Problems of emission and reception of gravitional waves

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The present status of the emission and reception of gravitational waves is reviewed. The emission, propagation, and interaction of gravitational waves with matter and with an electromagnetic field are physically described and mathematically formulated. The experiments performed on the search for gravitational waves of astrophysical origin are analyzed. Laboratory and cosmic sources of these waves and methods of their reception are described. Particular attention is paid to the proposed versions of the complete laboratory experiment.

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INTRODUCTION

The problem of emission and reception of gravitational waves is one of the most urgent and fundamentally important problems of the relativistic theory of gravitation and of all of modern physics. The interest in these problems has increased particularly in recent years, when the experimentalists obtained the first promising report of observable signals of astrophysical origin, which could be interpreted as gravitational waves. These communications have led to an extensive expansion of the theoretical and experimental research and by now more than 20 experimental groups in the world are carefully searching for gravitational radiation from outer space.

Reliable experimental proof of the very existence in nature of energy-carrying gravitational waves would serve as a strong foundation for the physical evaluation of many presently competing theoretical models in general relativity theory, and would help select those which agree better than others with reality. The discovery of gravitational waves would also provide a reliable basis for the development of the theory of quantization of gravitational fields. Gravitational radiation could find extensive use also in the solution of certain fundamental applied problems, such as the mastery of gravitation-wave astronomy and new communication channels. All modern experiments on gravitational waves are based on shaky theoretical ground. The problem is that in the relativistic theory of gravitation there is no clear-cut invariant definition of the concept of gravitational waves. Nor is the question of the definition of physically observable quantities completely clear. Yet this is in essence the basis of many experimental investigations. This situation in general relativity is due to the complexity of the nonlinear structure of Einstein's equations in the absence of a generally covariant expression for the energy of the gravitational field. It is obvious that the presence of gravitational radiation in space should not be connected mathematically with the choice of the coordinate system or, in other words, with the character of motion of the observer, as is the case of true physical quantities. However, in view of the equivalence principle, there is no covariant energy-momentum tensor in general relativity, and the employed energy-momentum pseudotensor depends entirely on the choice of the coordinate system. It follows therefore that the gravitational field at any point can be "annihilated" or "made" arbitrarily large by merely transforming the coordinates. The situation is alleviated somewhat by the fact that for weak gravitational waves the field equations admit of a well-founded linearization, for which the obtained wave equations point unequivocally to the existence of gravitational waves in nature. These waves are perturbations of the gravitational field detached from a source propagating in space with the fundamental velocity. The gravitational waves carry with them an energy that cannot be made to vanish by some coordinate transformation. In the linearized theory, the energy-momentum pseudotensor acquires the property of a true symmetrical tensor, with the aid of which it is possible to calculate rigorously all the energy characteristics of the gravitational radiation.

The main purpose of the present review is, on the one hand, to present in concise form the contemporary physical picture and mathematical description of the radiation processes, of the propagation and interaction of gravitational waves with matter and with an electromagnetic field, and on the other hand to discuss the published results on the experimental searches for gravitational radiation and to analyze new ideas in this field.

Gravitation has already been the subject of several reviews and monographs in the world's literature, but in this field of research every day brings new data and new ideas, both theoretical and experimental. Particularly many new proposals and experimental approaches to the solution of the problem of gravitational waves have been published in the last three years, and their exposition is urgently needed.

GRAVITATIONAL-FIELD EQUATIONS

From the least-action principle for the gravitational field and the material field it follows that[1,2]

$$R_{ik}-0.5g_{ik}R=\frac{8\pi G}{c^4}T_{ik}, \tag{1}$$

or, in mixed components

$$R_i^k-0.5R\delta_i^k=\frac{8\pi G}{c^4}T_i^k. \tag{2}$$

Equations (1) and (2) are the basic equations of general relativity theory, and were first derived by Einstein[1] in 1916. They determined the connection between the curvature of space-time and the distribution in it of matter and of fields. In these equations, R is the scalar curvature of space-time, R^k_{ik} is the Ricci tensor, with $R = g^{ik}R_{ik} = g^{il}g^{km}R_{iklm}$; G is the gravitational constant,[1] equal to 6.67×10^{-8} cm^{3}g^{-1}sec^{-2}; T_i is the material energy-momentum tensor, including matter and all types of fields, with the exception of energy and momentum of gravitational origin; δ^k_i is a unit tensor whose components are $\delta^k_i = 0$ at $i \neq k$ and $\delta^k_i = 1$ at $i = k$; R_{iklm} is the Riemann curvature tensor; g_{ik} is the metric tensor, which in general relativity has the meaning of the potentials of the gravitational fields.

Multiplying (2) by g_{ik} and contracting, we obtain

$$R = -\frac{8\pi G}{c^4} T, \tag{3}$$

where $T = T^i_i$. Summation over dummy indices is implied throughout. Taking (3) into account, Eq. (2) can now be represented in the form

$$R_{ik} = \frac{8\pi G}{c^4} (T_{ik} - 0.5 g_{ik} T). \tag{4}$$

Equation (4) is nonlinear, so that the superposition principle is generally not valid for gravitational fields. Only for weak fields does the equation become linear in first-order approximation and the superposition principle hold. In empty space $T_{ik} = 0$ and consequently $R_{ik} = 0$.

Einstein's equations in the weak-field approximation were first considered by Einstein himself.[1] The subsequent progress of the solution of these equations can be found in detail in Refs. 2—10.

Assume that the gravitational field in vacuum is quite weak and the space-time metric is practically Galilean. Then it is possible to choose in this space a reference frame in which the metric tensor $g_{\alpha\beta}$ differs little from the Minkowski tensor $\mathring{g}_{\alpha\beta}$, i.e.,

$$g_{\alpha\beta} = \mathring{g}_{\alpha\beta} + h_{\alpha\beta}. \tag{5}$$

Here $\mathring{g}_{\alpha\beta} = 1$ for $\alpha = \beta$; $\mathring{g}_{\alpha\beta} = 0$ for $\alpha \neq \beta$ and $\mathring{g}_{00} = -1$; $\mathring{g}_{0\beta} = 0$; $h_{\alpha\beta}$ are small corrections to the unit metric, i.e., $h_{\alpha\beta} \ll 1$. It is assumed that the partial derivatives of $h_{\alpha\beta}$ are of the same order of smallness as $h_{\alpha\beta}$; $h_{\alpha\beta,\mu} \sim h_{\alpha\beta,\mu\nu} \sim h_{\alpha\beta}$, so that their product can be neglected in comparison with the quantities themselves. The commas in the subscripts denote differentiation with respect to x^μ.

Introducing the new quantity $\Psi^k_i = h^k_i - 0.5 h \delta^k_i$ and choosing a harmonic coordinate system in which Ψ^k_i is a conserved quantity, we obtain

$$\partial \Psi^k_i / \partial x^k = 0. \tag{6}$$

By virtue of this condition, the expression for the Ricci tensor R^k_i takes the form

$$R^k_i = -0.5 \Box h^k_i. \tag{7}$$

The scalar curvature is therefore

$$R = -0.5 \Box h, \tag{8}$$

where $\Box h_{ik} = \mathring{g}^{lm} \partial^2 h_{ik} / \partial x^l \partial x^m$; $h = h^i_i$; $\Box$ is the d'Alembert operator of special relativity theory, given by

$$\frac{\partial^2}{\partial x^2_\alpha} - \frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2} = \Delta - \frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2};$$

Δ is the Laplace operator. Combining (2), (7), and (8), we obtain a field equation in standard form:

$$-0.5 \Box (h^k_i - 0.5 h \delta^k_i) = -0.5 \Box \Psi^k_i = \frac{8\pi G}{c^4} T^k_i, \tag{9}$$

or

$$\Box \Psi^k_i = -\frac{16\pi G}{c^4} T^k_i. \tag{10}$$

By substituting (7) in (4) we get

$$\Box h^k_i = -\frac{16\pi G}{c^4} (T^k_i - 0.5 \delta^k_i T). \tag{11}$$

We see that in the linear approximation Einstein's equations are wave equations for the potentials Ψ^k_i or h^k_i. The right-hand sides of these equations describe the sources of the gravitational field, and the left sides the potentials generated by these sources.

For vacuum, as already noted, $R^k_i = 0$ and consequently

$$\Box h^k_i = 0. \tag{12}$$

This is the usual wave equation, which is analogous to the corresponding wave equation $\Box \varphi = 0$ for the vector potential in electromagnetic field theory, where φ is one of the vectors A, E, or H. It is known from electrodynamics, in particular, that in the absence of charges the electromagnetic field propagates at the speed of light. Therefore, by virtue of the analogy between the wave equations for the gravitational and electromagnetic fields, it is assumed that gravitational waves should propagate in a space free of masses and their velocity should equal the fundamental velocity, i.e., that of light.

2. THE ENERGY-MOMENTUM CONSERVATION LAW IN GENERAL RELATIVITY

The solution of the problem of gravitational radiation in general relativity encounters tremendous difficulties,[1,2,6,11,12,14] due to the absence of an invariant definition of this concept. The central point in this problem is that of the invariance of the formulation of the energy-momentum tensor of the gravitational field. In special relativity theory, as we well know, the energy-momentum conservation law is given by the equation

$$\partial T_{ik} / \partial x^k = 0. \tag{13}$$

However, no such simple relation exists for the gravitational field. To write down the conservation law in the latter case it is necessary to add to the tensor T_{ik} also a special differential-geometry object t_{ik} (which is not a tensor) that describes the distribution and the displacement of energy and momentum of gravitational origin. Gravitation theory is the only theory in which there is no rigorous energy-momentum or angular-momentum tensor. The introduction of t_{ik} into the conservation law is carried out quite formally and artificially (see, e.g., the discussion of this problem in the books[2,9,10]).

The conserved quantity in relativistic theory of gravitation is an energy-momentum complex in the form

$$\frac{\partial}{\partial x^\nu} [\sqrt{-g} (T^{\mu\nu} + t^{\mu\nu})] = 0.$$

Here $t^{\mu\nu}$ is the gravitational-field energy-momentum pseudotensor, which is asymmetrical in the indices μ and ν and depends on the coordinate system. In very weak gravitational fields, however, when the metric of the space-time events is almost Galilean, $t^{\mu\nu}$ behaves like a symmetrical tensor. In this case it can be used for a rigorous description and calculation of physically observable quantities. This approach will be used below in all the calculations. The concrete form of the pseudotensor $t^{\mu\nu}$, expressed in terms of derivatives of $g^{\mu\nu}$, is[2,3,6]

$$t_i^k = \frac{\delta_i^k}{2\varkappa} G + \frac{c^4}{32\pi G} \cdot \frac{1}{\sqrt{-g}} \times \left[g^{ks} \left(\frac{\partial g_{ls}}{\partial x^m} + \frac{\partial g_{ms}}{\partial x^l} - \frac{\partial g_{lm}}{\partial x^s} \right) \frac{\partial (\sqrt{-g}\, g^{lm})}{\partial x^i} - g^{ls} \frac{\partial g_{ls}}{\partial x^m} \cdot \frac{\partial (\sqrt{-g}\, g^{km})}{\partial x^i} \right]. \tag{14}$$

3. GRAVITATIONAL WAVES

By a gravitational wave, from the mathematical point of view, one should understand the propagation of the space-time curvature.[4-7] Physically, a gravitational wave is the perturbation of the gravitational field, propagating with finite velocity and carrying energy. The lowest multipoles in electromagnetic waves are the electric and magnetic dipole radiations. Electromagnetic waves are transverse, namely, the vectors E and H of the electric and magnetic field intensities of the wave are mutually perpendicular and lie in a plane perpendicular to the wave-propagation direction. A similar situation is obtained in the case of gravitational waves in the tensor variants of gravitation theory. The lowest multipole of gravitational waves is in this case the quadrupole. However, if we take into account also the presently less prevalent point of view, which admits the existence of a scalar-tensor theory of relativistic gravitation, then monopole radiation exists in addition to quadrupole radiation. In all gravitation theories, however, including the scalar-tensor theory, the gravitational-radiation waves are transverse.

