Determination of lifetimes of nuclear excited states by the delayed-coincidence method (methods of analysis)

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In this review we discuss the most frequently used method of analysis of delayed-coincidence spectra in study of the lifetimes of excited states of nuclei, and also versions of programs developed in the Division of Nuclear Spectroscopy and Radiochemistry of the Laboratory of Nuclear Problems at the Joint Institute for Nuclear Research.

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INTRODUCTION

The delayed-coincidence method has an important place in study of the lifetimes of excited nuclear states. By means of this method it is possible to determine the average lifetimes of levels excited in nuclear reactions and in radioactive decay. The principle of the method lies in study of the time correlations between the moment of formation of an excited state and the moment of its decay. The moment of formation of the excited state is determined either by the α, e, $\beta^{(\pm)}$, or γ radiation populating this state or by a synchronization pulse from the accelerator. The moment of deexcitation is fixed by the time of appearance of the γ rays which remove the excitation of the state under study.

We will consider here methods of analysis of the time spectra of excited states which are obtained in decay of radioactive isotopes. In time measurements, as a rule, use is made of time-to-pulse-height converters, which permit analysis of lifetimes in the range from fractions of a nanosecond to several microseconds.

The most frequently used spectrometers and detectors and the minimum time resolution and limiting effective angles which characterize the system used are listed in the table. Use of one time system or another is determined by the expected lifetime of the excited state, by the type and energy of the measurements, and by the intensity of the cascade studied.

In this paper we shall discuss the basic methods of determining the lifetimes of excited nuclear states from analysis of delayed-coincidence spectra, and we shall dwell on the analysis methods used in the Division of Nuclear Spectroscopy and Radiochemistry of the Laboratory of Nuclear Problems at the Joint Institute for Nuclear Research.

ANALYSIS OF DELAYED-COINCIDENCE TIME SPECTRA

A number of different methods have recently been developed for mathematical analysis of delayed-coincidence yield curves[8–23] for the purpose of determining the

TABLE I. Comparative characteristics of time spectrometers.

Type of time spectrometer	Form of coincidence	Detector	Energy, keV	$2\tau_0$, nsec	Effective solid angle ÷ 4π	References
Scintillation (SC)	$\gamma-\gamma$	56 AVP [NaI (Tl)—NaI (Tl)]	(1170)—(1330) (511)—(511) (930)—(930)	0.80 1.30 0.132	<0.25	[1] [1] [2]
	$(e-e)$	XP 1021 (NE111—NE111)				
Combined	$\gamma-\gamma$	Ge (Li)—XP 1021 ↓ (1.9 cm³ stilbene planar)	(50)—(200—1000) (511)—(200—1000) (1332)—(200—1000) (10—1330)—(200—1000)	15.6 2.7 1.1 2.0	$<5\cdot10^{-2}$	[3]
	—	Ge (Li)— XP 1021 ↓ ↓ (35 cm³ coaxial) (stilbene)	(50)—(10—1332) (511)—(10—1332) (1332)—(10—1332) (10—1332)—(10—1332)	37.5 5.1 2.7 5.4	$<5\cdot10^{-2}$	[3]
Magnetic beta spectrometer (MBS) + SC	$e-\gamma$	XP 1020 (NE 104—NE 104)	(220)—(600—900)	0.5	$1\cdot10^{-2}$	[4]
MBS + MBS	$e-e$	XP 1020 (Naton 136—Naton 136)	(329)—(329)	0.85	$\sim 2.5\cdot10^{-3}$	[5]
Single-crystal scintillation time spectrometer	$\gamma-\gamma$ $(e-e)$ $(e-\gamma)$	XP 1020 (NE 104) 56 AVP [NaI (Tl)]	>10 >10	~ 20 ~ 1000	~ 1 ~ 1	[6] [7]

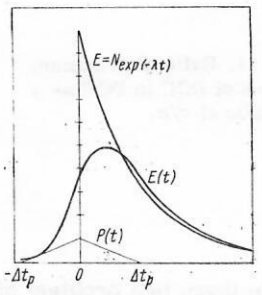

FIG. 1. Folding of exponential decay with finite instrumental time resolution.

exponential decay constant λ of the excited states of nuclei:

$$\lambda = 1/\tau; \quad \tau = T_{1/2}/\ln 2,$$

where τ is the mean lifetime of the level; $T_{1/2}$ is its half-life. In analysis of delayed-coincidence curves (DCC) two cases can be encountered: the case of exponential decay (one-component) and the case of many-component exponential decay (many-component).

For the simplest case of one component and on the assumption that the populating radiation is detected by one channel and the deexciting radiation by another, the probability density for appearance of the second radiation at a time t after the first radiation is

$$\left.\begin{aligned} f(t) &= 0 \quad \text{for} \quad t < 0; \\ f(t) &= \exp(-t/\tau)/\tau \quad \text{for} \quad t \geqslant 0. \end{aligned}\right\} \tag{1}$$

For an infinitely small time resolution of the apparatus, the time distribution of the number of delayed coincidences has the usual form of a radioactive-decay curve. Under real conditions the probability of detecting one delayed event at time t in a multichannel time analyzer has a definite distribution due to the finite time resolution.

The result of folding the exponential decay with the finite time resolution due to the apparatus is a change in the shape of the radioactive-decay curve as shown in Fig. 1. The mathematical expression for this process has the following form:

$$F(t) = \int_{-\infty}^{\infty} f(t') P(t-t')\, dt', \tag{2}$$

where $F(t)$ is the time distribution of pulses from the delayed radiation; $P(t)$ is the time distribution of the prompt coincidences.

By prompt-coincidence curve (PCC) we mean the experimental time distribution measured for a state whose lifetime is significantly less than the resolving time of the apparatus, $\tau \ll 2\tau_0$. The instrumental resolving time $2\tau_0$ is defined as the width of the PCC at half-height.

In the case of a one-component decay this representation has the form

$$F(t) = \int_{0}^{\infty} \exp(-t'/\tau)\, P(t-t')\, dt'/\tau. \tag{3}$$

In analysis of delayed coincidences it is important to know: a) the instrumental time resolution $2\tau_0$ and the shape of the PCC; b) the accidental-coincidence background; c) the number of exponential components in the DCC. The time resolution and the shape of the PCC are important in determination of short lifetimes comparable with the instrumental resolving time, $\tau \lesssim 2\tau_0$.

The parameters of the PCC when it is approximated by a normal distribution are given in Fig. 2. Here $2\tau_0$ is the width at half-height and σ is the standard deviation; $2\tau_0 = 2\sqrt{2\ln 2}\sigma (2\tau_0 \approx 2.35\sigma)$. Accurate knowledge of the accidental-coincidence background is particularly important in analysis of lifetimes $\tau \gg 2\tau_0$.

Knowledge of the number of exponential components in the DCC is one of the factors determining the choice of the method of analysis. In classification of methods of analysis of time distributions it is necessary to distinguish: A) methods using part of the statistical material; B) methods using all of the statistical material obtained in an experiment. The first group includes: a) the method of treating the exponential time distribution in the logarithmic representation; b) the method of treating the experimental distribution in the exponential representation; c) the method of areas,[9] which uses part of the areas of the DCC and PCC; d) the Fourier transform method,[19,20] which uses an extended part of the DCC distribution. The second group includes: a) the method of moments[8]; b) the method of folding (convolution) of the DCC and PCC[13,14]; c) the method of deconvolution by Fourier transform.[19]

1. METHODS OF GROUP A

Method of treatment of the experimental time distribution in the exponential representation

This method includes two different approaches: The first is the slope method (logarithmic representation of the number of coincidences), which is used in the case of one-component decay; the second is the direct-fitting method, which is applicable both to the case of one-component decay and to many-component decay. It reduces to fitting the sum of exponentials to the experimental time distribution.

