greater part of the periodic table (except for the region of light nuclei), and extend roughly from 20 to 70 MeV in excitation energy. At lower values of $E$ the probability of precompound emission is small, and in the region $E \gtrsim 100$ MeV it is more appropriate to use cascade models based on consideration of quasifree scattering of the incident particle by the individual nucleons of the nucleus.[13]

### Reactions induced by nucleons. Analysis of spectra.

Nucleon-nuclear reactions turn out to be the simplest and most convenient for comparison with the predictions of MPD. In fact, in this case we can choose with sufficient definiteness for the initial configuration the value $n_0 = 3$ and, in addition, there is no need to introduce additional parameters such as the probability of cluster formation (see Section 1).

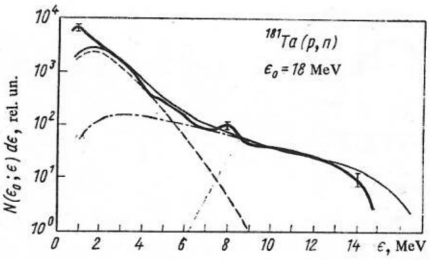


FIG. 17. Analysis of spectrum of proton charge-exchange reaction: All designations are the same as in Fig. 16; the heavy curve is the experiment of Ref. 14.

*Neutron spectra.* We shall attempt to decompose the experimentally observed neutron spectra $\sigma_{nn'}(\epsilon_0, \epsilon)$ into an equilibrium component $S_{n'}(\epsilon)$ and a preequilibrium component $N_{n'}(\epsilon)$:

$$\sigma_{nn'}(\epsilon_0, \epsilon) = k_1 N_{n'}(\epsilon) + k_2 S_{n'}(\epsilon). \tag{41}$$

As the equilibrium part we can take the "evaporative" spectrum calculated with the statistical theory and use of the Fermi-gas model for the density of excited states[1]:

$$S_{n'}(\epsilon) = C\sigma_r(\epsilon_0)\,\epsilon\sigma_{\text{inv}}(\epsilon)\,U^{-2}\exp(2\sqrt{aU}), \tag{42}$$

where $\sigma_r(\epsilon_0)$ is the reaction cross section; $C$ is some constant. If we limit ourselves to not very high excitation, Eq. (32) will be a good approximation for $N_{n'}(\epsilon)$.

By fitting the theoretical spectra to the experimental values by the $\chi^2$ method it is possible to obtain the constants $k_1$ and $k_2$. Such an analysis has been carried out for more than thirty nuclei at $\epsilon = 14.6$ MeV.[46–50] It turned out that the neutron spectra are very successfully described in the approximation (41). Furthermore, as a result of the considerable difference in shape of the spectra $N_{n'}(\epsilon)$ and $S_{n'}(\epsilon)$, the preequilibrium component can be reliably separated from the "evaporative" background. A typical example of the analysis is shown in Fig. 16. The values obtained for the constants $k_1$ and $k_2$, which determine the fraction of preequilibrium emission and are expressed in terms of the mean square of the matrix element $\langle |M|^2\rangle$, we shall discuss below.

It is interesting to note that a substantial contribution

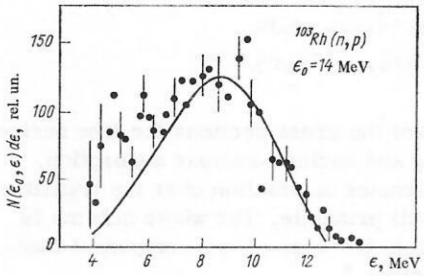


FIG. 18. Spectrum of protons from the reaction $^{103}Rh(n,p)^{103}Ru$: The curve was calculated with Eq. (31); the mean square of the matrix element was evaluated from the mean free path of a nucleon in the nucleus and on the assumption that the scattering cross section is independent of the nucleon energy, which gives $\langle |M|^2\rangle g^4 \sim 1$.[38]

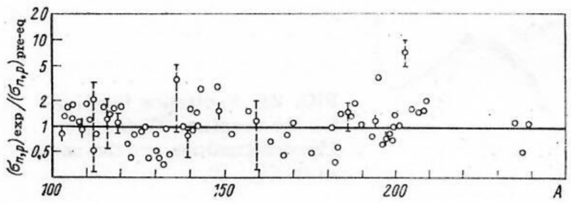


FIG. 19. Ratio of experimental and theoretical cross sections for the $(n,p)$ reaction at a primary-neutron energy $\epsilon_0 = 14$ MeV: The calculations were carried out for the same assumptions as in Fig. 18.

to the preequilibrium spectrum comes only from the simplest configurations with $n = 3, 5$, and 7. The fraction of preequilibrium emission exhausts for the most part the sum of these partial contributions, although statistical equilibrium sets in at values of the number of excitons larger by a factor of two ($\bar{n} \approx 14$). In other words, the results of the analysis do not depend on an exact knowledge of the upper limit of the summation in Eq. (32). The results of a similar investigation for the $(p, n)$ reaction are given in Fig. 17.

In the proton spectra the fraction of the preequilibrium component is substantially higher than in the neutron spectra. This is explained by the effect of the Coulomb barrier, which prevents emission of protons from weakly excited nuclei; in the case of neutrons, states with low $\epsilon$ produce the "evaporative" background. For $\epsilon_0 \approx (10-20)$ MeV the emitted-proton spectrum is practically completely determined by nonequilibrium processes (Fig. 18).

The dominant role of preequilibrium emission in $(n, p)$ reactions appears especially clearly in discussion of the cross-section ratio $\sigma_{np}/\sigma_{nn}$. When calculated by compound-nucleus theory, this ratio is greatly different from that measured experimentally, as the result of underestimation of $\sigma_{np}$. The discrepancy reaches three orders of magnitude in the region of heavy nuclei, which of course cannot be attributed to uncertainty in the model parameters and clearly indicates the existence of another reaction mechanism. If we assume that all protons are the products of preequilibrium decay the ratio of the experimental $(n, p)$ cross section to the theoretical value turns out to be close to unity over a wide range of mass numbers, $100 < A < 240$ (Fig. 19).

Let us consider how the result changes if we use various modifications of the exciton models. It was remarked above (see Fig. 12) that the difference in the models affects mainly the magnitude of the contribution of the preequilibrium component to the low-energy part of the neutron spectrum. In analysis of the complete spectrum in terms of Eqs. (41) and (42), this leads to different values of the level-density parameter $a$. In other words, by fitting Eq. (41) with the three parameters $k_1$, $k_2$, and $a$ to the experimental spectra we can extract the *single-particle state density parameter a*. Since the values of $a$ are rather well known from other independent experiments, obtaining values close to the experimental ones serves as a check of the *shape of the preequilibrium spectra*, which in principle permits a choice to be made between various versions of the exciton model.

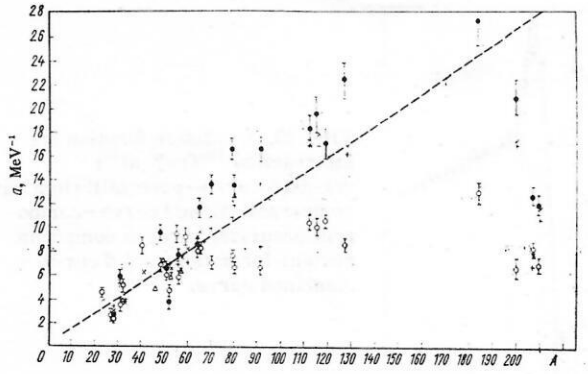


FIG. 20. Level-density parameters $a$ from analysis of neutron inelastic-scattering spectra, carried out without inclusion(⊥) and with inclusion(⊥) of preequilibrium emission[72]: For comparison we have shown data from analysis of neutron resonances (×—Ref. 73, +—Ref. 74) and the fluctuations of the cross section (Δ—Ref. 75); the dashed curve corresponds to $a = A/7.5$ MeV$^{-1}$.

An analysis of this type was carried out by Meister *et al.*[72] on the basis of the prompt-neutron spectra at a primary-neutron energy $\epsilon_0 = 14.6$ MeV (Fig. 20). The result shows that without inclusion of preequilibrium emission the values of $a$ obtained differ substantially from the average behavior $a = A/7.5$ MeV$^{-1}$ extracted from independent experiments.[73,74] The simple exciton models agree reasonably well with experiment, but, overestimating the contribution to the region of high excitations of the residual nucleus, they lead all the same to exaggerated values of the parameter $a$. Here it turns out that the best agreement with experiment is achieved in Blann's model (32), while for the hybrid model (35) the difference from the average behavior of $a$ is appreciable. The cause of this discrepancy in spectrum shape is due at least partly to the features of the interactions in the surface layer of the nucleus. These effects are for the most part taken into account in the geometry-dependent hybrid model[40,41] discussed above. As can be seen in the case of the reaction $(n + {}^{93}Nb)$ (Fig. 21), in accordance with our expectations this model gives a value for the parameter $a$ about 13% lower than Blann's model.

In the approximation of an equidistant spectrum of single-particle states the tail of the high-energy distribution of nucleons behaves as $U^{n_0-2}$ [see Eq. (34)].

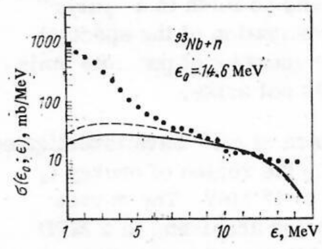


FIG. 21. Spectrum of neutrons emitted in the reaction $n + {}^{93}Nb$[47]: The solid curve is the preequilibrium component calculated according to the geometry-dependent hybrid model, Eq. (36)[76]; the dashed curve is the same according to Blann's model, Eq. (32).