Let us explain now the meaning of the gravitational waves, using as examples the relative acceleration of trial particles in the field of these waves.[4] Thus, in the case of a Newtonian field it is known that the gravitational force decreases with distance as r^{-2}. The relative acceleration of two test bodies in such a field decreases as r^{-3}. These are the well-known tidal forces. The quadrupole component of the gravitational field of a body decreases as r^{-4}, and the relative acceleration of test particles in this field decreases as r^{-5}. This is the conclusion of Newtonian theory.

The relativistic theory of gravitation yields a different statement for the case of a variable quadrupole component of the field. The theory states, as will be rigorously shown below, that at distances $r > cT$, i.e., in the wave zone, where c is the velocity of light and T is the characteristic time of variation of the quadrupole moment of the field source, the relative acceleration of the test bodies depends on the distance as r^{-1}. If we compare this dependence with the decrease of the relative acceleration of bodies in a field due to a static quadrupole source (as r^{-5}), then we can see that the relative acceleration of the bodies in these cases is negligibly small in comparison with the relative acceleration in a gravitational wave. This is precisely the basis of experiments aimed at detecting gravitational waves.

Plane waves. In a plane wave, the field varies in space along one direction, which for the sake of argument we take to be the axis $x^1 = x$. Equation (12) takes in this case the form

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2} \right) h_i^k = 0.$$

Obviously, the solution of this equation is the function $f(t - x/c)$, i.e.,[3,6] $h_i^k = F_{1i}^k(x - ct) + F_{2i}^k(x + ct)$.

We consider only one wave traveling in the positive direction of the x axis: $h_i^k = F_{1i}^k(x - ct)$. Taking the additional condition (10) into account, we have:

$$\partial \Psi_i^k / \partial x^k = \partial (h_i^k - 0.5 h \delta_i^k) / \partial x^k = 0,$$

or, as applied to our case,

$$\frac{\partial (h_1^1 - 0.5h)}{\partial x} + \frac{\partial h_1^0}{c\, \partial t} = 0; \quad \frac{\partial h_{2,3,0}^1}{\partial x} + \frac{\partial h_{2,3,0}^0}{c\, \partial t} = 0.$$

Since $\partial h_i^k / \partial x = -\partial h_i^k / c\, \partial t$, it follows that, integrating and discarding the constants (we are interested in the variable part of the field), we obtain $h_1^1 = h_1^0 + 0.5h$; $h_{2,3,0}^1 = h_{2,3,0}^0$; $\Psi_i^k = h_i^k$.

By the method of infinitesimally small coordinate transformations we can cause some of the components h_i^k to vanish and, in final analysis, obtain the results $h_2^2 = -h_3^3$; $h_2^3 \neq h_2^2$. Consequently, a gravitational wave is transverse and is determined by two components h_2^2 and h_2^3. In this case it is indeed analogous to a plane electromagnetic wave. The polarization of the gravitational waves is determined by a symmetrical tensor of rank 2 in the yz plane, the sum $h_2^2 + h_3^3$ of the diagonal components of which is equal to zero. The components of the metric tensor (5), which determines the space-time metric in which weak plane gravitational waves propagate, now take the form $g_{00} = -1$; $g_{11} = 0$; $g_{22} = 1 + h_{22}$; $g_{33} = 1 + h_{33}$; $g_{23} = h_{23}$. The remaining components g_{ik} vanish.

Let us calculate the energy flux density carried by a plane gravitational wave. To this end we use formula (14). For a wave propagating along the x axis, only the component t^{01} will differ from zero.[2,3] Calculation by means of formula (14) yields

$$t^{01} = -\frac{c^3}{32\pi G} \left[\left(\frac{\partial h_{22}}{\partial x} \cdot \frac{\partial h_{22}}{\partial t} + \frac{\partial h_{33}}{\partial x} \cdot \frac{\partial h_{33}}{\partial t} + 2 \frac{\partial h_{23}}{\partial x} \cdot \frac{\partial h_{23}}{\partial t} \right) - \frac{\partial^2}{\partial x\, \partial t} (h_{22}^2 + h_{33}^2 + 2h_{23}^2) \right],$$

but since $h_{ik} = f(x - ct)$, we have

$$\frac{\partial h_{ik}}{\partial x} - \frac{1}{c} \cdot \frac{\partial h_{ik}}{\partial t} = -\frac{h_{ik}}{c} = -\frac{\partial h_{ik}}{\partial x^0}.$$

Therefore

$$t^{01} = \frac{c^2}{16\pi G} \left[\dot{h}_{23}^2 + \frac{1}{4} (\dot{h}_{22} - \dot{h}_{33})^2 \right]. \tag{15}$$

Cylindrical waves. The exposition in this section follows Refs. 6 and 11–14. The first to investigate cylindrical gravitational waves were Einstein and Rosen,[13] and also Weber and Wheeler.[12] In the cylindrical coordinate system $x^i = x^i(r, \varphi, z, ct = T)$, a space-time interval can be represented in the form

$$ds^2 = \exp[2(\gamma - \psi)](-dT^2 + dr^2) + r^2 \exp(-2\psi)\, d\varphi^2 + \exp(2\psi)\, dz^2,$$

where the scale factors γ and ψ depend only on r and T. The gravitation Eq. (10) for a space with this metric is satisfied under the conditions

$$\psi'' + \frac{1}{r}\dot{\psi}' - \ddot{\psi} = 0; \; \gamma' = r[\psi'^2 + \dot{\psi}^2]; \; \dot{\gamma} = 2r\psi'\dot{\psi}.$$

The prime and the dot denote here partial differentiation with respect to r and ct, respectively. The solutions of the first of these equations are well known: they are cylindrical waves that can be represented at large distances from the source either in the form of a plane monochromatic wave

$$\Psi = A(\omega r)\cos\omega t + B(\omega r)\sin\omega t, \tag{16}$$

or in the form of a pulse

$$\Psi = C[(a - iT)^2 + r^2]^{-1/2} + C[(a + iT)^2 + r^2]^{-1/2}, \tag{17}$$

where a is an approximate measure of the width of the pulse. The wave (16) has a polarization described by an ellipse in a plane perpendicular to the wave propagation direction. One of the principal axes of the ellipse is parallel to the cylinder axis, and the other is perpendicular. This ellipse describes the distance from a certain central test particle from the set of test particles, which were at rest on a circle prior to the arrival of the wave. On going through the phase of the oscillation, the longer axis of the ellipse becomes the shorter one, and vice versa, but the axes do not rotate. Cylindrical waves have no other independent polarization state, as is the case, for example, in truly plane waves, in which the two polarization states (this will be demonstrated below) differ by rotation of the principal axes of the ellipse through 45° in the polarization plane.

An analysis of the energy-momentum pseudotensor t^{01} for the monochromatic wave (16) and for the gravitational pulse (17) shows[11,12] that although the energy density in the wave does not vanish everywhere, the total energy flux carried by the waves is equal to zero. Møller[11] uses this result, in particular, to conclude that a gravitational wave, as a physical phenomenon, possibly does not exist in nature. However, Weber[12] and Papapetrou[66] believe that this conclusion is not convincingly proved, since it is based on the value of the pseudotensor $t^{\mu\nu}$, which, as is well known, is not invariant to the coordinate transformation, and by a suitable choice of the coordinate system it is possible to assign it an arbitrary prescribed value at any point. To verify that the considered waves are not fictitious "coordinate" waves in space, but constitute a real physical wave process, it suffices to consider for this purpose the Riemann curvature tensor in the space occupied by the wave. It can be shown by direct calculation[12,66] that in this case some of the components R_{iklm} cannot be made to vanish by any coordinate transformation. This means that cylindrical gravitational waves produce a real curvature of space, which leads to different physical effects such as the change of the distance between neighboring infinitesimally small test particles. Consequently, a cylindrical gravitational wave should be assumed to exist objectively.

Spherical waves. The solution of this problem is the subject of work by Robinson and Trautman.[15–17] They obtained a class of exact Einstein solutions in empty space, using a special metric. Bartrum[18] has furthermore established that spherical gravitational waves can form a self-consistent system with electromagnetic waves, propagating together with them along the common trajectory. Kerr and Schild[19] also obtained a solution of the relativistic gravitation equation for the case of spherical waves, using a different metric than in Refs. 15–17. They have shown in particular, that these waves propagate along geodesics and have a zero dispersion.

4. THEORY OF RADIATION OF GRAVITATIONAL WAVES

Tensor theory of radiation. Equations (10), as already noted, coincide in form with the corresponding equations of electrodynamics in special relativity theory: $\Box A_\mu = -j_\mu$, where A_μ is the four-dimensional potential and j_μ is the current-density vector. It must therefore be assumed that the energy-momentum tensor $T_{\mu\nu}$ should play in gravitation the same role as the four-dimensional vector j_μ in electromagnetic theory, i.e., the tensor $T_{\mu\nu}$ should be the source of the gravitational field and consequently the source of the gravitational waves [gravitons with spin 2 (Refs. 14 and 20)]. By analogy with electrodynamics, Eq. (10) is solved by the retarded-potential method.

The solution for a weak field[1–3,6,8,14] produced by bodies moving with velocities $v \ll c$ is

$$\Psi_i^k = \frac{4G}{c^4}\cdot\frac{1}{R_0}\int (T_i^k)_{t-R_0/c}\frac{1}{R}\,dv,$$

where R_0 is the distance from the origin, for example from the center of the volume v occupied by the source of the field. After simple transformations, the equation goes over into

$$\Psi_{\alpha\beta} = \frac{2G}{c^4}\cdot\frac{1}{R_0}\cdot\frac{\partial^2}{\partial t^2}\int \mu x^\alpha x^\beta\, dv,$$

where μ is the density of the mass in the source. At large distances from the source, the gravitational wave can be regarded as plane, propagating in the direction of the x axis, and the energy flux for this wave can be found from formula (15):

$$t^{01} = \frac{G}{36\pi c^5 R_0^2}\left[\left(\frac{\dddot{D}_{22} - \dddot{D}_{33}}{2}\right)^2 + \dddot{D}_{23}^2\right]$$
$$= -\frac{c^2}{16\pi G}\left[\dot{h}_{23}^2 + \left(\frac{\dot{h}_{22} - \dot{h}_{33}}{2}\right)^2\right].$$

The dot denotes here differentiation with respect to t; $D_{\alpha\beta}$ is the quadrupole moment of the masses, equal to

$$D_{\alpha\beta} = \int \mu(3x^\alpha x^\beta - \delta_{\alpha\beta}x_\gamma^2)\, dv.$$

The intensity of radiation in an arbitrary direction n, where n is a unit vector with component n_α, into a solid-angle element $d\Omega$, is expressed by the relation[2,3,8]

$$dI = \frac{G}{36\pi c^5}\left[\frac{1}{4}(\dddot{D}_{\alpha\beta}n_\alpha n_\beta)^2 + 0.5\dddot{D}_{\alpha\beta}^2 - \dddot{D}_{\alpha\beta}\dddot{D}_{\alpha\gamma}n_\beta n_\gamma\right] d\Omega.$$

The total radiation in all directions is then $-d\mathscr{E}/dt = G\dddot{D}^2_{\alpha\beta}/45c^5$. The quantity $\dddot{D}^2_{\alpha\beta}$ is a 3-scalar, which can be represented as $\xi M^2 r^2 \omega^6$, where M is the system mass, r is the "radius" of the system, ω is the oscillation frequency, and ξ is the dimensionless parameter that characterizes the geometry and dynamics of the system. If we recognize now that $GM^2/r = E_0$ is the gravitational energy of the system, then the equation for the total radiation can be represented in the form

$$-d\mathscr{E}/dt = (\xi GM^2 c/r^2)(r\omega/c)^6 = \alpha E,$$

where α is a coefficient representing the fraction of the system energy that goes to gravitation radiation, and is equal to $[\xi GM/(r^2 c)](r\omega/c)^6$.

