Measurement of the neutron-neutron scattering length and the question of the charge dependence of nuclear forces

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Fiz. Élem. Chastits At. Yadra, 6, 347–392 (April–June 1975)

The experimental and theoretical methods used to determine the neutron-neutron scattering length are reviewed. A weighted mean value for the scattering length is extracted from the existing experimental data. The problem of the violation of charge independence and the breaking of charge symmetry of the nucleon-nucleon interaction is discussed on the basis of this value of a_{nn}.

PACS numbers: 21.30.

INTRODUCTION

One of the problems of nuclear forces is the dependence of the nucleon interaction on the charge state. The conjecture of charge independence and of charge symmetry of the nucleon-nucleon forces already arose in the thirties. It follows from the approximate equality of the binding energies of isobaric nuclei that the forces between different pairs of nucleons, i.e., between a proton and proton, neutron and proton, and neutron and neutron, are approximately the same. In order to compare the binding energies of isobaric nuclei, it is necessary to take into account the Coulomb energy of unpaired protons, which can be determined exactly only when the wave function of the nucleus is sufficiently well known. Unfortunately, even now information on the wave functions of nuclei is not satisfactory. The conjecture cannot therefore be fully confirmed in this way. Nevertheless, on the basis of the conjecture of charge independence the isospin formalism was developed and found wide application in nuclear and elementary-particle physics. Although the success of this formalism in the description of phenomena in nuclear physics is obvious, there remains the fundamental task of verifying experimentally the accuracy with which charge independence and charge symmetry of the nucleon-nucleon interaction are realized in nature.

A direct way of confirming the conjecture is to investigate the interaction of different pairs of nucleons in scattering experiments. But when measurements of *pp* and *np* scattering are compared, the contribution of Coulomb forces and other electromagnetic effects, which mask the effect of the purely nuclear forces, must be taken into account. Of course, when one speaks of charge independence and charge symmetry it is only the nuclear part of the total interaction that is meant. The extraction of the parameters of the interaction between hypothetical protons without charge from the scattering between two real protons is one of the tasks in the comparison of the forces between different nucleons. Whereas *pp* and *np* scattering can be directly observed with a high accuracy, investigation of neutron-neutron scattering is much harder because there are no neutron targets. Experimental data on the *nn* interaction have therefore appeared only in recent years. Although much attention has been devoted by experimenters to this problem during the last ten years, the quantitative data on the *nn* interaction at present available are as yet unsatisfactory.

The present review is devoted mainly to a discussion of the methods and results of the experimental investigation of the neutron-neutron interaction. Despite all their shortcomings, the results obtained in this field enable one to draw certain conclusions relating to the problem of the charge dependence of nuclear forces. In addition, the present state of the problem of calculating the parameters of the *pp* interaction for corresponding particles without charge is discussed.

1. MEASUREMENT OF THE NEUTRON-NEUTRON SCATTERING LENGTH

Neutrons can be scattered on neutrons by directing a beam of neutrons onto deuterium. The neutron in the deuteron is used as a target. This method can be used at high energies, when the binding energy of the deuteron can be ignored.

Here we shall consider experimental methods that enable one to observe the interaction of two neutrons at low energies only. We shall restrict ourselves in studying the problem to the use of the effective-range theory (see, for example, /1/). The parameters of this theory—the effective range and the scattering length a—depend on the interaction. The scattering length is very sensitive to small changes in the potential. Since the expected differences between the potentials for different charge states of interacting nucleons are small, comparison of the scattering lengths of different pairs of nucleons is a suitable and fairly sensitve method of testing the charge dependence of nuclear forces. In recent years, many people have therefore concentrated their attention on an experimental determination of the neutron-neutron scattering length and on the development of appropriate experimental and theoretical methods.

Since we are dealing with two identical particles, interaction is observed only in the singlet state, so that we are concerned with determining the singlet scattering length a_{nn}. This of course must be compared with only the singlet *np* scattering length.

General Method of Determining the Neutron-Neutron Scattering Length. The flux density of neutrons from reactors or other sources does not yet enable one to scatter neutrons directly on neutrons. Therefore, a_{nn} has hitherto been determined only in nuclear reactions in which two neutrons are formed. The interaction of these neutrons in the final state influences the spectrum of secondary particles and, thus, enables one to determine a_{nn}. The reactions used for this purpose are

$$d+n\to p+n+n; \tag{1}$$

$$^3\mathrm{H}+n\to d+n+n; \tag{2}$$

$$d+d\to p+p+n+n; \tag{3}$$

$$^3\mathrm{H}+d\to {}^3\mathrm{He}+n+n; \tag{4}$$

$$^3\mathrm{H}+{}^3\mathrm{H}\to {}^4\mathrm{He}+n+n; \tag{5}$$

$$\pi^-+d\to\gamma+n+n. \tag{6}$$

The essence of the method of determining a_{nn} is based on simple physical ideas. Suppose that the primary interaction of the initial particles takes place in a small volume whose diameter does not exceed the range of the nuclear forces. If the attraction between two neutrons is strong and the relative energy of the neutrons low, there is a high probability of a secondary interaction between these particles; this has become known as the "final-state interaction" (FSI). The FSI probability can be expressed by means of the neutron-neutron scattering amplitude. For the reaction amplitude as a whole, the principle of detailed balance enables one to conclude that it and the amplitude of the primary reaction must contain a factor corresponding to *nn* scattering. Under these assumptions, this factor appreciably increases the contribution of the amplitude under kinematic conditions for which the relative energy of the neutrons is low. In this case, one can use the effective-range theory to express the phase shift δ_0:

$$k\operatorname{ctg}\delta_0=-1/a_{nn}+r_{nn}k^2/2. \tag{7}$$

Here k is the relative momentum of the neutrons and r_{nn} the effective range. The energies being low, it is sufficient to take only the partial wave with $L=0$. Then the partial-wave method gives for the scattering cross section

$$\sigma_0(k^2)=4\pi\sin^2\delta_0/k^2. \tag{8}$$

From Eqs. (7) and (8)

$$\delta_0(k^2)=4\pi/[k^2+(-1/a_{nn}+r_{nn}k^2/2)^2]. \tag{9}$$

A factor of such form occurs in the cross section of all the processes (1)—(6). This FSI description was proposed independently by Migdal[2] and Watson[3] and is known in the literature as the Migdal-Watson (MW) approximation. The assumption that the reaction takes place in a small volume requires special consideration when the theory is applied to any reaction in which deuterons participate, because of the fairly extended structure of the deuteron.

Determining experimentally the dependence $\sigma_0(k^2)$, one can obtain a_{nn} and r_{nn}. For sufficiently small k and $|r_{nn}/a_{nn}|\ll 1$, the expression (9) simplifies to

$$\sigma_0(k^2)=4\pi/(k^2+1/a_{nn}^2). \tag{10}$$

Using Eq. (10), one can determine only the absolute value of a_{nn}, the sign remaining indeterminate. The expression (9) also gives two results. In practice, one of them can usually be excluded by the χ^2 test. Nevertheless, it is desirable to obtain additional information on the sign of a_{nn} (see below) by a different way.

The maximal value of σ_0 is attained at the momentum $k=0$. At $k^2=1/a_{nn}^2$, the cross section falls to half the maximal value. These circumstances are important for selecting the experimental conditions in the measurement of a_{nn}. It is necessary to choose the parameters of the experiment in such a way that the minimal relative energy of the two neutrons be as near zero as possible. Because the cross section of the reactions (1)—(6) is determined not only by the factors (9) or (10), one measurement of the cross section at low energy cannot give the required result. It is essential to measure the cross section in a certain range of k and determine a_{nn} from the relative profile of the spectrum. This range must include if possible k values from zero to a few times the value $k^2=1/a_{nn}^2$, at which σ_0 has decreased by a half.

When the cross section is fitted by means of Eq. (9), the parameters a_{nn} and the effective range r_{nn} are correlated. This is important in many determinations of a_{nn}, as we shall see below. At small k, the cross section depends relatively weakly on r_{nn}. At large k, other reaction mechanisms usually begin to particupate in the reactions (1)—(6), so that Eq. (9) is no longer suitable. For these reasons, it has not yet been possible to obtain a reliable value for r_{nn}.

The Migdal-Watson approximation can be assumed to be reasonable only under conditions in which the FSI of two neutrons is the predominant factor in the reaction amplitude and the amplitude of the primary reaction can be assumed constant. A completely satisfactory theory of the processes (1)—(6) does not yet exist. To understand the problems which arise in determining a_{nn}, let us consider here a somewhat simplified picture of three-particle reactions. The aim is to obtain information about the limits of applicability of the various approximations used in the determination of a_{nn}. The simplest reaction among the processes (1)—(5) is the breakup of a deuteron by a neutron. Let us consider this reaction as an example.

The solutions of Faddeev's equations[4] give a complete description of three-nucleon systems. A program for solving this complicated system of integral equations suitable for practical application has been written by Ebenhöh.[5] In these calculations, the dynamics of the three-nucleon system is completely described. The only approximation is in the use of simple models of the nucleon-nucleon interaction in the form of separable potentials with different form factors, for example, the Yamaguchi potential.[6] The first comparisons of Ebenhöh's results with experimental data on the $d(p,2p)n$ and $d(n,2n)p$ reactions[5,7] revealed good agreement for both the absolute cross section and the profile of the spectrum. At the present time, further experimental studies with higher accuracy are being made to test the applicability of this theory. Preliminary results

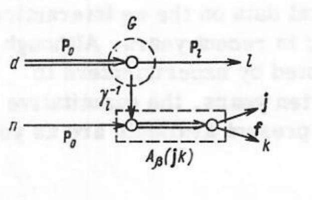

FIG. 1. Pole graph of deuteron breakup.

indicate that there are some small deviations between the theoretical and experimental spectra.[8] Such deviations depend on the form factor of the potential. For the determination of a_{nn} this is very important and requires careful investigation.

Hitherto, the complete theory has seldom been used to determine a_{nn}. The majority of a_{nn} measurements has been based on more or less crude models.

As a first approximation of Faddeev's theory, one can consider the pole approximation.[9] For the following discussion, it is helpful to present this approximation here in some detail. In the pole graph (Fig. 1), G is the deuteron breakup vertex. In the linear approximation in the effective range, one can write $G=\sqrt{8\pi\alpha_t}/(1-\alpha_t r_{ct}/2)$, and G is a constant that does not depend on the kinematics. We call γ_i^{-1} the propagator of the transferred nucleon; it has the form $\gamma_i^{-1}=\alpha_t^2+(\mathbf{p}_i-\mathbf{p}_0/2)^2$ ($\mathbf{p}_l$ is the momentum of particle l; $\mathbf{p}_0$ is the momentum of the incident particle; both in the center of mass system); α_t is related to the deuteron binding energy E_d and the nucleon mass m: $\alpha_t^2=-\mathbf{E}_d m/\hbar^2$.

The final-state interaction of particles j and k in the singlet or triplet state ($\beta=s,t$) is described by $A_\beta(jk)$, which has the form (linear approximation in the effective range)

$$A_\beta(jk)=[1/a_\beta-r_{0\beta}f^2_{jk}/2+if_{jk}]^{-1}.$$

Here, f_{jk} is the momentum of the relative motion of particles j and k; a_β and $r_{0\beta}$ are known parameters of the effective-range theory for the state β. The product of these three factors gives the matrix element of the pole graph:

$$T^S_{\beta ijk}=G\frac{A_\beta(jk)}{\gamma_i}=\frac{\sqrt{8\pi\alpha_t}\,(1-\alpha_t r_{0t}/2)^{-1}}{[1/a_\beta-r_{0\beta}f^2_{jk}/2+if_{jk}]\,[\alpha_t^2+(\mathbf{p}_i-\mathbf{p}_0/2)^2]_i}.$$

As yet, the matrix element has been written down only for one definite state of total spin S in the initial state, for one distribution of the three nucleons between particles j, k, and i, and for one spin state of the particles j and k interacting in the final state. In the total matrix element of the reaction one must take into account all possible states and distributions. This also includes antisymmetrization. In Refs. 10 and 11 it was shown that qualitative agreement with experiment can be achieved only if the pole approximation is used in the completely antisymmetrized form.