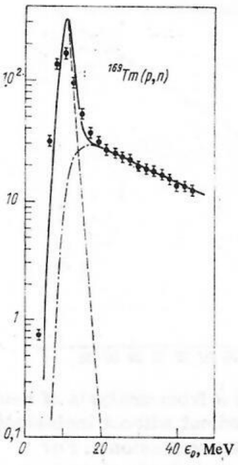


FIG. 22. Excitation function for the reaction $^{165}$Tm$(p,n)$[30]: dot-dash curve—preequilibrium curve component; dashed curve—component occurring through compound nucleus formation; solid curve—combined curve.

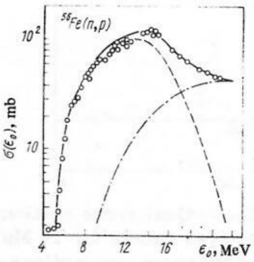


FIG. 23. Excitation function for the reaction $^{56}$Fe$(n,p)$[56]: The designations are the same as in Fig. 22.

In the case of nucleon-nuclear reactions we have $n_0=3$, but because of the *effect of shell structure* we can expect deviations of the effective value from $n_0=3$. A search for such effects was undertaken by Lee and Griffin[63] and Alevra *et al.*[69] in study of reactions for six isotopes of tin at bombarding proton energies $\epsilon_0=10$ and 14 MeV, and also for $^{51}V$ at $\epsilon_0=22$ MeV.[58] It turned out that for the nuclide $^{115}$Sn the effective value of the initial number of excitons is close to $n_0=4$, which agrees qualitatively with calculations of the density of quasiparticle states in the Nilsson scheme. However, an unambiguous interpretation of this fact is hindered by the influence of the pairing effect. Introduction of a gap of width $\Delta\approx 1$ MeV in the excitation energy for even proton and neutron systems also leads to a deformation of the spectra, which can be parametrized by a change in the effective value of $n_0$. In order to draw unique conclusions regarding the cause of the deviation of the exponent from $n_0-2$ it will be necessary to make further, more precise and systematic experimental studies.

### Excitation functions in reactions induced by nucleons.

All excitation functions have a characteristic form: a rapid rise at energies $\epsilon_0$ above the threshold of the reaction considered, arrival at a more or less distinct maximum, and then a comparatively slow drop due to opening of new reaction channels as $\epsilon_0$ increases (Figs. 22 and 23). It is easy to discern a direct relation between the slow falloff of the excitation function and the high-energy tail in the spectrum of emitted particles; study of the behavior of the cross sections for the simple reactions $(p,n)$, $(n,n')$, and so forth is a "purer" case in comparison with investigation of the spectral distributions, since here the question of possible emission of several particles does not arise.

Gadioli *et al.*[24] and Birattera *et al.*[56] have investigated simple excitation functions in the region of energy $\epsilon_0$ from the reaction threshold to 45 MeV. The curves given in Fig. 22 were calculated according to a MPD based on Eq. (30); the probabilities of intranuclear transitions were evaluated by averaging $\lambda_{\infty 11}$ over all possible particle-hole configurations [Eqs. (25) and (26)]. The model satisfactorily reproduces the shape of the excitation function, but to obtain agreement with experiment in absolute value it turns out to be necessary to introduce a constant factor of 0.3–0.7 which depends only on the target nucleus. It should be noted that here no additional assumption is made regarding the contribution of preequilibrium emission.

At low excitation energies to a rather good approximation $\lambda_+\sim E^2$, i.e., $\langle|M|^2\rangle\approx$const and, consequently, it is possible to use Eq. (31). Calculations according to Blann's model are compared with experiment in Fig. 23. Comparison of the results shown in Figs. 22 and 23 shows that in reactions with emission of a proton the role of preequilibrium emission is substantially greater than in reactions with a neutron in the exit channel.

### Emission of several particles.

Information on the multiplicity of particles produced is obtained from analysis of the excitation functions for $(p,xn)$ and $(p,p'xn)$ reactions. Their general shape is similar to that shown in Figs. 22 and 23, but for large values of $x$ the maximum is somewhat broadened. In terms of MPD the process is described as a sequence of events in which individual particles are emitted, all characteristics for emission of the $n$th particle being determined by the state of the system after emission of the $(n-1)$th particle. The Monte-Carlo method is particularly convenient for making such calculations.[17]

Several studies[24,77–79] have been devoted to excitation functions for reactions with several particles in the final state. All calculations have been made on the assumption that only the first particle is emitted in the preequilibrium stage. It is possible to reproduce the shape of the excitation and its absolute value if the fraction of preequilibrium emission is considered as a free parameter. Essentially, the authors of these stud-

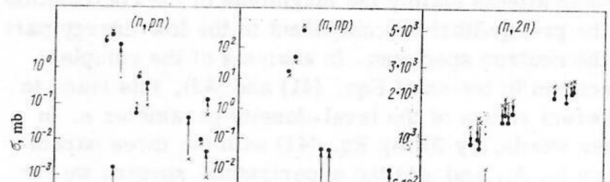


FIG. 24. Effect of preequilibrium emission on two-particle reaction channels, $\epsilon_0=14.6$ MeV[80]: The calculations without inclusion (×) and with inclusion (●) of preequilibrium emission for the same nucleus have been joined by an arrow.

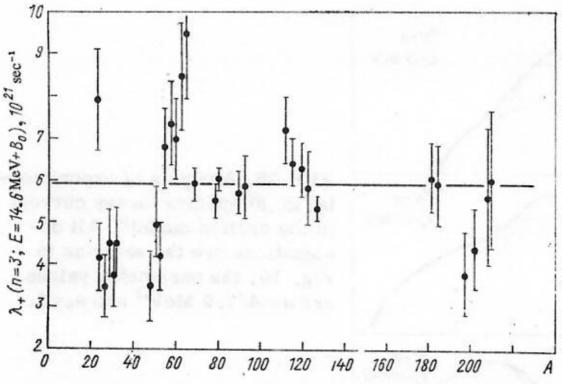

FIG. 25. Transition probability for three-quasiparticle states, obtained from analysis of $(n,n')$ spectra at $\varepsilon_0 = 14.6$ MeV[85]: The straight line corresponds to the average value of $\lambda_+$.

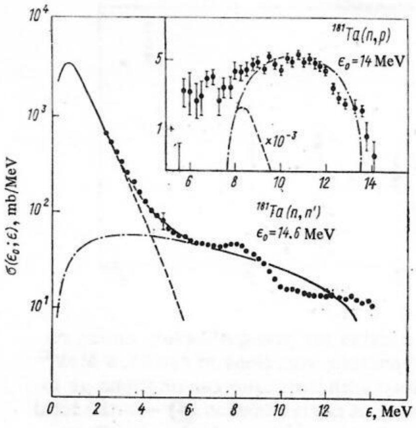


FIG. 26. Absolute calculation of nucleon spectra in nucleon-nuclear reactions in the tantalum nucleus[49]: The curves were obtained by solution of the master equation (1) by the Monte Carlo method; the effective matrix element was evaluated in the approximation of Eq. (24); the dashed curve and the dot-dash curve are the equilibrium and preequilibrium components; the solid curve is the sum of these curves.

ies are attempting to include in the preequilibrium component also the direct processes, whose role increases in the transition to functions with larger values of $x$. Experimental indications of direct processes were obtained by Miyano *et al.*[79]: For the reaction $^{209}Bi(p,p'3n)^{206}Bi$ in the region $\epsilon_0 \approx 35-50$ MeV a significant number of recoil nuclei were observed with a shorter mean range than would be expected from the hypothesis of capture of the incident proton by the nucleus.

Double nucleon emission has been studied in detail for reactions initiated by neutrons with energy 14 MeV.[51] In terms of the hybrid model (35) all possible paths for achieving the equilibrium state are followed, and the subsequent behavior of the system is described by the statistical theory of the compound nucleus. As shown by Kalbach—Cline,[51] the probability of successive emission of two particles in the preequilibrium stage amounts to less than 1% of the probability of formation of a three-quasiparticle doorway state. Thus, second-generation particles can be considered with high accuracy as emitted from the compound nucleus. Nevertheless preequilibrium processes exert a significant influence on the cross section for the reaction with emission of two nucleons through the emission of the first-generation particle, this being true in different degrees for neutrons and protons. As can be seen from Fig. 24, taking into account preequilibrium decay decreases the $(n,2n)$ cross section by about 15—20% and appreciably strengthens the $(n,pn)$ channel.

### Probabilities of intranuclear transitions obtained experimentally.

The quantity $k_1$ obtained from analysis of spectra with use of Eq. (41) provides absolute normalization of the preequilibrium spectrum $N_i(\epsilon)$. However, it is possible to pose the inverse problem: What value of the effective matrix element $\langle |M|^2\rangle$ or transition probability $\lambda_+$ corresponds to the absolute description of the spectral distributions? Particular interest is presented by the question of whether the transition probabilities between $n$-exciton states in different reaction channels will turn out to be identical—the assumption on which the exciton model is based.