The slope method.[9] Equation (3) leads to the following relations:

$$dF(t)/dt = \lambda [P(t) - F(t)] \tag{4}$$

and

$$d[\ln F(t)]/dt = -\lambda [1 + P(t)/F(t)]. \tag{5}$$

These relations determine two important aspects of this method:

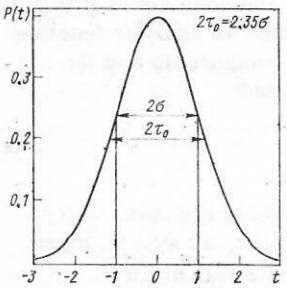

FIG. 2. Normalized PCC for the case of a normal distribution with $\sigma = 1$.

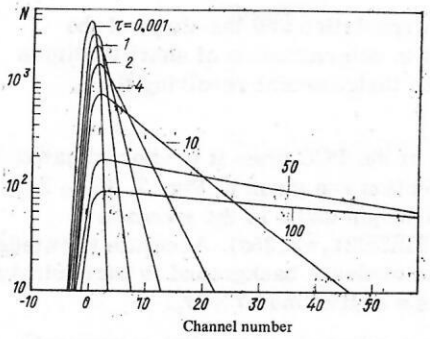

FIG. 3. Logarithmic representation of area-normalized PCC and DCC for the case of one-component decay with $\sigma=1$.

a) In the region where $F(t) \gg P(t)$, $d[\ln F(t)]/dt = -\lambda$, i.e., in the semilogarithmic representation, one obtains a straight line whose slope gives λ;

b) DCC and PCC curves normalized in area intersect at the maximum of $F(t)$, as can be seen from Fig. 3.

In analysis of the experimental data by the method of least squares[24] the number of coincidences in the logarithmic representation is approximated by a straight line $f(x_i) = a + bx_i$.

The mean life of the excited state is

$$\tau = -\sum_i N_i (x_i - \overline{x})^2 / \sum_i N_i (x_i - \overline{X})(y_i - \overline{Y}), \tag{6}$$

where x_i is the abscissa of channel i, $y_i = \log N_i$, and N_i is the number of coincidences after subtraction of the accidental-coincidence background:

$$\left.\begin{aligned} \overline{X} &= \sum_i (N_i / \sum N_i) x_i; \\ \overline{Y} &= \sum_i (N_i / \sum N_i) y_i. \end{aligned}\right\} \tag{7}$$

The statistical error is found from the expression

$$\Delta\tau = \tau^2 \Big/ \sqrt{\sum_i N_i (x_i - \overline{X})^2}. \tag{8}$$

Here we have not taken into account the contribution of the statistical error of the background.

It should be noted that for the one-component case the point of intersection of the DCC and PCC curves normalized in area can be used to evaluate τ if we make use of a nomogram which gives the ratios of the maximum value of the PCC as a function of τ/σ (Fig. 4). Values of τ obtained from the nomogram may be of interest as the initial value of the parameter in calculations of the lifetime, for example, by the convolution method.

Direct fitting method. This method, like the slope method, is applicable to the region of the experimental data where $F(t) \gg P(t) (\tau > 2\tau_0)$, and consists of fitting to the experimental time distribution an analytic function containing several exponential components and the accidental-coincidence background:

$$N(t) = \sum_k A_k \exp(-\lambda_k t) + B. \tag{9}$$

This permits analysis of more general cases, which are usually encountered in practice, as well as more correct determination of the statistical error.

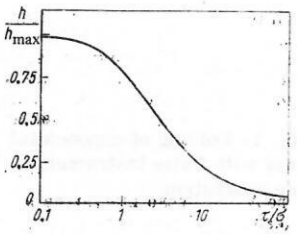

FIG. 4. Ratio of maximum values of DCC to PCC as a function of τ/σ.

In the methods discussed above there is a problem of choice of the point t_0 which determines the upper edge of the spectral region analyzed. Theoretical calculations[19] show that the amplitude of the exponential is distorted only by 5% at the point $t_0 = 2\tau_0$. However, investigation of the variation of χ^2 as a function of t_0 shows that fitting of the data to a pure exponential decay becomes correct only beginning with $t_0 = 4\tau_0$.

Newton's integral method

This method was proposed by Newton[9] and at the present time is only of historical interest. The principle of the method can be understood from Fig. 5 and the basic relation

$$\lambda = [F(A) - F(B)] \Big/ \int_A^B [F(t) - P(t)]\, dt, \tag{10}$$

where A and B are arbitrary points; $F(A)$ and $F(B)$ are the ordinates of the DCC at points A and B.

Fourier-transform method

Several papers[15,16] have been devoted to discussion of the cases of one-component and many-component exponential decay with use of a Fourier transform. In this approach the analysis of the time distribution is carried out after subtraction of the background.

In the work of Dumont *et al.*[19] the case of one-component decay is discussed and an analysis is made of the effect of the subtracted background on the lifetime values obtained.

The time distribution after subtraction of the background is written in the following form:

$$\langle N_{is} \rangle = S \exp(-t_i/\tau) \times R(i), \tag{11}$$

where N_{is} is the number of pulses in channel i, S is the amplitude of the exponential, and $R(i)$is the transfer function, for which we use the PCC.

The Fourier transform of this product is equal to the product of the Fourier transforms of the exponential and the PCC:

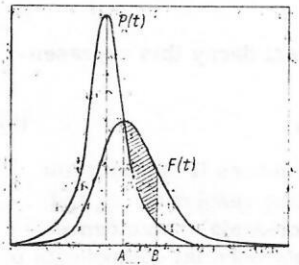

FIG. 5. Region of DCC and PCC distributions used in analysis by Newton's integral method.

$$FT[\langle N_{is}\rangle] = S \times FT[R(i)]/(1+2\pi\nu\tau j), \quad (12)$$

where $j=\sqrt{-1}$. This expression permits determination of the lifetime τ by two means: 1) with use of only that portion of the DCC where the DCC $\gg$ the PCC, and 2) with use of the complete set of data of the DCC and PCC.

In the first case the factor $FT[R(i)]$ in Eq. (12) disappears and the square of the modulus corresponds to a Lorentz distribution

$$|FT[\langle N_{is}\rangle]|^2 = S^2/(1+4\pi^2\nu^2\tau^2). \quad (13)$$

First the Fourier transform of an interval of the experimental spectrum is obtained. Then the transformed spectrum obtained is fitted with Eq. (13), the value of τ being determined as the fitting parameter. The accuracy in determination of τ is substantially affected by the value of the background subtracted. It turns out that incorrect subtraction of the background has an appreciable effect in the low-frequency region. However, if the fitting is carried out in the frequency region $\nu > 1/2\gamma$, where γ is the time range of the DCC used in the analysis, we find that the error due to incorrect background subtraction becomes negligible.

Gardner *et al.*[20] suggest a method utilizing the Fourier transform to determine the parameters N_k and λ_k in many-component decay:

$$E(t) = \sum_{k=1}^{n} N_k \exp(-\lambda_k t). \quad (14)$$

The principle of the method is that the sum (14) is expressed in the form of the Laplace integral equation

$$E(t) = \int_0^\infty \exp(-\lambda t)\, g(\lambda)\, d\lambda, \quad (15)$$

where $g(\lambda)$ is the sum of Dirac δ functions

$$g(\lambda) = \sum_{k=1}^{n} N_k \delta(\lambda - \lambda_k). \quad (16)$$

If $E(t)$ designates the experimental distribution of the DCC, then for $g(\lambda)$ the representation looks like a continuous spectrum with maxima at the points $\lambda = \lambda_i$. This method has important limitations which permit its use only to evaluate the number of components and values of λ_i, which can then be used as initial values of parameters in more accurate methods of analysis.