The total matrix element T of the reaction is given by

$$|T|^2=\frac{2}{3}|T^{3/2}|^2+\frac{1}{3}|T_t^{1/2}|^2+\frac{1}{3}|T_s^{1/2}|^2. \tag{11}$$

Here, $T^{3/2}$, $T_s^{1/2}$, and $T_t^{1/2}$ are the terms corresponding to the three-nucleon states with total spin $S=3/2$ and $1/2$. The state $S=1/2$ is split into two possible configurations in which the two nucleons j and k are in either the triplet or singlet state. The matrix elements T^S_β can be written down explicitly as[10,11]

$$\left.\begin{aligned}
T_s^{1/2}&=-\sqrt{3}\frac{\hbar^2}{m}\pi G\left[\frac{A_t(23)+A_s(23)}{\gamma_1}+\frac{A_t(31)+A_s(31)}{\gamma_2}+4\frac{A_s(12)}{\gamma_3}\right];\\
T_t^{1/2}&=\frac{\hbar^2}{m}\pi G\left[\frac{A_t(23)-3A_s(23)}{\gamma_1}-\frac{A_t(31)-3A_s(31)}{\gamma_2}\right];\\
T^{3/2}&=4\frac{\hbar^2}{m}\pi G\left[\frac{A_t(23)}{\gamma_1}-\frac{A_t(31)}{\gamma_2}\right],
\end{aligned}\right\} \tag{12}$$

if particles 1 and 2 are neutrons and 3 a proton.

Although the pole graph is only a first approximation, it enables one to describe all the important features of the reaction mechanism. The two-particle scattering matrix $A_\beta(jk)$, which describes the FSI of particles j and k, has an FSI maximum in the spectrum if f_{jk} attains a minimum. It follows from the equations for the total matrix element (11) and (12) that, depending on the kinematic conditions, one can observe maxima from all three particle pairs in one and the same spectrum.

The propagators γ_i contain a different dependence of the matrix element on the kinematics. A maximum appears in the spectrum when the absolute magnitude of the vector $\mathbf{p}_i-\mathbf{p}_0/2$ attains a minimum. The highest maximum is obtained for the case $\mathbf{p}_1-\mathbf{p}_0/2=0$. This means that one nucleon of the original deuteron preserves the initial motion, and the scattering occurs between only two particles, i.e., one has quasifree scattering. In the laboratory system, one nucleon of the deuteron target remains at rest.

If the Migdal-Watson approximation is used, one usually considers only one term, the one with A_s in (12), from the 11 terms in the total matrix element. The propagator is assumed constant. Use of the Migdal-Watson approximation requires the choice of kinematic conditions of the experiment under which the term with A_s in (12) is predominant and all the others can be regarded as a constant background. This condition can be satisfied best if f_{12} takes the value zero. Then A_s in (12) has the highest maximum, with width directly related to the scattering length.

The character of the approximation of this approach requires a careful testing of what distortions there can be of the FSI maximum. It follows from the kinematics that near the FSI peak the propagator γ_3 varies relatively little. But this does not exclude a strong dependence of the other terms of the matrix element on the propagators γ_1 and γ_2. Other distortions can arise from the tails of the FSI maxima of the other particle pairs. This is particularly dangerous in the reactions (4) and (5). In this case, the final-state interaction for ^3He+n and ^4He+n leads to excited states of the ^4He and ^5He systems at the corresponding relative energies. The maxima of these states may lie near the two-neutron FSI maximum.

To determine a_{nn}, the impulse approximation[12] (IA) has been used; in it, it is assumed that the incident neutron interacts with only one neutron of the target nucleus. The scattering between these particles is treated as scattering between free particles. Then the reaction cross section contains the nn scattering amplitude and therefore a_{nn}. The binding of the neutron in the target nucleus is taken into account by the approximate specification of the momentum distribution of this particle in the corresponding state. In this model, one ignores the influence of the remaining nucleons of the target nucleus on the two-particle interaction. Such an assumption is best satisfied if the distance between the nucleons in the target nucleus is large, as, for example, in the deuteron, and if the reaction takes place sufficiently rapidly, i.e., at high energies of the incident

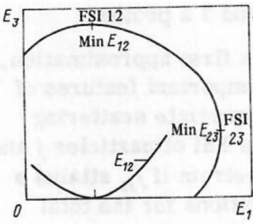

FIG. 2. Kinematic locus of three-particle reaction.

particles. The Born approximation (BA), which is described in detail in Ref. 13, provides a further possibility for extracting a_{nn} from experimental data. The expression describing the final state of the reaction contains the wave function of the relative motion of two neutrons. In the wave function, the phase shift δ_0 can be replaced by the parameters of the effective-range theory. Using the Born approximation, one can give the most complete description of the initial and final state of the reactions, but the fundamental restriction associated with it is the assumption that the interaction responsible for the reaction is a small perturbation.

As we have seen, the mechanisms of three-particle reactions are diverse. Usually, the experimental conditions are chosen in such a way that the FSI mechanism is predominant. But the role of other mechanisms cannot be completely eliminated. It is therefore very desirable to verify the applicability of this theoretical description of the reaction. This can be done by determining the neutron-proton (a_{np}) or proton-proton (a_{pp}) scattering lengths by the same methods as a_{nn} from mirror reactions. This approach has been called the "comparison procedure" and was discussed in detail by van Oers and Šlaus.[13] The comparison procedure is valid if the mechanisms of the mirror reactions are the same and the final states analogous, so that the same model describes both reactions. Strictly, such conditions are never satisfied. Nevertheless, investigation of mirror reactions enables one to obtain additional information about the reaction mechanism, and, therefore, interpret the experiments more reliably.

The reaction kinematics are more complicated for three than for two particles in the final state. The experimental methods for determining a_{nn} are based on the exploitation of the features of this kinematics. The methods of measurement can be divided into two groups: kinematically complete and incomplete measurements.

Let us begin by describing the complete experiments. The kinematics of three particles in the final state is determined by nine variables, i.e., by the absolute value of the momentum and two direction angles for each final particle. Because of the laws of conservation of momentum and energy, only five of these variables are independent. If two detectors are used to measure the directions and energies of two particles, six variables are determined. It follows that the energy of one particle depends on the other's. Such a dependence $E_1(E_3)$ is called a kinematic locus (Fig. 2). In Eq. (12) the indices 1 and 2 are used for the neutrons; the index 3, for the proton. If one of the neutrons and the proton (or another charged particle) are detected, one speaks of the kinematic locus $E_1(E_3)$. The position and form of the kinematic locus depend on the type of reaction, the energy of the incident particles, the nature of the detected particles, and the position angles of the detectors.

The expressions for the three-particle kinematics can be found, for example, in Ref. 14. The basic setup of a kinematically complete experiment is shown in Fig. 3. The particles leaving the target are detected by the detectors D1 and D3. The pulses from the detectors are amplified and transmitted to the inputs of a two-dimensional analyzer. Random coincidences are suppressed by a fast coincidence circuit. In the memory of the analyzer, the pulses from true events are added in channels corresponding to energies E_1 and E_3 lying on the kinematic locus. The physical information is contained in the density distribution of events over the kinematic locus.

The kinematics of all events detected in this manner is completely determined. This means that one knows the energy and direction of departure of the third particle and the relative energies of all the particle pairs. This considerably simplifies the choice of the experimental parameters and the analysis of the two-dimensional spectra. It is easy to find the conditions most suitable for revealing the FSI mechanism. If two final particles have relative energy zero, they appear as a single particle with the total mass and total energy. The kinematics in this case is the same as for two particles. On the basis of this, one can calculate the corresponding angles at which the detectors should be placed. When the relative energy of the two particles attains its minimum, the energy of the third is maximal. For example, at the maximum of the energy E_3 of particle 3 there is a minimum of the relative energy E_{12} of particles 1 and 2 (see Fig. 2). The final-state interaction is observed mainly at these points of the spectra. It is easy to see from Fig. 2 that the two-dimensional spectrum represented on the kinematic locus can be directly regarded as the reaction cross section as a function of the relative energy. This is precisely what we are interested in in a determination of the scattering length. At the point FSI12 (see Fig. 2), the cross section must have a maximum with width corresponding to the scattering length.

In kinematically incomplete experiments, a single detector is used to measure the spectrum of only one particle. Only three of the five independent kinematic variables are determined. One therefore obtains a spectrum integrated over the angle of the second particle. The limiting two-dimensional spectrum that contributes to the integration is the one in which the rela-

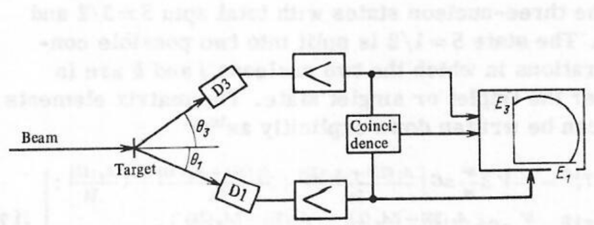

FIG. 3. Schematic arrangement of a kinematically complete experiment.

TABLE 1. Maximally allowed statistical error ΔN that must not be exceeded if the spectrum is to be distinguished for $a_{nn} = -16$ F and for $a_{nn} = -24$ F, as a function of the energy resolution ΔE (see Ref. 15).

ΔE, keV	0	200	400	800
ΔN, %	9	6	3,3	2,8

tive energy of the two undetected particles attains zero. If these particles interact in the final state, a maximum is obtained at the upper limit of the continuum. The height and width of this maximum depend on the scattering length.

This can be shown in the Migdal-Watson approximation. In Eqs. (12), the indices 1 and 2 denote neutrons. In the one-dimensional spectra, the charged particle 3 is detected. To obtain the one-dimensional spectrum of particle 3, one must integrate the differential cross section

$$\frac{d^5\sigma}{dE_3\, d\Omega_1\, d\Omega_3} \sim \rho\, |T|^2$$

with respect to Ω_1. Here, ρ is the phase-space factor; ρ and T depend on the kinematics. For a kinematically complete spectrum, ρ can be described by Ohlsen's method[14]:

$$\rho = m_1 m_2 m_3 p_1 p_3 \,[m_1 + m_2 + m_3\, (\mathbf{p}_3 - \mathbf{p}_0)\, \mathbf{p}_1 / p_1^2].$$

In this expression, the scalar product $(\mathbf{p}_3 - \mathbf{p}_0)\mathbf{p}_1$ depends on Ω_1. Therefore, ρ must be integrated. To show the dependence of the matrix element T on Ω_1, we use the expression (12) of the pole approximation. As we have already seen, in the Migdal-Watson approximation one considers only the term with A_s in (12). If it is assumed that all the remaining terms are small, it remains to integrate only this term with respect to the angle Ω_1:

$$\frac{d^3\sigma}{dE_3\, d\Omega_3} = \mathrm{const} \int \frac{\rho\, d\Omega_1}{|(1/a_s - r_{0s} f_{12}^2/2 + i f_{12})\,(\alpha_t^2 + (\mathbf{p}_3 - \mathbf{p}_0/2)^2)|^2}.$$

In the denominator, $\mathbf{p}_3$ does not depend on Ω_1, since in the three-particle system there are five independent kinematic variables: $\mathbf{p}_3$, $v_3 \varphi_3$, v_1 and φ_1. A simple argument based on energy and momentum conservation shows that the relative momentum f_{12} depends only on E_3 or $\mathbf{p}_3$, but not on Ω_1. Thus, the integral reduces to an integral over the phase space and the kinematically incomplete spectrum depends in a fairly simple manner on the relative momentum. The scattering length in fact determines the width of the maximum near the upper limit of the spectrum. Of course, this is true only in the Migdal-Watson approximation.

Both methods have frequently been used to measure a_{nn}. Under ideal experimental conditions (very good energy resolution, no background, and small statistical errors which can be ignored) and for an exact theoretical description of the reaction the two approaches are of equal value. But these conditions are not fulfilled.

In incomplete experiments, the continuum must have a sharp peak at the upper limit of the energy spectrum. Because of the limited experimental resolution, this sharp peak is deformed. The extraction of a reliable value of the scattering length from such a distorted spectrum requires a very careful folding with the resolution. This folding is made difficult by statistical errors. In Ref. 15, Davis *et al.* investigated what maximal statistical errors are possible for given energy resolutions if one is to distinguish two spectra for $a_{nn} = -16$ F and $a_{nn} = -24$ F. The results of these calculations are given in Table 1. The requirements on the experimental accuracy become much more stringent if it is necessary to determine a_{nn} with an accuracy of about 1 F.

The situation is simpler for kinematically complete measurements. Let us consider the projection of the two-dimensional spectrum onto the E_1 axis. The FSI maximum of particles 1 and 2, which as a function of E_{12} would have a width of 200–300 keV, extends over 1 MeV and more because of the slow variation of the relative energy at the position of the maximum. This broadening effect is very convenient, since for the experiment it is sufficient to have moderate energy resolution. The sensitivity of the result to folding with the resolution is appreciably less than in incomplete experiments. In addition, in the case of complete experiments information on the scattering length is contained separately in each half of the maximum.

