Coupled-channels method of calculations in the shell model with allowance for the continuous spectrum

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The main equations are given of a method that generalizes the traditional shell model by including states with one nucleon in the continuum. This improvement of the method can explain not only the behavior of the total cross section but also the resonance parameters. The coupling between the space of bound states and the space of scattering states is discussed for specific examples. The method is used to calculate numerically examples describing nucleon scattering on light nuclei and absorption of γ rays and muons by light nuclei.

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INTRODUCTION

In recent years, it has proved possible to obtain a good description of the properties of nuclei on the basis of the shell model; even in the cases when cluster structures play a role, the independent-particle model has been found to apply. At the present time, calculations with large configuration spaces are made on the basis of the shell model. In the traditional shell model for light nuclei, the basis wave functions of the nucleons are determined in a potential well of infinite depth. The nucleon-nucleon correlations not included in the average field are taken into account by a residual interaction. Such a description of nuclei enables one to obtain the structure of nuclear states but not directly determine a reaction or decay process. For their description the R-matrix theory has been developed. The boundary conditions at a certain radius a are used to take into account the coupling between the wave functions inside the nucleus ($r < a$) with those outside the nucleus ($r > a$). This enables one to determine the reduced widths of levels above the breakup threshold. Using the results of calculations of the shell model, one can obtain good results in many fields of nuclear physics.

Before nuclear reactions are calculated on the basis of the shell model in conjunction with the R-matrix theory, it is necessary to assume a definite reaction mechanism. In the case of the resonance mechanism, the reaction proceeds through two independent stages: first, the formation of an intermediate nucleus; second, its decay. For example, for the reactions $A(\gamma,n)B$ the $A+\gamma \rightarrow A^*$ excitation probabilities are first calculated and then the $A^* \rightarrow B+n$ decay probabilities. The probability of finding a definite reaction channel is then proportional to the product of the two intermediate probabilities. Here it is impossible to take into account simultaneously the direct part of the reaction (without the formation of an intermediate nucleus in the state A^*).

The direct reaction mechanism is assumed for the description of quasielastic knockout reactions at intermediate energies of the bombarding particles such as, for example, the reactions $A(p,2p)B$ at proton energies of a few hundred MeV. In this case, it is assumed that the proton is knocked out in the reaction directly as a result of quasielastic scattering. The relative probabilities for the excitation of different levels of the final nucleus B under the assumption of such a mechanism are proportional to the $A_{gs} \rightarrow B^* + p$ decay widths. In contrast to the reaction $A(\gamma,n)B$, in the framework of the shell model there is no possibility of taking into account the contributions of resonance processes. Thus, the undoubted shortcomings of calculations based on the traditional shell model are due to the need to introduce additional assumptions in order to describe the interaction of the discrete states with the continuum. Because of this, one cannot study interference effects associated with the simultaneous existence of different reaction mechanisms, nor study the form of resonances.

The considerable successes in the description of nuclear reactions on the basis of the shell model in conjunction with the R-matrix theory, on the one hand, and the undoubted intrinsic shortcomings of the model, on the other, have led to the development of a shell model with allowance for the continuum. The advantage of this model is that in it one can describe the structure of nuclei as accurately as by the ordinary shell model.[1] However, in contrast to the R-matrix theory, the interaction of discrete states with the continuum is taken into account in the same way as the interaction of the discrete states with one another. A system of A nucleons is described by an interaction operator which does not depend on whether or not one of the nucleons is in the continuum or in a bound state. When the shell model has been extended naturally in this way to include scattering states, one can treat the direct and the resonance parts of the reaction and their interference effects on an equal footing.

In this paper, we discuss generalizations of the traditional shell model, emphasizing primarily the typical differences from the ordinary shell model.

1. COUPLED-CHANNEL METHOD IN THE SHELL MODEL WITH CONTINUUM

Basic Equations. The point of departure of the shell model with continuum in the form presented here is the shell-model approximation for nuclear reactions formulated by Mahaux and Weidenmüller[2] and the projection formalism of Feshbach.[3] This model is an extension of the traditional shell model in the sense that in addition to the basis functions Φ_i of the discrete states one introduces the scattering functions χ_E^c. In the model discussed here, a restriction is made to states χ_E^c in which not more than one nucleon is in the continuum, while $A-1$ nucleons are in bound one-particle states. The wave function is represented in the form[2]

$$\Psi_E^c = \sum_{i=1}^{M} b_E^c(i)\,\Phi_i + \sum_{c'=1}^{\Lambda} \int_{\epsilon_{c'}} dE' a_E^c(E'_i c')\,\chi_{E'}^{c'}, \tag{1}$$

where $b_E^c(i)$ and $a_E^c(E'_i c')$ are the coefficients of the expansion with respect to bound and free functions with allowance for finitely many discrete states Φ_i and channels c; ϵ_c is the threshold energy for channel c.

By means of the projection operators P and Q, the total scattering wave function can be represented in the form

$$\Psi_E^c = \Psi = P\Psi + Q\Psi. \tag{2}$$

Here, P projects onto the space of scattering states with basis functions χ_E^c; Q projects onto the space of discrete states with basis functions Φ_i. The Q space is determined by the discrete states independently of their energy (bound states embedded in the continuum, or BSEC in the terminology of Mahaux and Weidenmüller).[2] If the representation (2) is used, the Schrödinger equation takes the form

$$(H_{PP}-E)\,P\Psi = -H_{PQ}Q\Psi; \tag{3a}$$

$$(H_{QQ}-E)\,Q\Psi = -H_{QP}P\Psi, \tag{3b}$$

and $H_{PP} \equiv PHP$, etc.

We define ξ as a solution of Eq. (3a) without inhomogeneous term, i.e., as the solution in P space:

$$(H_{PP}-E)\,\xi = 0; \quad Q\xi = 0 \tag{4}$$

with the boundary condition of the incoming wave in the entrance channel and outgoing waves in all the other channels. Then Eq. (3a) can be represented in the form

$$P\Psi = \xi - G_P^{(+)} H_{PQ} Q\Psi, \tag{5}$$

where

$$G_P^{(+)} = 1/(H_{PP}-E^{(+)}) \tag{6}$$

is the Green's function for the motion of a particle in P space. From (3b) and (5) we obtain the following expression for $Q\Psi$:

$$Q\Psi = -\frac{1}{H_{QQ}-E-H_{QP}G_P^{(+)}H_{PQ}} H_{QP}\xi. \tag{7}$$

It is expedient to exand $Q\Psi$ with respect to the eigenfunctions Φ_R of the operator H_{QQ}:

$$Q\Psi = -\sum_R B_R \Phi_R. \tag{8}$$

From (7), the expansion coefficients are

$$B_R = \sum_{R'} \left\langle \Phi_R \left| \frac{1}{H_{QQ}-E-H_{QP}G_P^{(+)}H_{PQ}} \right| \Phi_{R'} \right\rangle \langle \Phi_{R'} | H_{QP} | \xi \rangle. \tag{9}$$

The complete solution of the problem is obtained by means of Eqs. (5) and (8):

$$\Psi = \xi + \sum_R (w_R - \Phi_R)\,B_R, \tag{10}$$

where

$$w_R = G_P^{(+)} H_{PQ} \Phi_R \tag{11}$$

is a function which describes the coupling of one state in the Q space to the continuous scattering states in the P space. The function w_R contains only outgoing waves and describes the decay of the state Φ_R. From Eq. (11),

$$(H_{PP}-E)\,w_R = H_{PQ}\Phi_R. \tag{12}$$

Equations (4) and (12) have the general form

$$(H_{PP}-E)\,u = PI; \quad Qu = 0, \tag{13}$$

where

$$u = w_R; \quad I = H_{PQ}\Phi_R = H\Phi_R \tag{14}$$

or

$$u = \xi, \quad I = 0. \tag{15}$$

Therefore, the problem is to solve Eq. (13) for the two cases (14) and (15) and to invert the matrix in Eq. (9):

$$M_{RR'} = \langle \Phi_R | H_{QQ} - E | \Phi_{R'} \rangle - \langle \Phi_R | H_{QP} | w_{R'} \rangle. \tag{16}$$

whose eigenvalues are complex; $E_R - i\Gamma_R/2 - E$. The functions Φ_R are eigenfunctions of the operator H_{QQ} and are obtained by the usual diagonalization on the basis of the shell model using a finite-depth potential well of Woods-Saxon type.

Below, we describe a method of solving Eq. (13) which includes, besides nucleon scattering processes, processes of absorption of the type $A(\gamma, n)$ or capture of the type $A(\mu, n)$. In addition, we compare this method with that of Buck and Hill.[4]

Channel Representation. To solve Eq. (13) by the method of coupled channels, the many-particle wave functions u are expanded with respect to the channel wave functions u_c, which depend on the radial coordinate of only one nucleon:

$$u = \mathcal{A}_A \sum_c \Omega_c u_c(r). \tag{17}$$

Here, $\mathcal{A}_A$ is the operator of antisymmetrization of the last nucleon with respect to the $A-1$ nucleons of the target nucleus. The functions Ω_c depend on all the coordinates of the $A-1$ nucleons and also on the angular, spin, and isospin coordinates of the last nucleon:

$$\Omega_c = \frac{i^l}{r} \Phi_T(1, \ldots, A-1)\, Y_{lj}(\Omega_A, \sigma_A)\, \chi_\tau(\tau_A)\, (J_T \mathbf{j} | \mathbf{J}). \tag{18}$$

Here, $\Phi_T(1, \ldots, A-1)$ is the wave function of the target nucleus, which depends on the coordinates of $A-1$

nucleons; $(J_T\mathfrak{J}|J)$ is a Clebsch-Gordan coefficient; Y_{lj} and χ_τ depend on the coordinates of the orbital angular momentum, the spin, and the isospin of the last nucleon.