It is seen from the foregoing analysis that the lowest possible multipole radiative term in the potential $\psi_{\alpha\beta}$ is the quadrupole term. Dipole radiation cannot occur in gravitation theory, since the ratio of the gravitational charge to the inertial mass for all values is the same, and the dipole moment is equal to zero. The absence of a dipole term indicates, in particular, that spherical symmetrical pulsations of material systems cannot radiate gravitational waves in the tensor variant of the theory.

Scalar-tensor theory of radiation. At the present time general relativity is not confined only to the tensor variant of the theory, proposed and developed by Einstein. There exist many competing theories (see, e.g., the discussion and comparison of different general relativity theories in Refs. 21–24) which are in satisfactory agreement with experiment. One of these theories is the scalar-tensor theory proposed by Jordan[25] and by Brans and Dicke[26] and generalized in Refs. 4, 22, and 27–29, and is second only to Einstein's in popularity. In accordance with the scalar-tensor theory, bodies with spherical symmetry are capable of effectively emitting gravitational monopole radiation (gravitons with zero spin), a process forbidden in the pure tensor variant of the theory. The intensity of the monopole waves can reach, and in some cases even exceed, the intensity of the quadrupole waves that would be radiated by the same source if its sphericity were to be violated.[22,29]

We describe below the principal features of the scalar-tensor gravitational radiation. We follow mainly Ref. 35. The field equation in the scalar-tensor theory is given by

$$\Box\varphi = [8\pi/(3+2\delta)c^4]T; \quad \Box\varphi = \varphi^i_{;i}, \tag{18}$$

where $T = g_{ik}T^{ik}$; $\varphi = \varphi(\mathbf{r}, t)$ is the scalar field and is connected with a gravitational "constant" that depends on the coordinates and the time; δ is the interaction constant between the scalar and tensor fields, and is assumed equal to 6. The comma in the subscripts of (18) denotes partial differentiation. To solve the wave equation (18) it is assumed, just as in the tensor theory, that the gravitational radiation perturbs only weakly the pseudo-Euclidean metric, i.e., $g_{ik} = \mathring{g}_{ik} + h_{ik}$; $\varphi = \varphi_0(1+\psi)$, where $\varphi_0^{-1} = (3+2\delta)k/(4+2\delta) = \gamma k$; $\gamma \approx 1$; $k = k(\mathbf{r}, t) \sim \varphi^{-1}(\mathbf{r}, t)$. Here k has the meaning of the gravitational constant.

In a harmonic coordinate system we have

$$[h^k_i - 0.5\delta^{(k)}_i (h + 2\psi)]_{,k} = 0$$

and the wave equations (18) take the form

$$\left(\Delta - \frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\right)[h_{ik} - 0,5\mathring{g}_{ik}(h + 2\psi)] = \frac{16\pi}{c^4}\gamma k T_{ik}; \tag{19}$$

$$\left(\Delta - \frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\right)\Psi = -\frac{8\pi\gamma k}{(3+2\delta)c^4}T. \tag{20}$$

The solution of these equations far from the source, in the case of a plane wave propagating in the x direction, yields $h_{00} = h_{01} = h_{02} = h_{03} = 0$; $h_{22} + h_{33} = 2\psi$. These equations duplicate in their structure Eqs. (9)–(11). We can therefore conclude even from this that the scalar waves, like the tensor ones, are transverse and have a fundamental propagation velocity equal to that of light.

To calculate the energy carried by such a gravitational wave one uses the energy-momentum pseudotensor in the form

$$t^{ik} = k\varphi t_0^{ik} + t_\varphi^{ik}.$$

where t_0^{ik} is the pseudotensor used in the pure tensor variant of the theory and defined by (15). The additional pseudotensor t_φ^{ik} is due to the presence of the scalar field $\varphi(\mathbf{r}, t)$. In the case of a plane wave we have

$$t^{01} = \frac{c^2}{16\pi\gamma k}\left[\dot{h}^2_{23} + \frac{1}{4}(\dot{h}_{22} - \dot{h}_{33})^2 + (3+2\delta)\dot{\Psi}^2\right]. \tag{21}$$

The solution of the field equations (19) and (20) yields for h_{22}, h_{33}, h_{23} and Ψ the following expressions:

$$h_{23} = -\frac{2\gamma k}{3c^4 r_0}\ddot{D}_{23}; \quad h_{22} - h_{33} = -\frac{2\gamma k}{3c^4 r_0}(\ddot{D}_{22} - \ddot{D}_{33});$$

$$\Psi(r_0, t) = [2\gamma k/c^4(3+2\delta)]\int \frac{T(t - |\mathbf{r} - \mathbf{r}_0|/c)}{|\mathbf{r} - \mathbf{r}_0|}dv,$$

where $\mathbf{r}_0$ is the radius vector of the observation point. The intensity of pure tensor gravitational radiation in this theory coincides with the conclusions of L.D. Landau and E.M. Lifshitz.[2] The energy lost per unit time to scalar radiation is

$$-\left(\frac{d\mathscr{E}}{dt}\right)_\varphi = \frac{k\gamma}{4\pi c^5(3+2\delta)}\int\left[\frac{\partial}{\partial t}\int T(t')\,dv\right]^2 d\Omega. \tag{22}$$

Here $t' = t - |\mathbf{r}_0 - \mathbf{r}|/c$; $d\Omega$ is the solid-angle element on a sphere of radius r_0. A concrete calculation[37] shows that in certain physical processes the scalar radiation can greatly exceed in its intensity the tensor radiation. Among these processes are, in particular, the turbulent-vortical motion of matter in hot plasma, radially-symmetrical collapse of stars, pulsations of stars, etc. Thus, if a star executes three-dimensional radial harmonic oscillations of the type $R = R_0 + r_1\cos\omega t$, $r_1 \ll R_0$, where R_0 is the average radius of the star and r_1 is the amplitude of the oscillations, then such a star, in accord with (21) and (22), will emit only scalar waves with intensity

$$-\frac{d\mathscr{E}}{dt} = \frac{1}{20}\cdot\frac{k}{c^5(4+2\delta)}(Mr_2R_0)^2\omega^6.$$

It is appropriate to note here that a sufficiently accurate experimental investigation of the characteristics of the gravitational radiation (intensity and polarization) can establish uniquely the contribution of the scalar theory in general relativity theory. It is presently believed[38] that this contribution is about 10%.

5. SOURCES OF GRAVITATIONAL RADIATION

Astrophysical sources of quadrupole-mass type. Several reviews with detailed calculations on the intensity and polarization of gravitational radiation from different cosmic objects are presently available on astrophysical sources (see, for example, one of the latest detailed reviews).[40] To complete the picture of the discussed problem, however, we shall also note here the most characteristic astrophysical sources of gravitational radiation.

Binary stars. A system of two stars with masses m_1 and m_2, rotating about a common center of gravity with frequency ω, has a quadrupole moment and should therefore radiate harmonic gravitational waves[2,41] with power

$$P = \frac{32G}{5c^5} \cdot \frac{m_1^2 m_2^2}{(m_1 + m_2)} r^4 \omega^6,$$

where r is the distance between the centers of the stars. A value close to this was obtained for gravitational radiation also in Ref. 42. The fundamental radiation is at frequency equal to double the frequency of the revolution of the stars. According to the data of Ref. 41, the power of gravitational radiation from binary stars reaching the earth is 10^{-9}—10^{-10} erg/cm² sec, corresponding to a dimensionless amplitude $h \approx 10^{-20} - 10^{-21}$. For the record we point out that the sun—Jupiter system emits 450 watts of gravitational radiation, which is smaller by a factor 10^{24} than the power of the electromagnetic radiation.[7]

In the case of relativistic rotation of the binary-star component,[43] gravitational radiation is emitted in a broad frequency spectrum. The total radiation power is then $\mathcal{E} = GE^2\Omega^2/[c^5(1-\beta^2)]$, where E is the total energy of the system; Ω is the angular velocity of the star revolution; β is the relativistic velocity. The total radiation flux from all the 2×10^7 binary stars of our galaxy reaching the earth is $\sim10^{-7}$ erg/cm² sec.[41] It is appropriate to take notice here of the radiation from a binary system of stars in which one star passes close to the other star.[44] In this case the gravitational-radiation energy pulse amounts to $\Delta\mathcal{E} = (c^2m^2/M)(r_g/r)^{3,5}$, where r is the coordinate of the periastron, r_g is the gravitational radius of the system, m is the mass of the passing star, and M is the mass of the "resting" star. In the case of direct collision of stars we have $\Delta\mathcal{E} = 2\times10^{-2}m^2M^2c^2/(m+M)^3$.

In Ref. 45 it was shown that gravitational waves can be radiated from rapidly rotating nonspherical homogeneous "drops" of liquid. Thus, white dwarfs and neutron stars, with a revolution period 0.001 sec, can produce in the vicinity of the earth a short-duration gravitational flux of about 4×10^7 erg/cm² sec. An even greater radiation intensity should be produced by nonspherical explosions of giant astrophysical objects, for which the gravitational luminosity is $\mathcal{E} = L^{-1}E^2\tau^{-2}$, where $L = c^5/G = 3.63\times10^{59}$ erg/sec, E is the energy of the explosion, and τ is the characteristic time of the explosion. According to estimates by L.M. Ozernoi,[46] the explosion of a quasar with $E \sim 10^{59}$ erg and $\tau \approx 10^8$ sec yields $\mathcal{E} \approx 10^{45}$ erg/sec.

Collapse of stars and absorption of matter by a black hole. These processes give rise to gigantic flashes of gravitational radiation, the power of which can carry away several solar masses.[40,46–49] And the most powerful flash of gravitational radiation can result from collision[50] or coalescence[51] of black holes. Thus, the energy flux due to coalescence of two black holes can amount to $(2-\sqrt{2})mc^2$, where m is the mass of one black hole.

Laboratory sources of the quadrupole-mass type. The simplest quadrupole-mass emitter of gravitational waves on the laboratory scale is a rigid rod rotating about the center of gravity. Such a rod emits waves[52] of intensity

$$\mathcal{E} = \frac{32GI^2\omega^6}{5c^5} = 1.73\cdot10^{-59}I^2\omega^6 \text{ erg/sec},$$

where I is the moment of inertia of the rod and ω is the angular frequency of this rotation. If we take a rod of mass 10^4 g and length 1 m, made of the strongest material, then at its maximum revolution speed, when it is ruptured by the centrifugal tension, the calculated radiation power reaches 10^{-30} erg/sec, amounting to approximately 10 gravitons per year.

We now consider a linear quadrupole oscillator consisting of two masses executing harmonic motion along the straight line joining these masses. Such a system has a quadrupole moment and consequently is capable of emitting gravitational waves[52] of intensity $\mathcal{E} = GI^2\omega^6/(60\pi c^5)$, where I is the amplitude of the tensor of the quadrupole moment of the oscillator and ω is the oscillation frequency. The maximum value of the radiation power is also about 10^{-30} erg/sec.