A systematic analysis of about thirty neutron spectra measured in bombardment of nuclei by neutrons with energy $\epsilon_0 = 14.6$ MeV has been carried out by Hermsdorf *et al.*[81] and Marchuk and Kolesov.[82] These authors took into account corrections for particle emission and the distinguishability between neutron and proton systems in calculation of the density of states; $g = (6/\pi^2)(A/7.5)$ MeV$^{-1}$ according to Faccini and Saetta—Menichella.[73] The values of $\lambda_+$ obtained under these assumptions are shown in Fig. 25. As was expected, the transition probability is practically independent of the mass number. The average value obtained from these experiments is $\langle\lambda_+(n=3)\rangle = (5.9 \pm 0.7) \times 10^{21}$ sec$^{-1}$, which gives for the mean life of an $n$-quasiparticle state $t_3 = (1.7 \pm 0.2) \times 10^{-22}$ sec. From Eq. (17) we obtain an estimate of the mean square of the transition matrix element for $E = 22$ MeV $\langle|M|^2\rangle = (18 \pm 2)A^{-3}$ MeV.[2)]

A similar study of the $(n,p)$ reaction at $\epsilon_0 = 14$ MeV for nuclei in the region $A > 100$ has been carried out by Braga—Marcazzan *et al.*[38] Analysis of the experimental spectra was carried out with Eq. (31) and the same parameter values as in the previous analysis. It turned out that $\langle\lambda_+(n=3)\rangle = 4.9 \times 10^{21}$ sec$^{-1}$ or $\langle|M|^2\rangle = 15.2A^{-3}$ MeV$^2$. This is in good agreement with the results of analysis of the neutron channel and indicates the consistency of the MPD.

A similar value for the transition probability was obtained in study of the excitation functions in the $(p,n)$ reaction: $\lambda_+(n=3) \approx 6 \times 10^{21}$ sec$^{-1}$.[56]

If we evaluate theoretically the probability of nucleon—nucleon collisions in nuclear matter at $\epsilon_0 = 14$ Mev, then

---

[2)]It should be noted that in many studies the relation between $\lambda_+$ and $\langle|M|^2\rangle$ is established by means of an expression proposed by Williams,[30] which differs from the exact equation (17) by a factor 1/2. We shall everywhere use Eq. (17), appropriately renormalizing the results of other studies.

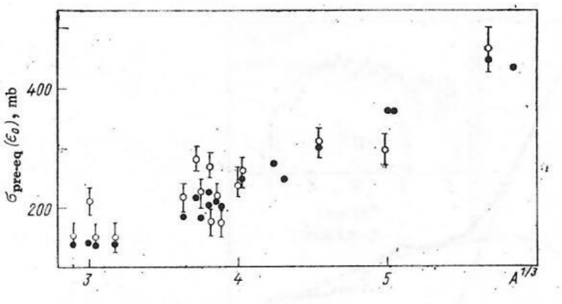


FIG. 27. Total cross section for preequilibrium emission of neutrons in neutron-nuclear reactions at $\epsilon_0 = 14.6$ MeV[83]: the results were obtained with the same assumptions as in Fig. 26 with two variants of normalization: (ɸ) —to the total cross section for inelastic interaction calculated with the optical model, i.e., without free parameters; (ɸ)—to the experimentally measured $(n,n')$ spectra.

from Eqs. (17), (21), and (27) we obtain $\lambda_+(n=3) = \lambda_{coll}/2 = 14\times10^{21}$ sec$^{-1}$, which is more than twice the experimental value of $\langle\lambda_+\rangle$. An independent estimate of $\langle\lambda_+\rangle$ is given by the optical model. For 14-MeV neutrons the imaginary part of the phenomenological optical potential is $W_{opt}\approx 4.5$ MeV for practically all values of the mass number.[82] Using Eq. (22), we have $\lambda_+(n=3) = 7\times10^{21}$ sec$^{-1}$. This good agreement is not accidental, since the optical-potential parameters were chosen so as to describe experiments on elastic scattering of nucleons.

An absolute value in agreement with experiment $\lambda_+(n=3,\ E=22\ \text{MeV}) = 7.3\times10^{21}$ sec$^{-1}$ is given by $\lambda_{coll}$ calculated for the average energy per exciton [see Eq. (24)]. However, for $\lambda_{coll}$ averaged over all allowed particle-hole configurations of the nucleus [see Eqs. (25) and (26) and Fig. 8)], the discrepancies with the measured values are large: $\lambda_+(n=3,\ E=22\ \text{MeV}) = 24\times10^{21}$ sec$^{-1}$.

The energy dependence of $\langle|M|^2\rangle$ has been studied in $(p,p')$ and $(p,n)$ reactions by means of the master equation.[43,51] Satisfactory reproduction of the experimental spectra is achieved for parametrization of the matrix element in the form $\langle|M|^2\rangle = kE^{-1}A^{-3}$ with a coefficient $k = 190\pm60$ MeV$^3$. For $E = 22$ MeV this gives $\langle|M|^2\rangle = (9\pm3)A^{-3}$ MeV$^2$, which agrees with the results given above.

An independent means of finding the widths of two- and three-quasiparticle states from an autocorrelation analysis of the fluctuations in the preequilibrium spectra has been suggested by Braga–Marcazzan and Milazzo–Colli.[75] However, such an analysis has not yet been carried out in practice.

The closeness of the experimental and theoretical values of $\lambda_+$ permits us to hope that MPD will be successful in predicting absolute spectra and cross sections. Calculations of this type have been carried out in a large number of target nuclei[49,83] in terms of MPD based on a Monte-Carlo solution of the master equation and approximation (24) and for evaluation of the cross section for interaction of excitons. The degree of agreement in the spectra can be seen from Fig. 26, and the combined results of the analysis for the cross section for preequilibrium emission of neutrons are

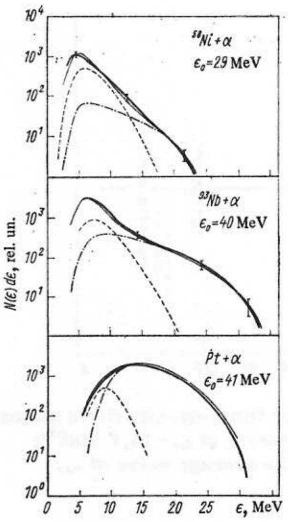


FIG. 28. Analysis of experimental $(\alpha,p)$ spectra (heavy curves) in the exciton model[14]: All designations are the same as in Fig. 16; the parameter values are $a = A/7.9$ MeV$^{-1}$ and $n_0 = 5$.

shown in Fig. 27. It can be seen that the absolute agreement of the calculations with experiment is rather convincing, and as expected this cross section turns out to be proportional to $A^{1/3}$.

The experimental data over a wide range of mass numbers and energy $\epsilon_0$ are quite satisfactorily reproduced also in terms of the hybrid model with use of the collision probability $\lambda_{coll}$ calculated according to Eq. (21).[39] It is possible to improve the description by determining $\lambda_{coll}$ in terms of the imaginary part of the optical potential.[41,76]

### Reactions induced by complex particles. Emission of nucleons.

We shall discuss here mainly reactions produced by $\alpha$ particles. From the spectra given in Fig. 28 it is evident that they cannot be described in terms of compound-nucleus theory. The relative contribution of preequilibrium emission of protons is rather large and, as in the case of nucleon-nuclear reactions, increases in the transition to heavier nuclei. It is interesting that these spectra can be interpreted by means of the MPD formalism developed above for nucleon-nuclear interactions with replacement of the initial number of excitons by $n_0 = 5$.[14]

As a result of the uncertainty in the quasiparticle

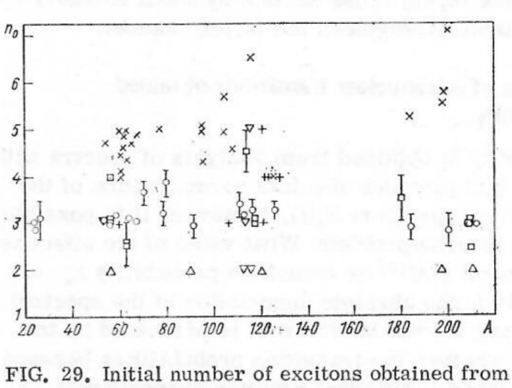


FIG. 29. Initial number of excitons obtained from analysis of the shape of spectra for reactions induced by nucleons (□—Ref. 14; △—Ref. 15; ▽—Refs. 58 and 62; ○—Ref. 46) and by $\alpha$ particles (×—Ref. 14; + —Ref. 69).

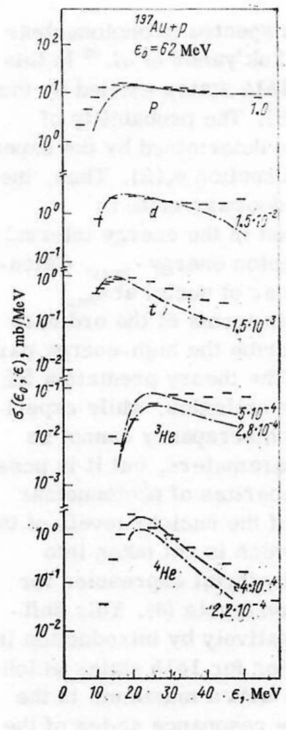

FIG. 30. Energy spectra of protons, deuterons, tritons, $^3$He, and $\alpha$ particles, obtained in gold nuclei bombarded by protons[15,31]: dot-dash curves—calculation of preequilibrium spectrum carried out by means of the master equation (1); the normalization factors are indicated [the factor $\gamma_i$, see Eq. (14)] for the complex-particle spectra relative to the proton spectrum.