To determine the channel wave functions u_c from Eq. (13) written in the form

$$(H-E)\,u = PI + H_{QP}u = I + Q\,(Hu - I), \tag{19}$$

we obtain the system of equations

$$\sum_{c'} (H_{cc'} - E\delta_{cc'})\,u_{c'}(r) = I_c + Q_{cc'}\Big(\sum_{c'} H_{cc'}u_{c'}(r) - I_c\Big)\,. \tag{20}$$

where

$$Q_{cc'} = \delta_{cc'} \sum_q \delta_{l_c j_c,\, l_q j_q} \varphi_q(r) \int dr' \varphi_q(r') \tag{21}$$

is the Q operator in the channel representation, which can be found by using the completeness condition for the one-particle functions with negative and positive energy. Further, φ_q are the radial functions of the bound one-particle states in the finite-depth (Woods-Saxon) potential of the shell model. They are normalized, $\int dr \varphi_q^2(r) = 1$.

Equation (20) is an equation for determining the one-particle functions $u_c(r)$. The right-hand side contains integral terms because of the presence of the Q operator, so that Eq. (20) is an integrodifferential equation. The system of equations (20) is solved as follows. First, one looks for a solution of the equation without the term containing the Q operator. The resulting function $\overline{u}$ has the correct boundary conditions. One then determines the solutions of the inhomogeneous equations

$$\sum_{c'} (H_{cc'} - E\delta_{cc'})\,u_{c'}^{(\mu)}(r) = \delta_{c\mu}\delta_{l_c j_c,\, l_q j_q}\varphi_q(r) \tag{22}$$

with outgoing waves. The number of equations and the number of functions $u^{(\mu)}$ is equal to the number of terms of the Q operator (21). The final solution u is obtained by adding the functions $\overline{u}$ and $u^{(\mu)}$ with suitable weights in order to guarantee orthogonality of the functions u and the functions of the Q space in accordance with Eq. (13). A numerical solution is obtained by means of Numerov's method.[5]

In our calculations, the interaction in the Hamiltonian

$$H = \sum_i \varepsilon_i :a_i^\dagger a_i: + \frac{1}{4}\sum_{ijkl} V_{ijkl} :a_i^\dagger a_j^\dagger a_k a_l: \tag{23}$$

is represented in the δ-functional form

$$V(\mathbf{r}_1 - \mathbf{r}_2) = V_0\,(a + bP_{12}^\sigma)\,\delta(\mathbf{r}_1 - \mathbf{r}_2). \tag{24}$$

Here, V_0, a, and b are parameters; P_{12}^σ is the spin exchange operator. Interaction in the form (24) leads to local potentials in Eq. (20).

Source Method. Nuclear reactions in which a nucleus goes over into an excited state higher than the breakup energy threshold when there is an interaction in addition to the operator (23) can also be described in the framework of the shell model with continuum if the additional interaction can be correctly treated in the Born approximation. The Hamiltonian of the perturbed system has the form

$$H_s = H + H_{\text{int}}, \tag{25}$$

where H_{int}, the operator of the additional interaction, can describe a process of absorption (for example, of γ rays) or a process of capture (for example, of muons) and also nuclear (d,p) or (p,p') reactions if the energy of the incident particle is high compared with the binding energy of the nucleons in the nucleus, and also nuclear reactions due to other particles (for example, electrons).

The Schrödinger equation for the perturbed system is

$$(H-E)\,\tilde{\Psi} = -H_{\text{int}}\tilde{\Psi}. \tag{26}$$

For $\tilde{\Psi}$ we have the equation

$$\tilde{\Psi} = \varphi - H_{\text{int}}\tilde{\Psi}/(H-E), \tag{27}$$

where φ is a solution of the equation without interaction:

$$(H-E)\,\varphi = 0. \tag{28}$$

In the Born approximation, (27) yields

$$\tilde{\Psi} = \varphi - \frac{1}{H-E}H_{\text{int}}\varphi. \tag{29}$$

The second term in the expression (29):

$$\Psi = -H_{\text{int}}\varphi/(H-E) \tag{30}$$

is the source of the interaction with the nucleus. Introducing the function

$$F = -H_{\text{int}}\varphi, \tag{31}$$

we obtain from (30)

$$(H-E)\,\Psi = F, \tag{32}$$

where φ includes the wave function of the target nucleus. The function Ψ has only outgoing waves, since the reaction is produced not by incoming nucleon waves but by the mechanism described in H_{int}.

Using the projection operators P and Q, one can separate states of the Q space in the same way as above. From (32),

$$(H_{PP} - E)\,P\Psi = -H_{PQ}Q\Psi + PF; \tag{33}$$

$$(H_{QQ} - E)\,Q\Psi = -H_{QP}P\Psi + QF. \tag{34}$$

By analogy with Eqs. (4), we define ξ_F as the solution in P space:

$$(H_{PP} - E)\,\xi_F = PF; \quad Q\xi_F = 0 \tag{35}$$

or

$$\xi_F = G_P^{(+)}F. \tag{36}$$

Then from Eqs. (33) and (34),

$$P\Psi = G_P^{(+)}F - G_P^{(+)}H_{PQ}Q\Psi; \tag{37}$$

$$Q\Psi = -\frac{1}{H_{QQ} - E - H_{QP}G_P^{(+)}H_{PQ}}\,(H_{QP}G_P^{(+)} - Q)\,F. \tag{38}$$

For the complete function $\Psi = P\Psi + Q\Psi$ we obtain

$$\Psi = G_P^{(+)}F + (G_P^{(+)}H_{PQ} - Q)\,\frac{1}{H_{QQ} - E - H_{QP}G_P^{(+)}H_{PQ}}\,(H_{QP}G_P^{(+)} - Q)\,F \tag{39}$$

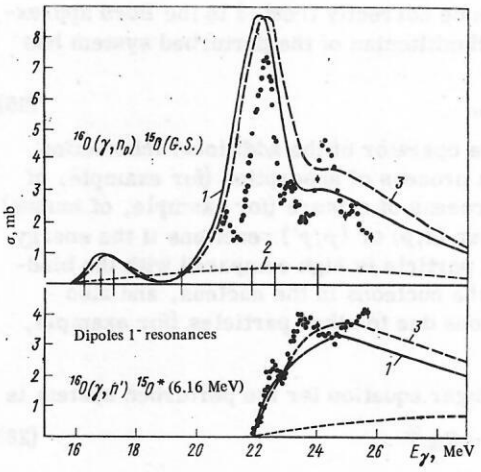

FIG. 1. Cross section of γ-absorption reaction on ^{16}O: 1) cross section calculated with the present model; 2) direct part of the reaction (without one-particle $d_{3/2}$ resonance); 3) results of calculation by Buck and Hill's method.[4] The calculation parameters are taken from Ref. 4. The vertical strokes indicate the positions of the resonances. The experimental points from Ref. 7 are given for comparison.

or, by analogy with Eq. (10),

$$\Psi = \xi_F + \sum_R (w_R - \Phi_R) B_R^{(F)}, \tag{40}$$

if we use Eqs. (9), (11), (16), and (36) and

$$B_R^{(F)} = \sum_{R'} \frac{1}{M_{RR'}} \langle \Psi_{R'} | H_{QP} \xi_F - F \rangle = \sum_{R'} \frac{1}{M_{RR'}} \langle \Phi_{R'} | H_{QP} G_F^{(-)} - 1 | F \rangle. \tag{41}$$

In contrast to Eq. (10), Ψ in Eq. (40) does not contain ingoing waves responsible for reactions. The inhomogeneous term F, a source, is coupled through H_{int} to both the P and the Q space. Therefore, $B_R^{(F)}$ consists of two terms: the first includes the coupling of the resonance states to the continuum through the nuclear interaction H_{QP} (channel-resonance scattering in γ absorption), as in the scattering of nucleons. The coupling of the continuum to the source is described by ξ_F. The second term in $B_R^{(F)}$ decribes the direct coupling of resonance states to the source. Such a term is absent in the scattering of nucleons, i.e., in the expression (9).

To perform numerical calculations, it is first necessary to determine F from Eq. (31) and then solve Eq. (13), in which u is determined by means of (14) and the relations

$$u = \xi_F, \quad I = F \tag{42}$$

instead of (15) in accordance with Eq. (35). Thus, the method of solution in the source case is the same as in the case described above.