Weber[52] considered the possibility of emission of gravitational waves by using mechanical stresses that are produced in a piezoelectric rod via electrostriction. He found that the radiation intensity is $\mathcal{E} = GP^2\lambda^2\pi^2/(120c^3)$, where P is the effective tensile stress in the rod and λ is the length of the gravitational wave corresponding to double the frequency of the electrostriction oscillation of the rod. If we take a crystal 50 cm long and emit with it gravitational waves of $\lambda = 1$ m, then at the threshold of crystal rupture the radiation power is about 10^{-13} erg/sec. However, the power delivered to the crystal in this case should be approximately 10^8 W. A similar conclusion is reached also in Ref. 53.

Grischchuk and Sazhin[54] considered a radiator in the form of a spherical shell. The oscillations of such a shell cause strong elastic stresses in its material, and this leads to the emission of gravitational waves. The maximum radiation power occurs when the shell oscillations are at resonance, and amounts to

$$\mathcal{E} = \frac{G}{c^3}(\rho c^2)^2\left(\frac{c_s}{c}\right)^6 r_0^2\lambda^2,$$

where ρ is the alternating part of the density of the material in the elastic wave, c_s is the speed of sound in the shell, r_0 is the radius of the shell, and λ is the length of the gravitational wave. Calculations by means of this formula for $r_0 \approx 10^3$ cm and $\lambda \approx r_0$ yield $\mathscr{E}_{\max} \approx 10^{-7}$ erg/sec.

The production of a solitary gravitational wave in a large asymmetrical explosion of matter was analyzed in Refs. 46 and 55. It was found[46] that the average energy carried away by the gravitational wave from the explosion zone is $\mathscr{E} = 3.8 \times 10^{-57} E^2/\tau^2$ erg/sec, where E is the total explosion energy and τ is the characteristic explosion time. Thus, the explosion of a 17-kiloton uranium bomb should produce gravitational radiation of approximately 10^{-4} erg/sec of duration 10^{-8} sec.[56]

A new approach to the emission of gravitational waves is being developed in Refs. 53 and 57–64. The gist of idea is to use microscopic and macroscopic quantum mass systems as generators of coherent gravitational radiation. On the basis of the analogy between electromagnetic and gravitational weak fields, it was demonstrated in these papers that the coherent oscillation of a large number of multipole moments of the masses of molecules, ions, or nuclei is capable of generating directed pulses of gravitational radiation.

To excite matter into the superradiant state, it is proposed to use laser pumping. This possibility is brought about by the fact that the distribution of the density of the electric charge in the medium is closely connected with the density distribution of the mass that carries these charges. Consequently, when a superradiant electromagnetic state is produced in a substance by laser pumping, stimulated coherent precession of the multipole masses is simultaneously produced. In the free state, however, such a system relaxes rapidly via the electromagnetic radiation channel. The ratio of the intensity of electromagnetic to gravitational radiation is then 10^{36}—10^{42}. To decrease significantly the probability of electromagnetic radiation, it is proposed to contain the entire radiator in an electromagnetic high-Q resonator, i.e., to "forbid" the electromagnetic radiation channel. Estimates show[53,57] that in this manner one can produce a gravitational-radiation pulse of about 10^{-10} W, where W is the electromagnetic power produced by the laser. The radiation intensity can be raised further by using the directivity of the radiation of large systems. As examples of such radiators, the following systems were considered in Ref. 58:

Ruby radiator. If the cross section of the ruby rod is $S = 10^5$ cm^2, the length is $l = 10^3$ cm, and the radiation is carried out at a gravitational wavelength $\lambda = 7 \times 10^3$ Å, then the gravitational-energy flux is approximately 10^{-55} erg/sec.

Molecular crystals. In this case one can count on a gravitational-radiation flux of about 10^{-47} erg/sec. By using crystals in which the active impurities are placed in individual "nests" with linear dimensions proportional to λ, then the gravitational flux at a focus with dimensions proportional to λ^2 is about 10^{-37} erg/sec.

Nuclear quadrupole systems. The gravitational flux from them can be 10^{-57} erg/sec.

In the general case, the intensity of coherent gravitational radiation of a sample containing N active particles is[58]

$$I = I_0 \frac{\lambda^2 N^2}{4S} \sin^2\theta \, th^2 \frac{h\omega}{2kT},$$

where I_0 is the intensity of the spontaneous gravitational radiation, λ and ω are the length and frequency of the gravitational radiation, θ is the angle between the radiation direction and the vector $\mathbf{K}_1 + \mathbf{K}_2$, $\mathbf{K}_1$ and $\mathbf{K}_2$ are the wave vectors of two laser beams that pump the medium, k is Boltzmann's constant, and T is the absolute temperature.

A system of N coherently pulsating particles with frequency ω is capable also of emitting scalar gravitational waves of intensity $I = 4G\kappa A^2\omega^6/[9c^5(1+6\kappa)]$, where $\kappa \sim 1$; the quantity A has the meaning of the amplitude of the pulsation of the density of the medium.

All the considered sources with coherent microscopic systems of particles produce, as we see, an exceedingly low radiation intensity. Kopvillem[60] sees a way out of this difficult position in the use of quantum macroscopic systems that are coherently interconnected. Thus, if we use as the radiating quantum macroscopic systems fluxoids, which are filamentary magnetic formations in type-II superconductors and constitute magnetic-field quanta, then the intensity of the gravitational radiation from them reaches 10^{-40} erg/sec.

Rotons and vortex filaments in superfluid helium can yield an even higher radiation intensity, reaching 10^{-20} erg/sec. A flux of the same order of magnitude can be obtained from a system of dislocations in a crystal, excited by ultrasound or by an electromagnetic field, and also by macroscopic quantized formations in ferroelectrics and ferromagnets, or finally in optical filaments produced in liquids under the influence of strong picosecond light pulses.

The principle of coherent emission of gravitational waves is the basis of various proposed devices[65,66] in which many plates are made to vibrate in phase. An interesting idea is developed in Ref. 67, in which it is proposed to emit gravitational waves by local elastic oscillations produced on the surface of piezoelectric crystals by electromagnetic microwaves. Seci *et al.*[68] make the important suggestion that pulsed gravitational radiation can be produced by ultrahigh-frequency acoustic oscillations generated in a semiconductor under the influence of a laser flash. The gravitational-wave power flux in this case can reach about 10^{10} erg/sec in the pulse.

The foregoing proposals for the emission of gravitational waves with the aid of quantum macroscopic systems are worthy of definite interest, since they point to hopeful ways of performing a complete gravitational experiment under laboratory conditions.

Radiation of waves at resonance between electromagnetic and gravitational waves. So far we have considered the emission of gravitational waves as a result of a nonstationary state of matter. There is one more radiation mechanism, connected with the presence of the electromagnetic-field energy-momentum tensor $T_{\mu\nu}$ in the gravitation wave equations (10). This means that under certain conditions an alternating electromagnetic field, just like a nonstationary state of matter, is capable in principle of emitting gravitational radiation. Indeed,[4,69] when electromagnetic waves (E,H) propagate in a field H_0, there is produced a stress tensor proportional to HH_0, which alternates in space and in time. This tensor is a source of gravitational waves. In other words, in a plane electromagnetic wave that travels, for example, along the x axis, we have $T_{22}=T_{33}$ by virtue of the symmetry of the properties of the fields E and H in vacuum. If furthermore the wave is so polarized that H coincides with the constant field H_0, which is directed along z, then additional components of the energy-momentum tensors are produced:

$$T_{33}=-\frac{HH_0}{4\pi}\sin(kx-\omega t); \quad T_{22}=\frac{HH_0}{4\pi}\sin(kx-\omega t).$$

These electromagnetic fields serve as a source of gravitational radiation of the same frequency and of the the same propagation direction. The amplitude of the gravitational wave should increase as it moves along the x axis in the space occupied by the constant magnetic field H_0. Moreover, by virtue of the equality of the propagation velocities of the gravitational and electromagnetic waves, a wave resonance is possible between them,[70] and is accompanied by a transfer of energy from one type of wave to the other.[70,71] This problem was first considered by M.E. Gertsenshteĭn.[70] His analysis was based on the wave equation (10) for the gravitational potential, in which the energy-momentum tensor of the electromagnetic field was taken in the form

$$\tau_h^i=\frac{1}{4\pi}\left[F^{il}F_{kl}-\frac{1}{4}\delta_k^i(F^{lm}F_{ml})\right].$$

Here F^{lm} is the tensor of the electromagnetic field.

In the general case τ_k^i consists of three terms: the square of the intensity of the constant electromagnetic field, the square of the alternating field, and an interference term representing the wave resonance. For the pure resonant state, Eq. (10) takes the form

$$\square\Psi_i^k=-\frac{4G}{c^4}\left[\mathring{F}^{kl}F_{il}-\frac{1}{4}\delta_i^k(\mathring{F}^{lm}F_{lm})\right]. \quad (23)$$

where $\mathring{F}^{ik}$ is the tensor of the external "magnetizing" field in which the electromagnetic wave propagates. If it is assumed that this magnetizing field is a constant magnetic field $\mathring{F}$, and the wave propagates in the x direction, then the ratio of the amplitude a of the gravitational wave to the amplitude b of the electromagnetic wave at resonance is given by[70]

$$|a/b|^2=\frac{G}{\pi c^2}\mathring{F}^2T^2,$$

where T is the time of propagation of the electromagnetic wave in the constant magnetic field $\mathring{F}$. If the fields $\mathring{F}$ are turbulent and random, as is the case for interstellar fields, and have a correlation radius R_0, over which the magnetic field $\mathring{F}$ can be regarded as constant, then

$$|a/b|^2=\frac{G}{\pi c^3}\mathring{F}^2R_0T.$$

The length over which half of the energy of the electromagnetic wave is transferred into the energy of the gravitational wave[4] is $L=c^4/(\lambda GH_0^2)$, where λ is the electromagnetic wavelength. Thus, for interstellar fields one can assume $\mathring{F}\sim10^{-5}$ Oe, $R_0\approx10$ light years, and $T=10^7$ years. The last expression then yields $|a/b|^2\approx10^{-17}$.

This process is reversible, i.e., gravitational waves are capable of generating electromagnetic waves.[72-74] The physical basis of this process[2] is that the gravitational field acts on the electromagnetic field by changing the dielectric constant and magnetic permeability of the vacuum: $\epsilon=\mu=(-g)^{-1/2}$. In particular, a gravitational wave of frequency ν changes ϵ in accordance with the law $\epsilon=\epsilon_0+\epsilon_1\times\cos(\mathbf{Kr}-\nu t)$, where $\epsilon_1\ll\epsilon_0$. When a gravitational wave propagates in a field H_0, the field is stretched and compressed because of the changes in the space-time metric, which are accompanied by the appearance of an alternating magnetic field $h(x,t)H_0$, where h characterizes the change of the metric in the gravitational wave. The field is in this case a source of electromagnetic waves. This process has been considered in many papers (see, e.g.,[20,71,72,76-79]). In particular, in Refs. 72 and 78 special attention was paid to the importance of the role of coherence in these processes.

Thus, if we estimate the coefficients of mutual energy transformation of the electromagnetic and gravitational waves upon satisfaction of the conditions of synchronization and polarization states in the entire interaction volume, then the value of the coefficient, as shown in Ref. 78, turns out to be larger by a factor of 10^{10} and more than the transformation without allowance for the synchronization conditions.[77] It is noted that the medium exerts a harmful effect on the mutual conversion of waves precisely because the coherence is violated (the electromagnetic waves propagate in the medium with lower velocity than the gravitational waves).