Inspection of the pole approximation in the form of Eq. (12) revealed a very complicated picture of the mechanism of three-particle reactions. Since incomplete measurements cover a much wider range of the kinematic variables than complete experiments, the use of simple approximations such as the Migdal-Watson theory to extract a_{nn} from one-dimensional spectra is problematic. In complete experiments, the kinematic region can be matched to the problem, and one can therefore give a preference to the reaction mechanism which is best described by the given approximation.

In complete experiments, the background from random coincidences is distributed more or less uniformly over the whole region between the E_1 and E_3 axes. It is not difficult to interpolate the background under the spectrum on the kinematic locus. In incomplete experiments, the background must be determined by a separate measurement. Thus, kinematically complete measurements have some important advantages over incomplete experiments, so that more reliable results are to be expected from them.

As we have already pointed out, the experimental spectra obtained by both methods require folding because of the finite energy and angle resolutions. At the present time, this problem is frequently solved by simulating the experiment by the Monte Carlo method. After these general considerations, let us turn to the individual reactions and the results.

Sign of the Neutron-Neutron Scattering Length and the Problem of the Dineutron. Here we consider the

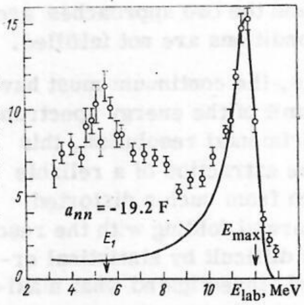

FIG. 4. One-dimensional spectrum of protons from the reaction $d(n,p)2n$ (Ref. 23); $E_n = 14$ MeV, $\vartheta_p = 4°$.

problem of obtaining additional information about the sign of the scattering length. It follows from the effective-range theory that an attractive potential corresponds to a positive scattering length if the potential allows the existence of a bound state of two nucleons. Conversely, a negative scattering length is obtained if an attractive potential does not lead to a bound state. A positive scattering length also corresponds to a repulsive potential. All the results of nuclear physics indicate that the forces between neutrons must be attractive. The question of the sign of the scattering length can be solved by proving experimentally the existence or absence of a bound state of two neutrons—the "dineutron".

The question of the existence of the dineutron has been studied experimentally by different methods. One can look for the dineutron in nuclear reactions in which two neutrons are formed in the final state; for example, $^3H(d, {}^3He)2n$, $^3H({}^3H, {}^4He)2n$, $d(\pi^-, \gamma)2n$. The existence of a bound state would lead to narrow lines in the ^3He, ^4He, or γ spectra of the corresponding reactions. In Refs. 16–21 such lines were not found. The upper limit of the probability of formation of a dineutron compared with the formation of a pair of free neutrons was found by Butler *et al.*[21] to be $5 \cdot 10^{-9}$. Jarmie *et al.*[17] found an upper limit $\sigma = 10^{-29}$ cm² for the dineutron production cross section.

Another method is the method of activation analysis. If a dineutron is formed in one of the reactions, it can be captured as a whole by nuclei. As a result, one obtains the isotope of the target nucleus with mass number increased by two. This reaction product differs from products of the capture of one neutron in its lifetime. Such experiments were made by Katase *et al.*[22] on ^{209}Bi and ^{27}Al nuclei. The expected ^{211}Bi and ^{29}Al isotopes were not observed.

None of these experiments confirmed the existence of the dineutron. It is now generally accepted that there is no bound state of two neutrons, so that the nn scattering length is assumed negative.

Determination of a_{nn} from the Reaction $dn \to pnn$. The first attempt to determine a_{nn} from the reaction $dn \to pnn$ was made by Ilakovac *et al.* in 1961.[23] They measured the one-dimensional spectrum of protons at an angle 4° (incomplete experiment), and the result is shown in Fig. 4. Theoretical interpretation of the spectrum by means of the Born approximation led to the value $a_{nn} = -22 \pm 2$ F. The specified error contains only the statistical errors. The same group repeated the experiment and obtained the slightly more accurate value $a_{nn} = -21.7 \pm 1$ F with measurements[24] at the angle 4.8°. Measurements of this kind have been made several times. The work of Voĭtovetskiĭ *et al.*[25] deserves especial attention. To interpret the experimental spectra, they used the method of graph summation of Komarov and Popova.[9] The pole graph and the triangular graph were taken into account in the interpretation of the spectra. The contribution of the remaining infinite series of graphs was taken into account by a constant that does not depend on the kinematics. The value $a_{nn} = -23.6^{+1.6}_{-2}$ F was obtained. The experiments of Prokof'ev *et al.*[26] and Skorodumov *et al.*[27] are in fact repetitions of Ref. 25. The results of these studies were, respectively, $a_{nn} = -23.2^{+1.8}_{-1.9}$ and $a_{nn} = -23.0^{+4.2}_{-3.6}$ F. Skorodumov *et al.* also compared their experimental spectra with calculations in the impulse and Born approximations. In these cases, the theory agrees best with the experiment if $a_{nn} = -15$ F.

The pole graph or the more general method of graph summation must give a more complete description of the three-nucleon system than the Migdal-Watson theory. Therefore, the approach of Voĭtovetskiĭ *et al.*[25] suggests itself. However, new calculations of the contribution of the triangular graph have recently appeared.[28,29] It has been found that it does not enable one to improve the description of the experiments at low energies; instead, it completely distorts the profile of the spectrum. It is only at relatively high energies (higher than 50 MeV) that the contribution of the triangular graph is reduced relative to the pole graph and transformed into a correction that improves the interpretation of the experiments. It is obvious that under such conditions at low energies the remaining terms of the infinite series of graphs cannot be taken into account in the form of a constant that does not depend on the kinematics, as was done in Ref. 25. Therefore, we cannot now assume that the results obtained on the basis of these approximations are correct. At the energies used to determine a_{nn}, the pole graph gives a cross section that is an order of magnitude greater than the experimental

TABLE 2. Results of measurements of a_{nn} for different energies E_n of the incident neutrons.[32]

E_n, MeV	a_{nn}, Fermi	
	Migdal-Watson	Impulse Approximation
8	$-37.0^{+4.0}_{-7.0}$	$-22.0^{+2.3}_{-2.5}$
10	$-27.0^{+2.5}_{-3.0}$	$-16.0^{+1.5}_{-2.0}$
13	$-30.0^{+2.5}_{-4.5}$	$-17.5^{+1.7}_{-1.7}$
14	$-26.0^{+4.0}_{-5.0}$	$-14.0^{+3.0}_{-3.0}$
23	—	$-13.5^{+2.0}_{-2.5}$
28	—	$-16.5^{+4.5}_{-7.0}$
Mean		-16.8 ± 1.0
Mean with values for 8 MeV excluded		-15.9 ± 1.1

TABLE 3. Results of measurements of a_{nn} by means of the reaction $d+n\rightarrow p_2+n+n_3$.

E_n, MeV	Angle, deg	Experimental Method	Theory	a_{nn}, F	r_{nn}, F	Literature
14	4	Incomplete	BA	-22 ± 2	2.84	[23]
14	4.8	»	BA	-21.7 ± 1	2.8	[24]
14	0	»	Graph .	$-23.6^{-2}_{-1.6}$	—	[25]
14	—	»	IA	-14 ± 3	—	[37]
			BA	-16 ± 3	—	
8—28	5	»	IA	-15.9 ± 1.1	2.40	[32]
	20		MW	-30.0		
14.1	—	»	[46]	(-23.78)	2.80	[38]
14	0	»	MW comp.	$-16.7^{-2.6}_{-3.0}$	2.63	[34]
14.1	0	»	BA	-16.2 ± 2.2	—	[36]
14.1	0	»	Graph .	$-23.2^{+1.8}_{-1.9}$	2.65	[26]
14.06	3.5	»	Graph .	$-23.0^{+3.6}_{-4.2}$	2.65	[27]
50	3.8	»	IA	-21.7 ± 1.2	2.60	[33]
14.1	4.0	»	BA	-19.3 ± 0.8	2.60—3.00	[30]
14.1	4.0	»	IA	-18.31 ± 0.22	2.84	[30]
14.5	30	Complete	[31]	(-25)	—	[39]
14.1	±67.5	»	BA	$-18.8^{+5.5}_{-11.9}$	—	[35]
18.4	—	»	MW	$-16.4^{+2.6}_{-2.9}$	2.5	[42]
14.3	20	»	Graph	(-25 ± 3)	2.8	[48]
14.5	30	»	[31]	(-23.78)	2.50	[40]
14.17	—	»	[46]	-16.0 ± 1.2		
			MW	-16.8 ± 1.3	2.86	[45]
130	—	»	MW	-17.1 ± 0.8	2.84	[47]
18.4	—	»	[5]	-16.1 ± 0.9	3.4 ± 0.6	[43]

The values of a_{nn} in parentheses are not the results of fitting but of a comparison of the experimental data with the theoretical data for fixed parameters.

one. In addition, there is an indication that the calculated FSI maximum is broader than in the measured spectra.[10]

Shirato *et al.*[30] made very careful measurements in a kinematically incomplete experiment. They devoted particular attention to accurate folding of the spectrum. In the theoretical interpretation, they used the impulse approximation and obtained the result $a_{nn}=-19.3\pm0.8$ F. In addition, a calculation on the basis of Faddeev's equations by Cahill's method[31] gave $a_{nn}=-18.31\pm0.22$ F. In my opinion, the accuracy of this experiment was overestimated. In accordance with the rules for estimating the statistical error when parameters are determined by the χ^2 test, the experimental uncertainty must be about ±1.0 F.

Bond[32] measured the proton spectra at different energies of the incident neutrons from 8 to 28 MeV and found that a_{nn} varies with the energy and in addition depends on the theoretical interpretation of the spectra. Bond's results are given in Table 2. It is interesting to note the systematic difference between the a_{nn} values obtained by means of the Migdal-Watson approximation and the impulse approximation. Bond assumes that only the results obtained by the latter are reliable. In Ref. 33, Stricker *et al.* report a measurement of a_{nn} at an energy 50 MeV of the incident neutrons. The impulse approximation was used to find $a_{nn}=-21.7\pm1.2$ F. It can be assumed that at such high energies the impulse approximation describes the experimental data well. Nevertheless, Stricker *et al.* could not obtain satisfactory agreement between experiment and calculation. Therefore, this result was ignored in the determination of the mean value of a_{nn} (see below).

The comparison procedure was used for the first time by Slobodrian *et al.*[34] to test the applicability of Migdal-Watson theory. They found that this approximation in the case of the $d(p,2p)n$ reaction gives $a_{nn}=-13$ F instead of the value -7.8 F obtained from direct measurements. Bearing in mind this distortion of the Migdal-Watson result, they extracted from the experimental data the value $a_{nn}=-16.7^{+2.6}_{-3.0}$ F. This work shows that the direct extraction of a_{nn} from experimental spectra by means of the Migdal-Watson theory does not lead to the correct result. Slobodrian *et al.* expressed the opinion that this may be due to the rather extended structure of the deuteron, which does not accord with the assumptions of the Migdal-Watson theory. But this argument obviously does not solve the problem. Despite the extended structure of the deuteron, the Migdal-Watson theory in the case of complete measurements gives a result that differs from the one obtained by Ebenhöh's calculations[1] by 0.5 F.

One should also mention the work of Grässler and Honecker.[35,36] In these experiments, they measured one-dimensional spectra of neutrons subject to the condition of a coincidence with protons. The kinematic conditions were chosen in such a way that a final-state interaction of two neutrons was favored while such an interaction between the neutrons and the proton was suppressed. Evaluation of these two experiments by means of the Born approximation gave $a_{nn}=-18.8^{+5.5}_{-11.9}$ F and $a_{nn}=-16.2\pm2.2$ F. The results of other investigations based on incomplete measurements are given in Table 3.[37,38]

In some studies, kinematically complete measurements were made of the reaction $d(n,2n)p$. Perrin *et al.*[39] were the first to communicate the results of such an experiment. The arrangement of such experiments is shown in Fig. 5.