Comparison with Buck and Hill's Method.[4] The mathematical formulation of the shell model with continuum described here is equivalent to Buck and Hill's method.[4] The differences are that in the latter the system of coupled equations (20) is solved with the spaces $P\Psi$ and $Q\Psi$ defined in such a way that the integral term containing $Q_{cc'}$ can be ignored. This is possible when the boundary between the spaces $P\Psi$ and $Q\Psi$ is the Fermi limit. This means that the Q space contains all closed shells; the P space, all the unclosed shells and the scattering states. For example, for ^{16}O Buck and Hill separate the shells into subspaces as follows: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$ into the Q space; $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, and the scattering states, into the P space. To do this, it is necessary to represent the discrete states of P space in the channel form

$$\Phi_i = \mathcal{A}_A \{w_c(r)\, \Omega_c\} \tag{43}$$

by analogy with the scattering states

$$\chi_E^c = \mathcal{A}_A \{u_c(r)\, \Omega_c\}. \tag{44}$$

Here w_c and u_c are radial wave functions with negative or positive energies. All the other notation is as in Eq. (17). The representation of the functions Φ_i in the form (43) without summation over the channel functions requires additional assumptions about the structure of the nuclei:

1) The ground state and the low-lying excited states of the target nucleus can be represented as hole states;

2) the states of the intermediate nucleus are pure one-particle-one-hole states.

If the nuclear structure is more complicated, then so is the representation of the discrete functions Φ_i with respect to channel functions more complicated than in Eq. (43). Therefore, in Buck and Hill's method the system of equations (20) with $Q_{cc'} = 0$ is solved only for the case (15), and the projection formalism, which provides the basis for analyzing resonance structure, is not used.

When Buck and Hill's definition of the spaces $P\Psi$ and $Q\Psi$ is used, neglect of the integral term in Eq. (20) leads to only small errors if (γ,n) and (γ,p) reactions on doubly magic nuclei are described. In the case of ^{16}O, neglect of the integral term means that the $1s$ shell is not taken into account in the calculation, i.e., the antisymmetry of the s waves of the continuum with $1s$ states is ignored. Since the $1s$ shell is 20 MeV from the $1p$ shell, the resulting errors are small (Fig. 1).

The representation of the wave functions Φ_i of the discrete states in channel form and the neglect of the integral term containing $Q_{cc'}$, whose influence depends on the particular nucleus being studied, restrict the use of Buck and Hill's method for the description of physical problems.

In the above formulation of the shell model with continuum, there are no restrictions on the structure of the bound states. Since the Q space contains all discrete states and the P space only scattering states, it is possible to include in the Q space states with complicated configurations obtained from calculations by the traditional shell model. However, the integral term in Eq. (20) cannot be ignored. It contains information about bound one-particle states and causes the cross section calculated for the case (15) (solution in P space)

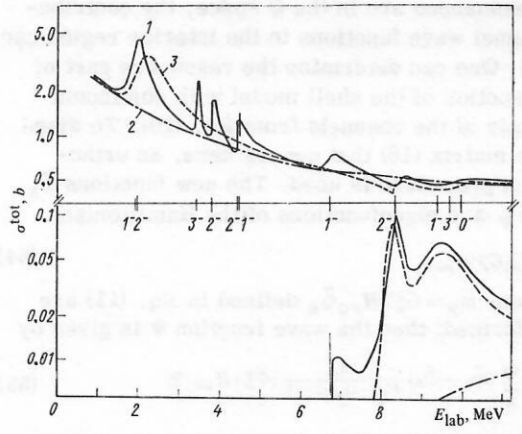

FIG. 2. Excitation function of elastic and inelastic $^{15}N+n$ scattering: 1) total cross section; 2) direct part of the reaction with inclusion of the one-particle $d_{3/2}$ resonance in the Q space; 3) direct part with inclusion of the one-particle $d_{3/2}$ resonance in the P space. The calculation parameters are given in Ref. 1. The vertical strokes indicate the positions of the resonances.

to have a structureless behavior. Resonance behavior of the cross section is obtained after application of the projection formalism, which also gives the structure of the resonances (see below).

2. TREATMENT OF ONE-PARTICLE RESONANCES

Definition of the $P\Psi$ and $Q\Psi$ subspaces. Above, the complete space of the functions Ψ was split into $P\Psi$ and $Q\Psi$ subspaces in such a way that $P\Psi$ contains the states of the continuum and $Q\Psi$ all the discrete states. One-particle resonances corresponding to quasistationary states above the breakup threshold can be included in either the $P\Psi$ or the $Q\Psi$ space.

From the point of view of the calculation of the total reaction cross section, it is simplest to include the one-particle resonances in the P space. The cross section calculated with the solution ξ in the P space in this case exhibits resonance behavior deriving from the one-particle resonances. Results of a calculation for the reaction $^{15}N+n$ are given in Fig. 2, which shows that the one-particle $d_{3/2}$ resonance is mixed with other resonances. Therefore, one cannot determine the wave function of resonances if the one-particle resonances are not included in the Q space. The space of functions in the traditional shell model differs from the Q space in the case of the ^{16}O nucleus by the state $1d_{3/2}$, which is included in the ordinary calculation instead of the one-particle $d_{3/2}$ resonance. It is well known from numerous calculations that the $1d_{3/2}$ state is mixed with other one-particle states, which corresponds qualitatively to the results in Fig. 2.

To determine the structure of resonances, it is expedient to include the one-particle resonances out to a certain finite cutoff radius R_{cut} in the Q space. Then the Q space of the shell model with continuum is the same as the space of functions in the traditional shell model. When formulated in this way, the shell models with and without continuum differ in that it is only in the former that the coupling of the Q space to the P space is taken into account directly.

The cross section obtained with the solution ξ in the P space when the one-particle resonance is included in the Q space up to some R_{cut} corresponds to curve 2 of Fig. 2. It is to be expected that in this case the cross section is structureless. Of course, the cross section calculated with the total wave function Ψ is independent of whether the one-particle resonance belongs to the P or the Q space. The cutoff method for one-particle resonances and also the determination of the resonance parameters will be considered below.

Cutoff Method for One-Particle Resonances. In order to treat one-particle resonances as bound states, the discrete one-particle wave functions φ^q, which can be normalized, are determined by Wang and Shakin's method:[6]

$$\varphi^q(r) = \varphi_{E,c}(r)\,\Theta(R_{cut}-r). \tag{45}$$

Here, $\varphi_{E,c}(r)$ describes the scattering of a nucleon with energy E in channel c. The cutoff function $\Theta(R_{cut}-r)$ has the form

$$\Theta(R_{cut}-r) = \begin{cases} 1 & \text{for } r < R_{cut}, \\ 0 & \text{for } r > R_{cut}. \end{cases} \tag{46}$$

The introduction of the function φ^q requires a modification of the wave functions of the continuum. The modified wave functions φ^p of the continuum orthogonal to φ^q satisfy the same orthogonality relations as $\varphi_{E,c}$:

$$\langle \varphi^q \mid \varphi^p_{E,c}\rangle = 0; \tag{47}$$

$$\langle \varphi^p_{E,c} \mid \varphi^p_{E',c'}\rangle = \langle \varphi_{E,c} \mid \varphi_{E',c'}\rangle = \delta_{cc'}\delta(E-E'). \tag{48}$$

The wave functions φ^p_E satisfy a Schrödinger equation of the form

$$(E - pHp)\,\varphi^p_{E,c} = 0, \tag{49}$$

where

$$pHp = (1 - |\varphi_q\rangle\langle\varphi_q|)\,H\,(1 - |\varphi_q\rangle\langle\varphi_q|). \tag{50}$$

Using Eq. (50), we can define the Q space as the space containing the bound one-particle states of the shell model and the states φ^q. All the states of the nucleus formed from these states can be called quasibound states embedded in the continuum (QBSEC).[1] The P space is defined by $P = 1 - Q$.

The function $\varphi_{E,c}(r)$ is a solution of the Schrödinger

TABLE 1. Influence of the cutoff radius R_{cut} on the position and width in the γ-absorption reaction on ^{16}O in the resonance region.

R_{cut} = 5 F		R_{cut} = 7.5 F		R_{cut} = 10 F		R_{cut} = 12.5 F	
E_R, MeV	Γ_R, MeV	E_R, MeV	Γ_R, MeV	E_R, MeV	Γ_R, MeV	E_R, MeV	Γ_R, MeV
19,55	1,04	19,55	1,04	19,55	1,04	19,55	1,04
21,88	2,26	21,89	2,18	21,87	2,14	21,85	2,16
22,80	1,72	22,62	1,50	22,55	1,58	22,55	1,48

TABLE 2. Shifts $(E_R - E_{exp}^{(sh)})$ of the positions and the widths Γ_R of positive-parity resonance states with $2p-2h$ structure of ^{16}N with allowance for hole-structure states of ^{15}N.