2. METHODS OF GROUP B

The moment method

In the moment method there are two approaches to determination of the lifetime: 1) from the relative moments of the DCC and PCC[8] and 2) from the moment of the DCC relative to its own centroid.[11,12]

The first approach was proposed by Bay,[18] who showed that the moments $M(F)$ for the DCC and $M(P)$ for the PCC are related by the expression

$$M_r(F) = \sum_{k=0}^{r} \frac{r!}{k!\,(r-k)!} M_{r-k}(P)\, M_k(f), \quad (17)$$

where $M_r(F)$ is the moment of order r for the DCC:

$$M_r(F) = \int_{-\infty}^{\infty} t^r F(t)\, dt. \quad (18)$$

For a one-component exponential decay (1),

$$f(t) = \exp(-t/\tau)/\tau.$$

The moments of first, second, and third order of one curve determine its centroid, width, and asymmetry. With this definition the moment of order r for $f(t)$ is expressed by the relation

$$M_r(f) = r!\tau^r. \quad (19)$$

Bay's method leads to the following relations between the lifetime τ and the moments of the DCC and PCC:

$$\tau = M_1(F) - M_1(P); \quad (20)$$

$$\tau = [M_2(F) - M_2(P)]^{1/2}; \quad (21)$$

$$\tau = [M_3(F)/2 - M_3(P)/2]^{1/3} \quad (22)$$

(the areas of the DCC and PCC have been normalized to unity).

The moment method of first order (20) gives a smaller statistical error. Therefore it is preferable to use just this method in determination of a lifetime in the region near 10^{-11} sec. However, this method depends strongly on the stability of the apparatus, its overload characteristics, and the identity of the energy intervals in obtaining the DCC and PCC.

In the second approach the first moment of the PCC is equated to zero, $M_1(P)=0$. The moments of second and third order of the PCC are designated as $M_2(P)=\Delta^2$ and $M_3(P)=\varepsilon$, and from Eq. (16) one obtains the moments $r=1,2,3$ for the DCC relative to the time zero:

$$M_1(F) = \tau; \quad (23)$$

$$M_2(F) = \Delta^2 + 2\tau^2; \quad (24)$$

$$M_3(F) = 3\Delta^2\tau + 6\tau^3 + \varepsilon. \quad (25)$$

Weaver and Bell[12] overdetermine the moments of order $r>1$ for the DCC relative to its own centroid $M_1(F)=\tau$:

$$N_r(F) = \int_{-\infty}^{\infty} (t-\tau)^r F(t)\, dt,$$

which gives for $r=1,2,3$ the following expressions:

$$N_1(F) = 0; \quad N_2(F) = \Delta^2 + \tau^2;$$
$$N_3(F) = 2\tau^3 + \varepsilon.$$

The problem of determining τ is solved by calculation of the centroid of the DCC and use of it as the point of zero time. In this case from the moment of third order we obtain

$$\tau = \{[N_3(F) - \varepsilon]/2\}^{1/3}.$$

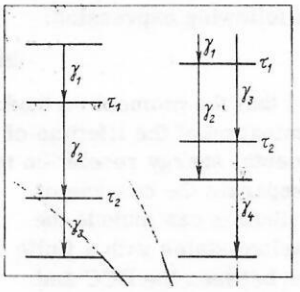

FIG. 6. Series and parallel cascades connecting states with lifetimes τ_1 and τ_2.

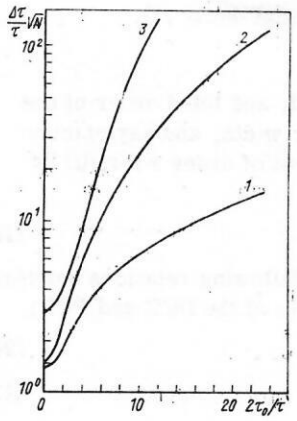

FIG. 7. Statistical errors $\Delta\tau\sqrt{N}/\tau$ obtained in analysis by the moment method for $2\tau_0 = 1$ nsec (Ref. 10) as a function of $2\tau_0/\tau$: 1—centroid-shift method; 2—second-moment method; 3—third-moment method.

The value $\varepsilon = M_3(P)$ characterizes the asymmetry of the PCC and is an insignificant correction in Eq. (24). In cases in which the PCC is not very symmetric but ε has a value not exceeding $N_3(F)/3$, it is not necessary to consider ε, since the error in this case does not exceed 10%.

It should be noted that in discussion of histograms of time distributions obtained by means of multichannel time analyzers (after subtraction of the accidental-coincidence background), the moment of order r is calculated as

$$M_r(F) = \sum N_i t_i^r / \sum N_i,$$

where t_i is the value of time channel i and N_i is the number of counts in channel i.

The preceding discussion refers to the case of one-component decay. In practice one may encounter cases of nonexponential decay in which the DCC contains the contribution of prompt coincidences or of several exponential components.

In the first case let us turn from consideration of $f(t)$ as given in Eq. (1) to consideration of

$$f(t) = \begin{cases} 0 & \text{for } t < t_0; \\ (1-\eta)\,\delta(t) + \eta \exp(-t/\tau)/\tau & \text{for } t \geqslant t_0, \end{cases} \quad (26)$$

where $(1-\eta)/\eta$ is the fraction of prompt coincidences. This leads to the following expressions for the moments of first and second order[11]:

$$M_1(t) = \eta\tau; \quad M_2(t) = (2\eta - \eta^2)\,\tau^2.$$

For the desired lifetime we obtain

$$\tau = M_1(f)\left\{1 + \frac{1}{2}\frac{M_2(f) - [M_1(f)]^2}{[M_1(f)]^2}\right\}. \quad (27)$$

The moment of first order of the DCC, as in the usual case, is determined from the following expression:

$$\eta\tau = M_1(F) - M_1(P). \quad (28)$$

However, it should be noted that the moment method does not permit unique determination of the lifetime of an excited state if the instrumental energy resolution is poor so that it is difficult to separate the cascade of interest. Thus, the DCC distribution can include the contribution of at least two excited states with a finite lifetime (Fig. 6). The relation between the DCC and PCC moments and the lifetimes τ_1 and τ_2 of the states is given by the following expressions:

$$M_1(F) - M_1(P) = \tau_1 + \tau_2; \quad (29)$$

$$M_2(F) - M_2(P) = \tau_1^2 + \tau_2^2; \quad (30)$$

$$M_3(F) - M_3(P) = 2\tau_1^3 + 2\tau_2^3 \quad (31)$$

for the consecutive cascades $[\gamma_1 - (\tau_1) - \gamma_2 - (\tau_2) - \gamma_3]$ and

$$M_1(F) - M_1(P) = \eta\tau_1 + (1-\eta)\,\tau_2; \quad (32)$$

$$M_2(F) - M_2(P) = \eta\tau_1^2 + (1-\eta)\,\tau_2^2; \quad (33)$$

$$M_3(F) - M_3(P) = 2\eta\tau_1^3 + 2(1-\eta)\,\tau_2^3 \quad (34)$$

for the parallel cascades $[\gamma_1 - (\tau_1) - \gamma_2]$ and $[\gamma_3 - (\tau_2) - \gamma_4]$, where η and $(1-\eta)$ are the contributions to the DCC from states with lifetimes τ_1 and τ_2, respectively. It should be noted that Eq. (32) leads to the case of one-component decay with an admixture of the prompt component (28) for $\tau_2 \ll \tau_1$.

The statistical errors obtained in analysis by the moment method have been analyzed by Sundström[10] and by Weaver and Bell.[12] For the condition of equality of the number of coincidences in the DCC and PCC we obtain[13]:

$$\Delta\tau/\tau = \sqrt{N}\sqrt{2}[(\sigma/\tau)^2 + 1]^{1/2}; \quad (35)$$

$$\Delta\tau/\tau = \sqrt{N}\sqrt{3/2}[(\sigma/\tau)^4 + (\sigma/\tau)^2 + 3/2]^{1/2}; \quad (36)$$

$$\Delta\tau/\tau = \sqrt{N}\sqrt{5/6}[(\sigma/\tau)^6 + (3/2)(\sigma/\tau)^4 + (9/2)(\sigma/\tau)^2 + 56/6]^{1/2} \quad (37)$$

for the first, second, and third moments, respectively.

The results of calculation of the statistical errors are given in Fig. 7, from which we can see that the method of first moments gives the least statistical error in determination of short lifetimes.