Neutrons from the reaction $^3H(d,{}^4He)n$ are incident at right angles on the target D_0, a deuterium scintillator. In this scintillator, the protons from the breakup of the deuteron trigger two neutron spectrometers D_1 and D_2, which must detect two neutrons in coincidence by the time-of-flight method. To realize the FSI condition at low relative energies, the two neutron detectors

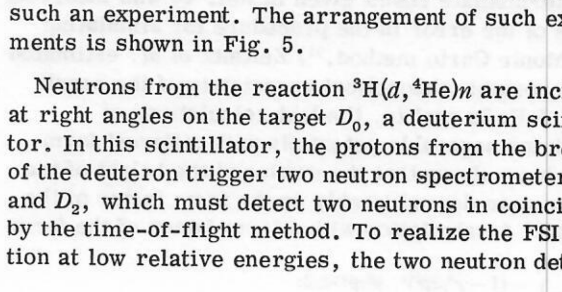

FIG. 5. Schematic arrangement of kinematically complete experiment for determining a_{nn} from the reaction $d(n,2n)p$ with detection of two neutrons in coincidence: d is the deuteron beam; T is the tritium target; D_0 is the deuterium target in the form of a deuterium scintillator for detecting recoil protons; D_1 and D_2 are neutron detectors with scintillators and fast photomultipliers; S_1 and S_2 are neutron shields.[42]

must be placed at almost the same angle. In the experiment of Perrin *et al.* they were next to one another ($\Delta v=0$). The principal difficulties of such an experiment derive from the presence of only a weak and unfocused neutron beam and from the need to suppress the background. Final results were given in Ref. 40. Perrin *et al.* compared the experimental data with solutions of Faddeev's equations obtained by Cahill *et al.*[31] on the basis of the Yamaguchi potential. They found that the value $a_{nn}=-23.7$ F gives a better agreement with the experiment than the value $a_{nn}=-16$ F.

In their kinematically complete experiment, Bouchez *et al.*[40] had great difficulties with the background and the unsatisfactory statistics. In addition, they compared their data with the preliminary variant of Cahill's calculations, which at that time did not yet give the correct absolute cross section. Bouchez *et al.* noted that the calculations give cross sections that are twice the experimental ones. It should be noted that Cahill's calculations in the final variant and Ebenhöh's calculations agree fully and give the correct cross section.[41] Thus, this result cannot be regarded as fully reliable.

The second measurement of this type was made by Zeitnitz *et al.*[42] In this first study, the experiments were interpreted by the Migdal-Watson approximation. The experimental data were simulated by the Monte Carlo method. The value $a_{nn}=-16.4^{+2.6}_{-2.9}$ F was obtained. Later, Zeitnitz *et al.* repeated their experiments under improved conditions. The data were interpreted on the basis of Faddeev's equation by means of the program written by Ebenhöh.[5] The final result $a_{nn}=-16.1\pm0.9$ F of this work with the effective range $r_{nn}=3.4\pm0.6$ F was communicated at the Los Angeles Conference in 1972.[43] (The intermediate result given in Ref. 44 was incorrect because of the error in the procedure for simulating by the Monte Carlo method.[41]) Zeitnitz *et al.* estimated that the possible theoretical uncertainty of the result was ±0.5 F. In reality, the last calculations of Ebenhöh for separable potentials with different form factors have shown that the width and the height of the FSI maximum depend weakly on the form factor of the potential.[7] A calculation with a form factor of the form

$$g(p^2)\begin{cases} =(1-p^2/2\beta^2)^2, & p^2/\beta^2<2;\\ =0, & p^2/\beta^2\geqslant 2\end{cases}$$

(β^2 is the propagation parameter of the nuclear forces) gives better agreement with the experimental spectra than the Yamaguchi potential. When this form factor is used, the scattering length is increased to $a_{nn}=-16.5$ F.

At the Los Angeles Conference in 1972, two other studies of this kind were reported. Breunlich *et al.*[45] measured the two-dimensional spectrum at energy 14.17 MeV of the incident neutrons. Assuming the effective range $r_{nn}=r_{np}=2.86$ F known, and using the theory of Aaron *et al.*[46], they obtained $a_{nn}=-16.0\pm1.2$ F, while using the Migdal-Watson approximation $a_{nn}=-16.8\pm1.3$ F. McNaughton *et al.*[47] communicated the preliminary result $a_{nn}=-17.1\pm0.8$ F from measurements at an energy of 130 MeV of the incident neutrons. The Migdal-Watson theory and the model of quasifree scattering were used to interpret this experiment.

A different attempt to determine a_{nn} by means of a kinematically complete measurement was undertaken by Saukov *et al.*[48] Unfortunately, their result $a_{nn}=-25\pm3$ F cannot be regarded as reliable. The statistics of the measurement were clearly inadequate, and the background was subtracted in an arbitrary manner. For the theoretical interpretation, Saukov *et al.* used the pole approximation in the form of the Chernukhin-Shuvalov expressions.[49] These do not take into account correctly the antisymmetrization of the three-nucleon problem.[10]

The results of determining a_{nn} from the reaction $dn\to pnn$ are given in Table 3. It follows from Table 3 that the values of a_{nn} are divided into two groups. Some are grouped around the value -23 F, the others about -16 F. The first group consists basically of kinematically incomplete experiments. But values around -16 F were also obtained by this method. The results of the incomplete measurements are contradictory. The graphical method and Migdal-Watson theory always gave a_{nn} around -23 F. The Born and impulse approximations gave results in both regions. Bond pointed out the large difference between the results obtained by means of Migdal-Watson theory and the impulse approximation. Bearing in mind that the experimental errors and the folding have a large influence on the result as well as the theoretical interpretation, it is highly desirable to investigate the reasons for these discrepancies separately.

If we leave out of account the measurements of Refs. 40 and 48, which have already been criticized above, the kinematically complete experiments give a clearer picture. The five values of a_{nn}, obtained by different theoretical methods and at different energies, including the measurement of Ref. 47 at 130 MeV, agree to within ±0.5 F. In addition, these results include one obtained by means of Faddeev's theory,[43] the most reliable one. One can therefore conclude that with a high probability the value of a_{nn} in the range between -16 and -17 F is more reliable than the other value near -23 F. In the determination of the mean value of a_{nn} (see below), the values near -23 F were ignored.

Determination of a_{nn} from the Reaction $^3\mathrm{H}+n\to d+n+n$. The mechanisms of this reaction can be represented in the form of the graph in Fig. 6. There are two possible ways of realizing the interaction of two neutrons in the final state (the graphs in Figs. 6(a) and 6(b)). The third graph (Fig. 6(c)) corresponds to interaction of a neutron with the deuteron in the final state.

Ajdačić *et al.*[50] were the first to attempt to extract the scattering length from this reaction. From the deuteron spectrum measured at the angle 0° they obtained $a_{nn}=-18\pm3$ F by the Born approximation. A

FIG. 6. Pole graphs of the reaction $^3\mathrm{H}+n\to d+n+n$.

similar experiment made by Fushini *et al.*[51] gave $a_{nn} = -17 \pm 2$ F. In this case, the Migdal-Watson approximation was used to interpret the spectrum.

Adam *et al.*[52] made a third measurement of a_{nn} by means of the reaction $^3\mathrm{H} + n \to d + n + n$. In this case, they investigated influence of different wave functions of the triton and the $2n$ system used in the Born approximation on the result. They found that the form of the wave function is important. Averaging of the data obtained by Adam *et al.* gives the mean value $a_{nn} = -14.7$ F. The error of this result, due mainly to the theoretical uncertainty, is ± 6 F. The values of a_{nn} obtained from investigation of the reaction $^3\mathrm{H} + n \to d + n + n$ are given in Table 4.

To verify the suitability of this reaction for determining a_{nn}, the mirror reactions $^3\mathrm{He}(p, d)2p$ and $^3\mathrm{He}(n, d)np$ were investigated. Antolković *et al.*[53] found that the Migdal-Watson approximation gives too broad maxima in the spectra. The Born approximation was also found to be unsuitable for describing correctly the experimental data because of the uncertainty of the ^{3}He wave function. On the basis of other measurements at 30 and 46 MeV, Chang *et al.*[54] concluded that the Migdal-Watson approximation is not suitable for describing incomplete experiments, but they successfully applied this approximation to kinematically complete measurements of the same reactions. They explained this circumstance by the fact that in kinematically complete experiments the FSI mechanism can be selected more rigorously. Finally, Harbison *et al.*[55] also concluded that at 30.5 and 49.5 MeV the Migdal-Watson theory gives too broad maxima compared with experiment. Agreement with experiment could be achieved by introducing a correction due to the long-range Coulomb interaction between the protons and deuterons in the final state. Such long-range forces are incompatible with the assumptions made in the Migdal-Watson approximation. If the Coulomb interaction really is the reason for the discrepancy, measurement of a_{nn} by means of the $^3\mathrm{H} + n$ reaction must give the correct result, since there are no Coulomb forces in this case.

As yet, the discrepancy between the experimental spectra of the $^3\mathrm{He}(p, d)2p$ and $^3\mathrm{He}(n, d)np$ reactions and the results of theoretical calculations in accordance with different models is still too large for one to be able to apply the comparison procedure to this group of mirror reactions. It would be very desirable to confirm the above values of a_{nn} by means of kinematically complete experiments.

Determination of a_{nn} from the Reaction $dd \to ppnn$. In the reaction $dd \to ppnn$, four particles are formed in the final state. If the usual method of two-dimensional measurements in the E_1E_2 plane is used, the kinematically allowed events occupy a certain area bounded by the kinematic locus of the events for which two neutrons depart with relative energy zero. This locus is in fact the locus of the three-particle reaction $dd \to pp(2n)$. The width of the event distribution must contain information about a_{nn}. Preliminary data on such an experiment were published by Witsch *et al.*[56] If was found that the observed events were strongly concentrated on the three-particle kinematic locus at the place corresponding to a low relative energy of not only two neutrons but also two protons. This means that the reaction mechanism is similar to the two-particle reaction $dd \to (2p)(2n)$. The statistics of this experiment were still inadequate to extract the value of a_{nn}. The same experiment was made by Assimakopoulos *et al.*,[57] who were able to determine the value $a_{nn} = -15.5 \pm 1.1$ F at $E_d = 16.0$ MeV and an angle 20°. Unfortunately, Assimakopoulos *et al.* did not state what experimental method was used to obtain this result.

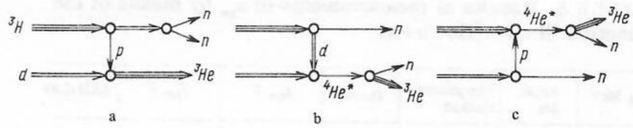

FIG. 7. Pole graphs of the reaction $^3\mathrm{H} + d \to {}^3\mathrm{He} + n + n$.

The experimental data on this reaction are as yet inadequate. A detailed theoretical analysis of the reaction mechanism has not yet been made. Therefore, the value of a_{nn} obtained in Ref. 57 can be regarded as only a preliminary result.

Determination of a_{nn} from the Reaction $^3\mathrm{H} + d \to {}^3\mathrm{He} + n + n$. The investigations Refs. 58–64 were devoted to determining a_{nn} from the reaction $^3\mathrm{H} + d \to {}^3\mathrm{He} + n + n$. In Refs. 62–64 kinematically complete measurements are presented; in the others, kinematically incomplete ones. A large number of studies have analyzed the reaction mechanism. They have investigated the mirror reactions $^3\mathrm{He} + d \to {}^3\mathrm{H} + p + p$ and $^3\mathrm{He} + d \to {}^3\mathrm{He} + n + p$. It is found that the mechanism of these reactions is fairly complicated and depends strongly on the kinematic conditions of the experiment. It is therefore hard to analyze the spectra obtained and not all attempts to extract the value of a_{nn} were successful. From the experimental point of view, the reaction $^3\mathrm{H} + d$ is much more advantageous than the preceding reactions. It can be studied directly with a strong and well focused beam of charged particles obtained from an accelerator.

A general theory of this reaction does not yet exist. Hitherto only individual features of the mechanism have been considered, and on the basis of such a crude approach attempts were made to extract the parameters of the effective-range theory. To analyze the different possible reaction mechanisms, we give their pole graphs (Fig. 7). There are three possibilities of forming final states with two neutrons. For the determination of a_{nn}, the graph of Fig. 7(a) is the most interesting. Here, two neutrons interact in the final state.

TABLE 4. Results of measurements of a_{nn} by means of the reaction $^3\mathrm{H} + n \to d + n + n$.

E_n, MeV	Angle, deg	Experimental Method	Theory	a_{nn}, F	r_{nn}, F	Literature
14.4	0	Incomplete	BA	−18±3	—	[50]
15.1	0	»	MW	−17±2	2.5	[51]
13.95	0	»	BA	−14.7±6 *	—	[52]

* Average result of six values obtained by means of different wave functions for ^{3}H and for the $2n$ system.

TABLE 5. Results of measurements of a_{nn} by means of the reaction $^3H + d \rightarrow {}^3He + n + n$.