J	$E_{exp}^{(sh)}$, MeV	$(E_R - E_{exp}^{(sh)})$, keV	Γ_R, keV
0+	1.030	−45	19
1+	0.868	−92	39
1+	1.823	−13	13
2+	2.589	−7	8
3+	1.474	−100	0.2

equation near the resonance energy $E=E_R$. The discontinuity of the function $\varphi^q(r)$ at R_{cut} leads to additional terms in the Schrödinger equation:

$$(H_0-E)\,\varphi^q = X\varphi_{E_R,c}, \tag{51a}$$

where

$$X=\frac{\hbar^2}{2m}\left[\frac{d}{dr}(\delta(r-R_{cut})\varphi_{E_R,c})+\delta(r-R_{cut})\frac{d}{dr}\varphi_{E_R,c}\right]. \tag{51b}$$

Diagonalizing the matrix M [see Eq. (16)], we obtain additional Wronskian-type determinants, which follow from the relation

$$\langle\Phi\,|\,X\,|\,\Psi\rangle=\frac{\hbar^2}{2m}(\Phi\Psi'-\Phi'\Psi). \tag{52}$$

A change of R_{cut} alters the relation between the direct and the resonance part of the reaction. Therefore, the calculated positions and widths of the resonances have a weak dependence on R_{cut}. The deviations of the numerical results for the $^{16}O(\gamma,n)$ reaction in the region of the giant resonance when different values of R_{cut} are used are given in Table 1. The calculated cross section does not depend on R_{cut} (see above).

Wave Function, Energy, and Width of a Resonance. The properties of resonances are determined in the first place by the behavior of the resonant-state wave function within the nucleus and are almost independent of the properties of the various channels in the exterior region. This is discussed in detail by Mahaux and Saruis[8] and is the basis of the description of resonances in the ordinary shell model. In this approximation, all the one-particle wave functions allowed for are bound. The wave functions and energies of the resonances are determined by the eigenfunctions and eigenvalues of the Hamiltonian of the system of A nucleons. The eigenvalues are real. The interaction between the resonance states and the continuum is introduced by means of the R-matrix theory. The widths of the states above the breakup threshold contain the overlap integrals.

$$\langle\Psi_A\,|\,\Psi_{A-1},\,\Psi_1\rangle, \tag{53}$$

where Ψ_A is the wave function of the nucleus consisting of A nucleons.

The shell model is generalized naturally to take into account the continuum by including the one-particle resonances by the cutoff method in the space of bound states. The new Q space and the space of functions of the traditional shell model are identical. Since the one-particle resonances are in the Q space, the contribution of channel wave functions in the interior region can be ignored. One can determine the resonance part of the wave function of the shell model with continuum independently of the channels from Eq. (10). To diagonalize the matrix (16) that occurs here, an orthogonalization procedure is used. The new functions $\tilde{\Phi}_R=\sum_{R'}Q_{RR'}\Phi_{R'}$ are eigenfunctions of the Hamiltonian

$$H_{QQ}-H_{QP}G_P^{(+)}H_{PQ}. \tag{54}$$

The functions $\tilde{w}_R=G_P^{(+)}H_{PQ}\tilde{\Phi}_R$ defined in Eq. (11) are also transformed; then the wave function Ψ is given by

$$\Psi=\xi+\sum_R(\tilde{w}_R-\tilde{\Phi}_R)\frac{1}{E_R-i\Gamma_R/2-E}\langle\tilde{\Phi}_R^*\,|\,H_{QP}\,|\,\xi\rangle \tag{55}$$

with allowance for Eq. (10). Here, ξ describes the part of the reaction which does not proceed through intermediate states. It corresponds to the direct part of the reaction and does not lead to narrow resonances in the cross section. The sum over R contains all processes occurring through discrete intermediate states. The wave functions $\tilde{\Phi}_R-\tilde{w}_R$ describe discrete resonance states, including their decay into the P space. The amplitude, a measure of the probability of occurrence, is maximal for $E=E_R$, so that $\tilde{\Phi}_R=\sum a_{RR'}\Phi_{R'}$ can be interpreted as the wave function of the resonance state. The matrix element $\langle\tilde{\Phi}_R^*\,|\,H_{QP}\,|\,\xi\rangle$ gives the probability of formation of the resonance state Φ_R from the entrance channel.

The eigenvalues $E_R-i\Gamma_R/2$ of the operator (54) are complex, E_R giving the position and Γ_R the width of the resonance. In contrast to the traditional shell model, the resonances automatically have a finite lifetime. Their widths are obtained by diagonalizing the Hamiltonian (54) from the matrix elements

$$\langle\Psi_A\,|\,H_{QP}\,|\,\Psi_{A-1},\,\Psi_1\rangle. \tag{56}$$

Therefore, instead of the overlap integrals (53), the interaction between the resonance states and the continuum is here taken into account directly.

Different definitions of the widths in the framework of the traditional shell model and the shell model with continuum lead in certain cases to different results. These are most pronounced when the overlap integrals (53) vanish but the matrix elements (56) are nonzero. As an example, we consider here transitions between positive-parity levels with structure $2p-2h$ in ^{16}N and levels of 1h configuration in ^{15}N. They are zero in the traditional shell model, but not when the continuum is taken into account. Numerical results for some resonance states are given in Table 2. In the traditional shell model, states with complicated structure are coupled to the entrance channel of one-hole structure only through states with the structure $1p-1h$:

$$1h\Rightarrow 1p-1h. \tag{57}$$

In the case with the continuum, the coupling is more complicated: states with both $1p-1h$ and $2p-2h$ structure are directly coupled to the entrance channel:

$$\begin{array}{l} 1h\Rightarrow 1p-1h; \\ \searrow\ 2p-2h. \end{array} \tag{58}$$

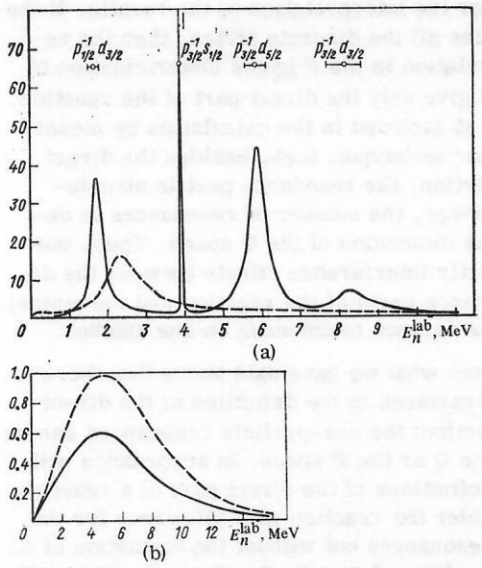

FIG. 3. Spectrum of neutrons in the reactions $^{16}O(\mu^-,\nu n_0)^{15}N_{g.s}$ (continuous curves) and $^{16}O(\mu^-,\nu n_2)^{15}N_{3/2^-}$ (dashed curves). 1^- resonance states are excited in the intermediate ^{16}N nucleus by transitions of first degree of forbiddeness. Calculation made without imaginary part of the potential: a) resonance and direct part of the reaction; b) direct part of the reaction (without one-particle $d_{3/2}$ resonance).

If the possibility of the coupling (58) is curtailed by parity or isospin selection rules, it may reduce to the simple coupling (57). This occurs, for example, in the (γ,n) reaction.

In calculations based on the traditional shell model the two low-lying states $1/2^-$ and $3/2^-$ are usually not described by hole states. Widths calculated for states of the type $2p-2h$ are not zero if only configuration mixing is taken into account for the ^{15}N states.

3. DESCRIPTION OF NUCLEAR REACTIONS IN THE SHELL MODEL WITH CONTINUUM

Shell Model with Continuum as Generalization of the Traditional Shell Model. The shell model with continuum is a natural generalization of the ordinary shell model. In numerical calculations, one first makes calculations of ordinary type in the shell model with a finite-depth potential in order to determine the wave functions Φ_T of the target nucleus and the functions Φ_R of the intermediate nucleus. The wave functions Φ_T enter Ω_c [Eq. (18)], and the functions Φ_R are basis functions of the expansion of $Q\Psi$ in accordance with Eq. (8). The modification goes beyond the traditional shell model because, after the ordinary calculations, the interaction of the Q and the P space is taken into account directly by the coupled-channel method. Because results obtained by the ordinary shell model are used, the wave functions are, on the one hand, automatically antisymmetric with respect to all A nucleons and, on the other hand, spurious solutions in discrete states can be eliminated by the usual methods. A shortcoming of the shell model with continuum formulated in this way is that one can describe only reactions with one nucleon in the continuum in accordance with the original equation (1).

Below, we discuss the differences between the shell models with and without continuum and we also discuss the effect on the numerical results of allowing for the interaction of states in the continuum.

Absolute Value of the Cross Section. It is well known that calculations in the ordinary shell model give too large values of the cross section for dipole resonances and for processes of μ capture by light nuclei. Various approximations of the shell model with allowance for the continuum[4,10-13] exhibit the same tendency. Therefore, a complex potential is introduced in order to reduce the cross section.

The difference between the results obtained by the two models for light nuclei arises from the different assumptions:

1) about the average potential (in ordinary calculations one obtains the wave functions Φ_T and Φ_R in an harmonic-oscillator potential; in calculations with continuum, in the Woods-Saxon potential);

2) about the size of the discrete-state space, which in calculations hitherto with allowance for the continuum has been smaller than in the ordinary calculations;

3) about the reaction mechanism.