Integral transform method

Sparrman[17,18] has proposed a method based on the Laplace integral transform of the two curves (DCC and PCC):

$$L(P) = \int_0^\infty \exp(rt)\,P(t)\,dt; \quad (38)$$

$$L(F) = \int_0^\infty \exp(rt)\,F(t)\,dt. \quad (39)$$

If A_F and A_P are the total numbers of counts in the respective curves DCC and PCC, then $C = A_F/A_P$ is the normalization coefficient.

Numerical values of $L(P)$ and $L(F)$ are easy to obtain by numerical integration. The choice of r is arbitrary except for the one condition $r < \lambda$. Using the well known expression

$$F(t) = C\int_0^t P(t')\,\lambda\exp[-\lambda(t-t')]\,dt',$$

we can obtain an expression for λ as a function of $L(F)$, $L(P)$, and r:

$$L(F) = \int_0^\infty \exp(r)\,C\,dt \int_0^t P(t')\,\lambda\exp[-\lambda(t-t')]\,dt'.$$

After integration we have

$$L(F) = \lambda C L(P)/(\lambda - r)$$

and hence

$$\tau = (1 - CL(P)/L(F))/r.$$

The standard deviation in determination of τ is:

$$S_T = \frac{C}{r}\frac{L(P)}{L(F)}\left[\left(\frac{S_L(P)}{L(P)}\right)^2 + \left(\frac{S_L(F)}{L(F)}\right)^2 + \left(\frac{S_0}{C}\right)^2\right]^{1/2}, \quad (40)$$

where

$$S_L^2(F) = \sum_{n=0}^{N} \exp(2rn)\, F(t_n);$$

$$S_L^2(P) = \sum_{n=0}^{N} \exp(2rn)\, P(t_n);$$

$$S_c/C = (1/A_F + 1/A_P)^{1/2}.$$

The method discussed permits taking into account instrumental drift in comparison of the DCC and PCC. On the assumption that the DCC is shifted by a units, two calculations are carried out with r_1 and r_2. In this case we have the following expressions:

$$\exp(r_1 a) L_1(F) = \lambda L_1(P)/C(\lambda - r_1);$$
$$\exp(r_2 a) L_2(F) = \lambda L_2(P)/C(\lambda - r_2),$$

and with the choice $r_2 = -r_1$ we obtain

$$\lambda = r_1\left[1 - \frac{1}{C^2}\frac{L_1(P)}{L_1(F)}\frac{L_2(P)}{L_2(F)}\right]^{-1/2}.$$

A comparative analysis of the statistical errors of the Sparrman integral method and the first moment method with $C = 1$ is given in Fig. 8.

Method of deconvolution (Fourier)

The second means of using the Fourier transform, proposed by Dumont *et al.*,[19] permits use of the entire set of experimental data in the DCC and PCC. In this case also we proceed from the expression

$$[FT(\langle N_{is}\rangle)]_\nu = S\,[FT(R(i))]_\nu/(1 + 2\pi\nu\tau j),$$

where $[FT(\langle N_{is}\rangle)]_\nu$ and $[FT(R(i))]_\nu$ are the Fourier transforms for the DCC and PCC after background subtraction, obtained by numerical integration, for frequency ν. For various values of ν, in this way, we have a set of corresponding transforms of the DCC and PCC.

The square of the modulus of the ratio of $[FT(\langle N_{is}\rangle)]_\nu$ to $[FT(R(i))]_\nu$ (the deconvolution) corresponds to a Lorentz distribution

$$\left|\frac{[FT(\langle N_{is}\rangle)]_\nu}{[FT(R(i))]_\nu}\right|^2 = \frac{S^2}{1 + 4\pi^2\nu^2\tau^2}. \quad (41)$$

In fitting this distribution to the square of the modulus of the transformed experimental DCC and PCC, τ is determined as the fitting parameter. As in the variant with the Fourier transform, the accuracy in determination of τ is considerably affected by the value of the subtracted background, particularly in the low-frequency region. Therefore, it is reasonable to limit the range of frequencies used in the fit to the region $\nu > 1/2\gamma$ where it is possible to neglect the contribution of the background to the statistical error. This form of the calculation does not permit us to exclude the possibility of a shift in the DCC and PCC during the experiment.

Folding (convolution) method

We previously showed that the form of the DCC (see Fig. 1) is the result of folding an exponential distribution with a finite time resolution due to the apparatus:

$$F(t) = \int_{-\infty}^{\infty} f(t')\, P(t - t')\, dt'. \quad (42)$$

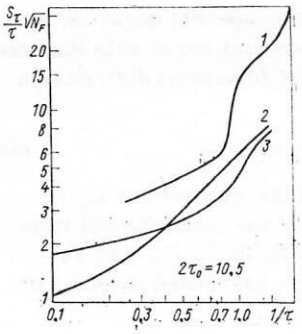

FIG. 8. Comparison of statistical errors $S_\tau\sqrt{N_F}/\tau$ obtained with Sparrman's analysis method and the first-moment method[17]: 1—integral transform according to Sparrman with correction of instrumental shift; 2—centroid-shift method; 3—integral transform according to Sparrman without correction for instrumental shift.

The general principle of folding methods in determination of the lifetime is that τ is obtained as the parameter in fitting Eq. (42) to the experimental DCC distribution. These methods are used to determine lifetimes in a large time range, both for $\tau \geq 2\tau_0$ and for $\tau \leq 2\tau_0$. A number of authors[13–15,21,22,25–27] describe various methods of folding. Some of these studies use an analytic approximation of the PCC, $P(t)$, as a triangle,[25] a Gaussian,[13] a modified Gaussian,[28] two exponentials,[27] and other studies[14,15,21] use the experimental PCC for $P(t)$.

Folding with a Gaussian. Malmskog[13] considers the case of one-component decay

$$f(t) = \begin{cases} N(k)\exp[-\lambda(t-k)] & \text{for } t \geq k; \\ 0 & \text{for } t < k. \end{cases}$$

The total number of counts in the DCC $A = \int_k^\infty N(k) \times \exp[-\lambda(t-k)]\,dt$ permits obtaining the value of the parameter $N(k)$: $N(k) = A\lambda$.

To take into account the finite time resolution, we approximate the instrumental transfer function by a Gaussian distribution $\exp\{-[(t-u)/\sigma]^2/2\}/\sigma\sqrt{2\pi}$. The number of coincidences which should fall in the time interval u, $u + du$ but as a result of the transfer function of the apparatus will be recorded in channel t is given by the expression

$$A\lambda \exp[-\lambda(u-k)]\exp\{-[(t-u)/\sigma]^2/2\}/\sigma\sqrt{2\pi}.$$

Thus, integration over u gives the total number of counts in channel t. On taking into account a constant background in each channel, one obtains an analytic expression describing the number of coincidences in the channels:

$$F(t) = \int_k^\infty A\lambda \exp[-\lambda(u-k)]\frac{1}{\sigma\sqrt{2\pi}}\exp\{-[(t-u)/\sigma]^2/2\}\,du + B. \quad (43)$$

The parameters in fitting this expression to the experimental data for the DCC are the area A of the DCC, the decay constant $\lambda = 1/\tau$, the standard deviation of the Gaussian distribution σ, the channel giving the zero time, and the accidental-coincidence background level B.

Folding with the experimental PCC. In this method

the DCC distribution of a one-component decay is described as the fold of an exponential curve with the true value of k and the experimental (discrete) distribution of the PCC (see Fig. 1)[14,15]:

$$F_i = F(N, \lambda, t_i) = N \int_0^\infty P(t-t_i) \exp(-\lambda t)\, dt. \quad (44)$$

This means that each value of the exponential E_i is distributed over the channels of the multichannel time analyzers in accordance with the PCC, P_j. The result of the convolution $E_i * P_j$ is expressed by sums of the type:

$$F_1 = P_1E_1;\ F_2 = P_2E_1 + P_1E_2;\ F_3 = P_3E_1 + P_2E_2 + P_1E_3. \quad (45)$$

The values of N and λ are determined as the parameters in fitting the expressions (45) to the experimental DCC distribution. It is important to note that the method is very sensitive to the relative instrumental time shifts of the DCC and PCC, as a result of the fact that the convolution curve F_i^{conv} is compared in each channel with the experimental values of the DCC, F_i^{exp}.