E_d, MeV	Angle, deg	Experimental method	Theory	a_{nn}, F	r_{nn}, F	Literature
32.5 40	6 25	Incomplete	MW comp.	−16.1±1	3.2	[58]
29.8 31.9	6 8	»	MW comp.	−16.5±1	3.1	[59]
11	6 20	»	BA comp.	−17.5±3 −19.25±2	2.6	[60]
19.9 * 22.0 *	5	»	MW comp.	−10.9		[61]
83	2.5 7.5	»	MW comp.	−15.5±1.1 −12.7±0.8 −11.3±0.8	3.2±0.4 3.4±0.4	[63]
13.43	29	Complete	MW	−16.0±1.0	2.67	[62]
8.45	0	»	MW	−16.2±1.2		[64]

* Energy of tritium nuclei

One cannot ignore the graphs of Figs. 7(b) and 7(c). These graphs describe different possibilities of interaction of one of the neutrons with ^3He in the final state. The probability of this mechanism depends on the relative energy of the neutron and ^3He. Maxima appear if the relative energy corresponds to one of the excited states of the system ^4He. This effect has an additional influence on the profile of the spectra and can completely mask the FSI effect of the two neutrons. One can find kinematic conditions of the experiment under which excitation of the levels of the system ^3He do not play a significant role.

The results of determining a_{nn} from the reaction $^3H + d \rightarrow {}^3He + n + n$ are given in Table 5. The incomplete measurements cover the range of incident deuteron energies from 11 to 83 MeV. In all experiments, the ^3He spectra were measured at small angles. In two experiments[58,60] the measurements were made at angles up to 20–25°. In Ref. 60, Larson showed how strongly the reaction yield and the profile of the spectra change with increasing angle (Fig. 8). The maximum at the upper limit of the spectrum disappears in the range of angles from 10–20°. At the energy 83 MeV, the reaction cross section falls between 2.5° and 7.5° by almost an order of magnitude. This behavior of the cross section cannot

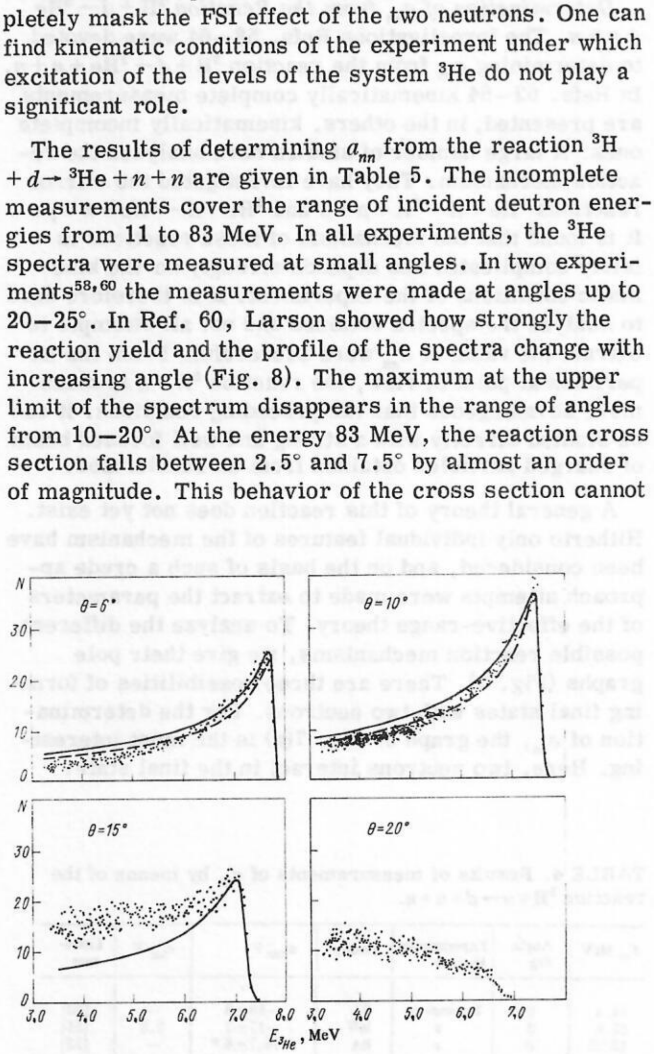

FIG. 8. Spectra of ^3He from the reaction $^3H + d \rightarrow {}^3He + 2n$ for different angles.[60]

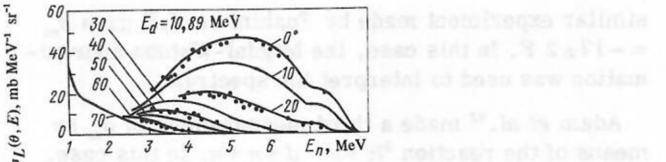

FIG. 9. Spectra of neutrons from the reaction $^3H + d \rightarrow {}^3He + 2n$ for different angles.[69]

be described by either Migdal-Watson theory or the Born approximation.

In all these investigations, attempts were made to verify the theoretical interpretation in mirror reactions. Baumgartner *et al.*[58] and Gross *et al.*[61] obtained from the reaction $^3He(d, {}^3H)2p$ the correct value of the pp scattering length a_{pp} but their results for a_{nn} from the reaction $^3H(d, {}^3He)2n$ differ strongly (see Table 5). This brings out clearly the influence of excited ^4He states on the result. It follows from the kinematics that the relative energy of the ^3He and the neutron is a function of $E_{^3He}$. This means that at the appropriate places the ^3He spectrum has peaks due to the ^4He states. If the energy of the incident particles is chosen infelicitously, such a peak will lie near the FSI maximum of the two neutrons and distort its profile. If the energy of the incident particles is too low, the ^4He states are not excited. If the energy is sufficiently high, such peaks lie far from the end of the spectrum at low ^3He energies and do not affect the width of the neutron FSI peak. This is the difference between the results of Baumgartner and Gross. Baumgartner's measurements were made with incident deuterons of 32.5 and 40 MeV, while Gross's measurements were made with incident tritons of 19.9 and 22.0 MeV. A simple calculation shows that at the latter energies the maximum of the ^4He state at 22 MeV is near the end of the spectrum. Thus, Gross's result must be assumed erroneous.

Slobodrian *et al.*[59] and also Bachelier *et al.*[63] determined the vertex form factor from the reaction $^3He(d, {}^3H)2p$ in order to obtain complete agreement between the experimental spectrum and Migdal-Watson theory for the correct value of a_{pp}. They took into account this form factor when extracting a_{nn} from the reaction $^3H(d, {}^3He)2n$. This approach was criticized by other authors since it assumes complete charge symmetry between these two reactions. Such symmetry cannot hold because in the case of final-state interaction of the ^3He and neutron a resonance appears at the energy 1.9 MeV in ^4He. The $^3H + p$ system has resonances at 200 keV and 3 MeV, corresponding to the ^4He levels at 20 and 22 MeV.

More than twenty investigations have been made of the mechanism of the reactions $^3He(d, {}^3H)2p$ and $^3He(d, {}^3He)pn$. The aim of some of them was to obtain information about the excited states of the ^4He nucleus.[65,66] In other investigations, a study was made of the possibility of obtaining the parameters of the effective-range theory for the pp and np interactions. Contradictory conclusions were obtained. Some authors obtained correct parameters,[67] others concluded that these parameters cannot be measured by means of such reactions.[68]

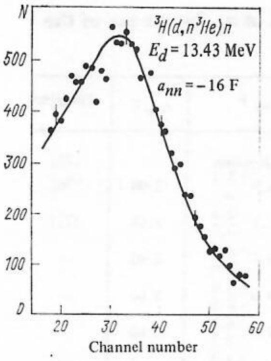

FIG. 10. Spectrum of the reaction $^3H + d \rightarrow {}^3He + 2n$ obtained in the kinematically complete experiment of Grötzschel *et al.*[62] The figure shows the projection of the two-dimensional spectrum onto the neutron-energy axis; the continuous curve is the result of fitting by means of Migdal-Watson theory for $a_{nn} = -16$ F.

This discrepancy obviously arises because of the strong dependence of the reaction mechanism on the kinematic conditions of the experiment. For the case of kinematically complete measurements, this was clearly shown by Assimakopoulos *et al.*[67] By choosing appropriately the conditions, they succeeded in observing a manifestation of the FSI mechanism, the mechanism of quasielastic scattering, and the excitation of levels of the ^{4}He nucleus in a more or less pure form. As a result, they obtained the correct value for a_{pp} and a_{np}. This study clearly showed the advantage of kinematically complete measurements.

The first kinematically complete experiment for determining a_{nn} by means of the reaction $^3H(d,n)^3He\ n$ was based on the same careful choice of suitable kinematic conditions.[62] The angles of the neutron and ^{3}He detectors were chosen in such a way that the relative energy of the two neutrons attained zero. To prevent distortion of the spectrum by the excitation of ^{4}He levels, both angles were taken fairly large ($\vartheta_{^3He} = 29°$, $\vartheta_n = 65.4°$). The last condition follows from the work of Poppe *et al.*[69] and Jarmie *et al.*,[70] who measured the spectrum of the neutrons from the reaction $^3H(d,n)^3He\ n$ and found that the lines corresponding to excitation of the ^{4}He

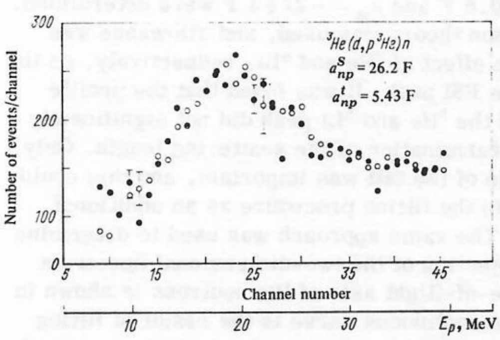

FIG. 11. Spectrum of the reaction $^3He + d \rightarrow {}^3He + n + p$ obtained in the kinematically complete experiment of Kühn *et al.*[71] The figure shows the projection of the two-dimensional spectrum onto the proton-energy axis; ○○○ experimental data; ●●● results of simulation of the experiment by the Monte Carlo method.

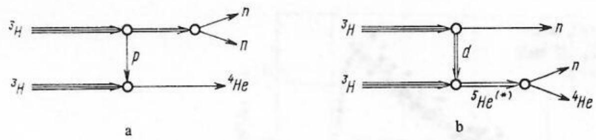

FIG. 12. Pole graphs of the reaction $^3H + {}^3H \rightarrow {}^4He + 2n$.

levels disappear rapidly when the detection angle is increased (Fig. 9). Under such conditions, it proved possible in Ref. 62 to achieve excellent agreement between calculations by the Migdal-Watson theory and the experimental spectrum (Fig. 10). This approach was additionally confirmed in a study of the spectra of the reactions $^3He(d,p)^3\ p$ and $^3He(d,p)^3He\ n$ under the same kinematic conditions.[71] It also proved possible to describe these spectra completely in the framework of Migdal-Watson theory and thus extract the correct scattering lengths a_{pp} and a_{np} (Fig. 11). In these investigations, the behavior of the propagator in the graph of Fig. 7(a) was also taken into account. The calculations showed that it can be assumed constant in the region of the spectrum in which the scattering length is deduced. In addition, it was shown that excitation of the ^{4}He levels does not in reality distort the spectra.

In a kinematically complete experiment Jeremie *et al.*[64] chose other kinematic parameters. Both neutrons were detected in coincidence at the angle 0°. Using Migdal-Watson theory, they obtained $a_{nn} = -16.2 \pm 1.2$ F. One can conclude that, although a theoretical description of the reaction $^3H + d$ as a whole does not yet exist, in specially selected cases certain approximations such as, for example, the Migdal-Watson theory enable one to measure a_{nn} and obtain reliable results.