The choice of different average potentials and different function spaces in the calculations based on the two models affects the absolute value of the cross section, but the choice is not unique. The main difference between the calculations in the shell model with and without continuum arises because an assumption must be made about the reaction mechanism in the latter. Only the resonance part of the reaction is taken into account in the description of γ absorption and μ capture. As can be seen from Figs. 1 and 3, the ignored direct part makes only a small contribution to the reaction in either case. One cannot therefore expect that calculations by the model with continuum will differ strongly from the results of an ordinary calculation without allowance for the direct part, provided the parameters of the shell model do not differ from one another.

One of the reasons why the theoretical cross section is too large may be that in the calculation of the wave function of the initial and the final nuclei, which differ by one nucleon, no allowance is made for the dependence of the average potential on the mass number. If the parameters of the shell model are different in the initial and the final nuclei, the overlapping of the wave functions of the initial and the final nuclei and, there-

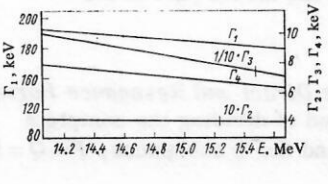

FIG. 4. Widths Γ as function of the incident-particle energy for four 0^+ resonance states of ^{16}N in the reaction $^{15}N+n$ with allowance for two $1/2^-$ and $3/2^-$ levels of ^{15}N: The vertical strokes indicate the position of the corresponding resonances ($E_R^{(4)}=10.1$ MeV). Calculation made without imaginary part of the potential and including the one-particle $d_{3/2}$ resonance in the Q space.

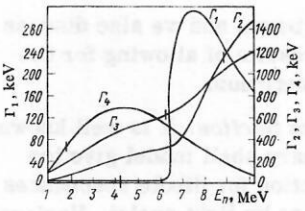

FIG. 5. Widths Γ as a function of the incident-particle energy for four 1^- resonance states of ^{16}N in the reaction ^{15}N+n with allowance for two $1/2^-$ and $3/2^-$ levels of ^{15}N: The vertical strokes indicate the positions of the corresponding resonances. Calculation made without imaginary part of the potential and with the one-particle $d_{3/2}$ resonance in the Q space.

fore, the absolute value of the (γ,n) cross section are smaller than for equal parameters. It is perfectly possible that the complex potential is partly due to the difference between the structures of the initial and the final nuclei, this being expressed by different values of the parameters in the case of the shell model. We have made numerical calculations relating to this problem.

Resonance Parameters. The wave function, energy, and width of resonances are obtained in the shell model with continuum by diagonalizing the Hamiltonian (54). The number and position of the resonances, and also their widths, are determined exactly in the model. This is important, especially for narrow resonances and resonances which differ strongly from the Breit-Wigner profile. The number of resonances corresponds to the number of states obtained from the ordinary calculation in the shell model for the intermediate nucleus. Allowance for the continuum shifts the position and also slightly changes the wave function of the resonances from the original values taken from the ordinary calculation. These effects are discussed in detail below.

The calculated widths of the resonances depend weakly on the energy E of the incident particle in the region of the resonances themselves. Some numerical examples are given in Figs. 4 and 5. The widths of the 0^+ states with structure $2p-2h$ of the ^{16}N nucleus with allowance for the $1/2^-$ and $3/2^-$ levels of the ^{15}N nucleus with hole structure are virtually independent of the energy in the considered region ($\approx$15 MeV, Fig. 4). The widths of the 1^- states with structure $1p-1h$ of the same nucleus with allowance for the $1/2^-$ and $3/2^-$ states of ^{15}N depend more strongly on the energy in the region of the resonances themselves as well, because they are near the one-particle resonances (see Fig. 5).

The results of calculation in the shell model with continuum differ therefore by a certain energy dependence of the widths and positions of the resonances from the results of the ordinary calculation using R-matrix theory. In the ordinary calculations, the resonance widths are calculated at one energy, and the resonance is assumed to have a symmetric profile, of Breit-Wigner type. Therefore, the resonance profiles which follow from the model with continuum differ from those obtained by the traditional method even without allowance for interference with the direct part of the reaction.

Interference between the Direct and Resonance Parts of the Reaction. The method of dividing the complete function space into the P and the Q subspaces, $P+Q=1$, is important for the interpretation of the results. If the Q space contains all the discrete states, then the results of a calculation in the P space (determination of the solution ξ) give only the direct part of the reaction. If the Q space is included in the calculation by means of the projection technique, then, besides the direct part of the reaction, the resonance part is also described. Moreover, the number of resonances is determined by the dimension of the Q space. Thus, one can study directly interference effects between the direct and resonance parts of the reaction and the mutual influence of the various resonances on one another.

It follows from what we have said above that there is a certain arbitrariness in the definition of the direct part of the reaction: the one-particle resonances can be put in either the Q or the P space. In accordance with the ordinary definitions of the direct part of a reaction, one must consider the reaction with allowance for the one-particle resonances but without the formation of an intermediate nucleus. Accordingly, it is necessary to include the one-particle resonances in the P space. However, in the ordinary calculations of resonance reactions in the shell model, the one-particle resonances are put into the Q space. One determines the part of the resonance reaction that proceeds through the formation of an intermediate nucleus in the eigenstates of the Hamiltonian H_{QQ}.

The interference between the direct and resonance parts of the reaction for three 0^+ states of ^{16}N with structure $2p-2h$ in the reaction ^{15}N+n is shown in Fig. 6. The one-particle resonance is put into the Q space in these calculations. The results in the elastic channel are given in Fig. 6a for different direct parts. The same resonances, but in the inelastic channel, which leads to the excitation of the ^{15}N nucleus in the $3/2^-$ state at 6.3 MeV, are given in Fig. 6b. Here only in one of the given cases does the direct part not vanish. As can be seen from Fig. 6, the interference effects

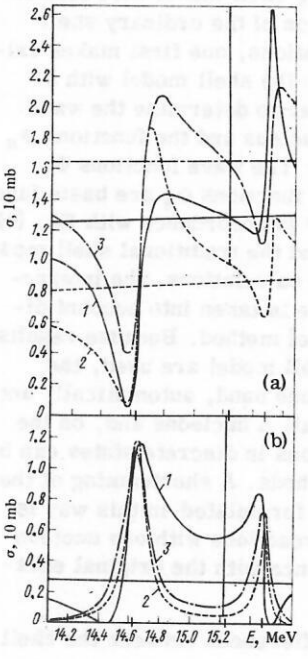

FIG. 6. Influence of the neglect of channel coupling in the continuum or the strength of the direct part of the reaction on the profiles of three 0^+ resonances in the reaction ^{15}N+n with allowance[8] for two hole-type states of ^{15}N: a) elastic scattering ^{15}N$(n,n)^{15}$N$_{1/2^-}$; b) inelastic scattering ^{15}N$(n,n')^{15}$N$_{3/2^-}$; 1) results which are exact in the model ($V^P_{cc'}\neq 0$); 2) results with the approximation $V^P_{cc'}=0$; 3) with the approximation $V^P_{cc'}=V_0\delta_{cc'}$. Calculation without imaginary part of the potential.

TABLE 3. Influence of the approximations $V^P_{cc'} = 0$ and $V^P_{cc'} = V_0\delta_{cc'}$ on the positions E_R and widths Γ_R of 0^+ resonance states with $2p-2h$ structure in neutron scattering on ^{15}N with allowance for two hole-type states of ^{15}N.

E_{sh}, MeV	E_R, MeV			Γ_R, keV		
	$V^P_{cc'} \neq 0$	$V^P_{cc'} = 0$	$V^P_{cc'} = V_0\delta_{cc'}$	$V^P_{cc'} \neq 0$	$V^P_{cc'} = 0$	$V^P_{cc'} = V_0\delta_{cc'}$
14.63	14.60	14.65	14.63	94	72	94
15.24	15.24	15.24	15.24	0.2	0.2	0.2
15.48	15.48	15.49	15.48	37	37	60

between the direct and resonance parts determine the resonance profiles. Only in the case of a vanishing direct part does one observe a symmetric profile of the curves.

The resonance widths are obtained in the model discussed here by diagonalizing the operator (54). One can therefore determine the widths in the case of an asymmetric resonance profile with the same accuracy as for symmetric resonances. In Fig. 6a, the width of the resonance with a strongly expressed interference minimum corresponds approximately to the distance of the minimum from the maximum.

Discussion of the Approximation $V^P_{cc'} = 0$ in the Model. If the one-particle resonances are taken out of the P space, one would then expect a small effect of the direct coupling of the continuum channels on the resonance behavior of the cross section. Therefore, Mahaux and Saruis[8] introduce the approximation

$$V^P_{cc'} \equiv \langle \chi^c_E | V | \chi^{c'}_{E'} \rangle = 0. \tag{59}$$

The physical meaning of this approximation is that one ignores the coupling of the direct reaction channels. Accordingly, there are no direct contributions to the reaction in the inelastic channels. Figure 6 shows the influence of the approximation (59) for the case of the reaction $^{15}N + n$ with three 0^+ resonances of structure $2p-2h$ in the energy range from 14.0 to 15.5 MeV. To the cross section obtained with the solution with $V^P_{cc'} \neq 0$ that is exact in the framework of the model, there correspond the curves 1. In the case of the approximation (59), the curves 2 are obtained. As we would expect, the direct part of the reaction is reduced in the elastic channel and disappears in the inelastic channel. The interference picture is changed in both channels. Additional calculations with the approximation

$$V^P_{cc'} = V_0\delta_{cc'} \tag{60}$$

are shown by the curves 3. In this case too the direct part of the reaction is zero in the inelastic channel and is greater in the elastic channel than for the exact solution with $V^P_{cc'} \neq 0$.