Boström and Olsen[14,15] propose a corresponding procedure for determination of the instrumental shift between the DCC and PCC. They find the sum of the least squares from the following expression:

$$S = \sum_i p_i (F_i^{conv} - F_i^{exp})^2,$$

where p_i is the weight, which is expressed in terms of the number of true coincidences F^{exp} and a number of accidental coincidences F^{acc} in this way:

$$p_i \approx (F^{exp} - 2F^{acc})^{-1}.$$

First one calculates three values of S for three arbitrary shifts Δt (for example, +1, 0, and −1 channel) between the PCC and DCC. The minimum of the function S obtained in plotting the parabola $S = \text{const}(\Delta t)^2$ passing through these three values determines the real value of the shift $(\Delta t)_{min}$ between the DCC and the PCC and permits a correct value to be found for λ.

To improve the accuracy in determination of λ the minimization procedure for S is repeated for other smaller sets of three values of the artificial shift Δt near the location of the previous value $(\Delta t)_{min}$, taken as a zero time. In the case of admixture of a prompt component to the DCC, the method is not used, particularly in the region of small lifetimes ($\tau \ll 2\tau_0$). In treatment of many-component exponential decays, folding with the experimental PCC is greatly complicated.

Kaczorowski and Pietrzyk[21] have extended the applicability of the convolution method in treatment of DCC containing one component from the right-hand side of the DCC, one from the left-hand side, admixture of a prompt component, and a constant background. Proceeding from the folding function

$$f(t) = A_0 \int_0^\infty P(t-t') \exp(-t'/\tau)\, dt', \quad (46)$$

it is possible to determine the number of coincidences $F(t_i)$ in the time channel t_i:

$$F(t_i) = \int_{t_i}^{t_{i+1}} f(t)\, dt$$

or

$$F(t_i) = A_0 \int_0^\infty \exp(-t'/\tau)\, P(t_i - t')\, dt',$$

where

$$P(t_i - t') = \int_{t_i}^{t_{i+1}} P(t-t')\, dt$$

corresponds to the experimental PCC after background subtraction. The integration region is determined by the region (t_1, t_2) in which the PCC differs from zero, and the theoretical expression for the right-hand part of the DCC $F_R(t_i)$ has the form:

$$F_R(t_i) = A_0 \int_{\max(0,\ t_i - t_2)}^{t_i - t_1} \exp(-t/\tau)\, P(t_i - t)\, dt. \quad (47)$$

Overdetermination of the variable $y = t_i - t$ leads to the expression

$$F_R(t_i) = A_0 \exp(-t_i/\tau)\, S(t_i), \quad (48)$$

where the function $S(t_i)$ has the form

$$S(t_i) = \int_{t_1}^{\min(t_i,\ t_2)} G(y)\, dy = \int_{t_1}^{\min(t_i,\ t_2)} \exp(y/\tau)\, P(y)\, dy,$$

where

$$S(t_{i+1}) = S(t_i) + \int_{t_i}^{t_{i+1}} G(y)\, dy \quad \text{for } t_i < t_2; \quad (49)$$

$$S(t_i) = \text{const} \quad \text{for } t_i \geqslant t_2. \quad (50)$$

It follows from Eq. (50) that in the region $t_i \geqslant t_2$ the theoretical function (48) has an exponential form. On integration of the function $G(y) = \exp(y/\tau)P(y)$ over y it is possible to use a procedure which permits use of the experimental PCC distribution. For this purpose $G(y)$ is approximated by the second-order polynomial $ay^2 + by + c$.

For each time region which includes the three channels t_{i-1}, t_i, and t_{i+1}, a set of values of the coefficients a, b, and c is determined from the equation $\exp(y/\tau)P(y) = ay^2 + by + c$ (for $y = t_{i-1}, t_i, t_{i+1}$), where the experimental values of the PCC are used for $P(y)$.

The integral $C = \int_{t_i}^{t_{i+1}} G(y)\, dy$ is determined as the average value of the integrals which are obtained for the function $G(y)$ found in the interval (t_{i-1}, t_i, t_{i+1}) and the function $G(y)$ found in the interval (t_i, t_{i+1}, t_{i+2}).

This leads for expression (29) to the analytic relation

$$S(t_{i+1}) = S(t_i) + [-G(t_{i-1}) + 13G(t_i) + 13G(t_{i+1}) - G(t_{i+2})]/24.$$

In a similar way we determined the theoretical expression for the left-hand part of the DCC, $F_L(t_i)$:

$$F_L(t_2 - t_i) = A_0 \exp(-t_i/\tau_L) \int_{t_2}^{\min(t_i,\ t_1 - t_2)} \exp(-t/\tau_L)\, P(t_2 - t)\, dt, \quad (51)$$

where $i = 0, 1, 2, \ldots$ and the integral is calculated as was described above.

Thus, the number of coincidences in the channel t_i of the DCC containing the left and right exponential components $F_L(t_i)$ and $F_R(t_i)$, the prompt admixture $P(t_i)$, and the constant background T is given by the following expression:

$$N(t_i) = A_0 \left[J_R \frac{F_R(t_i)}{A_R} + J_L \frac{F_L(t_i)}{A_L} + (1 - J_R - J_L) \frac{P(t_i)}{A_P} \right] + T, \quad (52)$$

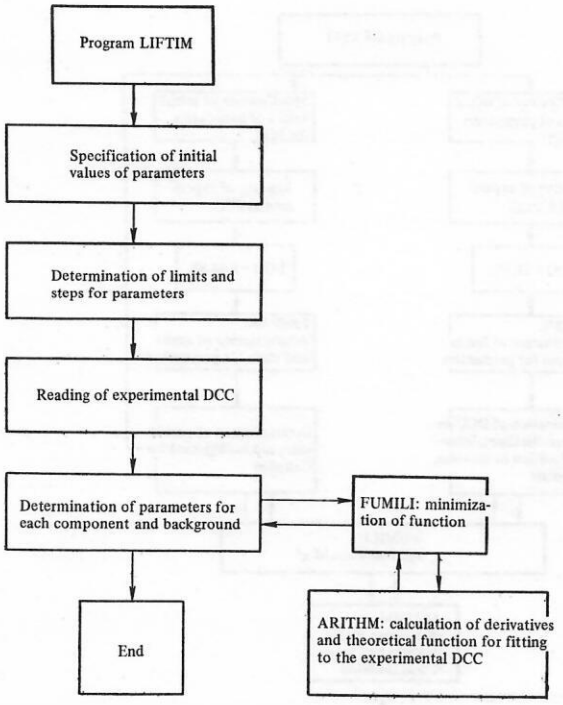

FIG. 9. Logic scheme of the program LIFTIM.

where A_0 is the total number of counts in the DCC after background subtraction; A_R, A_L, and A_P are the areas of the decay components; J_R, J_L, and J_P are the weights corresponding to them.

The lifetimes τ_R and τ_L are determined as parameters in minimization of the function

$$\chi^2 = \sum_{t_i=t_3}^{t_4} \frac{[N(t_i) - N^{\exp}(t_i)]^2}{N(t_i)},$$

where t_3 and t_4 are the first and last channels of the experimental DCC distribution.

Analysis of DCC by means of the fitting program FUMILI

In our work on determination of the lifetime of excited levels we used the method of fitting an analytic expression describing a given decay to the experimental DCC. We used the fitting program FUMILI,[29] written in the FORTRAN language, which was included in the library of standard programs for the BÉSM-6 and CDC-6200 computers at the Joint Institute for Nuclear Research. This program minimizes the function $\chi^2/2$, which depends on the parameters shown:

$$\frac{\chi^2}{2} = \frac{1}{2} \sum_{i=1}^{N} \left(\frac{y_{i\,\exp} - y_i(x_i(1), \ldots x_i(k), A(1), \ldots A(M))}{\Delta y_i} \right)^2; \qquad (53)$$

where $y_{i\,\exp} \pm \Delta y_i$ are the experimental values to which are fitted the theoretical function y_i, which depends on several coordinates x and on the fitting parameter $A(i)$; k is the number of coordinates which describe one experimental point; M is the number of parameters; N is the number of experimental points.