configuration of the initial state in the case of a reaction induced by a complex particle, an additional parameter $n_0$ appears. A number of workers have attempted to determine this parameter experimentally from analysis of the high-energy part of the spectrum of emitted particles [see Eq. (34)]; these results are shown in Fig. 29. The results show that, for the case of reactions induced by protons, the values of $n_0$ are distinctly grouped near the expected value $n_0=3$, while in reactions induced by $\alpha$ particles $n_0=5$ and the dispersion is noticeably greater. The existing data on interaction of nuclei with $^3$He corresponds to a parameter value $n_0\approx 3.6-4.0$. Experiments carried out with beams of deuterons and $^6$Li nuclei lead to values $n_0\approx 2-4$ and 6–8.[12] The general appearance of the excitation function of $(\alpha,p)$ and $(\alpha,xpyn)$ reactions is the same as in reactions induced by nucleons.[14,16,65,78,86,87]

Analysis of the absolute values and shape of the excitation functions and spectra of emitted particles permits evaluation of the square of the matrix element for intranuclear transitions. According to the data of Kalbach–Cline,[43] which are based on study of eleven nuclei in the region $54<A<124$ at energies $E=34$ and 44 MeV, $\langle|M|^2\rangle=1450E^{-1}A^{-3}$ MeV$^2$. The dependence of $\langle|M|^2\rangle$ on mass number is confirmed in an analysis of the $(\alpha,n)$ excitation function in the nuclei $^{51}$V, $^{181}$Ta, and $^{197}$Au, but here no important energy dependence was observed in the interval $E\approx 25-50$ MeV.[65]

Treatment of $\alpha$-particle elastic scattering by means of an optical potential at $E\approx 35-50$ MeV leads to a mean free path $\Lambda_\alpha\approx 0.4F$.[88] For $E=40$ MeV this corresponds to $\lambda_{\rm coll}=1\times10^{23}$ sec$^{-1}$. Setting $n_0=4$ and using Eqs. (17) and (27), we have $\lambda_+(n=4)=2\lambda_{\rm coll}/5=4\times10^{22}$ sec$^{-1}$, which is very close to the value $5\times10^{22}$ sec$^{-1}$ found from the estimate of $\langle|M|^2\rangle$ given above.[43]

The small value of $\Lambda_\alpha$ indicates that the interaction of the $\alpha$ particle occurs mainly in the surface layer of the nucleus. For this reason particular interest is presented by study of the $(\alpha,\alpha')$, $(\alpha,p)$, and $(\alpha,n)$ channels with inclusion of the reaction geometry.

**Emission of complex particles.**

As was noted above, emission of complex particles is included in MPD at the price of an additional parameter which can be introduced in two ways: either by means of a condensation coefficient $\gamma_i$ [see Eq. (14)] or through the probability of "preparation" of the particles [see Eq. (15)].

The results shown in Fig. 30 are an example of the first approach. Values of $n_0$ and of the parameters in $\lambda_+$ were chosen on the basis of the proton spectrum and used without change for other charged particles. In the figure the dominant role of preequilibrium processes in the complex-particle emission channels is clearly seen. The discrepancy with experiment in the energetic part of the spectrum indicates the contribution of direct reaction mechanisms.

The second approach was developed by Milazzo-Colli and colleagues.[32,33,89] A combined analysis of the spectra, excitation functions, and cross sections for $(n,\alpha)$ and $(p,\alpha)$ reactions in heavy nuclei in the energy region $\epsilon_0\leq 20$ MeV permitted extraction of the probability of preparation of the $\alpha$ particle $\varphi$. These values are shown in Fig. 31 for various nuclei, and in Fig. 32 we have shown the accuracy obtainable by this means in describing the $\alpha$-particle spectra. It is interesting to note that the same quantity $\varphi$ enters into the expression for the probability of spontaneous $\alpha$ activity of even–even nuclei:

$$W_\alpha=\varphi\,(T_0^\alpha/2\pi\hbar)\,(4/g),$$

where $T_0^\alpha$ is the transmission coefficient calculated with the optical model and $g/4$ is the density of single-particle states of the $\alpha$ particle in the nucleus. The values of $\varphi$ found from $W_\alpha$ and plotted in Fig. 31 agree reasonably well with the results of analysis in the MPD, which is considered an argument in favor of introducing this parameter. The physical picture of preparation of

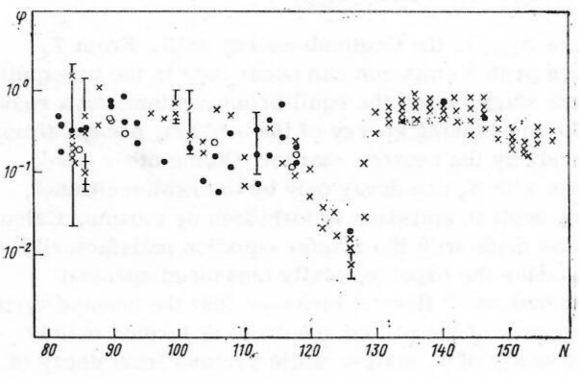

FIG. 31. Probabilities of "preparation" of $\alpha$ clusters, obtained from analysis of $(p,\alpha)$ and $(n,\alpha)$ reactions (respectively ○ and ●): ×—similar values extracted from the probability of $\alpha$ decay of heavy nuclei.[89]

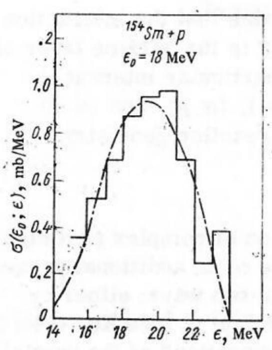


FIG. 32. Energy spectrum of $\alpha$ particles emitted in a proton-nuclear reaction[33]: histogram—experiment; curve—calculation with exciton model with $\varphi = 0.36$.

$\alpha$ particles corresponds in principle to description of the $(n, \alpha)$ reaction as a process of $\alpha$-particle knockout.[70,71,90]

**Open questions of the MPD.**

Nucleon-nuclear reactions are the traditional field of application of MPD. The success of the simple exciton models has stimulated extension of the preequilibrium approach to other processes. In particular, special interest is presented by reactions in which there appear certain nuclear states which are separated in view of their specific nature. Among such states are first of all isospin-analog states, giant-dipole-resonance levels, and so forth.

The question of the *role of isospin* in preequilibrium decay reactions was raised by Kalbach-Cline *et al.*[54] The $(p, p')$ reaction was studied at $\epsilon_0 = 14.0$ and $17.8$ MeV in the separated neutron-excess isotopes[118,120,124] Sn. Generalization of the exciton model to this case was made by analogy with the compound-nucleus theory of Robson[91]: One considers two components of the states, with larger isotopic spin values $T_> = T_0 + 1/2$ and smaller values $T_< = T_0 - 1/2$ ($T_0$ is the isospin of the target nucleus), and the probabilities of formation of the components are determined by the squares of the corresponding Clebsch—Gordan coefficients. The density of states with $T_<$ is given by the total density of excited states, while for $T_>$ one takes the density of isobaric-analog states at an energy $E - \Delta_{\text{Coul}}$:

$$w(n, E, T_<) = w(n, E);$$
$$w(n, E, T_>) = w(n, E - \Delta_{\text{Coul}}),$$

where $\Delta_{\text{Coul}}$ is the Coulomb energy shift. From $T_<$ states proton emission can occur only in the preequilibrium stage, since the equilibrium system, as a result of the low binding energy of the neutron, decays almost entirely by the neutron channel. On the other hand, states with $T_>$ can decay only by the proton channel, since neutron emission is forbidden by isospin. Calculations made with the master equation satisfactorily reproduce the experimentally measured spectral distributions.[54] Here it turns out that the preequilibrium component of the proton spectrum is formed mainly from decay of $T_<$ states, while protons from decay of $T_>$ states contribute to the equilibrium part of the spectrum. It should be emphasized that inclusion of isospin in the scheme of the exciton model is a rejection of the hypothesis of identity of all states with given $n$.

Investigation of the neutron spectra in photonuclear reactions was undertaken by Luk'yanov *et al.*[92] In this case the doorway states are $1p1h$ states excited by the bremsstrahlung spectrum $F(E)$. The probability of interaction with a nucleus was determined by the experimental photoabsorption cross section $\sigma_\gamma(E)$. Thus, the spectrum of excitation of the doorway state is $F(E)\sigma_\gamma(E)dE$, which is specified in the energy interval from zero to the maximum photon energy $\epsilon_{\gamma\max}$. Calculations carried out for a number of nuclei at $\epsilon_{\gamma\max} = 14$ and 28 MeV showed that in terms of the ordinary MPD it is not possible to describe the high-energy parts of the photoneutron spectra. The theory predicts a 5% contribution of preequilibrium emission, while experiment gives about 20%.[92] This discrepancy cannot be removed by variation of the parameters, but it is possible to explain the detailed properties of photonuclear reactions by collectivization of the nuclear levels of the dipole resonance—an effect which is not taken into account in the simplest combinatorial expression for the density of exciton excitation levels (8). This deficiency can be corrected qualitatively by introduction into Eq. (8) of a modulating factor for $1p1h$ states which would have a resonance shape with a maximum in the region of location of the dipole resonance states of the nucleus. In the first stage of discussion it is natural to assume that this factor is proportional to $\sigma_{\gamma\exp}$, and the proportionality coefficient $K$ can be taken from comparison of the neutron spectra with experiment. Then

$$\tilde{w}(1p, 1h, E) = K\sigma_{\gamma\,\exp}(E)\, w(1p, 1h, E). \tag{43}$$

The fact that the nucleus is formed as the result of a coherent transition to the doorway $1p1h$ state already has been taken into account by use of the experimental value of $\sigma_\gamma$ in determination of the excitation energy. It is clear that, as the result of the condition (43), in the subsequent intranuclear transitions the probabilities $\lambda_-(n=4)$ and $\lambda_0(n=2)$ will be enhanced, returning the system to the initial two-quasiparticle state. Thus, the physical meaning of the modulation (43) is that now the system lives a comparatively long time in $1p1h$ states. As a result a relative enhancement occurs in the channel for neutron emission directly from the initial dipole-excitation state to the continuum.