It follows from these results that the resonance profiles differ between the approximations (59) and (60) because the direct part of the reaction is changed. It should however be pointed out that in the considered energy range there are other levels of a different spin besides the 0^+ levels, so that the profiles in Fig. 6 are not observed experimentally, but only the superposition of many levels. The approximations (59) or (60) have little influence on the positions and widths of the resonances (Table 3). Both approximations are suitable for obtaining these quantities.

4. INTERACTION BETWEEN THE P AND Q SPACES

Effective Hamiltonian in One Subspace. The generalization of the shell model presented in this paper consists of taking into account directly the influence of states with one nucleon in the continuum on the properties of nuclei. One can study this influence in the formulation of the model presented here. The effective Hamiltonian in Q space is

$$H^{\text{eff}}_Q = H_{QQ} + H_{QP}\frac{1}{E - H_{PP}}H_{PQ}. \tag{61}$$

Diagonalization of the first term, H_{QQ}, gives the results of the ordinary calculation with allowance for only bound one-particle states (with finite-depth potential). If the coupling between the Q and P spaces is taken into account by the second term of Eq. (61), the bound states above the breakup threshold go over into resonance states with widths and positions which follow from the diagonalization of H^{eff}_Q. The influence of the second term will be discussed below in numerical examples. In addition, the shell model with continuum can be regarded as an extension of an ordinary calculation of DWBA type in the sense that the resonance part is taken into account as well as the direct part of the reaction. In the shell model with continuum described here, the direct part of the reaction corresponds to the solution of the problem in the P space, in which the effective Hamiltonian is

$$H^{\text{eff}}_P = H_{PP} + H_{PQ}\frac{1}{E - H_{QQ}}H_{QP}. \tag{62}$$

In the description of nuclear reactions, for example, the scattering of nucleons by light nuclei, the first term of Eq. (62) corresponds to the direct part of the reaction. Using the ordinary definitions of the direct part of the reaction, it is best in this case to put the one-particle resonances into the P space. The coupling of the P to the Q space, i.e., the second term of (62), leads to the formation of a compound nucleus and enables one to describe resonances microscopically. The complete solution of the problem contains both an almost smooth behavior of the cross section corresponding to the direct part and a resonance structure of the cross section corresponding to processes in which a compound nucleus is formed. The influence of the size of the Q space on the excitation function of the reaction will be discussed below in numerical examples.

TABLE 4. Shifts $(E_R - E_{sh})$ of the positions and the widths Γ_R of 1^- resonance states with allowance for N states of the target nucleus in neutron scattering on ^{15}N.

E_{sh}, MeV $N=0$	(E_R-E_{sh}), MeV		Γ_R, keV		E_{sh}, MeV $N=0$	(E_R-E_{sh}), MeV		Γ_R, keV	
	$N=2$	$N=4$	$N=2$	$N=4$		$N=2$	$N=4$	$N=2$	$N=4$
1.92	−0.11	−0.27	330	330	7.18	−0.91	−0.86	600	550
4.38	−0.22	−0.02	2	20	9.29	−0.44	−0.44	1170	1110

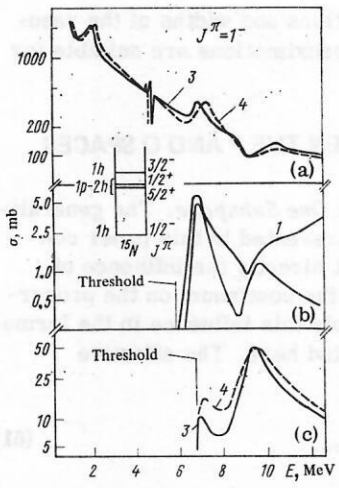

FIG. 7. Influence of the number of allowed for channels on the excitation function in neutron scattering on ^{15}N through 1$^-$ resonance states: a) elastic scattering $^{15}N(n, n_0)^{15}N_{1/2^-}$; b) inelastic scattering $^{15}N(n, n_1)^{15}N_{5/2^+}$ (1) and $^{15}N(n, n_2)^{15}N_{1/2^+}$ (2); c) inelastic scattering. 1), 2), and 3) are the results of calculation with allowance for four states of ^{15}N: 1/2$^-$ (0 MeV), 3/2$^-$ (6.32 MeV), 5/2$^+$ (5.27 MeV), 1/2$^+$ (5.30 MeV); 4) results of calculation with allowance for only the two states 1/2$^-$ and 3/2$^-$ of ^{15}N. Calculation made without imaginary part of the potential and with inclusion of the one-particle $d_{3/2}$ resonance in the Q space.

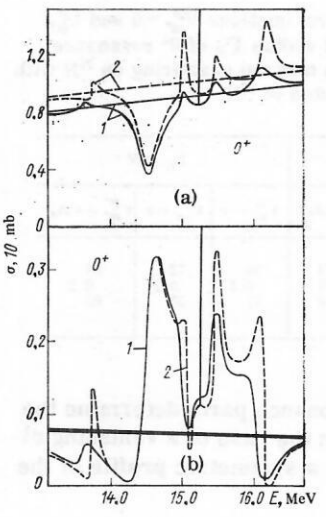

FIG. 8. Influence of the number of allowed for channels on the excitation function in neutron scattering on ^{15}N through 0$^+$ resonance states: a) elastic scattering $^{15}N(n, n_0)^{15}N_{1/2^-}$; b) inelastic scattering $^{15}N(n, n_1)^{15}N_{5/2^+}$; 1) results of calculation with allowance for seven states of ^{15}N: 1/2$^-$ (0 MeV), 5/2$^+$ (5.27 MeV), 1/2$^+$ (5.30 MeV, 3/2$^-$ (6.32 MeV), 5/2$^+$ (7.15 MeV), 3/2$^+$ (7.30 MeV), and 3/2$^+$ (7.57 MeV) 2) results of calculation with allowance for only the four lowest states, 1/2$^-$, 5/2$^+$, 1/2$^+$, and 3/2$^-$, of ^{15}N. Calculation made without imaginary part of the potential and with inclusion of the one-particle $d_{3/2}$ resonance in the Q space.

Influence of the Dimension of the P Space on the Cross Section. In order to study the influence of the dimension of the P space on the resonance behavior of the cross section, we made calculations for the reaction $^{15}N(n,n)^{15}N$ with different numbers of channels. In Table 4 and Fig. 7 we give numerical results of a calculation of the positions and widths of some 1$^-$ resonance states with $1p-1h$ structure. The calculations were made with allowance for different numbers N of states of the ^{15}N nucleus:

$N=0$; calculation in the ordinary shell model without continuum;

$N=2$; the hole states $(1p_{1/2})^{-1}$ and $(1p_{3/2})^{-1}$ of ^{15}N together with s and d waves of the continuum are included in the calculation. In this case five lj channels are taken into account;

$N=4$; besides those of the case $N=2$, the 5/2$^+$ and 1/2$^+$ states of the target nucleus ^{15}N near 5.3 MeV together with the p and f waves of the continuum are taken into account. In this case there are 10 lj channels.

The results show (see Fig. 7) that when the P space is enlarged some levels are shifted further from the position obtained in the calculation without continuum, whereas others get nearer to the original position. This behavior of the positions of the resonances when the number of channels taken into account is increased corresponds to what one would expect from the point of view of the ordinary shell model. The widths of the resonances also change when the P space is enlarged, as can be seen from Table 4.

The results of a calculation of the reactions $^{15}N(n,n)^{15}N_{1/2^-}$ and $^{15}N(n,n')^{15}N_{5/2^+}$ with the formation of an intermediate ^{16}N nucleus in some 0$^+$ states with $2p-2h$ structure near an excitation energy of 17 MeV are given in Table 5 and Fig. 8. The calculations were made with allowance for different numbers N of states of the final nucleus, as in the foregoing example:

$N=1$; only the elastic channel, i.e., the 1/2$^-$ ground state of ^{15}N is allowed for;

$N=2$; two hole-structure 1/2$^-$ and 3/2$^-$ states of ^{15}N are taken into account;

$N=4$; besides the two hole states, 5/2$^+$ and 1/2$^+$ states with $2h-1p$ structure near 5.3 MeV are taken into account (dashed curves);

$N=7$; in addition to those of the case $N=4$, three 5/2$^+$, 3/2$^+$, and 7/2$^+$ states at 7.2, 7.3, and 7.6 MeV with $2h-1p$ structure are taken into account (continuous curves).