Ya.B. Zel'dovich[71] presented a unified approach to the mutual transformation of gravitational waves (GW) and electromagnetic waves (EMW) within the framework of their complete energy transformation. He has shown that in the ideal case of coherence, the transformations GW $\rightleftharpoons$ EMW have an oscillatory character and the complete EMW → GW → EMW cycle occurs over a length $x_0\approx c^2/(H_0\sqrt{G})\approx10^{24}/H_0$, where x_0 is in centimeters and H_0 is in gausses. In the real case the coherence is disturbed by the atoms, electrons, and neutrons contained in the space in which the wave transformation takes place. In vacuum, however, in view of the nonlinearity of electrodynamics (especially at short wavelengths), the coherence is also limited. If we start from the condition of "pure" coherence, then diffraction of EMW transformed into GW in a field H_0 along a path x_0 is[71,80] $\alpha=GH_0^2R^2/c^4$.

Thus, for pulsars we can assume $H_0=10^{13}$ Oe and $R=10^6$ cm, and obtain $\alpha\approx10^{-11}$. In the laboratory scale we have respectively $H_0=10^5$ Oe, $R=10^3$ cm, and $\alpha\approx10^{-33}$. A similar value of α was obtained in Ref. 78 for the case when a giant light pulse from a laser is used.

The problem of radiation of gravitational waves by standing electromagnetic waves in resonators was considered from a somewhat different point of view.[54] The basis of this analysis, as before, was Eq. (1). For distances $R > l$, where l are the characteristic dimensions of the source, the solution of (23) is expressed in the form of the integral

$$\Psi_{ik} = -\frac{4G}{c^4 R} \int \tau_{ik}\, dv.$$

The flux density of a gravitational wave in the wave zone ($R \gg \lambda$), determined from an energy-momentum pseudotensor, depends quadratically on $\dot{\Psi}_{\alpha\beta}$, and consequently increases with increasing frequency ω of the electromagnetic-field oscillations. This increase continues until $\lambda \approx c/\omega \approx l$ and the flux density of the gravitational energy in the wave zone reaches the value $G\epsilon^2\lambda^4/c^3R^2$, where ϵ is the average energy density in the radiating system. With further increase of the frequency ω, the radiation-coherence condition will be violated.

The electromagnetic-field energy-momentum tensor $T_{\mu\nu}$ is a quadratic function of E and H, so that a standing electromagnetic wave of frequency ω emits gravitational waves of double the frequency. If there are two standing waves characterized by ω_1 and ω_2, then in addition to the radiation at the frequencies $2\omega_1$ and $2\omega_2$ there will be emitted also gravitational waves at the sideband frequencies $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$. A wave of frequency ω in the presence of an external constant field is capable, as noted above, of radiating, gravitational waves of the same frequency ω. In principle, an alternating electromagnetic field in a resonator produces quadrupole oscillations of its walls and elastic stresses in the walls, and this can contribute to the gravitational radiation. This contribution, however, is essentially small in comparison with the direct contribution from the electromagnetic field.

In the case of a spherical resonator, the gravitational-energy flux density averaged over the period and entering a solid-angle element $d\Omega$ is equal to

$$\frac{dI}{R^2\, d\Omega} = \frac{G}{c^3 R^2}\, \mathcal{E}^2 r_0^2 \lambda^2 \sin^4\theta.$$

where the angle θ is measured from the polar axis, which coincides with the field direction in the resonator; λ is the length of the gravitational wave; r_0 is the radius of the spherical resonator; $\mathcal{E}$ is the density of the electromagnetic energy in the resonator. It is seen from the foregoing formula that there is no gravitational radiation in the direction of the poles. If we assume, for example, $\mathcal{E} = 10^{10}$ erg/cm^3 and $\lambda = r_0 = R = 10^3$ cm, then the power radiated in the equatorial plane is approximately 10^{-7} erg/sec.

Assume now that a uniform magnetic field is present in the resonator together with the electromagnetic wave. In this case the radiation flux density is $G\mathcal{E}\mathcal{E}_H r_0^4/(c^3R^2)$, where $\mathcal{E}_H$ is the energy density of the constant magnetic field. For a rectangular electromagnetic resonator with characteristic dimensions l_1, l_2, and l_3, the energy flux density in the gravitational wave propagating from the resonator in the x-axis direction (x is chosen along the resonator length l_1) is

$$\frac{dI}{R^2\, d\Omega} = \frac{G}{c^3 R^2}\, \mathcal{E}^2 \left(\frac{l_2}{\lambda}\right)^2 \left(\frac{l_3}{\lambda}\right)^2 \lambda^4.$$

The flux can be increased by using a series of N identical resonators arranged in tandem so as to satisfy in them the conditions for coherent radiation, i.e., the oscillations of the electromagnetic field in the neighboring resonators should be shifted $\pi/2$ in phase. The amplitudes of the gravitational waves add up at the observation point, and the resultant amplitude is proportional to N, while the energy flux density is proportional to N^2. By way of an example we consider a resonator with dimensions $l_1 = 1$ cm, $l_2 = l_3 = 10^3$ cm, $\lambda = 4$ cm, and $\mathcal{E} = 10^{10}$ erg/cm^3. Then at the upper limit, at $R = 10^3$ cm, the energy flux density in the gravitational wave is 10^{-13} erg/cm^2 sec. If we take 10^3 such resonators and ensure coherent-emission conditions in them, then the flux in a focal spot of approximate area 1 cm^2 can reach 10^{-2} erg/cm^2 sec.

Gravitational waves can also be emitted by an electromagnetic wave packet moving in a toroidal closed waveguide.[81] The calculated radiation power is

$$W \approx \frac{c^2 \mathcal{E}^2 G}{c^5 R^{4/3} (c\tau)^{2/3}},$$

where $\mathcal{E}$ is the total energy of the electromagnetic wave packet, R is the radius of the torus, and $c\tau$ is the width of the packet. For $R = 10^2$ cm and $c\tau = 10$ cm we have $W \approx 10^{-4}\, \mathcal{E}^2$ erg/sec.

Photon-graviton transformations. A natural consequence of the interaction of gravitational and electromagnetic waves, which was considered in the preceding section, is the conversion of photons into gravitons in external electromagnetic and gravitational fields. A detailed analysis of graviton-photon and photon-graviton transformation is given in Refs. 20 and 82—84. The differential cross section of this process[20] in the charge field of a particle is

$$d\sigma = \frac{GQ^2}{2c^4}(1 + \cos^2\theta)\, \mathrm{ctg}^2(\theta/2)\, d\Omega,$$

where Q is the charge in whose field the photon is converted into a graviton, and θ is the angle between the propagation directions of the photon and the graviton. The conversion of a photon into a graviton in an external magnetic field or electric field[83,85] is characterized by a total cross section $\sigma = 64\pi GRW/c^4$, where R is the photon path in the field and W is the total energy of the field in the volume V. For $H \approx 10^4$ Oe and $V = 1$ m^3 we have $\sigma \approx 10^{-36}$ cm^2. A similar result was obtained in Ref. 86.

If photons are made to pass perpendicular to the direction of the electric field in a parallel-plate capacitor,[20] then the maximum differential cross section for photon-graviton conversion is

$$d\sigma = \frac{2\varkappa E^2 \hbar^2 d^2}{(2\pi)^2 c^2 q^2} \sin^2(bq/2\hbar) \sin^2(cq/2\hbar)\, d\Omega. \tag{24}$$

Graviton production is most favored in the plane of the capacitor plates, at an angle $\theta = \pi/2$ to the direction of the photon motion. The following notation is used in (24): $\varkappa = 8\pi G/c^2$ is the Einstein gravitational constant; a, b, and c are the dimensions of the capacitor; q is the photon momentum. It can be seen from (24) that to obtain a cross section $\sim 10^{-30}$ cm^2 the field intensity in the capacitor must be $E=10^{10}\ a\lambda$, where a is the photon path in the electric field of the capacitor, and λ is the photon wavelength.

In the case of photon propagation in a magnetic field, the cross section for photon-graviton conversion is

$$d\sigma=\frac{\varkappa B^2\hbar^2z^2}{(2\pi)^2c^2q^2}\sin^2(aq/2\hbar)\sin^2(bq/2\hbar)\,d\Omega.$$

This relation was derived for the case when the magnetic field is directed along the x axis, the photons propagate in the z direction, and the gravitons move along y. Here B is the magnetic field intensity. To obtain the cross section 10^{-30} cm² the magnetic field intensity must be $H=10^{10}/z\lambda$. If the magnetic field is assumed to be $z\approx10^{10}$ cm long and the photon wavelength is ~10^5 cm, then this cross section can be reached at $H=10^{-5}$ Oe. These are field and distance scales prevalent in astrophysics. In the interior of stars there also exist strong magnetic fields in which photons and γ quanta move, and they can generate gravitational radiation. De Sabbata *et al.*[82] estimated the energy "pumping" in a laser beam of 10^3 W power in a 10^5 G magnetic field along a ray 10^4 cm long and obtained $|a/b|^2\approx10^{-25}$. The flux of gravitons having energy 1 eV should in this case be approximately 10^{-3} cm^{-2} sec^{-1}.

The possibility of production and absorption of gravitons by photons in the earth's gravitational field was considered in Refs. 82 and 87–89. Calculations[82,88] show that the conversion of photons of energy 1 eV has a cross section 10^{-37} cm². The graviton flux that can be obtained in this process is $N\approx\sigma n_0/V$, where n_0 is the photon flux and V is the volume of the Earth. If we assume $n_0=10^{28}$ cm^{-2} sec^{-1} (laser beam), then $N\sim10^{-36}$ graviton/cm² sec.

The flux of gravitons produced in photon-photon interactions in the earth's field can reach $N=Rn_0^2/cV$, where $R\approx10^{-43}$ cm⁵. For $n_0=10^{28}$ cm^{-2} sec^{-1} we have $N=10^{-24}$ cm^{-3} sec^{-1}. These calculations demonstrate, unfortunately, the tremendous difficulties of generation of gravitons of measurable intensity with the aid of photon beams. Only in strong gravitational fields can the effect become noticeable. Thus, in quasar fields one can expect the graviton yield due to electromagnetic radiation to be approximately 10^{28} erg/sec.

The photoproduction of gravitons is, as already indicated, a reversible process. Therefore, if fluxes of gravitational waves do exist in outer space, then they should be converted in magnetic fields into photons of the same wavelength.

Radiation of relativistic particles. A charged particle moving in a magnetic or electric field is capable of radiating gravitational waves from two sources: the mass tensor of the particle itself, and the tensor of the electromagnetic field of the particle. In essence, graviton emission as a result of the electromagnetic part of the charge is analogous to electromagnetic synchrotron radiation. Within the framework of the weak gravitational field, it can be found[80] that a particle having a mass m and the elementary unit of electric charge e, moving in a magnetic field with energy $\mathcal{E}$, emits gravitational waves of intensity

$$-\frac{d\mathcal{E}}{dt}=\frac{39}{8}\cdot\frac{GM^2c}{a^2}\cdot\frac{\mathcal{E}}{\mathcal{E}_0},$$

where $\mathcal{E}_0=Mc^2$ is the self-energy of the particle and a is the radius of curvature of the particle orbit. In the ultrarelativistic case, the particle emits gravitational waves predominantly forward, and in the wave zone the waves are transverse.[80,90–92]

We note here, for comparison, the ratio of the synchrotron gravitational and electromagnetic radiations[20,93]:

$$\left(\frac{d\mathcal{E}}{dt}\right)_{\rm gr}\Big/\left(\frac{d\mathcal{E}}{dt}\right)_{\rm el}=\frac{13}{4}\cdot\frac{GM^2}{e^2}.$$

For radiation by an electron, this ratio is 10^{-43}.