It turned out that modulation at a level of 5–7% is sufficient to explain the 20% contribution of the pre-equilibrium component to the photoneutron spectrum.[92] It was apparently in photonuclear reactions that the need first arose to introduce into MPD some general structural features.

*Capture of $\mu$ mesons* has been discussed in terms of MPD by Eramzhyan and Toneev[93] and Kozlowski and Zglinski.[94] The specific properties of these reactions, at least for light nuclei, are that the transition occurs with formation of the daughter nucleus not in the ground state but in excited states which are isospin analogs of the giant photonuclear resonances of the target nucleus. Therefore one of the questions which arises in study of the characteristics of $\mu$ capture is determination of the excitation spectrum of the daughter nucleus. This problem has been solved in various ways,[92,93] with various goals in mind: Luk'yanov *et al.*[92] used the exciton model to analyze the effect of the excitation spectrum in $\mu$ capture on the shape of the energy distributions of emitted neutrons, while Eramzhyan and Toneev[93] emphasize the possibility of obtaining absolute agreement with experiment for the neutron and proton spectra. The general conclusion which follows from these studies is that application of the ideas on MPD to nuclear reactions of this type is unquestionably useful; in such work it is necessary to take into account the gross structure of the excited states.

*Reactions induced by heavy ions* are one of the most attractive applications of MPD. The interpretation of a considerable number of phenomena in heavy-ion physics in terms of compound-nucleus theory encounters serious difficulties,[95] and there are direct experimental indications of the importance of nonequilibrium processes.[96,97] However, the uncertainties arising in replacement of the bombarding particle by a nucleus hinder the direct use of the simple relations of the exciton model. This refers first of all to the definition of the doorway state. It is unclear what to choose as the initial number of excitons, and the absence of corresponding measurements of the spectral distributions does not permit $n_0$ to be obtained experimentally. In addition, at least for the population of the initial state, the assumption of equal probability of all energetically accessible states is violated. This is due to the fact that in the process of fusion, the nucleons of the nucleus, which have a significant momentum of their own, may be bound with the bombarding nucleus (or on the other hand with the target nucleus) be very different means, within the limits allowed by the conservation laws and the Pauli principle. As a result the distribution in the energy of the doorway state will more likely be random among all possible energies than an equal-probability distribution.

Thus, at the present time use of MPD in reactions induced by heavy ions still has a qualitative nature. Actually we have only an indication of phenomena in which the systematic use of the nonequilibrium approach can provide interesting results. Among the most promising applications of MPD[96–99] we can mention: *the fusion of two nuclei*, where the preequilibrium mechanism may turn out to be the main channel of energy dissipation; *the competition of fission and particle emission*—here it is necessary to take into account the possibility of emission of a fast particle in the preequilibrium stage before analyzing the ratio of the fission and evaporative widths; *the effect of critical angular momentum*, which was discussed until recently without taking into account the fact that a high-energy preequilibrium particle can carry away a large angular momentum; *many-nucleon transfer reactions*, which must be discussed in terms of the same ideas as the fusion process; *the behavior of the high energy part of the spectra and excitation functions*—this must be determined mainly by preequilibrium emission.

## 3. MODELS OF PREEQUILIBRIUM DECAY AND THE GENERAL METHODS OF NUCLEAR-REACTION THEORY

### Stationary approaches.

One of the first attempts to go beyond the framework of the statistical theory of the compound nucleus was made by Izumo,[4] who considered the mechanism of partial equilibrium in which only several "valence" nucleons are excited and the core of the nucleus remains inert. The development of exciton models is directly related to analysis of the intermediate structure of nuclei and to the concept of hallway states.[3)] We recall that Griffin's work[5] is based on statistical treatment of hallway states. Grimes *et al.*[60,61] proposed to describe also preequilibrium decay in terms of the statistical theory, not for compound states, but for hallway states.

The question of the relation of exciton models to *the unified theory of nuclear reactions* has been raised by Feshbach.[100] In the unified theory of nuclear reactions[101] the problem of interaction, for example, of a nucleon with a nucleus described by a total Hamiltonian $H$:

$$H\Psi=E\Psi, \tag{44}$$

is solved by separation from the complete wave function $\Psi$ of the wave function in the open channels $P\Psi$ by means of projection operators $P$ and $Q$: $P+Q=1$, $PQ=QP=0$, $P^2=Q^2=1$. Then instead of Eq. (44) we will have a system of coupled equations

$$(E-H_{PP})P\Psi=H_{PQ}Q\Psi; \tag{45}$$

$$(E-H_{QQ})Q\Psi=H_{QP}P\Psi \tag{46}$$

or

$$\left(E-H_{PP}-H_{PQ}\frac{1}{E-H_{QQ}}H_{QP}\right)P\Psi=0; \tag{47}$$

$$\left(E-H_{QQ}-H_{QP}\frac{1}{E^{+}-H_{PP}}H_{PQ}\right)Q\Psi=H_{QP}\Psi_0^{(+)}. \tag{48}$$

Here we have used the designations $H_{PP}=PHP$, $H_{QQ}=QHQ$, and so forth; $\Psi_0^{(+)}$ is the eigenfunction of the operator $H_{PP}$; $E^{(+)}$ denotes $E+i\eta$ with $\eta\to+0$.

The transition amplitude $\mathcal{T}$ for scattering of the incident particle is found from consideration of the asymptotic form of $P\Psi$. As can be seen from Eq. (47), $\mathcal{T}$ will contain a part slowly varying with energy, due to scattering of the particle by the average field $H_{PP}$, and a part which resonates on passage of the energy $E$ through the eigenvalues of the operator $H_{QQ}$:

$$(E_S-H_{QQ})\Phi_S=0. \tag{49}$$

This is actually Eq. (46) with the coupling with the open channels neglected, i.e., $\Phi_S$ can be identified with the stationary states of the compound nucleus. We can use the well known technique to write the transition amplitude in the following form[100]:

$$\mathcal{T}=\mathcal{T}_{\text{pot}}+\left\langle\Psi_f^{(-)}\middle|H_{PQ}\frac{1}{E-H_{QQ}-W_Q}H_{QP}\Psi_0^{(+)}\right\rangle, \tag{50}$$

---

3)Here and subsequently we mean by hallway state an intermediate state of any order, and not only a doorway or entrance state.

where $\mathcal{T}$, the potential scattering matrix, is defined by the relation

$$2\pi i \mathcal{T}_{\text{pot}} = 1 - \langle \Psi_f^{(-)} | \Psi_0^{(+)} \rangle,$$
$$W_Q = H_{QP} \frac{1}{E^{(+)} - H_{PP}} H_{PQ}. \qquad (51)$$

In the case of an isolated resonance the second term in Eq. (50) corresponds to a typical Breit-Wigner shape of the resonance amplitude.

Inclusion of hallway states of any order or, what amounts to the same thing, expansion of the wave function $\Phi_S$ in states of increasing complexity in terms of unified nuclear theory, is accomplished by representation of the projection operator $Q$ in the form $Q = \sum_i^n d_i + q$.[102] Here the projection operators of hallway states of $i$th order $d_i$ satisfy a "valuable" condition which we shall write in terms of the effective Hamiltonians: $H_{d_i d_j} = 0$ if $|i-j| > 1$; $H_{d_i q} = 0$ for $i \neq n$; $H_{d_i P} = 0$ for $i > 1$, and we have assigned to the operator $q$ all states with complexity higher than $n$ (the designations have the meanings $H_{d_i d_j} = d_i H_{QQ} d_j$, etc.). We can then generalize Eq. (50) as follows[100]:

$$\mathcal{T} = \mathcal{T}_{\text{pot}} + \sum_i^n \mathcal{T}_i + \mathcal{T}_q;$$
$$\mathcal{T}_i = \Big\langle \Psi_f^{(-)} | H_{Pd_i} \frac{1}{E - H_{d_i d_i} - W_i} H_{d_i d_{i-1}} \cdots$$
$$\cdots H_{d_2 d_1} \frac{1}{E - H_{d_1 d_1} - W_1} H_{d_1 P} \Psi_0^{(+)} \Big\rangle;$$
$$W_l = H_{d_l d_{l+1}} \frac{1}{E^{(+)} - H_{d_{l+1} d_{l+1}} - W_{l+1}} H_{d_{l+1} d_l}.$$

The sequence of operators $H_{d_l d_{l+1}}$ describes traversal of a chain of hallway states with increasing complexity. The effect of the continuum appears through the operators $H_{Pd_i}$ and through $W_l$ as the result of its coupling with the open channels [see Eq. (51)]. The propagators of the corresponding states contain the quantity $W_l$, whose imaginary part determines the width of the possible decay of the hallway state of the next order $l+1$: $\Gamma(l+1, l)$. If we introduce the eigenfunctions $\varphi_l$ of the operator $H_{d_l d_l} + W_l$, then

$$\Gamma(l, l-1) = \frac{2\pi}{\hbar} |\langle \phi_l | H_{d_l d_{l-1}} \varphi_{l-1} \rangle|^2.$$

Assuming further that the phases of the matrix elements between the hallway states are randomly distributed, after statistical averaging over energy we obtain

$$\langle |\mathcal{T}_i|^2 \rangle = \frac{2\pi}{\hbar D_i} \cdot \frac{\Gamma_{fi}}{\Gamma_i} \cdot \frac{\Gamma(i, i-1)}{\Gamma_{i-1}} \cdot \frac{\Gamma(i-1, i-2)}{\Gamma_{i-2}} \cdots \frac{\Gamma(2, 1)}{\Gamma_{i-2}} \Gamma(1, 0), \qquad (52)$$

where $D_i$ is the average distance between hallway states of $i$th order; $\Gamma_i$ is the total averaged width of the hallway states; $\Gamma_{fi}$ is the width for decay to a final state from the $i$th hallway state; $\Gamma(1, 0)$ is the width for formation of the first hallway state from the initial state. The structure of Eq. (52) clearly recalls the form of the simple exciton models. Feshbach[103] presented preliminary results of a calculation with Eq. (52), using Griffin's assumptions (equidistant level scheme, constancy of transition matrix elements). The shape of the energy spectra of the emitted particles is reproduced quite well.