Determination of a_{nn} from the Reaction $^3H + {}^3H \rightarrow {}^4He + n + n$. As yet, the reaction $^3H + {}^3H \rightarrow {}^4He + n + n$ has been used to determine a_{nn} only twice.[72,73] Although as many as six nucleons participate in this reaction, the theoretical interpretation must be simpler than in the case of the reactions with the participation of five nucleons discussed above. The particles in the initial state are identical. Therefore, only the singlet interaction is possible. Thus, the spins of the initial and final states are uniquely determined. If the kinds of particles in the final state are fixed by the experimental conditions, it is sufficient to consider only the two pole graphs that describe the possible reaction mechanisms (Fig. 12). The graph of Fig. 12(a) corresponds to the interaction of two neutrons in the final state. The graph of Fig. 12(b) represents a reaction taking place through

TABLE 6. Results of Measurements of a_{nn} by Means of the Reaction $^3H + {}^3H = {}^4He + n + n$

E_t, MeV	Angle, deg	Experimental method	Theory	a_{nn}, F	r_{nn}, F	Literature
22	5	Incomplete	MW	−16.69±0.51	2.84 *	[72]
	8	»	MW	−17.4±1.8	2.4±1.5 **	
		»	comp.			
1.39	10	Complete	MW	−18.1±0.8	2,84	
		»	BA	−15,0±1.0	2.7	[73]

* $r_{nn} = r_{pp} = 2.84$
** Both parameters a_{nn} and r_{nn} are included in the fitting procedure

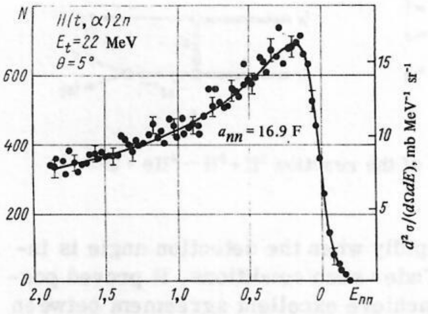

FIG. 13. Spectrum of ^{4}He in the reaction $^3H+^3H \to {}^4He+2n$.[72] The relative energy of the neutrons is plotted along the abscissa.

the intermediate formation of the unstable system ^{5}He. It is well known that the ground state of ^{5}He is a $p_{3/2}$ state with energy 0.95 MeV. This means that as a result of the interaction between the neutron and the α particle at relative energy 0.95 MeV there must appear a maximum of the cross section. The first excited level of ^{5}He is a $p_{1/2}$ state at 2.5 MeV with width 1.5 MeV. In the interpretation of the spectra, it must be borne in mind that the excitation of these ^{5}He states can distort the FSI maximum of two neutrons. In Table 6 the main data and the results of two measurements of a_{nn} by means of the reaction $^3H+^3H$ are given.

In Ref. 72, Gross *et al.* made kinematically incomplete measurements of the ^{4}He spectra of the reactions $^3H(^3H, {}^4He)2n$, $^3He(^3H, {}^4He)np$, and $^3He(^3He, {}^4He)pp$. All the spectra could be well described by Migdal-Watson theory (Fig. 13). The values found for the np and pp scattering lengths were $a^S_{np} = -21.5 \pm 2.3$ F and $a_{pp} = -7.52 \pm 0.22$ F, in good agreement with the results of direct measurements. In the fitting of the theory to the

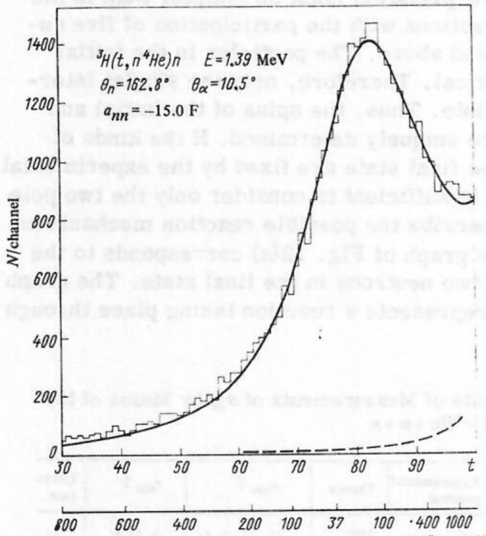

FIG. 14. Spectrum of the reaction $^3H+^3H \to {}^4He+2n$ obtained in the kinematically complete experiment of Ref. 73. The figure shows the projection of the two-dimensional spectrum onto the neutron time-of-flight axis. The relative energy of the two neutrons is plotted along the lower abscissa.

TABLE 7. Results of measurements of a_{nn} by means of the reaction $\pi^- + d \to n+n+\gamma$.

E_γ, MeV*	Experimental method	Theory	a_{nn}, F	r_{nn}, F	Literature
—	Incomplete	Watson[81]	No dineutron	—	[75]
—	»	The same	$-15.9^{+7.4}_{-\infty}$	2.65	[76]
120–131.5	»	» »	$-15.1^{+2.5}_{-3.3}$	2.65	[77]
122–131.5	»	» »	$-19.1^{+3.8}_{-5.9}$	2.65	—
124–131.5	»	» »	$-19.0^{+4.0}_{-6.6}$	2.65	—
126–131.5	»	» »	$-17.0^{+4.3}_{-7.7}$	2.65	—
125–131.5	»	Bander[82]	$-13.1^{+2.4}_{-3.4}$	—	[21]
120–131.5	»	The same	$-11.2^{+1.9}_{-2.6}$	—	
—	Complete	Bander[82]	-16.4 ± 1.3	2.65	[78]
—	»	The same	-18.42 ± 1.53	2.65	[79]**
—	»	» »	-16.4 ± 1.6	2.65	[80]**

* Energy range of γ rays for which fitting was made
** Evaluation of data of the same experiment of Haddock *et al.*[78]

experimental spectra, the correlation between a_{nn} and the effective range r_{nn} was important. Therefore, Gross *et al.* determined a_{nn} for different values of r_{nn}. In addition, they used the comparison procedure, determining a certain "correction" factor from the spectrum of the reactions $^3He(^3He, {}^4He)pp$ and applying this factor to find a_{nn}. The results of Ref. 72 are given in Table 6.

The only kinematically complete experiment on the reaction $^3He(^3He, {}^4He)pp$ was made by Kühn *et al.*[73] The neutron spectrum was measured by the time of flight. The kinematic parameters satisfied the conditions of the FSI mechanism optimally. In the two-dimensional spectrum there was a fairly strong peak from the ground state of ^{5}He (the graph of Fig. 12b) at the appropriate position. The tail of this peak evidently affected the FSI peak of the two neutrons. To understand the influence of the effect of formation of ^{5}He on the determination of the scattering length, the reactions $^3H(^3He, p^4He)n$ and $^3He(^3He, p^4He)p$ were studied under analogous kinematic conditions. In the reaction $^3He(^3He, p^4He)p$ at the position of ^{5}He the nucleus ^{5}Li was formed in the ground state at relative energy 1.95 MeV. From the two-dimensional spectra of these reactions the scattering length $a_{pp} = -7.6 \pm 0.6$ F and $a_{np} = -21 \pm 3$ F were determined. Migdal-Watson theory was used, and allowance was made for the effect of ^{5}He and ^{5}Li, respectively, on the profile of the FSI peak. It was found that the profile of the tail of the ^{5}He and ^{5}Li peak did not significantly affect the determination of the scattering length. Only the amplitude of the tail was important, and this could be included in the fitting procedure as an additional parameter. The same approach was used to determine a_{nn}. The projection of the two-dimensional spectrum onto the time-of-flight axis of the neutrons is shown in Fig. 14. The continuous curve is the result of fitting based on Migdal-Watson theory; the dashed curve is the relatively small ^{5}He admixture. In addition, the spectrum was calculated by the Born approximation. To within the experimental errors, the results of the two interpretations agreed. The good agreement between experiment and theory evidently justifies the approach

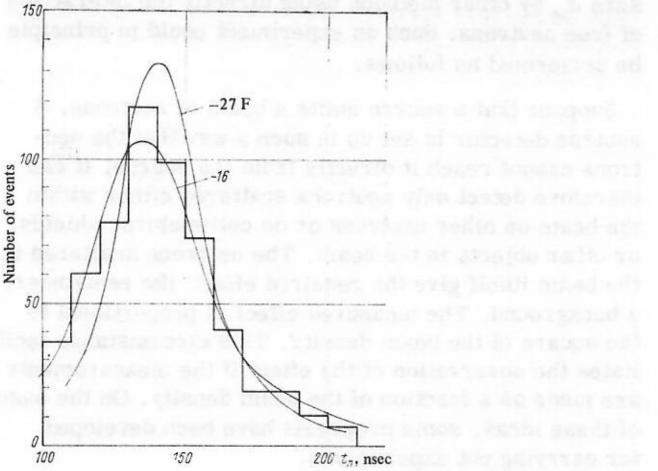

FIG. 15. Spectrum of neutrons from the reaction $\pi^- + d \to 2n + \gamma$ (Ref. 78). The smooth curves are the results of calculations with the parameters $a_{nn} = -16$ F and $a_{nn} = -27$ F.

and confirms the validity of the values obtained for a_{nn} (see Table 6).

In both investigations it was shown that Migdal-Watson theory describes the experimental spectra well. The correction in the case of Ref. 73 for the influence of ^{5}He production on the FSI maximum was insignificant and completely justified. The interpretation of these experiments was comparatively clear and unambiguous. Therefore, the results obtained from the analysis of the reaction $^3H + {}^3H$ can be assumed to be among the most reliable values of a_{nn}.

Determination of a_{nn} from the Reaction $\pi^- + d \to \gamma + n + n$. The determination of a_{nn} by means of observation of final-state interaction of neutrons from the reaction $\pi^- + d \to \gamma + n + n$ is frequently regarded as the most reliable method. In this reaction there are only two strongly interacting particles in the final state. Therefore, the description of the final state deals solely with the interaction of two neutrons, so that the theoretical uncertainty of the result is here least. On the other hand, the experimental difficulties are rather serious. The intensity of the π^- beams at the disposal of the experimenters have so far been weak. As a result, the statistics of all measurements so far made by this method are unsatisfactory.

Attempts to measure a_{nn} by means of the reaction $\pi^- + d$ have been made in several incomplete experiments and in one complete one. In the incomplete measurements, only the spectra of the γ rays were measured; in the complete one, the spectra of the gamma rays and both neutrons in coincidence were measured.

The reaction was discovered by Panofsky and his collaborators in 1951 by measuring the γ spectrum of this process.[71] In the same year they discussed for the first time the possibility of extracting information on the interaction between two neutrons.[75] But because of the inadequate resolution of the γ spectrum that they used, it was only possible to conclude that the probability of formation of a dineutron with positive binding energy was less than 25% of the total reaction cross section. The method of measuring γ spectrum was subsequently improved elsewhere.[76,77] The results of these investigations are given in Table 7. It can be seen that the spread of the values obtained and the experimental errors are comparatively large. In addition, the result depends on the energy range of the γ spectrum used to determine a_{nn}.[77]

The kinematically complete experiment of Haddock *et al.*[78] was widely acclaimed and is regarded as one of the most important and successful measurements of a_{nn}. The π^- mesons are incident on a target of liquid deuterium. A meson is captured at zero energy, and

TABLE 8. Comparison of Results of Measurements of a_{nn}*

Reaction	Experimental Method	Theory	a_{nn}, F	$\pm\Delta a_{nn}$, F	Literature
$d(n, p)2n$	Incomplete	BA	−22.0	2	[23]
	The same	BA	−21.7	1	[24]
	» »	Graph.	−23.6	+2 −1.6	[25]
	» »	IA	−14.0	3	[37]
		BA	−16.0	3	
	» »	IA	−15.9	1.1	[32]
		MW	−30.0	—	
	» »	[46]	−23.78	—	[38]
	» »	MW comp.	−16.7	+2.0 −3.0	[34]
	» »	BA	−16.2	2.2	[36]
	» »	Graph.	−23.2	+1.8 −1.9	[26]
	» »	Graph.	−23.0	+3.6 −4.2	[27]
	» »	IA	−21.7	1.2	[33]
	» »	IA	−19.3	0.8	[30]
	» »	[31]	−18.31	1.0	[30]
	Complete	BA	−25.0	—	[39]
	»		−18.8	+5.5 −11.9	[35]
	»	MW	−16.4	+2.6 −2.9	[42]
	»	Graph.	−25.0	3.0	[48]
	»	[31]	−23.78	—	[40]
	»	[46]	−16.0	1.2	[45]
		MW	−16.8	1.3	—

Reaction	Experimental method	Theory	a_{nn}, F	$\pm\Delta a_{nn}$, F	Literature
$d(n, p)2n$	Complete	MW	−17.1	0.8	[47]
	»	[5]	−16.1	0.9	[43]
$^3H(n, d)2n$	Incomplete	BA	−18.0	3	[50]
	The same	MW	−17.0	2	[51]
	» »	BA	−14.7	6	[52]
$d(d, pp)2n$	Complete	?	−15.5	1.1	[57]
$^3H(d, {}^3He)2n$	Incomplete	MW comp.	−16.1	1	[58]
	The same	MW comp.	−16.5	1	[59]
	» »	BA comp.	−18.4	1.4	[60]
	» »	MW comp.	−10.9	—	[61]
	» »	MW comp.	−15.5	1.1	[63]
			−11.3	0.8	
			−12.7	0.8	
	Complete	MW	−16.0	1.0	[62]
	»	MW	−16.2	1.2	[64]
$^3H(t, {}^4He)2n$	Incomplete	MW	−16.69	0.51	[72]
		MW comp.	−17.4	1.8	—
			−18.1	0.8	—
	Complete	MW, BA	−15.0	1.0	[73]
$d(\pi^-, \gamma)2n$	Incomplete	[81]	−15.9	+7.4 −∞	[76]
	»	[81]	−17.5	4.8	[77]
	»	[82]	−12.2	2.6	[21]
	Incomplete	[82]	−16.4	1.3	[78]
	»	[82]	−18.42	1.53	[79]
	»	[82]	−16.4	1.6	[80]