An effect inverse to that considered above is the influence of a gravitational field, particularly a gravitational wave, on electromagnetic radiation of relativistic charges. These effects were considered for a weak gravitational field in Refs. 94–105. Thus, analysis shows[94,95] that if a gravitational wave propagates along the x axis and a relativistic charge moves in the accelerator yz plane along a circle of radius R_0, then the synchrotron electromagnetic radiation dI of such a charge differs from the radiation dI_0 of this charge in "vacuum" (in the absence of the gravitational wave) in the ratio

$$dI/dI_0=-\frac{1}{6}\left(\frac{R}{R_0}\right)^2\left(\frac{\mathcal{E}}{mc^2}\right)^4h\sin\nu(t-x/c),$$

where $\mathcal{E}$ is the kinetic energy of the particle; h is the deviation of the metric in the gravitational wave from the metric of a flat world; ν is the frequency of the gravitational wave; R is the distance from the center of the orbit to the point where the synchrotron radiation is observed. This ratio is valid for distances given by $R_0\sqrt{1-\beta}/h$, where β is the particle velocity.

At an approximate electron energy 500 MeV we have $R/R_0\approx100$ and $h\sim10^{-18}$, which is equivalent to the amplitude of a gravitational wave reaching the earth from a binary star having a revolution period of approximately two hours; dI/dI_0 changes 0.15% during the period of the wave.[94]

An interesting problem was considered in Ref. 96 concerning the interaction of a plane gravitational wave with a beam of relativistic particles that move with equal acceleration along or across the propagation direction of the gravitational wave. It is obvious that by virtue of the spatial anisotropy of the gravitational wave these charge fluxes will interact differently with the gravitational wave and will have different radiation intensities. This phenomenon can serve as the basis for various gravitational radiation detectors.

Maxwell's equation for the electromagnetic-field tensor F_{mn} in a gravitational field is[96,97]

$$\frac{\partial}{\partial x^k}\left(\sqrt{-g}\,g^{im}g^{kn}F_{mn}\right)=\frac{4\pi}{c}j_lg^{il}\sqrt{-g}.$$

Solution of this equation shows that charges moving relativistically in the propagation direction of the gravitational wave do not "feel" this wave, and radiate electromagnetic waves equally well in the presence and in the absence of the wave. However, charges that move perpendicular to the gravitational-wave propagation have an additional electromagnetic radiation. In the relativistic case, the radiation due to h_{ik} can reach intensities commensurate with the electromagnetic radiation of an accelerated charge.

To detect a gravitational wave that reaches the earth from a binary star, Dimanshteĭn[96] proposes to use two particle accelerators, one directed along the propagation of the gravitational wave and the other oriented perpendicular to it. By measuring the difference between the synchrotron radiation of the particles in these accelerators, it would be possible to draw definite conclusions concerning the presence or absence of a gravitational wave in space.

Several recent papers (see, e.g., Ref. 106) have been devoted to the emission of gravitational waves by tachyons. We shall not dwell, however, on these papers, since the very existence of such exotic particles as tachyons in nature is highly problematical.

Refs. 20, 41, 87, and 107–109 consider effects connected with the appearance of gravitons in annihilation processes. Thus, in the annihilation of an electron-positron pair there should be produced a pair of gravitons[20,87] with a cross section

$$d\sigma_0=\frac{m^2k^4}{(64\pi)^2}\beta\,d\Omega\text{ — nonrelativistic case;}$$
$$d\sigma_\infty=\frac{P_0^2k^4}{2\,(64\pi)^2}(3\sin^2 2\theta+2\sin^4\theta)\,d\Omega\text{ — ultrarelativistic case.}$$

Here β is the relative velocity of part of the annihilating pair; m is the electron mass; $k=\sqrt{2\kappa}$; P_0 is the energy of the particles in the c.m.s. Somewhat different cross sections were obtained for this process in Refs. 41 and 107.

In the case when the products of electron-positron annihilation are a graviton and a photon,[20] the cross section is

$$d\sigma_0=\frac{e^2k^2}{16\,(4\pi)^2}\sin^2\theta\,(P^3/P_0^3)$$
$$\times(1+2\sin^2\theta)\,d\Omega\text{ — nonrelativistic case;}$$
$$d\sigma_\infty=\frac{e^2k^2}{(16\pi)^2}(1+\cos^2\theta)\,d\Omega\text{ — ultrarelativistic case.}$$

Here e is the electron charge and P is the particle momentum.

Finally, the exchange Compton effect, in which a charged particle absorbs a photon and emits a free graviton,[20] has a cross section $\sigma\sim e^2k^2(E/k_1)^2$, where E is the photon energy and k_1 is the photon wave vector.

A somewhat different expression for the same process is given in Ref. 107. Thus, for the low-energy process,

$$d\sigma=Ge^2\,\mathrm{ctg}\,\theta/2\,(\cos^4\theta/2+\sin^4\theta/2)\,d\Omega.$$

We present also a formula for the cross section for Compton scattering of a low-energy graviton with $\omega\ll m$ by an electron and a scalar particle[87] (the cross sections are equal):

$$d\sigma=G^2m^2\,(\mathrm{ctg}^4\,\theta/2+\sin^4\theta/2)\,d\Omega,$$

where θ is the graviton scattering angle.

6. RECEPTION OF GRAVITATIONAL WAVES

Interaction of gravitational waves with matter. This interaction can result in the conversion of the gravitational-wave energy into other forms of energy or into action on other types of motion. One of the convenient methods of receiving gravitational waves is to register the motion of test particles in the field of the gravitational wave. Let us examine this method in greater detail. In the absence of the gravitational field and of other external fields, test particles move inertially, (see, e.g., Ref. 2), i.e., $du^i=0$, where $u^i=dx^i/ds$ is the 4-velocity.

In the presence of a gravitational field, this equation takes the more general form

$$Du^i=du^i-\Gamma^i_{kl}\,du^k\,dx^l,$$

where Du^i is the covariant differential. It follows from this equation that free motion of particles in a gravitational field proceeds along the geodesics

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds}=0,$$

where s is the length of the arc along the geodesic, and $ds^2=g_{ik}\times dx^i\,dx^k$. The quantities Γ^i_{kl} are Christoffel symbols, which characterize the intensity of the gravitational field; g_{ik} is the potential of this field. Now the determination of the trajectory of a test body in a gravitational-radiation field reduces to the solution of the equations for geodesic lines. A detailed solution of this question is considered, for example, in Ref. 110. We present here a somewhat abbreviated variant of this analysis.

Assume that an assembly of test particles moves along the geodesic lines $x^\alpha(s)$. We consider one parametric family of such curves $x^\alpha(s,v)$, where the variation of the parameter v corresponds to a transition from one geodesic to another. To describe the motion of the particles one usually employs two vectors[14]:

$$u^\alpha(s,v)=\partial x^\alpha/\partial s;\quad \eta^\alpha(s,v)=\partial x^\alpha\,dv/\partial v.$$

The first is the vector tangent to the geodesic, and the second is an infinitesimally small vector, perpendicular to the tangent, of the displacement of one particle relative to the other. The quantity $D^2\eta^\alpha/ds^2$ is the geodesic deviation and is a measure of the relative acceleration of two infinitely close particles as they move along neighboring geodesics. In the theory of gravitation, the equation for the deviation of the geodesics[20] is

$$D^2\eta^\gamma/ds^2+R^\alpha_{\beta\gamma\delta}u^\beta u^\delta\eta^\gamma=0. \quad (25)$$

It follows from this equation that the relative acceleration of two closed particles is determined completely by the components of the curvature tensor of the gravitational field, i.e., by the Riemann tensor. Consequently the gravitational field perturbing the Riemann tensor changes by the same token also the relative acceleration of the two close particles. It is the observation of this acceleration which yields information on the passage of a gravitational wave. If, for example, two particles are coupled with each other by an elastic element, say a spring, then the gravitational wave will cause this element to oscillate. When the frequency of the gravitational wave coincides with one of the harmonics of the natural frequencies of the elastic element, a resonance is produced in the system and makes it possible to accumulate the amplitude of the oscillations over many periods of the gravitational wave, and thus to register very weak waves. Analogously, a gravitational wave can cause elastic oscillations in a piezoelectric crystal and produce in it, via the strain, a measurable emf.[52] Equation (25) does not take into account the elastic element (the interaction of the test particles) in the system. To take this interaction into account it is necessary to introduce in the right-hand side of the equation the interaction force F^{μ} (Ref. 52), i.e.,

$$\frac{\partial^2\eta^{\mu}}{\partial s^2}+R^{\mu}_{\alpha\beta\gamma}u^{\alpha}u^{\beta}\eta^{\gamma}=\frac{1}{c^2}\frac{\delta}{\delta v}\cdot\frac{F^{\mu}}{m}dv. \tag{26}$$

We consider now two cases, when a plane gravitational wave is incident on a system of particles that do not interact with one another and on a system of interacting particles. In the first case, solving (26) without the right-hand side and taking h_{ik} into account, we obtain (see, e.g., Ref. 4):

$$ds^2=dx^{1^2}-(1+h_{22})\,dx^{2^2}-(1-h_{22})\,dx^{3^2}-2h_{23}\,dx^2\,dx^3.$$

It follows therefore that the distance between two particles along the wave propagation direction does not change with time. However, the particles located in a plane perpendicular to the wave-propagation direction experience the largest relative acceleration. If we write down the equation of motion of the particles in this plane, taking one of the particles to be the origin of a polar coordinate system ($x^1=r\sin\theta$; $x^2=r\cos\theta$), then we obtain

$$s=r\,(1-h_{22}\cos 2\theta+0.5h_{23}\sin 2\theta).$$

This relation demonstrates once more the transverse character of the gravitational waves. The components h_{22}, h_{33}, h_{23} determine two polarization states of the gravitational wave, the difference between which is that the components of the stresses in the gravitational wave are rotated through 45° (details on polarization of gravitational waves can be found in Refs. 33, 36, 111, and 112). These two polarization states are shown schematically in Fig. 1. In the case of a scalar gravitational wave, there is only one type of linear polarization, in which the test particles in the wave experience only symmetrical radial accelerations (see Fig. 1). It is this picture of the action of gravitational waves on the test particles which determines in principle the possible methods of receiving such waves by measuring the accelerations or distances between them. Figure 2 illustrates by way of example a system[36] consisting of a radiator (binary star) and a receiver (quadrupole antenna).