With increase of the primary-particle energy ($\varepsilon_0 \approx 30-50$ MeV) one can expect that the average value of the energy transferred in the first interaction event is insufficient to transfer the incident particle to a bound state. In this case, as the result of successive collisions, components of target-nucleus states of higher and higher complexity will be excited. The configurations produced of a system with a particle in the continuum will make a large contribution to the preequilibrium decay spectra. The theory taking into account *excitation of complex states through direct interactions* was formulated by Rodberg in connection with the discussion of intermediate structures of nuclei.[104]

In this approach the configuration space of the complete Hamiltonian $H = H_0' + V = H_0' + \sum_i^A v_{i0}$ is divided into two subspaces $P_N + P_{NR} = 1$, where the projection operator $P_R$ assures the presence in the continuum of bound states responsible for the appearance of the resonances in the system. The nonresonance part of the $\mathcal{T}$ matrix for scattering of a nucleon by a nucleus consisting of $A$ nucleons can be expanded in the multiplicity of the collisions,

$$\mathcal{T}_{NR} = \sum_{i=1}^{A} t_{i0} + \sum_{j>i} t_{i0} \frac{P_{NR}}{E^{(+)} - H_0} t_{j0} + \ldots, \qquad (53)$$

where $t_{i0}$ is the total operator for scattering of the incident particle by a nucleon of the target nucleus in some effective optical potential $V_0 (H_0 = H_0' + V_0)$:

$$t_{i0} = V_{i0} + V_{i0} \frac{P_{NR}}{E^{(+)} - H_0} t_{i0} \text{ with } V_{i0} = v_{i0} - V_0/A.$$

The first term in Eq. (53) has the nature of a single-particle operator and limits the possible excitations to $1p1h$ components. The second term, which describes double collisions, leads to excitation of $2p2h$ states of the target nucleus. If we neglect interference terms, the reaction cross section can be represented as the sum of the partial cross sections for excitation of the various $n$-quasiparticle states.

Investigation of direct inelastic scattering with excitation of isolated final states is usually carried out in the approximation of single scattering with use of the distorted-wave method. The excited states can be described in terms of collective models or in the microscopic approach.[105] In the first case, to evaluate the dependence of the transition probability on the multipolarity, it is convenient to use a sum rule.[106] The microscopic description requires knowledge of the phases and contributions of the $1p1h$ components of the wave function of the high-lying states. Averaged reaction cross sections can be calculated on the assumption of a statistical distribution of phases.[107–109] It turns out that the theoretical shape of the energy spectra is close to that given by the exciton model for a state $n = 3$. It has also been possible to compare the structure observed in the experimental spectra with the nonuniformity of the distributions in the quasiparticle energies. The emitted-particle energy spectra calculated in the single-collision approximation are somewhat softer than the experimental spectra, which indicates the need of taking into account the next terms in the expansion (53). According to the estimate by Ive[109] the contribution of double collisions at $\epsilon_0 \approx 15$ MeV amounts to about 10%.

## Time considerations.

Use of time-dependent theory is quite natural for description of the process of establishing dynamic equilibrium. The appropriate methods were developed in terms of nonequilibrium statistical physics. Here a basic role was played by the work of N.N. Bogolyubov,[110] who introduced for the first time the concept of different stages of the time evolution process, formulated the initial system of integro-differential equations for description of the process—the so-called Bogolyubov-Born-Green-Kirkwood-Ivon hierachy of equations, and gave the first mathematically rigorous derivation of the Boltzmann equation.

One of the basic problems arising in derivation of equations which are irreversible in time, for example, the master equations or Boltzmann equations, consists in solution of the problem of how to introduce irreversibility into the initial dynamic equations. For the latter we usually take the Liouville equation, which when written for the density matrix $\rho(t)$ coincides in form with the classical equation of motion

$$\partial\rho/\partial t = (i/\hbar)\,[\rho,\ H]. \tag{54}$$

In Bogolyubov's method, irreversibility is introduced by choice of appropriate boundary conditions for the Bogolyubov-Born-Green-Kirkwood-Ivon hierachy of equations (the principle of damping of correlations). If we limit discussion only to elements of the density matrix which are diagonal in the energy representation and, in addition, introduce a gross-structure density matrix $\rho_c$, summing $\rho$ over states $\Delta n$ near $n$,

$$\rho_c(n,\ t)\,\Delta n = \sum_{n\subset\Delta n} \langle n'\,|\,\rho(t)\,|\,n'\rangle,$$

then the function $\rho_c(n,t)$, which is classically interpretable as the probability of observing the system in state $n$, will satisfy the equation

$$\frac{\partial\rho_c(t)}{\partial t} = \sum_{n'} G_n W_{nn'}\rho_c(n',\ t) - \sum_{n'} G_{n'} W_{n'n}\rho_c(n,\ t). \tag{55}$$

Here $W_{nn'}$ is the transition probability per unit time from a state in the region $\Delta n'$ to one of the states of the interval $\Delta n$; $G_{n'}$ is the total number of states in the interval $\Delta n'$. It is evident that the master equation used above is a particular case of Eq. (55).

The derivation of generalized master equations[4)] from Liouville's equation has been given by many authors and by various methods: Van Hove used the assumption a random distribution of phases at the initial moment of time and also the condition of diagonal singularity[111]; Zwanzig used the method of projection operators[112]; Prigogine and Resibois[113] used diagram technique. A review of these questions can be found in Ref. 114. Application of the methods of statistical mechanics to an infinite system of Fermi particles and, in particular, the equations of relaxation of a Fermi gas (39) are well explained in the book by Pines and Nozieres.[115]

In regard to the quantum description of the time development of a nuclear reaction, the successes here have been modest. In the series of studies by Yoshida[116] the wave-packet technique has been developed for discussion of experimental results on measurement of the lifetime of compound-nucleus states by the blocking method. An interesting application of the methods of statistical physics to the process of establishment of equilibrium in a nuclear reaction and a discussion of dynamic effects in the statistical theory of nuclear reactions have been given by Weidlach.[117] An attempt to generalize this formalism to preequilibrium decay was made by Reif *et al.*[118] with employment of the method of nonequilibrium statistical operators.[119]

The model of preequilibrium emission developed by S. T. Belyaev and his colleagues[120–122] deserves special attention. The relaxation process is described in the time-dependent Hartree—Fock approximation. In contrast to ordinary MPD, which consider only statistical collision relaxation, this model considers the coherent effect of relaxation of the self-consistent field. In fact, fast capture of an incident nucleon by a nucleus leads to a change in the self-consistent field, depending on the initial channel. At the time of the adjustment of the self-consistent field the nucleons of the nucleus are in some variable potential which creates particle-hole states. If among these states there is a component in the continuum, then the mechanism considered—*the splash mechanism*—will provide a contribution to preequilibrium emission.[120]

Mathematically the problem reduces to solution of Eq. (54) with replacement of the Hamiltonian $H$ by the self-consistent Hamiltonian $H_c$. At the initial moment (before capture of the particle) the system is in a stationary state $[H_c^0, \rho^0] = 0$. Capture of a nucleon produces a change in the density matrix $\rho(t) = \rho^0 + \delta\rho(t)$, which depends on the addition of a nucleon $\delta\rho'(t)$ and on the excitation of collective modes

$$\delta\rho(t) = \sum_\alpha a_\alpha \rho^\alpha \exp(i\omega_\alpha t) + \delta\rho'(t).$$

After linearization of the equation for $\rho$ with respect to $\rho^{(0)}$ we obtain

$$i\,\frac{\partial}{\partial t}\,\delta\rho = [H_c^{(0)},\ \delta\rho] + [\delta H_c,\ \rho^{(0)}], \tag{56}$$

where $\delta H_c$ depends on $\delta\rho$. It is still necessary to specify the initial conditions for $\rho(0)$ and $\delta\rho(0)$ which characterize the capture process and to take into account the principle of correlation damping in the boundary conditions. In solution of Eq. (56) Belyaev and his colleagues[120–122] retain the two-particle correlations. In the limiting case of an infinite uniform Fermi system the solution of Eq. (56) gives the master equation for the population number (39).[121]

Like all MPD, the splash mechanism will give a substantial excess over the "evaporative" background only in the high-energy part of the spectra. A characteristic distinction of the mechanism discussed is the memory of the quantum numbers of the entrance channel

4)The generalized master equation, in contrast to Eq. (55), has an explicit dependence on time in the right-hand side of the equation and consequently characterizes non-Markov evolution processes.

of the reaction, which naturally leads to appearance of anisotropy in the angular distribution of emitted particles. In addition, in capture of a particle not only is the equilibrium average field perturbed, but also the pairing potential is distorted, which is equivalent to appearance of an external field which can "splash" a correlated pair of nucleons. It is remarkable that such pairs will be correlated with each other (preferentially equal energies and opposite directions) but not with the incident particle.[120]

It should be noted that description of a nucleus by means of quantum kinetic equations is in essence a problem of self-consistent optical potential and permits interesting generalization of the discussion of collective excitations in nuclei.[122]

## CONCLUSION

It can be said that at the present time models of preequilibrium decay have completely received rights of citizenship. MPD are widely used in interpretation of nuclear reactions and provide another way of looking at some ideas which had already become familiar. In particular, we are discussing the arbitrariness of separating reactions into direct and equilibrium decay: It is necessary to trace the entire evolution with time of the complexity of quasiparticle states, since each stage contributes to the experimentally measured quantities.