*** In the calculation of the mean value a_{nn} the results obtained from one experiment by different theoretical methods were taken into account as independent results. If one experiment gave rise to several publications with different results differing only by different evaluation of the experimental data, only the last result was taken into account.**

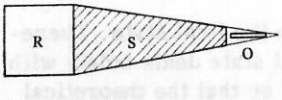

FIG. 16. Measurement of a_{nn} by means of a reactor in space (Ref. 85): R is the reactor, S the shield, D the detector.

the γ ray and the neutron pair depart in one plane. The momentum of the γ ray is opposite to the total momentum of the neutrons. It is clear that the FSI condition is satisfied best at small angles between the directions of the neutrons. These conditions dictated the geometry of the experiment. The apparatus made it possible to measure the energy and the angles of departure of the γ ray and both neutrons. Measurements were made of the one-dimensional spectra of the neutrons under the condition of a coincidence between the γ ray and both neutrons with a definite angle between the neutrons. One of the spectra obtained is shown in Fig. 15. The first evaluation of the experimental data gave $a_{nn} = -16.4 \pm 1.3$ F. Later, one of the group, Nygren, reevaluated the data[79] and obtained $a_{nn} = -18.42 \pm 1.53$ F. At the conference on few-nucleon systems at Los Angeles in 1972 Haddock's group published the final result $a_{nn} = -16.4 \pm 1.6$ F of a repeated analysis of the data given in Ref. 80. The many years of work on the evaluation of the experiment indicate the great difficulties in this procedure.

The experimental spectra of Refs. 75–77 were interpreted theoretically on the basis of the theory of Watson and Stuart[81] which was a forerunner of the general FSI theory of Migdal and Watson. A different theory based on the impulse approximation was developed by Bander.[82] In this theory one takes into account not only the first-order impulse approximation but also terms corresponding to multiple scattering. It was shown that processes of higher order can be ignored at the level of experimental accuracy at present achieved. Bander's theory was used in Refs. 21 and 78–80.

Shklyarevskiĭ[83] described the reaction $\pi^- + d$ on the basis of the nonrelativistic graph technique. He took into account the pole graph and the triangular graph. The contribution of the infinite series of multiple rescattering graphs was found to be negligibly small. The result of his calculations is very close to Bander's results.

Comparison of the results of these papers shows that the accuracy of kinematically incomplete measurements is not satisfactory. The potential possibilities of exact determination of a_{nn} by means of the reaction $\pi^- + d$ in kinematically complete experiments have evidently not yet been exhausted, and it would therefore appear to be desirable to repeat such an experiment on pion beams of higher intensity.

Proposals for Measuring a_{nn} by Other Methods. Above, an attempt was made to review all known results of measurements of the scattering length a_{nn} on the basis of the final-state interaction of two neutrons in nuclear reactions. The principal shortcoming of this method is the imperfect theoretical description of the reaction. It would therefore be very desirable to measure a_{nn} by other methods using directly the interaction of free neutrons. Such an experiment could in principle be performed as follows.

Suppose that a source emits a beam of neutrons. A neutron detector is set up in such a way that the neutrons cannot reach it directly from the source. It can therefore detect only neutrons scattered either within the beam on other neutrons or on collimators, shields, or other objects in the beam. The neutrons scattered in the beam itself give the required effect; the remainder, a background. The measured effect is proportional to the square of the beam density. This circumstance facilitates the observation of the effect if the measurements are made as a function of the beam density. On the basis of these ideas, some proposals have been developed for carrying out experiments.

Moravcsik[84] suggested that an underground nuclear explosion could be used for this purpose. According to his estimates, the attainable error in the neutron-neutron scattering cross section in the energy range from 20 keV to 2 MeV is approximately 10%. Although it is impossible to obtain neutron fluxes greater than from nuclear explosions, the difficulties in the carrying out of such an experiment are obvious.

A different experiment was proposed by Bondarenko *et al.*[85] Here the neutron source is a reactor launched into space. Thus, one realizes the most favorable conditions for suppressing the background. The arrangement of the reactor, detector, and shields is shown in Fig. 16. The neutron counting rate in the detector can be represented by

$$N = \varkappa Q^2 \bar{\sigma}_{nn} s \bar{\epsilon},$$

where Q is the number of neutrons emitted by the reactor; $\bar{\sigma}_{nn}$ is the neutron-neutron scattering cross section averaged over the energy; s is the area of the detector; $\bar{\epsilon}$ is the mean efficiency of the detector; and $\varkappa$ is a factor that depends on the neutron spectrum of the reactor, the angular distribution of the neutrons, and the geometry of the apparatus. It is easy to see that the determination of the factor $\varkappa$ is not a simple problem and that the accuracy of the result depends on its solution.

Such an experiment is the purest and clearest method of measuring the neutron-neutron scattering cross section that one could conceive of. Bondarenko *et al.* made detailed estimates of the effect and the background as a function of the power of the reactor and the other parameters of the experiment. Assuming that the neutron flux from the reactor under a pulsed regime can reach $Q = 8.6 \cdot 10^{17}$ neutrons in one burst, the effect must have an order of magnitude of 100 neutrons/pulse.

Dickinson *et al.*[86] have analyzed in detail the measurement of a_{nn} by means of a pulsed reactor with flux density $\Phi_n = 0.9 \cdot 10^{17}$ neutrons cm^{-2} sec^{-1}. In this plan, the detector is directed along the vacuum axial channel of the reactor, where the neutrons are scattered on other neutrons. Under certain geometrical conditions of the experiment, the neutron detector should count 275 neutrons per pulse from the reactor. The neutron-neutron scattering amplitude can be determined with an error as low as ±2–3%.

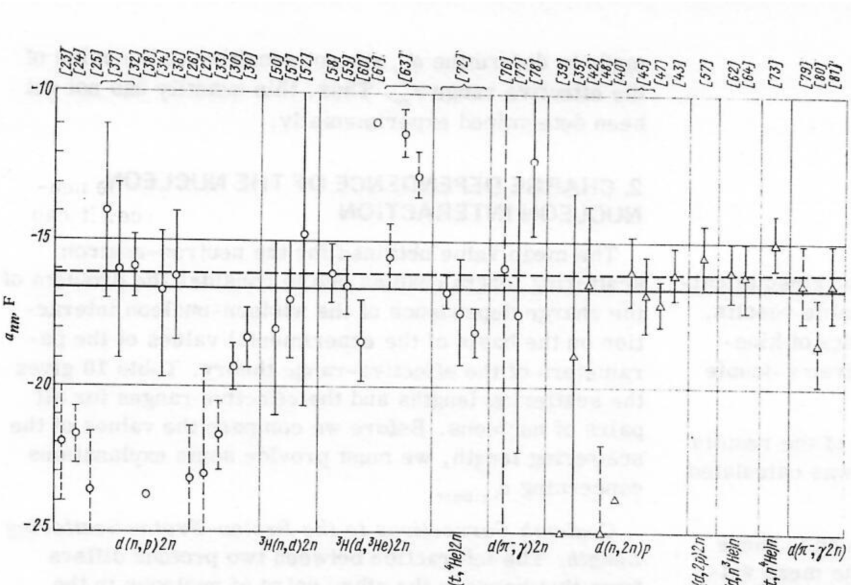

FIG. 17. Results of measurements of a_{nn}: — mean value $\bar{a}_{nn}$ obtained by weighted averaging of the results by variant 3; - - - weighted average of only the kinematically complete measurements.

As an addition to the other indirect measurements of a_{nn}, such experiments of a direct type would be good. The accuracy of such experiments should be at the least not worse than ± 1 F. The implementation of the experiments proposed by Bondarenko *et al.* and also by Dickinson *et al.* depends mainly on the realization of powerful pulsed reactors, which as yet are not at the disposal of experimenters.

Comparison of the Existing Experimental Results. Let us here compare the experimental results and obtain a mean value of a_{nn}, which is to be regarded as the most reliable value of a_{nn}. In Table 8 the known results of determining a_{nn} are given. The results are represented graphically in Fig. 17.

The measurements were made in different ways in different reactions. In the interpretation of the experiments, very diverse theoretical methods were used. Since 1961, a large number of data have been obtained. It is to be hoped that a reliable mean value $\bar{a}_{nn}$ can be extracted from them. Of course, not all results are of equal value. Some must be excluded in the determination of the mean value. These cases have been considered above.

Let us first verify whether the set of results for a_{nn} chosen for the determination of the mean value $\bar{a}_{nn}$ (see Table 8) corresponds to the conditions of the normal distribution. One can assume that the results a_{nni} were obtained independently of one another. Figure 18 shows the sequence of results. The ordinate scale is chosen in such a way that

$$I=\frac{1}{s\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\exp\left[-(a_{nn}-\bar{a}_{nn})^2/(2s^2)\right]da_{nn},$$

i.e., the integral of the normal distribution is transformed into a straight line. It can be seen that the sequence of values a_{nni} fits well onto a straight line. The upward kink near $a_{nn}=16$ F indicates that the maximum of the distribution is sharper than a normal distribution's.

To calculate the mean value, it is necessary to take into account the different significance of the individual results, which depends on the experimental accuracy, the reliability of the method of measurement, and the reliability of the theoretical interpretation. These different factors cannot be taken into account numerically without a certain arbitrariness. Therefore, the mean value $\bar{a}_{nn}$ was calculated in different ways:

1) It was assumed that all results have the same weight.

2) It was assumed that p_i of a result is equal to the reciprocal of the square of its error: $p_i \sim 1/\Delta a^2_{nni}$. (Here we ignore the fact that the errors given by different authors do not have a uniform nature. Some authors give only statistical errors, some include also systematic experimental errors, and in some cases theoretical uncertainties are also included).

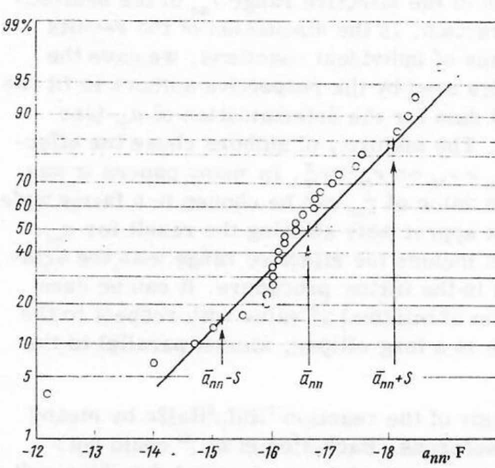

FIG. 18. Normal distribution verification of the sequence of results of measurements of a_{nn}. The sum of the weights of the results $\sum_{i=1}^{i} 1/\Delta a^2_{nni}$ to measurement i is plotted along the ordinate (100% corresponds to the sum of the weights of all measurements; (Δa_{nni} is the error of the result)

TABLE 9. Mean values of the scattering length a_{nn}.

Variant	1	2	3	4
$\bar{a}_{nn}$	−16.50	−16.80	−16.61	−16.15

3) The discussions above have shown that kinematically complete measurements give more reliable results, and therefore in the third variant the results of kinematically complete measurements were given a double weight.

4) Finally, the weighted mean value $\bar{a}_{nn}$ of the results of kinematically complete measurements was calculated separately.

Table 9 gives the mean values $\bar{a}_{nn}$ obtained in these ways. It can be seen from the table that the mean values $\bar{a}_{nn}$ found by the variants 1, 2, and 3 agree with a spread less than ±1%. The mean value $\bar{a}_{nn}$ obtained from the kinematically complete measurements (variant 4) is appreciably lower than the value obtained from the complete set of measurements.

An additional weight of the kinematically complete measurements was taken into account in variant 3. I regard the result of this variant, $\bar{a}_{nn} = -16.61$ F, as the currently most reliable value of the neutron-neutron scattering length. The standard deviation of the given distribution of a_{nni} values is $S = \pm 1.45$ F.