The program LIFTIM. This program is intended for analysis of time spectra by the direct-fitting method for one-component decay and the general case of many-component decay in the presence of a background and is characterized by the following expression:

$$Y(t) = \sum_{k=1}^{n} A_k \exp(-\lambda_k t) + B, \qquad (54)$$

where n is the number of exponential components; A_k is the intensity of each component; B is the accidental-coincidence background. The fitting parameters are λ_k, A_k, and B. When the program LIFTIM is used, it is necessary to specify initial values of the parameters and the number of components obtained by means of a preliminary analysis, for example, in a graphical analysis.

A preliminary analysis is made of the value and shape of the distribution of the accidental-coincidence background. After finding an initial value of the background, the parameters A_k and λ_k are determined approximately. These values are specified in the program as initial values. In the case when the background is obtained independently, B is assumed constant, $PL(\text{b.g.}) = 0$.

The logic scheme of this program is shown in Fig. 9. In Fig. 10 we show an experimental one-component spectrum of delayed coincidences, obtained in measurement of the lifetime of the 148-keV level of ^{79}Kr.[30] The analysis was made for the spectral region indicated in Fig. 10 $[P(t) \ll F(t)]$. As a result of the analysis we obtained a value $T_{1/2} = 81.2 \pm 3.2$ nsec, which is in good agreement with the well known results of Bleck *et al.*[31] $T_{1/2} = 77.7 \pm 1.5$ nsec.

To check the program LIFTIM in the case of two-component decay with background, we shall analyze an artificial two-component spectrum with background, obtained by means of the program SPECTR. This program generates a statistical spectrum of delayed coincidences based on a random-number generator with use of a specified analytic expression.

The artificial spectrum, the region used in the slope analysis, the parameters used in generation of the spectrum, and the parameters obtained after the fit are shown in Fig. 11. This program was used to analyze the time spectra obtained in study of isomeric

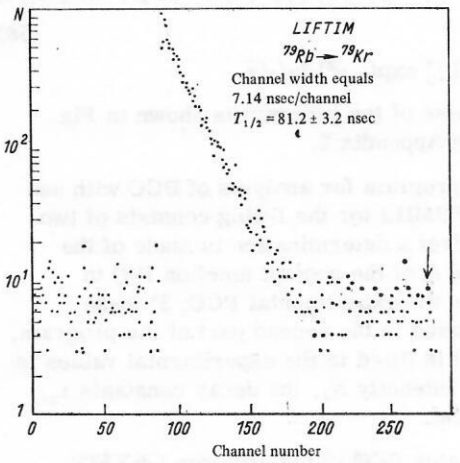

FIG. 10. Delayed-coincidence curve for the decay ^{79}Rb → ^{79}Kr.[30]

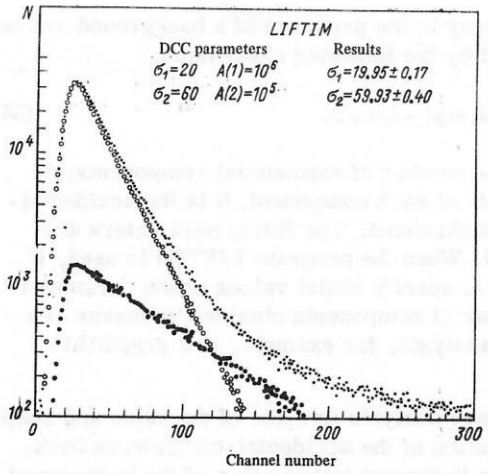

FIG. 11. Artificial two-component spectrum and results of analysis by the program LIFTIM.

states in the time range 60–700 nsec.[30] The text of the program is given in Appendix 1.

The program GEXFIT. This program is written in the FORTRAN language and is intended for analysis of time spectra by the method of fitting a function of the fold of a many-component decay with a prompt curve to the experimental DCC in the presence of a background with use of an analytic expression for the DCC. The mathematical fold of the functions

$$E(t)=\sum_{k=1}^{n} N_k \exp(-\lambda_k t) \tag{55}$$

and

$$P(t)=\exp\{-[(t-t_0)/\sigma]^2/2\}/\sigma\sqrt{2\pi} \tag{56}$$

with addition of the background B gives an analytic function

$$F(t)=\sum_{k=1}^{n} N_k \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty \exp(-\lambda_k t)\exp\{-[(t-t_0)/\sigma]^2/2\}\,dt+B, \tag{57}$$

which is fitted to the experimental DCC. This expression can be rewritten in a form more convenient for calculation[22,26]:

$$F(t_i)=\sum_{k=1}^{n} N_k \exp(-\lambda_k t_i+\lambda_k^2\sigma^2/2)\,[1+ERF(t_i/\sigma\sqrt{2}-\lambda\sigma/\sqrt{2})]/2, \tag{58}$$

where $ERF(x)=2\int_0^x \exp(-u^2)\,du/\sqrt{\pi}$.

The logic scheme of the program is shown in Fig. 12 and the text in Appendix 2.

The complete program for analysis of DCC with use of the program FUMILI for the fitting consists of two main parts: 1) First a determination is made of the parameters σ and t_0 of the prompt function $P(t)$ in fitting Eq. (56) to the experimental PCC; 2) these parameters are used in the second part of the program, in which Eq. (58) is fitted to the experimental values in order to find the intensity N_k, the decay constants λ_k, and the background.

The logic operator BØØ in the program GEXFIT separates these two branches of the program. The quantities of interest to us in the program GEXFIT,

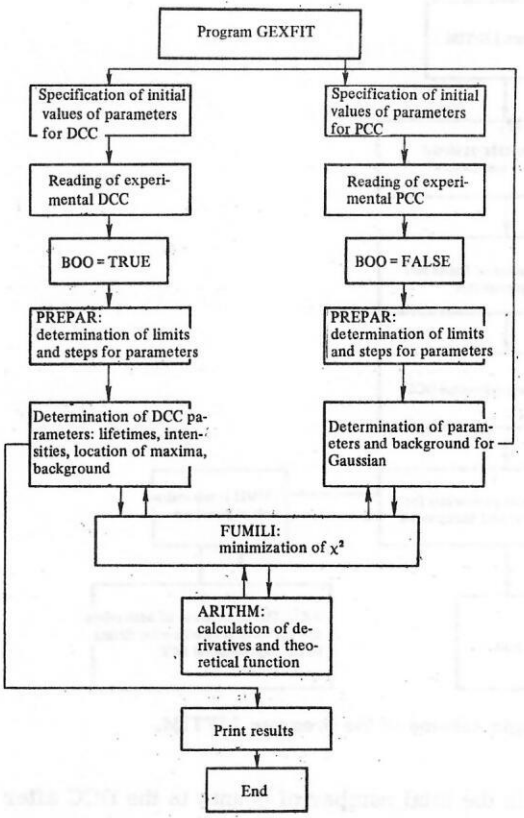

FIG. 12. Logic scheme of the program GEXFIT.

whose values must be specified, were designated as follows: For fitting the PCC:

CHAN—the value of a time channel;

MAXN—the number of experimental data in the PCC used in the fit;

FØN, CENTRE, FWHM—initial values for the background, the location of the maximum, and the half-width of the PCC.

For fitting the DCC:

CNAN—value of a time channel in the DCC;

MAXN—number of experimental data in the DCC used in the fit;

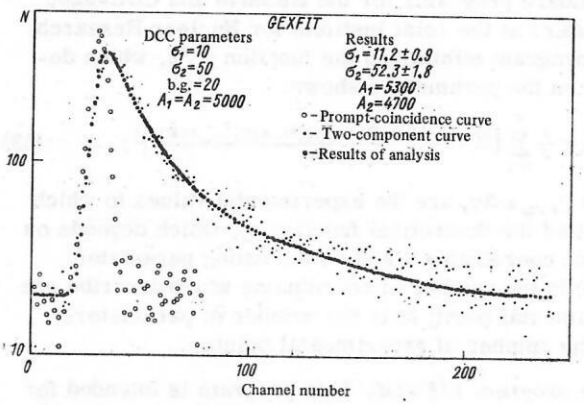

FIG. 13. Artificially obtained PCC and two-component DCC and results of analysis by the program GEXFIT.