The simple exciton models satisfactorily reproduce the shape of the energy spectra and excitation functions for the various reaction channels. Satisfactory agreement with experiment is achieved for the absolute value of the characteristics considered. Analysis by means of the exciton model leads to reasonable estimates of such nuclear quantities as the lifetime of a three-quasiparticle state.

It is a very attractive idea to transfer the concepts of preequilibrium decay, first developed with application to nucleon-nuclear reactions, to other phenomena and processes. Particular interest is presented by heavy-ion physics, including problems of the fusion and fission of nuclei, and also processes occurring through excitation of several separate states: photo- and electro-disintegration of nuclei, $\mu$ capture, absorption of stopped $\pi$ mesons, and so forth. Only the first steps have been taken in this direction, and we can be confident that these questions will receive further development. There will undoubtedly also be a broadening of the list of phenomena in which preequilibrium emission acquires substantial importance. Already at the present time interesting connections have been noted with the fragmentation of nuclei induced by high energy particles.[123] The ideas of preequilibrium emission also extend to elementary-particle physics as a generalization of the statistical theory of multiple production, which has turned out to be necessary for explanation of the production of secondary particles with large transverse momenta.[124]

In spite of all the successes of MPD, their current level of development is still far from perfected. Since they are phenomenological, they can be counted on only for a semiquantitative description. Further development of MPD requires consideration of more realistic nuclear schemes and a deep understanding of the physical meaning of the basic assumptions of the model. In this connection it is particularly important to establish the correspondence between MPD and the general methods of nuclear-reaction theory. Another important and necessary aspect is the continuation of experimental studies with the purpose of obtaining more precise and systematic data.

*Note added in proof.* After this review had been completed, a number of interesting papers devoted to preequilibrium decay of excited nuclei appeared. In particular, Bunakov and Nesterenko,[125] and Agassi *et al.*[126] have given a derivation of the master equation from the microscopic description of a nuclear reaction and have traced the connection with other theoretical approaches, and questions of the asymmetry of the angular distribution in the preequilibrium component have been discussed by Gudima and Toneev[127] and Mantzouranis *et al.*[128]; steadily increasing attention is being given to preequilibrium processes in study of reactions induced by heavy ions.[129–132]


1. V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).
2. F. E. Bertrand and R. W. Peelle, Phys. Rev. **C8**, 1045 (1973).
3. R. Serber, Phys. Rev. **72**, 1114 (1947).
4. K. Izumo, Progr. Theor. Phys. **26**, 807 (1961); Nucl. Phys. **62**, 673 (1962).
5. J. J. Griffin, Phys. Rev. Lett. **17**, 478 (1966).
6. M. Blann, Phys. Rev. Lett. **21**, 1357 (1968).
7. G. D. Harp, J. M. Miller, and B. J. Berne, Phys. Rev. **165**, 1166 (1968).
8. G. D. Harp and J. M. Miller, Phys. Rev. **C3**, 1847 (1971).
9. Proceedings of the International Conference on Nuclear Physics (Munich, Aug. 27—Sept. 1, 1973), Vol. 1, 2. Amsterdam-London-New York, 1973.
10. Proceedings of the Third International Seminar on Interactions of Fast Neutrons with Nuclei (November 5—9, 1973, Gaussig, GDR). Zentralinstitut für Kernforschung 271, Dresden, 1974.
11. Proceedings of the Europhysics Study Conference on Intermediate Processes in Nuclear Reactions (Aug. 31—Sept. 5, 1972, Plitvice Lakes, Yugoslavia). Berlin-Heidelberg—New York, 1973.
12. Proceedings of the International Symposium on Neutron-Induced Reactions (1974, Smolenice, CSSR). Acta Phys. Slov. **25**, N2—3 (1975).
13. V. S. Barashenkov and V. D. Toneev, Vzaimodeĭstvie vysokoénergeticheskikh chastits i yader s yadrami (Interaction of High-Energy Particles and Nuclei with Nuclei), Moscow, Atomizdat, 1972.
14. C. K. Cline and M. Blann, Nucl. Phys. **A172**, 225 (1971).
15. C. K. Cline, Nucl. Phys. **A193**, 417 (1972).
16. I. Rabanský, P. Obložinský, and E. Běták, Nucl. Phys. **A205**, 545 (1973).
17. K. K. Gudima, G. A. Ososkov, and V. D. Toneev, Yad. Fiz. **21**, 260 (1975) [Sov. J. Nucl. Phys. **21**, 138 (1975)].
18. S. Karlin, First Course in Stochastic Processes, Academic Press, 1966. Russ. transl., Mir, 1971.
19. T. Ericson, Adv. Phys. **9**, 425 (1960).
20. V. M. Stavinskiĭ, Fiz. Élem. Chastits. At. Yadra **3**, 832 (1972) [Sov. J. Part. Nucl. **3**, 417 (1972)].
21. V. M. Strutinski, Comptes Rendus du Congres International de Physique Nucleaire, 1958, Paris, p. 617.
22. M. Böhning, Nucl. Phys. **152**, 529 (1970).
23. F. C. Williams, Jr., Nucl. Phys. **A166**, 231 (1971).