We discuss now a second variant of the action of a gravitational wave on test particles, when the latter are interconnected by an elastic element.[33,52,113] For the case of two pointlike masses of a harmonic oscillator and a plane wave, Eq. (26) can be transformed into[33,52]

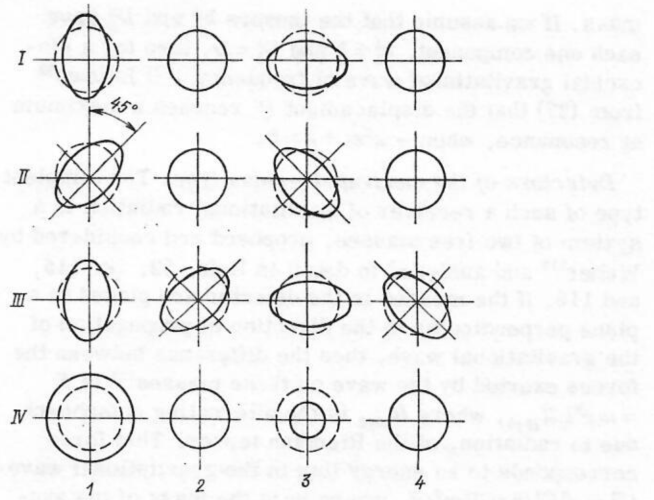

FIG. 1. Displacement of test particles in the field of a plane gravitational wave incident perpendicular to the plane in which the particles are located. The dashed circles represent the position of the particles prior to the arrival of the wave; 1, 2, 3, 4) sequence of the positions of the particles that differ in phase by 90°; I) tensor-type wave of one linear state of polarization; II) second independent state of polarization; III) circular polarization; IV) scalar-type wave.

$$\frac{d^2\xi^{\mu}}{dt^2}+\frac{D^{\mu}_{\alpha}}{m}\cdot\frac{d\xi^{\alpha}}{dt}+\frac{k^{\mu}_{\alpha}\xi^{\alpha}}{m}=-c^2R^{\mu}_{0\alpha 0}r^{\alpha}, \tag{27}$$

where ξ^{μ} characterizes the relative displacement of the test particles under the influence of the wave; the tensors k^{μ}_{α} and D^{μ}_{α} describe respectively the elastic properties of the "spring" and the damping in the spring; r^{α} is the distance between particles. The components of the Riemann tensor for a plane gravitational wave propagating in the direction of the z axis can be expressed in terms of the perturbation of the metric tensor h_{ik} in the following form[40]:

$$R_{x0x0}=-R_{y0y0}=-0.5\ddot{h}_{+}(t-z/c);$$

$$R_{x0y0}=R_{y0x0}=0.5\ddot{h}_{\times}(t-z/c).$$

All the remaining components are equal to zero. Here h_{+} and $h_{\times}$ are dimensionless functions representing the instantaneous amplitude of the wave in two mutually orthogonal states of polarization. We note here[40,52,114] that the relative force produced between the two test particles in the field of the gravitational wave and starting the oscillations of these masses is equal to $f_j=-\sum_k mc^2R_{j0k0}x^k$, where x^k is the component of the length vector between the masses of the quadrupole made up of the test particles; m is the total quadrupole

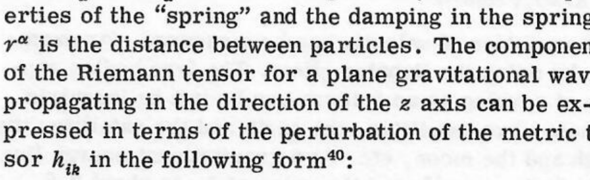

FIG. 2. System consisting of a radiator and receiver of gravitational wave: I) radiator, binary star with mass M; II) receiver, quadrupole antenna with mass m; 1) circular wave polarization, 2) linear wave polarization.

mass. If we assume that the tensors k^{μ}_{α} and D^{μ}_{α} have each one component, $k^1_1=k$ and $D^1_1=D$, then for a sinusoidal gravitational wave of frequency ω it follows[52] from (27) that the displacement ξ^{μ} reaches a maximum at resonance, when $-\omega^2 m+k=0$.

Detectors of the quadrupole-mass type. The simplest type of such a receiver of gravitational radiation is a system of two free masses, proposed and considered by Weber[115] and analyzed in detail in Refs. 52, 78, 115, and 116. If the masses in the detector are placed in a plane perpendicular to the direction of propagation of the gravitational wave, then the difference between the forces exerted by the wave on these masses[115] is $F=mc^2 l_0 R_{i0j0}$, where R_{i0j0} is the alternating component, due to radiation, of the Riemann tensor. This force corresponds to an energy flux in the gravitational wave $t^{01}=c^3F^2/8\pi G l_0^2 m^2\omega_0^2$, where m is the mass of the system, l_0 is the unperturbed distance between its component masses, and ω_0 is the frequency of the gravitational wave. The force leads to a change in the distance between the masses, given by $l=l_0+hl_0$, where h is the amplitude of the gravitational wave. It follows therefore that the relative displacement of the masses is $\Delta l/l=h(t)$. If the line joining the masses is not orthogonal to the wave vector of the gravitational wave, then the displacement Δl must be multiplied by a factor $\sin^2\theta$. The quantity Δl can be measured by an optical, electric, or radar method. Under the influence of the gravitational wave, the bodies in the binary system will experience also a change in their relative velocity,[78] by an amount

$$\Delta v=l_0\sqrt{8\pi G t^{01}/c^3}.$$

This variation of velocity can be measured, for example, by using the Doppler effect. The free bodies capable of serving as an antenna can be two heliocentric stations, two satellites, the earth and the satellite, the earth and the moon, etc. At a gravitational energy flux 10^{-2}–10^{-4} erg/cm² sec, the value of Δv is about 2.5×10^{-11} cm/sec. It is still difficult to measure such quantities on cosmic-space scales, but under laboratory conditions this quantity is relatively easy to measure.

The next modification of this type of detector is a system consisting of two bodies coupled by an elastic element, say a spring, a magnetic field, etc. Such an almost free system of masses is capable of resonant reception of gravitational waves, and this can greatly increase the reception sensitivity. The amplitudes of the displacements at resonance can reach $\xi=0.5\,Ql_0h$. The maximum power transmitted to the antenna by the gravitational wave[52,115] is

$$P_{\max}=m^2c^4(R^{\mu}_{0\alpha0}l^{\alpha})^2/8D_{in},$$

where l^{α} is the distance between the masses and D_{in} is the energy internally dissipated in the spring. The maximum amplitude of the oscillations is obtained when the conditions $l=\lambda/2$ is satisfied,[117] where λ is the acoustic wavelength in the spring. If there is no damping in the system, then the cross section for absorption of the gravitational wave by the antenna at resonance is $\lambda^2/4\pi$ (Ref. 33). However, any system is subject to damping characterized by a quality factor Q. Therefore the absorption cross section is[33,52]

$$\sigma=15\pi Gmr^2Q\beta^2/(8\omega c),$$

where mr^2 is the quadrupole moment of the antenna, $\beta=2\pi/\lambda$ is the wave vector of the gravitational wave, and ω is the frequency of the gravitational wave. The absorbed power is $P=\sigma t^{01}$. The real value of Q is usually 10^6, so that the sensitivity of the considered antennas is characterized by the value 2×10^5 erg/cm² sec (Ref. 116), which makes it possible to measure a distance variation $\Delta l_0=2\times10^{-23}$ cm. For such a sensitive reception, however, the nongravitational noise in the antenna must be "suppressed" to a level $F_{\rm mech}=2\times10^{-14}$ dyne.

In Refs. 118 and 119 there was proposed a design for detectors of low-frequency gravitational radiation, in which the masses are suspended in superconducting chambers with the aid of a magnetic field. Under ideal conditions (without thermal noise and external interference) these detectors are capable of measuring gravitational radiation of low and medium frequency with energy flux 10^{-20} erg/cm² sec.

V.V. Braginskiĭ and V.V. Rudenko[120] have proposed a quadrupole-mechanical resonant detector consisting of two pendulums suspended from a common base in a vacuum chamber on thin quartz ribbons. The detector is designed to receive gravitational waves at frequencies of several hertz. The detector has a high figure of merit $Q=10^9$, making it sensitive to a gravitational flux of 3×10^{-4} erg/cm² sec (for a synchronous signal-separation time on the order of days).

V.N. Mironovskiĭ[121] considered the possibility of measuring gravitational waves with the aid of a torsion pendulum suspended in vacuum on a long thin quartz filament. The deflection of the ends of the pendulum from the equilibrium position under the influence of the gravitational radiation is $\delta=l\sqrt{Gtt^{01}/c^3}$, where l is the lever arm of the pendulum and t is the duration of the observation. This detector can measure a deflection δ on the order of 10^{-15} cm. Antennas of the dumbbell type were also proposed,[122] in which the connecting element can be a thin elastic rod. Refs. 123 and 124 consider an almost free antenna in the form of a square "doughnut." These types of antennas, as claimed by their authors,[122–124] can have very high sensitivity.

Refs. 125 and 126 consider a simple but not very sensitive low-frequency quadrupole detector, comprising either one[125] or two[126] dumbbells rotating about a common center of gravity. If the plane of rotation of the dumbbell is orthogonal to the direction of propagation of the gravitational wave, then the wave exerts on the dumbbell a torque[125]

$$M=4m\omega R^2\sqrt{8\omega G t^{01}/c^3}\,\sin 2\varphi\cdot\sin\omega t,$$

where R is the "dumbbell level arm," ω is the frequency of the gravitational radiation, and φ is the angle between the straight line joining the masses of the dumbbells and one of the directions of the wave polarization. The gravitational wave will cause the dumbbell to execute torsional oscillations or beats, if it rotates. The sensitivity of this detector is about 1 erg/cm² sec. In the case of two dumbbells[126] arranged perpendicular to each other and rotating with frequency $\omega/2$, the linear swing of the relative beat can reach about 10^{-12} cm at a gravitational energy flux 10^{-6} erg/cm^2 sec. A similar antenna was proposed also by A.D. Sakharov (cited in Ref. 6). In his variant the dumbbell does not rotate, but at resonance with the gravitational wave longitudinal forced oscillations are excited in the dumbbell from the outside. The gravitational wave twists the dumbbell in one direction at the instant when it is in tension, and in the other direction when it is in compression, i.e., the dumbbell experiences angular accelerations under the influence of the wave.

A gravitational detector consisting of two charged test bodies was considered by G.A. Lupanov.[86] He imagined a charged parallel-plate electric capacitor, whose plates execute periodic oscillations under the influence of the gravitational wave and radiate a transverse electromagnetic wave which can be measured by means of an ordinary electromagnetic antenna. The gravitational-wave power transformed by the detector into an electromagnetic wave is

$$P = 4.9\cdot 10^{-20}\frac{l^2}{\nu^2}\, g t^{01},$$

where ν is the frequency of the gravitational wave, l is the distance between the plates of the capacitor, and g is the gain of the antenna.

The next class of detectors of the quadrupole type consist of solid bars or cylinders. The gravitational radiation, interacting with the solid bar, produces elastic oscillations, which can be transformed into an electric signal either directly, if the detector is made of piezoelectric material,[52,115,127] or with the aid of piezoelectric pickups secured to the antenna. Detectors of this type come in many modifications. Thus, a detector of a piezo-crystal type was first considered by Weber.[52,115] He has shown that the effective cross section of such an oscillator, absorbing gravitational waves in a band $\Delta\nu = \nu_0/Q$, is equal to $\sigma = 20[r_g/\lambda]QL^2$, where $r_g = 2Gm/c^2$ is the gravitational radius of the oscillator, λ is the resonant acoustic wavelength of the oscillator, and L is its linear dimension along the longitudinal axis. The displacement of the points relative to the mass center of the detector reaches its maximum at $kL/2 = n\pi/2$, where k is the wave vector of the acoustic wave in the crystal and n are odd integers. The power absorbed by the detector from the gravitational wave is[52]

$$P = 10^{-25}\omega^{-1} V Q t^{01},$$

where ω is the frequency of the gravitational wave, V is the volume of the rod (cylinder), and Q is its quality factor.

Braginskiĭ *et al.*[128] and Misner *et al.*[6] have shown that the main difficulty in the reception of gravitational signals by these detectors lies in the thermal noise that is present in solids. The minimum power that can be registered, for example, by a cylindrical detector in the case of a harmonic gravitational wave of known frequency and phase is

$$P_{\min} = 2kT\theta^2(\alpha, m)\, M\, \tau\, (m-1),$$

where θ is the tabulated confidence factor of the result for a sampling consisting of m measurements; α is the reliability of registration; τ is the time of one measurement; M is the mass of the detector.