FØN, ZERØ—initial values for the background and zero time of the DCC;

ICØM—number of exponential components;

RLAM(I), AINT(I)—initial values for the decay constant and intensity (area) of the exponential component I.

Artificial prompt- and delayed-coincidence spectra for a two-component decay, obtained by means of the generator SPECTR, and the corresponding parameters used in the generation and obtained as a result of the fit, are given in Fig. 13. After determination of the parameters of the DCC, the program SPECTR generates statistical delayed-coincidence spectra for each component on the basis of the parameters obtained in the fit.

It is necessary to mention that in the case when a prompt component is present in the DCC the program GEXFIT is used with a small change: In the analytic expression (57) for the DCC a term containing the prompt component is added:

$$F(t)=\frac{1}{2}\sum_{k=1}^{n}N_k\frac{1}{\sigma\sqrt{2\pi}}\int_0^\infty \exp(-\lambda_k t)\exp\{-[(t-t_0)/\sigma]^2/2\}\,dt + \frac{M}{\sigma\sqrt{2\pi}}\exp\{-[(t-t_0)/\sigma]^2/2\}+B. \qquad (59)$$

In this case in analysis of the DCC the intensity of the prompt component is also a variable parameter.

There are no fundamental difficulties in changing the form of the program for cases in which other analytic expressions are used for the PCC. For example, for an asymmetric Gaussian:

$$P_1(t)=\frac{1}{\sigma_1\sqrt{2\pi}}\exp\{-[(t-t_0)/\sigma_1]^2/2\}+\text{b.g. for}\quad t\geqslant t_0;$$
$$P_2(t)=\frac{1}{\sigma_2\sqrt{2\pi}}\exp\{-[(t-t_0)/\sigma_2]^2/2\}+\text{b.g. for}\quad t<t_0$$

it is necessary to fit individually $P_1(t)$ to the right-hand part and $P_2(t)$ to the left-hand part of the experimental PCC and to take into account in fitting the expression $F(t)$ three parameters instead of the two for the PCC. The number of mathematical expressions for $F(t)$ is doubled, but the principle of obtaining values of the lifetime as parameters of the fit is not changed.

Folding of the experimental PCC with an exponential decay. Several authors[14,15,21] have shown two methods of direct use of the experimental PCC in analysis of DCC by the folding method. By means of the program FUMILI for fitting, it is possible also to analyze DCC which contain one exponential decay and an accidental-coincidence background with account being taken of the distribution due to folding the experimental PCC.

Proceeding from the Eqs. (45) we have obtained a general expression describing the number of coincidences in a running channel k in the DCC:

$$F_k=\begin{cases} N_0\sum_{i=1}^{k}P_i\exp[-\lambda(k-i)\,\delta]+\text{b.g. for}\quad k<Q; \\ N_0\sum_{i=1}^{Q}P_i\exp[-\lambda(k-i)\,\delta]+\text{b.g. for}\quad Q\leqslant k\leqslant q, \end{cases} \qquad (60)$$

where Q and q are the extreme channels of the PCC and DCC in the direction of extension of the DCC; δ is the value of a channel.

It is interesting to note that, as in the work of Kaczorowski and Pietrzyk,[21] for channels in the region $Q\leqslant k\leqslant q$, i.e., in the region $P(t)\ll F(t)$, the number of coincidences F_k has a purely exponential drop:

$$F_k=N_0\exp(-\lambda k\delta)\sum_i P_i\exp(-\lambda i\delta)=\text{const}\,N_0\exp(-\lambda k\delta).$$

Expression (60), which was obtained for each channel k with allowance for the values of the experimental PCC, is fitted to the experimental DCC with variation of the parameters N_0, λ, and the background.

For many-component decay (M components) the expressions (60) take the following form:

$$F_k=\begin{cases} \sum_{j=1}^{M}N_{0j}\sum_{i=1}^{k}P_i\exp[-\lambda_j(k-i)\,\delta]+\text{b.g. for}\quad k<Q; \\ \sum_{j=1}^{M}N_{0j}\sum_{i=1}^{Q}P_i\exp[-\lambda_j(k-i)\,\delta]+\text{b.g. for}\quad Q\leqslant k\leqslant q. \end{cases}$$

The choice of the program for analysis of the time spectra depends on the specific conditions of the measurements: the instrumental time resolution $2\tau_0$, the lifetime τ of the states, the statistical material obtained during the experiment, and the background level.

Trial of the programs developed has shown that in the lifetime region $\tau>2\tau_0$ the program LIFTIM works efficiently, and in determination of lifetimes in the region $\tau\leqslant 2\tau_0$ it is necessary to use the more universal program GEXFIT, which permits reliable results to be obtained in any of the time ranges considered.

In conclusion the authors express their indebtedness to D. Horvath of the Central Institute for Physics Research in Budapest for his constant interest in this work and for valuable remarks.