It can be seen from the results presented that the number of channels allowed for has a large influence on the resonance parameters if the corresponding spectroscopic factors are zero ($N=1$ and 2). The number of allowed for channels has a greater influence on the direct part of the reaction than on the resonance part in the case of allowance for "allowed" (from the point of view of R-matrix theory) transitions. The resonance structure in the cases $N=4$ and 7 is more clearly expressed in the inelastic channel, for which the spectroscopic factors of 0$^+$ levels do not vanish.

In these examples the P space influences the resonance parameters because the resonance levels are shifted, as a rule, to lower energies by the allowance

TABLE 5. Positions E_R and widths Γ_R of 0$^+$ resonance states with allowance for N states of the target nucleus and neutron scattering on ^{15}N. In cases $N=1$ and 2 only hole-type states of ^{15}N are taken into account. The positions E_{sh} of the resonance states are obtained on the basis of the ordinary shell model.

E_{sh}, MeV	E_R, MeV				$\frac{1}{2}\Gamma_R$, keV			
	$N=1$	$N=2$	$N=4$	$N=7$	$N=1$	$N=2$	$N=4$	$N=7$
13.73	13.73	13.73	13.72	13.59	3.3	21.0	38.0	98.0
14.63	14.63	14.60	14.50	14.50	35.0	92.0	107.0	171.0
15.10	15.09	15.09	15.05	15.00	1.3	4.2	28.0	43.4
15.24	15.24	15.24	15.24	15.23	$6\cdot10^{-3}$	0.2	0.7	0.8
15.48	15.47	15.48	15.47	15.42	11.7	37.0	48.5	44.5
16.25	16.24	16.34	16.15	16.03	1.7	10.0	78.0	139.0

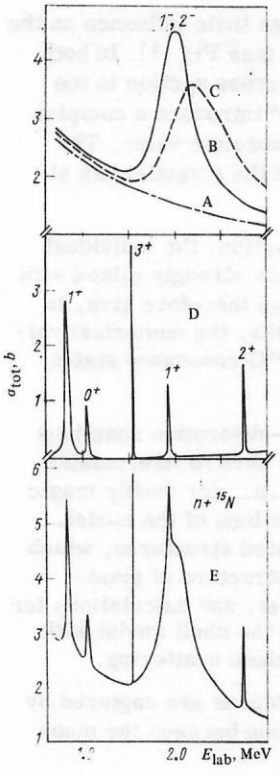

FIG. 9. Influence of the size of the Q space on the resonance structure of the $^{15}N+n$ reaction cross section. Calculation made without imaginary part of the potential: A) empty Q space; B) Q space containing only the quasibound state $1^-d_{3/2}$; C) Q space containing negative-parity states with structure $1p-1h$; D) Q space containing positive-parity states with structure $2p-2h$. (The direct part of the reaction, including negative-parity waves, is not shown); E) Q space containing all the states of C and D.

for the coupling between the P and Q spaces. However, it is not possible to formulate a rule expressing the shifts and widths as a function of the P-space dimension. The structure of the resonance states, i.e., their wave function $\tilde{\Phi}_R$, can be calculated numerically in the same way as in ordinary calculations of the model. The additional mixing of the states because of their coupling to the continuum is small, i.e., the real part of the resonance wave functions differs little from the eigenfunctions Φ_R of H_{QQ}. Only an imaginary part of the wave function is obtained additionally. The influence of the P space on the resonance profiles is shown in Fig. 6 and has already been discussed.

Influence of the Dimension of the Q Space on the Cross Section. We study the influence of the size of the Q space on the cross section for the reactions $^{15}N(n,n)^{15}N$ and $^{15}N(p,n)^{15}N$.

The results of the calculation for the reaction $^{15}N(n,n)^{15}N$ are shown in Fig. 9. The allowed for hole states of the ^{15}N target nucleus are directly coupled to states of the intermediate nucleus with both the structure $1p-1h$ and the structure $2p-2h$. The calculations for positive-parity states with $2p-2h$ structure were made as follows. The level scheme without allowance for the continuum was compared with the experimental. As usual, the experimental energies E_{exp} were used then instead of the theoretical E_{sh}. The two values differed by not more than 1 MeV, but this difference is greater than the energy shifts obtained by taking into account the coupling of the states to the continuum (see Table 2). At this stage, no attempt was made to improve the theoretical energies or eliminate spurious states. The only important thing is that the resonance levels of ^{16}N with $2p-2h$ structure and positive parity have nonvanishing widths Γ_R with respect to the levels of the ^{15}N target nucleus with hole structure, although the one-particle spectroscopic factors for these transitions vanish. Nonvanishing widths Γ_R are obtained in this case because the resonance levels are coupled to the p and f waves of the continuum. The cross section of the reaction $^{15}N(n,n)^{15}N$ (see Fig. 9) was obtained for different dimensions of the Q space.

The results of the calculation for the reaction $^{15}N(p,n)^{15}O$ are given in Fig. 10. In the calculation, 19 resonance states near the investigated energy range were chosen. Two proton and two neutron channels were taken into account, i.e., two hole-structure states for each nucleus ^{15}N and ^{15}O. The spectroscopic factors of all these resonances with respect to these states vanish.

As can be seen from Fig. 10, the seven resonances in the energy range from 7.5 to 8.5 MeV lead to two dominant peaks in the excitation function. A similar behavior is observed in the experimental cross section,[14] though here the individual theoretical resonances were not identified in the experimental cross section. In addition, the negative-parity resonances were not included in this case in the calculation. It follows from the two considered examples that the size of the Q space has a large influence on the reaction excitation function, determining the resonance behavior of the cross section.

5. APPLICATION OF SHELL MODEL WITH CONTINUUM TO THE DESCRIPTION OF NUCLEAR REACTIONS

Use for Different Types of Reaction. The shell model with continuum enables one to describe nucleon scattering on nuclei at energies comparable with the binding energy of nucleons in the nucleus. The total wave function of the system is antisymmetric with respect to all A nucleons of the system, i.e., with respect to the $A-1$ nucleons of the target nucleus and the one nucleon in a scattering state or the A nucleons of the intermediate nucleus. The interaction operator contains four terms:

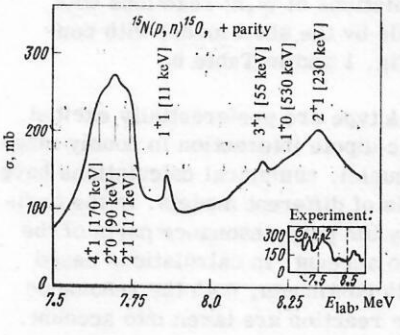

FIG. 10. Influence of the size of the Q space on the resonance structure of the $^{15}N+p$ reaction cross section. Calculation made without imaginary part of the potential. Only positive-parity resonance states with structure $2p-2h$ of ^{16}O taken into account.

$$H = H_{QQ} + H_{QP} + H_{PQ} + H_{PP}, \quad (63)$$

which characterize the types of interaction. The cross section contains the direct parts of the reaction determined by H_{PP} and the resonance parts determined by H_{QQ} and H_{QP} and H_{PQ}. Numerical examples are given below.

Reactions due to interaction with other particles are described by the source method. Examples of calculations are given below.

Scattering of Low-Energy Nucleons. The operator which describes the scattering of nucleons at low energy (a few MeV) on nuclei is the operator (63). Examples of numerical calculations of the scattering of low-energy nucleons on light nuclei are given in Figs. 2 and 4–10 and in Tables 2–5. When low-energy nucleons are scattered, the individual levels are excited nonselectively. All configurations in which the state of one or two nucleons differs from the configuration of the target nucleus can be excited. Examples of the excitation of resonance states with $2p-2h$ structure in the case of nucleon scattering on a nucleus in a state with structure $1h$ are shown in Figs. 4, 6, and 8–10 and in Tables 2, 3, and 5. Since as a rule the level density is high, individual resonances are seldom observed. In the majority of cases, the resonances overlap (see Fig. 10). Irrespective of the profiles of the individual resonances or their overlapping, numerical values of the resonance parameters can be obtained exactly in the framework of the model. In order to identify the individual resonances in the experimental data, it is necessary to measure the angular distribution at the corresponding energies of the resonances.

(γ,n) Reactions. In (γ,n) reactions, all A nucleons are in bound states in the entrance channel. A level of the intermediate nucleus is excited by the ingoing photon wave. It "decays" into the exit channels with one nucleon in the continuum. The Hamiltonian of the complete system has the form

$$H_s = H + H_\gamma, \quad (64)$$

where H is the interaction Hamiltonian of the bound nucleons in the nucleus; H_γ describes the interaction between the electromagnetic field and the nucleons in the nucleus. For dipole transitions,

$$H_\gamma = D, \quad (65)$$

where D is the dipole operator. The Hamiltonian (64) has the form of the operator (25), so that the problem can be solved by the source method. The results of some numerical calculations of (γ,n) reactions with dipole transitions made by the shell model with continuum are given in Fig. 1 and in Table 6.