The exact standard distribution corresponding to the values obtained for $\bar{a}_{nn}$ and S is shown in Fig. 18 by the straight line. It is shifted slightly to the right of the true distribution. This is due to the mean value $\bar{a}_{nn}$, which was found subject to the condition of a double weight of the kinematically complete measurements. Finally, one can calculate the value -16.61 ± 0.54 F for the 95% confidence interval of this mean value. This confidence interval includes the mean value obtained from the kinematically complete measurements.

We should also make some remarks concerning the determination of the effective range r_{nn} of the neutron-neutron interaction. In the discussion of the results found by means of individual reactions, we gave the effective range used by the respective authors to fit the experimental data for the determination of a_{nn} (see Tables 3–6). The majority of authors chose the effective range $r_{nn} = r_{pp}$ or $r_{nn} = r^S_{np}$. In many papers it was noted that the value of r_{nn} can be chosen in a fairly wide range without appreciably altering the result for a_{nn}. Some authors include the effective range with the scattering length in the fitting procedure. It can be seen that the region of minimal χ^2 value with respect to the $r_{nn} - a_{nn}$ plane is a long ellipse, almost parallel to the r_{nn} axis.

In their study of the reaction $^3H(d, {}^3He)2n$ by means of 83-MeV deuterons, Bachelier *et al.*[63] could not succeed in interpreting the experimental data by a suitable choice of the parameters r_{nn} and a_{nn} (see Table 4). The results of Ref. 63 were not taken into account in the determination of the mean value $\bar{a}_{nn}$.

Finally, we can establish that the experiments so far made to determine a_{nn} are not sensitive to the value of the effective range r_{nn}. Thus, this quantity has not yet been determined experimentally.

2. CHARGE DEPENDENCE OF THE NUCLEON-NUCLEON INTERACTION

The mean value obtained for the neutron-neutron scattering length enables one to consider the problem of the charge dependence of the nucleon-nucleon interaction on the basis of the experimental values of the parameters of the effective-range theory. Table 10 gives the scattering lengths and the effective ranges for all pairs of nucleons. Before we compare the values of the scattering length, we must provide some explanations concerning $a_{nn(\mathrm{theor})}$.

Coulomb Corrections to the Proton-Proton Scattering Length. The interaction between two protons differs from that between the other pairs of nucleons in the first place by Coulomb forces. Therefore, the values of the scattering length cannot be directly compared with the scattering lengths of the other nucleon pairs. The problem of subtracting the scattering length for protons without charge, i.e., for neutrons, on the basis of the experimental scattering lengths a_{pp} has been considered on many occasions. The complete necessary information on this subject can be found, for example, in the paper of Sher, Signell, and Heller.[88] Apart from the ordinary Coulomb forces, the correction includes the effect of vacuum polarization, interaction between the charge and the magnetic moment, and the interaction between the magnetic moments. The finite sizes of nucleons are also taken into account. An additional correction refers to the difference between the proton and neutron masses. After these corrections, one obtains the purely nuclear interaction, as it is expected between two neutrons. Sher *et al.* made detailed calculations for very different forms of the nuclear potentials. They used the Hamada-Johnston potential and various continuous potentials with hard core and a potential with soft core.[90] The value obtained for the scattering length depends to a very small degree on the form of the potential. The uncertainty in the value of $a_{nn(\mathrm{theor})}$ lies within the range ± 0.2 F.

Charge Symmetry of the Nucleon-Nucleon Interaction. The values of $a_{nn(\mathrm{theor})}$ and $a_{nn(\mathrm{exp})}$ give in Table 10 indicate a possible small breaking of the charge symmetry of the nucleon-nucleon interaction. The two values agree within the limits of the errors. However, in the subtraction of the Coulomb correction only phenomenological potentials of local type were used. It is known however that one cannot exclude contributions of a nonlocal character in the nuclear interaction. The question

TABLE 10. Parameters of the effective-range theory for the $^1S_0$ state.

Scattering length, F	Effective range, F	Literature
$a^S_{np} = -23.715 \pm 0.0015$	2.73 ± 0.03	[87]
$a_{pp} = -7.823 \pm 0.01$	2.794 ± 0.015	[87]
$a_{nn\ \mathrm{theor}} = 17.06 \pm 0.2$	2.84 ± 0.03	[87–89]
$a_{nn\ (\mathrm{exp})} = 16.61 \pm 0.54$	?	

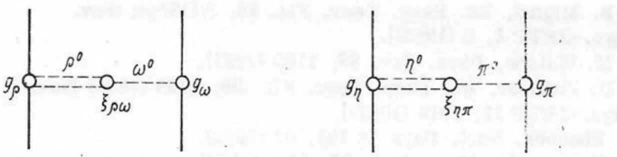

FIG. 19. Breaking of charge symmetry due to mixing of the π^0 with the η^0 meson and the ω^0 with the ρ^0 meson.

arises of how such contributions affect the result of the Coulomb correction. This problem was studied by Kumpf,[91] who showed that the nonlocal part in the nuclear potential increases the absolute value of $a_{nn(\text{theor})}$ as a function of the form factor of the nonlocal part. The increase can take values from zero to a few Fermis. Thus, the difference between $a_{nn(\text{exp})}$ and $a_{nn(\text{theor})}$ increases, and the indication of a possible breaking of charge symmetry becomes more substantiated.

A possible breaking of charge symmetry following from a discrepancy between $a_{nn(\text{exp})}$ and $a_{nn(\text{theor})}$ means that the interaction between two protons is slightly stronger than between two neutrons. This conclusion contradicts the results of the treatment of the Coulomb energies of mirror nuclei. For example, the binding energy of tritium is 0.764 MeV greater than that of ^{3}He. If charge symmetry does hold, this energy corresponds to the Coulomb energy of ^{3}He. The Coulomb energy of ^{3}He can be determined from the charge form factor[92] or from the wave function of this nucleus.[93] The two methods give the same result independently, namely, 0.64 ± 0.02 MeV. If one believes these data, the purely nuclear interaction leads to a binding energy of the nucleons in ^{3}He that is 0.12 MeV lower than in tritium. This means that the forces between two protons are slightly less than between two neutrons.

When considering this disagreement, one must bear in mind that the scattering lengths contain information about the behavior of the nuclear forces only in the $^1S_0$ state of two nucleons. But the wave functions of the nuclei include other states. Since we are here concerned with fairly subtle effects, one could imagine that the breaking of charge symmetry in the other states has the opposite sign to that of the state $^1S_0$. But at the present time there are neither experimental nor theoretical indications about this.

Henley and Keliher[94] and also Okamoto and Pask[93] investigated possible reasons for a breaking of charge symmetry on the basis of the meson theory of nuclear forces. If identical nucleons exchange only uncharged mesons (π^0 and ρ^0), their interaction is charge symmetric. Breaking of charge symmetry in exchange forces arises as a result of mixing of a π^0 meson with an η^0 meson or of a ρ^0 meson with an ω^0 meson (Fig. 19). Mixing of ρ^0 and ω^0 mesons is observed experimentally. Since the ρ^0 and ω^0 and also the π^0 and η^0 mesons have different isospins, the mixing of these particles leads to a violation of isospin conservation. The amplitudes corresponding to the diagrams of Fig. 19 lead to a correction to the nucleon-nucleon potential that is asymmetric under the replacement of protons by neutrons and vice versa. In Ref. 94 the change of the scattering length due to this effect was estimated. It was found that the order of magnitude of such a correction can completely explain the difference between $a_{nn(\text{exp})}$ and $a_{nn(\text{theor})}$. However, the sign of this correction as yet remains undetermined, since the sign of the coupling constant ratio g_ρ/g_ω is unknown.

Violation of the Charge Independence of the Nucleon-Nucleon Interaction. Whereas the breaking of charge symmetry is small and experimentally not yet a fully confirmed effect, there is no doubt that the charge independence is violated. The discrepancy between the *np* and *nn* scattering lengths far exceeds the limits of the experimental and theoretical errors. The first attempt to explain this discrepancy was made by Schwinger[95] on the basis of direct electromagnetic effects. Assuming the presence of a nuclear potential of Yukawa type and point distributions of the charge and magnetic moment in the nucleons, he was able to describe completely the difference between the values of $a_{nn(\text{theor})}$ and a^S_{np}. Later it became known that the charge and magnetic moment of the nucleons do not have a point distribution but one with a radius of about 0.8 F. In addition, it became clear that the nuclear potential has a hard core. Under these conditions the electromagnetic effects become small corrections which partly compensate one another.[96] At the present time, the difference between the *np* and *nn* interactions is explained by indirect electromagnetic effects, i.e., effects which are directly related to the nuclear interaction but undergo definite modifications depending on the charge state of nucleons and mesons.

The most important effect of this type is the difference between the masses of charged and uncharged π and ρ mesons. The forces between identical nucleons (*nn*, *pp*) differ from the forces between different nucleons (*np*) by the fact that identical nucleons exchange only uncharged π^0 and ρ^0 mesons, whereas in the case of the *np* interaction charged mesons ($\pi^\pm$, $\rho^\pm$) can also be exchanged. The difference between the masses of the charged and uncharged meson leads to a difference between the exchange forces. By means of the difference between the masses of the $\pi^\pm$ and π^0 mesons[97]

$$\Delta m(\pi) = 4.6043 \pm 0.0037 \text{ MeV}$$

one can explain about half the difference Δa between a^S_{np} and a_{nn}, i.e., $\Delta a(m(\pi)) \approx -3.5$ F. At the present time it is expected that the other half of the discrepancy can be explained by the difference between the masses of the charged and uncharged ρ-mesons, which are partly responsible for the short-range part of the nuclear interaction. The difference between the masses of the $\rho^\pm$ and ρ^0 mesons has not yet been established experimentally. According to the most recent data,[97] the masses of the ρ mesons are

$$m(\rho^{\pm,0}) = 770 \pm 5 \text{ MeV}$$

The experimental error does not enable one to detect a difference. But to explain the remaining discrepancy $\Delta a(m(\rho))$ one only requires $\Delta m(\rho) \approx -2$ MeV. The negative sign means that $m(\rho^0) > m(\rho^\pm)$. This difference is compatible with the experimental data and field-theory calculations.

Apart from effects due to the meson mass difference, the influence of other indirect electromagnetic effects has also been studied, i.e., radiative corrections to the meson-nucleon coupling constants. Studies have also been made of mixing of mesons due to electromagnetic forces, the simultaneous exchange of mesons and photons, and the different lifetimes of the charged and uncharged mesons. Compared with the meson mass difference, these effects are not so important, and therefore they can be ignored. The effects of breaking of charge symmetry and charge independence are discussed in more detail by Henley.[98]

CONCLUSIONS

In this review we have considered methods of measuring the neutron-neutron scattering length and discussed the results of such measurements on the basis of allowance for the interaction between two neutrons in the final state. We have compared the results obtained by means of different nuclear reactions and on the basis of different theoretical approaches. Although many questions in the description of three-particle reactions remain unresolved, one can say that the majority of the values obtained agree within the limits of the errors. It should however be pointed out that the values of a_{nn} in the region of -23 F cannot be regarded as reliable. These results, like some others, were ignored in the determination of the weighted mean scattering length $\overline{a}_{nn}$. The experiments so far made do not enable one to determine the effective range of the nn interaction.

The mean value $\overline{a}_{nn}$ obtained can be regarded as sufficiently reliable to draw a conclusion concerning the violation of charge independence and the breaking of charge symmetry of the nucleon-nucleon interaction. The difference between the np and nn scattering lengths, $\Delta a = a_{np} - a_{nn} \approx -7$ F, can be regarded as definitely established. It indicates that the force between the neutron and the proton in the $^1S_0$ state is approximately 2.5% greater than between two neutrons. This difference can be mainly explained by indirect electromagnetic effects, namely, the difference between the masses of charged and uncharged π and ρ mesons. With regard to the breaking of charge symmetry, definite conclusions cannot be drawn with the same certainty. First, the subtraction of $a_{nn(\mathrm{theor})}$ from the experimental value of a_{np} entails uncertainties because of the nonlocal contributions to the nuclear potential and, second, the accuracy of the mean value of a_{nn} is not yet satisfactory. A possible breaking of charge symmetry could be a consequence of mixing of mesons with different isospins.

The knowledge of the exact value of the neutron-neutron scattering length enables one to detect fairly subtle effects concerning the nature of nuclear forces. A further increase in the accuracy of the measurements requires not only an improvement of the experimental techniques but also the perfection of the theoretical description of three-particle reactions. It would be very desirable to make a direct measurement of a_{nn} in beams from powerful neutron sources.

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Translated by Julian B. Barbour