APPENDIX 1

```
      PROGRAM GEXFIT

      COMMON/PLU/PLU(100)/SIGMA/SIGMA(100)/,G/G(100)
     X/R/R(100)/DA/DA(100)
      COMMON/DATA/YD(500)
      COMMON/BLOCK/BOO, ICOM, M, CHAN, SIG
      COMMON/A/A(100)1/4 PL/PL(100)/EXDA/EXDA(1500)
     X/X/X(10)/NED/NED(2)
      COMMON/AU/AU(100)/AL/AL(100)/DF/DF(100)
      LOGICAL BOO
      DIMENSION AINT(5), RLAM(5)
      EQUIVALENCE(MAXN,NED (1)), (CENTRE,RLAM(3), A(3))
C     CONSTANTS FOR FUMILI
      IT = 1 $ NI = 2 $ N2 = 1 $ N3 = 40 $ NED(2) = 3 $ EPS = 0.01
C     CHANNEL WIDTH AND MAXIMAL CHANNEL NUMBER FOR THE
C     PROMPT SPECTRUM
C     INITIAL PARAMETER VALUES FOR FITTING THE
C     PROMPT SPECTRUM
C     CANAL = 44.4 PIC
C     PRELUCRARE E ———E—
C     ORAER OF THE PARAMETERS-MAXN, FON, CENTRE, FWHM, CHAN
C     CHAN = 1.
      READ 60, AMAXN, FON, CENTRE, FWNM, CHAN
      MAXN = AMAXN
   60 FORMAT(5F10,2)
C     COMPOSITION OF DATA BLOCKS
      READ 101,(YD(J), J = 1, MAXN)
      Y = —FLOAT(MAXN)+FON
      DO 1 I = 1, MAXN
      Y = Y+YD(I)
    1 CONTINUE
      UPS2PI = 1./SQRT(6.28318531)
      A(1) = Y*2.355*UPS2PI/FWNM
      A(2) = 2.727/FWNM**2
      A(4) = FON
      M = 4
      BOO = FACSE.
      PRINT 102
      PRINT 103, MAXN, CHAN
      PRINT 104, FON, CENTRE, FWNM,Y
      PRINT 101, (YD(I), I = 1, MAXN)
      CALL PREPAR (M, MAXN)
      CALL FUMILI (S, M, N1. N2, N3, EPS, AKAPPA, ALAMBD, IT, MC)
      CALL ERORF(M)
      FWNM = SQRT (2.727/A(2))

      FON = A(4)
      Y = A(1) *FWNM/2.355/UPS2PI
      PRINT 105
      PRINT 104, FON, CENTRE, FWNM, Y
```

```
      SIG = FWNM/2.355 CHAN
    C CHANNEL WIDTH, NUMBER OF CHANNELS AND COMPONENTS
    C FOR THE LIFETIME SPECTRUM
    C INITIAL PARAMETER VALUES FOR FITTING THE
    C LIFETIME SPECTRUM
      READ 70, AMAXN, FON, ZERO, AICOM, CHAN
   70 FORMAT (5F10.2)
      READ 80, RLAM(1), RLAM(2)
   80 FORMAT(2F10.5)
      MAXN = AMAXN
      READ 101, (YD(I),I = 1,MAX)
      SUM = 0.
      DO 20 I = 1, MAXN
      SUM = SUM+YD(I)
   20 CONTINUE
      AINT(1) = 0.5 × SUM
      AINT(2) = 0.5 × SUM
      ICOM = AICOM
    C  COMPOSITION OF DATA BLOCKS
      M = 2 × (WCOM+1)
      A(ICOM+1) = ZERO
      DO 2 I = 1, ICOM
      A(I) = RLAM(I)
      A(ICOM+I = AINT(I)
    2 CONTINUE
      A(M) = FON
      BOO = .TRUE.
      PRINT 106
      PRINT 103, BGXN, CHAN
      PRINT 107, FON, ZERO, (AINT(I), RLAM(I), I = 1,ICOM)
      PRINT 101, (YD(I), I = 1, MAXN)
      CALL PREPAR(M, MAXN)
      CALL FUMILI (S, M, N1, N2, N3, EPS, AKAPPA, ALAMBD, IT, MC)
      CALL ERRORF(M)
      PRINT 108
      DO 10 I = I, ICOM
      J = I 1COM+1
      AINT(I) = A(J)
   10 CONTINUE
      PRINT 107, A(M), A(ICOM+1), (RLAM(I), AINT(I), I = 1,ICOM)
  101 FORMAT(10F8.1)
  102 FORMAT(//LOX 43H INPUT DATA FOR FITTING THE PROMPT
      SPEC TXRUM//)
  103 FORMAT(6HMAXN = 16,5X, LIHCH. WIDTH =,E12.4/)
  104 FORMAT(12HBACKGROUND = ,F9.2,5 X, 15HPEAK LOCATION =,
  XR9.2,5 X,6HFWHM =,F9.2,5 X.6HAREA = ,E12.4//)
  105 FORMAT(//20X,29HFITTED PARAMETERS OF GAUSSIAN/)
  106 FORMAT(//10X,
  × 45H INPUT DATA FOR FIMMING THE LIFETIME SPECTRUM//)
  107 FORMAT(12HBACKGROUND = ,F9.2,6HXERO =,F9.2//
  ×,27HDECAY PATES AND INTENSITIES/(2E14.4)//)
  108 FORMAT(26HEFITTED LIFETIME PARAMETERS//)
      END

      SUBROUTINE PREPAR (M, MAXN)
      COMMON/DATA/YD(500)
      COMMON/AU/AU(100)/AL/AL(100)/PL/PL(100)/A/A(100)/
  XEXDA/EXDA(1500)
      COMMON/X/X(10)
      DO 3 I = 1,M
      AI = A(I) $ PL(I) = 0.1 × AI $ AL(I) = 0.1* PL(I)
      AU(I)1000. = AI
    3 CONTINUE
      DO 4I = 1,MAXN
      J = 3 × I — 2 $ Y = YD(I) $ EXDA(J) = Y
      IF(Y, LE.64.)EXD(J I) = 8.
      IF(Y. GT.64.) EXDA(J 1) = SQRT(Y)
      EXDA(J,2) = I
    4 CONTINUE
      RETURN
      END

      SUBROUTINE ARITHM(Y)
      COMMON/A/,A(100)/DF/DF(100)/X/X(10)
      COMMON/BLOCK/BOO, ICOM, M, CHAN, SIG
      EQUIVALENCE(A(1), B), (A(2),C), (A(3), D)
      LOGICAL BOO
      IF(BOO) GO TO 50
    C FITTING THE PROMPT SPECTRUM
      X1 = X(1)—D $ EK = EXP(—C × X1 × × 2) $ Y = B * EK*A (4)
      DF(1) = EK $ DF(2)= —B × X1 × × 2 × EK $ DF(3) = 2. × B ×
      × C × X1 × EK
      DF(4) = 1. $ RETURN
    C FITTING TNE LIFETIME SPECTRUM
   50 T = CHAN * (X(1) — A(ICOM+1))
      Y = 0. $ DTO = O. $ UPS = 1./SIG $ TPS = T × UPS
      GAUSS = 0.3989423 × EXP(— 0.5 × TPS ×× 2)
      DO 70 K = 1, ICOM
      RLAM = A(K) $ RINT = A(ICOM+K+1) $ RLT = RLAM × T
      RLS = RLAM × SIG $ RLS2 = RLS ×× 2 $ X = TPS — RLS
      X1 = 0.70710678 × ABS(Z) $ IF (Z.GE.4.) FIP1 = 2.
      IF(Z.LT.4..AND.Z.GE.O.) FIP1 = 1.+ERF(X1)
      IF(Z.LT.O..AND.Z.CT.—6.5) FIP1 = ERFC(X1)
      IF(Z.LE.—6.5) FIP1 = 0.
      IF(FIP1.GT.O.) FIP1 = FIP1 × EXP(0.5 × RLS2 — RLT)
      FKO = RINT × RLAM × FIP1 $ Y = Y+FKO $ GI = GAUSS × RINT
      DF(K) = FIP1 × PINT × (1.—RLT+RLS2) × 0,5 — RLS
      DTO = DTO+RLAM × (FKO — UPS × GI)
      DF(ICOM+K+1) = RLAM × FIP1 × 0.5
   70 CONTINUE
      Y = 0.5 × Y+A(M) $ DF(ICOM+1) = DTO $ DF(M) = 1.
      RETURN $ END
```

APPENDIX 2

```
      PROGPAM LIFTIM

      COMMON/PLU/PLU(100)/SIGMA(100)/G/G(100)
  X/R/R(100)/DA/DA(100)
      COMMON/A/A(100)/PL/PL(100)/EXDA/EXDA(1500)/X/X(10)
  X/NED/NED(2)
      COMMON/AU/AU(100)/AL/AL(100)/DF/DF(100)
      DIMENSION AXN(300), WE(300)
      N1 = 2 $ N2 = 1 $ N3 = 30 $ M = 4 $ IT = 2 $ EPS = 0.001
      NED(2) = 3 $ NDA = 0 $ NDAT = 6
   30 CONTINUE
      NDA = NDA+I
      READ 20, AMAX, (A(I), I = 1, M)
   20 FORMAT(8FIO.5)
      IMAX = IFIX(AMAX) $ NED 1 = IMAX
      DO 2I I = 1,M
      PL(I) = 0.1 × A(I) $ AL(I) = PL(I) $ AU(I) = 10. × A(I)
   21 CONTINUE
      READ 2, (E(J), J = 1, IMAX)
    2 FORMAT (10F8.1)
      PRINT 1, (E(J), J = 1, 1MAX)
    1 FORMAT (10FN.1)
      DO 50 J = 1, IMAX
      AJ = FJOAT(J) $ AXN(J) = AJ
   50 CONTINUE
      DO 6 I = 1, IMAX
      WE(I) = SQRT(E(I)) $ KJ = I × 3 — 2 $ EXDA(KJ) = E(1)
      EXDA (KJ+1) = WE(I)
      EXDA(KJ+2) = AXN(I)
    7 CONTINUE
      CALL FUMILI (S, M, N1, N2, N3, EPS, AKAPPA, ALAMBD, IT, MC)
      CALL ERRORF(M)
      TAUI = ALOG(2.)/A(3)
      PRINT 10, TAU1
   10 FORMAT(//3X, 5HTAU1 =,E20,5)
      IF(NDA.LT.NDAT) GO TO 30
      STOP
      END

      SUBROUTINE ARITNM(Y)
      COMMON/A/A(100)/DF/DF(100)/X/X(10)
      DF(1) = EXP (—A(3) × X(1))
      DF(2) = EXP(—A(4) × X(1))
      DF(3) = —A(1) × X(1) × DF(1)
      DF(4) = —A(2) × X (1) × DF(2)
      Y = A(1) × DF(1) × A(2) × DF(2)
      RETURN
      END
```


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