[24]E. Gadioli, E. Gadioli Erba, and P. G. Sona, Nucl. Phys. **A217**, 589 (1973).
[25]A. V. Ignatyuk and Yu. V. Sokolov, Yad. Fiz. **16**, 277 (1972) [Sov. J. Nucl. Phys. **16**, 155 (1973)].
[26]A. V. Ignatyuk and Yu. V. Sokolov, Yad. Fiz. **17**, 723 (1973) [Sov. J. Nucl. Phys. **17**, 377 (1973)].
[27]F. C. Williams, Jr., Nucl. Phys. **133**, 33 (1969).
[28]F. C. Williams, Jr., A. Mignery, and M. Blann, Nucl. Phys. **A207**, 619 (1973).
[29]A. G. Dovbenko and A. V. Ignatyuk, Neĭtronnaya fizika (Neutron Physics), Materials of the All-Union Conference, Kiev, May 24–28 1971, Vol. 1, Kiev, Naukova Dumka, 1972, p. 94.
[30]F. C. Williams, Jr., Phys. Lett. **B31**, 184 (1970).
[31]I. Ribansky and P. Oblozinsky, Phys. Lett. **B45**, 318 (1973).
[32]L. Milazzo-Colli and G. M. Braga-Marcazzan, Nucl. Phys. **A210**, 297 (1973).
[33]L. Milazzo-Colli *et al.*, Nucl. Phys. **A218**, 274 (1974).
[34]P. Obložinský, I. Ribanský, and E. Běták, Nucl. Phys. **A226**, 347 (1974).
[35]M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).
[36]K. Kikuchi and M. Kavai, Nuclear Matter and Nuclear Reactions. North-Holland Publishing Company, Amsterdam, 1968.
[37]M. Blann, Phys. Rev. Lett. **27**, 337 (1971).
[38]G. M. Braga-Marcazzan *et al.*, Phys. Rev. **C6**, 1398 (1972).
[39]M. Blann and A. Mignerey, Nucl. Phys. **A186**, 245 (1972).
[40]M. Blann, Phys. Rev. Lett. **28**, 757 (1972).
[41]M. Blann, Nucl. Phys. **A213**, 570 (1973).
[42]F. Reinhold, D. Seeliger, and K. Seidel, Jahresbericht Zentralinstitut für Kernforschung 243, TU Dresdan, 1972, p. 27.
[43]C. Kalbach-Cline, Nucl. Phys. **A210**, 590 (1973).
[44]V. M. Bychkov *et al.*, Neĭtronnaya fizika (Neutron Physics), Materials of the Second All-Union Conference on Neutron Physics, Kiev, 1973, Vol. 1, Obninsk, 1974, p. 316.
[45]P. Decowski, W. Groschulski, and A. Marcinkowski, in Ref. 10, p. 71.
[46]D. Seeliger and K. Seidel, International Conference on Study of the Nucleus, Kulingsborn, GDR, 1971, Abstracts, Part 1, p. 39.
[47]D. Hermsdorf *et al.*, Conference on Nuclear Structure Study with Neutrons (Budapest, Hungary, 1972). Plenum Press, New York and London, 1974.
[48]D. Seeliger, K. Seidel, and D. Wohlfarth, Neĭtronnaya fizika (Neutron Physics), Materials of the All-Union Conference, Kiev, May 24–28, 1961, Vol. 1, Kiev, Naukova Dumka, 1972, p. 243.
[49]D. Seeliger *et al.*, Neĭtronnaya fizika (Neutron Physics), Materials of the All-Union Conference, Kiev, 1973, Vol. 2, Obinsk, 1974, p. 269.
[50]D. Hermsdorf *et al.*, in Ref. 9, Vol. 1, p. 514.
[51]C. Kalbach *et al.*, in Ref. 9, Vol. 1, p. 522.
[52]A. J. Kennedy *et al.*, Phys. Rev. **C5**, 500 (1972).
[53]J. Wiley, Nucl. Phys. **A242**, 1 (1973).
[54]C. Kalbach-Cline, J. R. Huizenga, and H. K. Vonach, Nucl. Phys. **A222**, 405 (1974).
[55]D. Seeliger, K. Seidel, and A. Moister, in Ref. 10, p. 97.
[56]C. Birattari *et al.*, Nucl. Phys. **A201**, 579 (1973).
[57]E. Gadioli *et al.*, Nuovo Cimento A22, 547 (1974).
[58]S. M. Grimes *et al.*, Phys. Rev. **C8**, 1770 (1973).
[59]V. V. Verbinski and W. R. Burrus, Phys. Rev. **177**, 1671 (1969).
[60]S. M. Grimes *et al.*, Phys. Rev. **C3**, 645 (1971).
[61]S. M. Grimes *et al.*, Phys. Rev. **C4**, 607 (1971).
[62]S. M. Grimes *et al.*, Phys. Rev. **C7**, 343 (1973).
[63]E. V. Lee and J. J. Griffin, Phys. Rev. **C5**, 1713 (1972).
[64]W. W. Bowman and M. Blann, Nucl. Phys. **A131**, 513 (1969).
[65]I. Ribanský, P. Obložinský, and E. Běták, in Ref. 10, p. 48.
[66]R. Caplar and P. Kulisic, in Ref. 9, Vol. 1, p. 517.
[67]G. M. Braga-Marcazzan and L. Milazzo-Colli, Lett. Nuovo Cimento **6**, 357 (1973).
[68]L. Milazzo-Colli and G. M. Braga-Marcazzan, Phys. Lett. **B38**, 155 (1972).
[69]A. Alevra *et al.*, Nucl. Phys. **A209**, 557 (1973).
[70]L. Glowacka *et al.*, in Ref. 9, Vol. 1, p. 519.
[71]L. Glowacka *et al.*, in Ref. 10, p. 109.
[72]A. Meister *et al.*, in Ref. 10, p. 85.
[73]U. Faccini and A. Saetta-Menichella, Energia Nucl. **15**, 54 (1968).
[74]A. V. Malyshev, Plotnost' urovneĭ i struktura atomnykh yader (Level Density and Structure of Nuclei), Moscow, Atomizdat, 1969.
[75]G. M. Braga-Marcazzan and L. Milazzo-Colli, Energia Nucl. **15**, 186 (1968).
[76]R. Reif, K. Seidel, and H. Seidemann, in Ref. 12.
[77]C. Birattari *et al.*, Nucl. Phys. **A166**, 605 (1971).
[78]J. E. Bisplinghoff *et al.*, in Ref. 9, Vol. 1, p. 516.
[79]K. Miyano, M. Sekikawa, T. Kaneko, and M. Nomoto, Nucl. Phys. **A230**, 98 (1974).
[80]K. Seidel, D. Seelinger, and A. Meister, in Ref. 12.
[81]D. Hermsdorf *et al.*, Zentralinstitut für Kernforschung 277, TU Dresden, 1974.
[82]G. I. Marchuk and V. E. Kolesov, Primenenie chislennykh metodov dlya rascheta neĭtronnykh secheniĭ (Use of Numerical Methods for Calculation of Neutron Cross Sections), Moscow, Atomizdat, 1970.
[83]D. Seelinger, K. Seidel, and V. D. Toneev, in Ref. 10, p. 63.
[84]A. Chevarier *et al.*, Nucl. Phys. **A231**, 64 (1974).
[85]C. Cline, Nucl. Phys. **A174**, 73 (1971).
[86]M. Blann and F. M. Lanzafame, Nucl. Phys. **A142**, 559 (1970).
[87]A. Demeyer *et al.*, J. Phys. **31**, 874 (1970).
[88]G. Igo, L. F. Hansen, and T. F. Gooding, Phys. Rev. **131**, 337 (1963).
[89]G. M. Braga-Marcazzan and L. Milazzo-Colli, in Ref. 10, p. 39.
[90]M. Kozłowski *et al.*, Nucl. Phys. **A187**, 177 (1972).
[91]D. Robson, Phys. Rev. **137**, B535 (1961).
[92]V. K. Luk'yanov, V. A. Seliverstov, and V. D. Toneev, Yad. Fiz. **21**, 992 (1975) [Sov. J. Nucl. Phys. **21**, 508 (1975)].
[93]R. A. Eramzhyan and V. D. Toneev, Communication JINR, E4-8865, 1975.
[94]T. Kozłowski and A. Zglinski, Phys. Lett. **B50**, 222 (1974).
[95]V. S. Barashenkov *et al.*, Fiz. Élem. Chastits. At. Yadra **5**, 479 (1974) [Sov. J. Part. Nucl. **5**, 192 (1974)].
[96]V. V. Volkov, Fiz. Élem. Chastits. At. Yadra **6**, 1040 (1975) [Sov. J. Part. Nucl. **6**, 420 (1975)].
[97]V. M. Nikolaev and V. A. Shchegolev, Yad. Fiz. **16**, 469 (1972) [Sov. J. Nucl. Phys. **16**, 263 (1973)].
[98]M. Blann, Nucl. Phys. **A235**, 211 (1974).
[99]K. K. Gudima, A. S. Il'inov, and V. D. Toneev, Preprint JINR, R7-7915, Dubna, 1974.
[100]H. Feshbach, Rev. Mod. Phys. **46**, 1 (1974).
[101]H. Feshbach, Ann. Phys. (N.Y.) **5**, 357 (1958); **19**, 287 (1962).
[102]H. Feshbach, A. K. Kerman, and R. H. Kemmer, Ann. Phys. (N.Y.) **41**, 230 (1967).
[103]H. Feshbach, in Ref. 9, Vol. 2, p. 631.
[104]L. S. Rodberg, In: Intermediate Structure in Nuclear Reactions. Edited by H. P. Kennedy and R. Schrils, Kentucky, 1968, p. 65.
[105]V. G. Solov'ev, Fiz. Élem. Chastits, At. Yadra **3**, 770 (1972) [Sov. J. Part. Nucl. **3**, 390 (1972)].
[106]M. B. Lewis, Phys. Rev. **C9**, 1878 (1974).
[107]R. Reif, Zentralinstitut für Kernforschung 283, TU Dresden, 1974, p. 588.
[108]R. Reif, in: Ref. 12, p. 208; H. Iwe, R. Reif, and C. Riedel, Nucl. Phys. **A183**, 105 (1972).
[109]H. Iwe, Zentralinstitut für Kernforschung 283, TU Dresden, 1974, p. 576.
[110]N. N. Bogolyubov, Problemy dinamicheskoĭ teori v statisticheskoĭ fizike (Problems of Dynamical Theory in Statistical Physics), Moscow, Gostekhizdat, 1946.
[111]L. Van Hove, Physica **21**, 517 (1955); **23**, 441 (1957).
[112]R. Zwanzig, Physica **30**, 1109 (1964).
[113]I. Prigogine and P. Resibois, Physica **27**, 629 (1961).
[114]F. Haake, in: Springer Tracts in Modern Physics **66**, 98 (1973).

[115]D. Pines and F. Nozieres, Quantum Theory of Liquids, W.A. Benjamin, Russ. transl., Moscow, Mir, 1967.
[116]S. Yoshida, Phys. Lett. **B45**, 324 (1973). K. Yazaki and S. Yoshida, Nucl. Phys. **A232**, 249 (1974).
[117]W. Weidlich, Z. Phys. **222**, 403 (1969); W. Weidlich and K. Dietel, Nucl. Phys. **A205**, 367 (1973).
[118]R. Reif, G. Röpke, and H.-E. Zschau, in Ref. 12.
[119]D. N. Zubarev, Neravnovesnaya statisticheskaya termodinamika (Nonequilibrium Statistical Thermodynamics), Moscow, Nauka, 1971.
[120]S. T. Beljaev (Belyaev) and B.A. Rumyantsev, Phys. Lett. **B53**, 6 (1974).
[121]B. A. Rumyantsev and S.A. Kheifits, Yad. Fiz. **21**, 510 (1975) [Sov. J. Nucl. Phys. **21**, 267 (1975)].
[122]S. T. Beljaev (Belyaev), B.A. Rumyantsev, and V.T. Dmitriev, in: Ref. 9, Vol. 1, p. 556.
[123]V. I. Bogatin *et al.*, Communication JINR I-8715, Dubna, 1975.
[124]M. I. Gorenstein, V. P. Shelest, and G.M. Zinovjev, Preprint JINR, E1-8811, Dubna, 1975.
[125]V. E. Bunakov and M.M. Nesterenko, in: Materialy desyatoĭ zimneĭ shkoly LIYaF po fizike yadre i élementarnykh chastits (Materials of the Tenth Winter School on Nuclear and Elementary Particle Physics at The Leningrad Institute of Nuclear Physics), Vol. 1, Nauka, 1975, p. 297.
[126]D. Agassi, H.A. Weidenmüller, and G. Mantzouranis, Phys. Reports **C22**, 147 (1975).
[127]K. K. Gudima and V. D. Toneev, Communication JINR E4-9489, Dubna, 1975.
[128]G. Mantzouranis, D. Agassi, and H.A. Weidenmüller, Phys. Lett. **B57**, 223 (1975).
[129]W. Nörenberg, Z. Phys. **A273**, 251 (1975).
[130]L. C. Moretto *et al.*, Phys. Lett. **B58**, 31 (1975).
[131]L. C. Moretto and J.S. Sventik, Phys. Lett. **B58**, 26 (1975).
[132]C. Y. Wong, J.A. Maruhn, and T.A. Welton, Nucl. Phys. **A253**, 469 (1975).

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