Structures of $1p-1h$ type are preferentially excited by the electromagnetic dipole interaction in doubly magic nuclei. For these nuclei, numerical calculations have been made on the basis of different models. In the ordinary shell model, only the pure resonance parts of the reaction are taken into account. In calculations based on the shell model with continuum, both the resonance and direct parts of the reaction are taken into account. The fraction of the direct part has little influence on the total cross section, being small (see Fig. 1). In both cases, the absolute value of the cross section is too large. Therefore, Buck and Hill[4] introduce a complex potential in order to reduce the absolute value. The meaning of the imaginary part of the potential has already been discussed.

Because of the Coulomb interaction, the individual resonance states are more or less strongly mixed with respect to isospin. In Table 6, we therefore give, in addition to the positions and widths, the numerical values of the purity of some of the ^{16}O resonance states with respect to the isospin.

Hitherto, all calculations of γ-absorption reactions have been made for states of the excited intermediate nucleus with structure $1p-1h$, i.e., for doubly magic nuclei. This is not due to restrictions of the model. Calculations with more complicated structures, which enable one to elucidate the fine structure of giant resonances of doubly magic nuclei, and calculations for nonmagic nuclei can be made by the shell model with continuum, as in the case of nucleon scattering.

Capture of Muons by Nuclei. Muons are captured by nuclei through the weak interaction between the muon and proton:

$$\mu^- + p \to n + \nu. \quad (66)$$

In practice, the muon is captured only by the lowest shell (K shell) of the atom. As a result of the capture, an energy greater than 100 MeV is liberated and distributed over the particles in the final state. A large fraction of the energy (from 50 to 90 MeV) is transferred preferentially to the emitted neutrino. The residual energy is transferred either directly to a neutron, which then leaves the nucleus, or leads to the excitation of the nucleus. If the excitation energy is higher than the breakup threshold, then neutrons may be emitted. The (μ,n) reaction can therefore proceed through either the direct or resonance mechanism, like the (γ,n) reaction.

The μ-capture Hamilton can be represented in the form

$$H_s = H + H_\mu, \quad (67)$$

where H is the Hamiltonian of the nucleus, and H_μ is the weak interaction between the nucleons of the nucleus and the leptons. The operator (67) has the form of the operator (25), so that in this case too the process can be described by the source method.

The interaction operator H_μ consists of the sum of the operators of the vector, renormalized axial vector,

TABLE 6. Positions E_R, widths Γ_R, and probability $W(T=1)$ of resonance states with structure $1p-1h$ in the γ-absorption reaction by the ^{16}O nucleus.

E_R, MeV	Γ_R, MeV	$W(T=1)$, %	E_R, MeV	Γ_R, MeV	$W(T=1)$, %
13.13	0.22	99	19.51	0.80	99
16.27	0.80	1	21.82	2.10	99
16.87	1.27	32	22.57	1.56	5
17.26	0.88	69	23.97	2.50	98

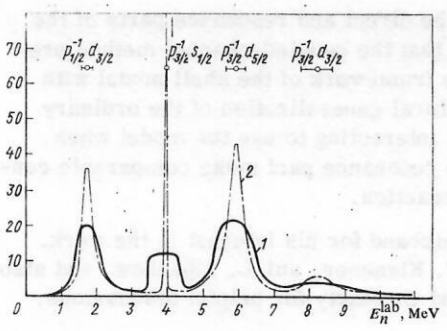

FIG. 11. Neutron spectrum shown in Fig. 3 a with allowance for spreading (1) and without spreading (2) and formation of the final nucleus in the ground state.

induced scalar, and weak magnetic interactions. As Morita and Fujii[15] show, H_μ can be decomposed into a sum of spherical tensor operators of different degrees j and parity π:

$$H_\mu = \sum_j W^{j\pi}, \tag{68}$$

where j is the spin of the excited nucleus in the case of μ capture by a doubly even nucleus. The representation (68) enables one to classify transitions according to different degrees of forbiddenness, as in the case of β decay.

Transitions of first forbiddenness in the case of μ capture by ^{16}O lead to ^{16}N levels with $j^\pi = 0^-, 1^-, 2^-$. These states with the structure $1p-1h$ are analogous to the states of the giant dipole resonance of the ^{16}O nucleus.

Some results obtained on the basis of the shell model with continuum for μ capture by ^{16}O are shown in Figs. 3 and 11. As in the excitation of dipole resonances, the direct part is small in the case of μ capture. As an example, we show the cross section of μ capture by ^{16}O through 1^- levels with structure $1p-1h$ of the intermediate ^{16}N nucleus. In this case, the direct part is from 5 to 10% and has a maximum at an energy of the neutrons between 4 and 5 MeV. Because the direct part of the reaction is small, interference effects between the direct and the resonance parts have almost no importance for μ capture. Therefore, the assumption of a pure resonance mechanism for neutron energies up to about 10 MeV is good. As regards the absolute value of the cross section, it can be discussed in exactly the same way as in the case of γ absorption.

A feature of μ-capture processes is that there is no possibility of detecting the neutrino. When a neutrino is emitted, a recoil momentum is imparted to the nucleus, this leading to an additional broadening of the resonances. Since the direction in which the neutrino is emitted is not known, it is necessary to add statistically the direction of the recoil momentum and the momentum of the neutrons. A spreading with respect to the energy of the neutrons is obtained. It increases with increasing energy of the emitted neutron (Fig. 12) and is greater for light than for heavy nuclei (Table 7).

The influence of spreading on the resonance structure

TABLE 7. Spreading ΔE of the neutron energy in μ capture by different nuclei at a neutron energy of about 5 MeV.

Target nucleus	ΔE, keV	Target nucleus	ΔE, keV
^{12}C	700	^{40}Ca	250
^{16}O	550	^{208}Pb	50

of the neutron spectrum is shown in Fig. 11 for μ capture by ^{16}O. The spreading of the energy restricts the spectroscopic conclusions obtained from the study of μ capture by nuclei.

Capture of other particles, for example, pions, can be described like muon capture or γ-ray absorption. The theory of the capture of particles that interact strongly with nucleons of the nucleus is more complicated than that for particles captured solely as a result of the electromagnetic or the weak interaction.

Interaction of a Nucleus with Particles of Intermediate Energy. A feature of the interaction of nuclei with fast particles with an energy of several hundred MeV is that the fast particles have a short wavelength, so that they can interact basically with only one nucleon in the nucleus. The momentum transferred to the nucleus is decisive for the outcome of the reaction. If the reaction products are detected under kinematic conditions such that the momentum transfer is small, resonance processes may be important as well as the direct processes.

The incident particle is most frequently a fast proton. It differs from the nucleons of the target nucleus by having a short wavelength compared with that of the nucleon bound in the nucleus. Therefore, the incident proton can be treated separately from the A nucleons of the target nucleus. As in capture processes, the theory contains an additional operator of an interaction (between the fast particle and the nucleus).

As an example, let us consider a knockout process with separation of the final-nucleus states. This process is described on the basis of the ordinary shell model under the assumption of a direct reaction mechanism. It is assumed that the nucleon is knocked out as a result of a quasielastic collision of the incident particle with a nucleon in the nucleus. Calculations are made of the probabilities of a transition for the formation of the residual nucleus in different excited states, $A \to (A-1)^* + p$. For the knockout of 1s nucleons, a momentum transfer $q \approx 0$ makes the main contribution. It is therefore expected that in this case resonance processes with excitation of levels above the breakup threshold, $A \to A^* \to (A-1)^* + p$, are important as well.

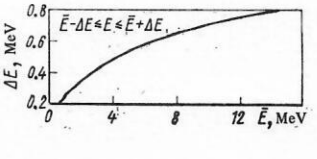

FIG. 12. Spreading ΔE of the neutron energy due to the recoil momentum imparted by the emitted neutrinos as a function of the mean energy $\bar{E}$ of the neutrons in the reaction $^{16}O(\mu, \nu n)^{15}N$.

It is not possible to distinguish these processes experimentally from direct knockout processes. On the basis of the shell model with continuum it is possible to calculate both contributions to the reaction, and also their interference. Calculations of this kind have so far been made only for (e,ep) reactions on the basis of Balashov's model.[16]

When fast particles interact with nuclei, levels of the final nucleus are excited selectively. Therefore, study of these reactions can give information about nuclear structure. In many cases, it is necessary to take into account both direct and resonance processes and their interference. We intend to make calculations of this type on the basis of the shell model with continuum.

CONCLUSIONS

The results presented here show that the shell model with continuum in conjunction with the coupled-channel method enables one to obtain a unified description of problems relating to nuclear structure and nuclear reactions if one considers reactions with one nucleon in the continuum. The method enables one to calculate not only the cross section but also the resonance parameters, which are obtained by diagonalizing the interaction operator. At the same time, one can determine the structure of resonances as accurately as in calculations by the ordinary shell model. The eigenvalues of the energy are complex after allowance has been made for the coupling of resonance states to the continuum. The imaginary part determines the width and the real part the position of the resonance.

In the description of reactions with one nucleon in the continuum one can obtain the strength of the direct part of the reaction, so that one can study directly interference between the direct and resonance parts of the reaction. We see that the coupled-channel method presented here in the framework of the shell model with continuum is a natural generalization of the ordinary shell model. It is interesting to use the model when the direct and the resonance part make comparable contributions to the reaction.

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