It is appropriate to note here that Weber[129] made extensive use in his experiments of a cylindrical monodetector made of aluminum, of length 1.5 m, diameter 60 cm, and mass 1.5 tons. The frequency of the fundamental mode of the detector was 1661 Hz. The oscillations were transformed into electric signals with the aid of piezoelectric pickups secured to the surface of the cylinder in its central part. The detector had $Q = 10^5$, and its sensitivity at room temperature, for the reception of the peak gravitational radiation, was approximately 10^6 erg/cm^2 sec, which corresponded to a decrease of the cylinder length by 10^{-15} cm. If such an antenna is used at temperatures 4 °K, then a gravitational-radiation flux 10^2–10^3 erg/cm^2 sec can be observed. In coherent reception of waves, with a signal separation time 10^7 sec, the sensitivity of the detector can be 10^{-3}–10^{-4} erg/cm^2 sec.[52,130]

A somewhat different detector construction was used by Braginskiĭ *et al.*[131,132] A solid aluminum cylinder of diameter 20 cm, length 64 cm, and mass 40 kg had additional cantilever rods ("horns") which were secured to the end of the cylinder and converged towards its center. In the gap between the "horns" were mounted removable capacitive displacement pickups. The detector had $Q = 10^5$ and the fundamental frequency was 4 kHz. This detector was sensitive to a gravitational flux that produced a cylinder contraction of 2×10^{-14} cm.[132] The "horns" in Braginskiĭ's detector served to transmit the displacement of the end of the cylinder to a measuring capacitor. Velay and Filler (cited in Ref. 40) proposed to use a resonant long rail several kilometers long for the same purpose. Such a rail behaves like an absolutely rigid rod between resonant frequencies.

Modifications of the ideas of Braginskiĭ *et al.*[131] were suggestions to measure the displacement of the ends of cylindrical antennas with the aid of a piezoelectric pickup,[133] a modified electric capacitor,[134,135] an inductive pickup,[136] and superconducting magnetic coils.[137,138] These improvements make it possible to measure a displacement of 10^{-17} cm.

An original indirect method of measuring transverse oscillations an aluminum cylinder under the influence of gravitational radiation was proposed by Kopvillem and co-workers.[139] The gist of the idea is that a lithium-niobate single crystal is glued to one of the ends of the cylinder, and monochromatic electromagnetic radiation (light from a laser source) is passed through the crystal. The elastic oscillations produced in the single crystal under the influence of the gravitational wave lead to scattering of the light. The light passing through the crystal experiences Rayleigh scattering at the fundamental frequency ω_0 and Mandel'shtam-Brillouin scattering at the sideband frequencies $\omega_0 \pm \omega$, where ω is the fundamental mode of the detector. In this way it is possible to measure a gravitational flux on the order of 10^{-12} erg/cm^2 sec. It should be noted by way of a critical remark, however, that Kopvillem *et al.*[139] disregarded in their calculation the perturbation due to the reaction of the light flux on the detector oscillations. Allowance for this circumstance decreases the indicated sensitivity by several orders of magnitude.

In concluding this section, we mention also the suggestions[88,140–142] to use the earth and the moon as massive detectors. These cosmic bodies have quadrupole moments and are capable of resonating with gravitational waves of approximate frequency 10^{-3} Hz. At the same time, the earth and the moon have local inhomogeneities, in which gravitational waves can also be absorbed[142,143] and acoustic oscillations can be excited. However, as shown by calculations,[142] the seismic noise on the earth and on the moon is very strong and exceeds by many orders of magnitude the expected useful signals due to the gravitational waves.

Microscopic and macroscopic quantum systems as gravitational-wave detectors. A number of papers[57–60,144–146] have proposed and analyzed a new class of detectors, in which gravitational radiation excites coherently a system of quadrupole moments in matter, such as atoms, molecules, and macroscopic quantum systems (such as fluxoids and superconductors, vortex filaments and rotons in superfluid helium, etc.). Two modes of gravitational-radiation reception are possible here. In the first[57–60] the gravitational wave interacts with the mass quadrupoles of an ensemble of particles and causes them to precess, giving rise simultaneously to precession of the electric multipoles at the same frequency. This process is accompanied by excitation of the medium to a superradiant electromagnetic state. The subsequent deactivation of the medium should be accompanied by emission of photons, which can be registered by the usual photoelectronic technique. It is obvious that such a detector must be carefully screened against external electromagnetic disturbances.

In the second mode of registration of gravitational radiation[57,58] it is proposed first to transfer the medium with the quadrupole moment into a superradiant state by electromagnetic pumping, and then "de-excite" the stored energy by simultaneous application of gravitational and laser radiation to the system. At definite phase relations and wave-vector directions of the gravitational and electromagnetic waves, the excited system will release its excess energy intensively. This regime of registration of the gravitational waves is seen to be analogous in many respects to the reception of weak harmonic electromagnetic signals with the aid of quantum amplifiers. Estimates show[145,146] that in any complete laboratory experiment employing both a transmitter and a receiver of gravitational waves of this type, operating in the pulsed regime, the photon flux from the receiving part amounts under the most optimal conditions to 10^{-5} sec^{-1} within a pulse time 10^{-7}–10^{-8} sec.

V.I. Vysotskiĭ and V.I. Vorontsov[147] propose to detect gravitational waves using induced Mössbauer lines.

The foregoing suggestions are quite interesting from the point of view of performing a complete laboratory experiment, but their technical realization is exceedingly difficult. It suffices to indicate that the cross section for excitation of the ammonia molecule by a gravitational wave is 10^{-60} cm^2 (Ref. 33). The cross section for excitation of the hydrogen molecule is of the same order.[148]

Detectors based on gravitational-electromagnetic resonance. We have considered above the effect of gravitational-electromagnetic resonance from the point of view of gravitational radiation, and indicated the conditions for the mutual conversion of their energies. This effect served in recent years as the basis for suggesting many new detectors intended primarily for the reception of gravitational radiation in the microwave band. Thus, a variant of a detector using a Michelson interferometer is analyzed in Refs. 149–151. A light beam propagating in two mutually perpendicular directions inside a gravitational wave experiences different phase shifts. The physical cause of this phenomenon is that the gravitational radiation changes the refractive index of the vacuum in accordance with the formula $n=1+0.5\, h_{\alpha\beta}n^\alpha n^\beta$, where n^α is a unit vector directed along the light beam. The relative difference between the optical lengths of light beams propagating along and across the propagation direction of the gravitational wave $\Delta l/l=0.5\, h_{\alpha\beta}n^\alpha n^\beta$, where l is the unperturbed length of the interferometer arm. Consequently, the gravitational wave should cause periodic displacements of the interference fringes in the instrument. The sensitivity of such a detector is determined by the relation[149]

$$\Delta l/l=\frac{1}{2\omega}\sqrt{16\pi G t^{01}/c^3}.$$

At a flux $t^{01}=1$ erg/cm^2 sec we have a ratio $\Delta l/l = 8\times10^{-17}$, which is measurable at the present time. The sensitivity of the detector can be increased further by lengthening the observation and brought up to 10^{-4} erg/cm^2 sec.

Vodyanitskiĭ and Dimanshteĭn[76] and Heintzmann[152] have considered the perturbation of the electromagnetic wave in the field of a gravitational wave for the case when these waves propagate in strictly opposite directions. They found that the electromagnetic wave undergoes strong amplitude modulation at frequencies $\omega_0\pm\omega$, where ω_0 is the frequency of the electromagnetic wave and ω is the frequency of the graviational wave. The effect of the modulation can be measured by means of a typical electromagnetic antenna. The power received by the antenna is

$$P\approx\frac{cs}{4\pi}\cdot\frac{E^2(\omega_0-\omega)^2}{16\omega_0^2}h_{\alpha\beta}^2,$$

where E is the intensity of the electric field in the electromagnetic wave and s is the effective area of the antenna.

The interaction of a gravitational wave with electric charges was analyzed in Refs. 94 and 102, and it was shown that the radiation characteristics of the charges are altered in the wave. It is proposed to use this circumstance to detect gravitational radiation. Thus, if a gravitational wave of intensity 10^{-9} erg/cm^2 sec and a period of two hours is incident on an electron accelerator with electron energy of approximately 500 MeV, then the intensity of the electromagnetic synchrotron radiation of the electrons in the accelerator, as already noted, will change by 0.15% over a half-cycle of the wave.[94] The gravitational radiation can cause oscillations of laser-emission frequencies,[153] and this can be used to detect the gravitational waves.

Braginskiĭ and Menskiĭ[154] have proposed a method of recording gravitational waves with the aid of a packet of electromagnetic waves "circulating" in a closed superconducting toroidal waveguide. Under the influence of the gravitational waves incident perpendicular to the plane of the waveguide, the energy of the packet changes in a ratio $\Delta\mathcal{E}/\mathcal{E} \approx \Delta l/l$, where l is the length of the packet. A detailed calculation of this process[156] shows that

$$\Delta\mathcal{E}/\mathcal{E} = 0.5h_0\Omega t \sin 2\varphi_0 = t \sin 2\varphi \sqrt{64\pi G t^{01}/c^3},$$

where h_0 is the amplitude of the gravitational wave, Ω is its frequency, and φ_0 is the angular coordinate of the point at the instant $t=0$. This detector is capable of recording gravitational fluxes on the order of 10^{-2} erg/cm^2 sec. A further modifications of this detector are a rectangular waveguide loop[157] with reflecting mirrors at the corners and a straight waveguide with ideally reflecting mirrors at the ends.[156] If the mirrors in the straight waveguide are placed not strictly perpendicular to the waveguide and not parallel to each other, then it is possible to produce in the waveguide conditions under which the length of the electromagnetic packet varies systematically with time under the influence of the gravitational wave. The maximum sensitivity of the detector in the case of resonant reception of the gravitational wave is approximately the same as for the ring-type detector.

Halpern[156] has also analyzed methods of receiving gravitational waves with the aid of waveguides containing electromagnetic fields. Thus, if a constant electromagnetic field is present in the waveguide, then a gravitational wave will be generated. The fraction of the gravitational energy transformed in the waveguide into the electromagnetic-wave energy[155] is

$$\alpha = Gc^{-4}(E^2 + B^2)h_0 ct,$$

where $E^2 + B^2$ is the energy density of the static electromagnetic field in the resonator. If electromagnetic oscillations are already present in the resonator, then the gravitational wave will change either their energy or their phase. If the waveguide is in the form of a thin toroid along which a monochromatic electromagnetic wave propagates, then a gravitational wave penetrating through the waveguide will produce in the latter new electromagnetic-field quanta of frequency $(n \pm 2)\,\Omega/2$, where Ω is the frequency of the gravitational wave. Detection of the amplitude and phase of the new quanta will yield complete information on the gravitational wave. Finally, if two electromagnetic waves with frequencies ω_1 and $\omega_2 = \omega_1 \pm \Omega$ are present in such a waveguide, then a gravitational wave will amplify the intensity of one of the waves (the weaker one) at the expense of the other wave. The rate of equalization of the energy will be $\sim h_0 Q$, where Q is the resonator figure of merit. Ref. 155 gives estimates of the sensitivities of all these methods to the gravitational fluxes and it is indicated that they can reach 10^{-4} erg/cm^2 sec.

Other types of gravitation detectors. The passage of gravitational radiation in space changes the dielectric constant and the magnetic permeability of the space.[2] Sladskiĭ[157] proposes to use this effect to receive gravitational radiation with the aid of a magnetic coil carrying direct current. The gravitational radiation changes the magnetic permeability in the interior of the coil, and consequently modulates the magnetic flux. Its ac component can be measured with sensitive magnetometers.

An antenna consisting of a liquid-filled figure-eight tube was proposed by Press.[40] A gravitational wave is capable of propelling the liquid along the tube in such an antenna. This antenna is equivalent to a magnetic-loop antenna in electromagnetism. A modification of the proposed antenna might be one made up of a superconducting wire, in which electrons are driven instead of the "liquid." The electrons would produce a weak alternating current of the same frequency as the wave. The possibility of using ordinary and superconducting metals to detect gravitational waves of medium frequencies $\omega \ll \omega_0$, where $\omega_0 = 10^{11}$ Hz is the natural frequency of the electron oscillations in the metal and corresponds to the binding energy in the Cooper pair, was analyzed in Refs. 158 and 159. Under the influence of a quasi-static gravitational wave, the electrons in the metal move like a gas in a gravitational field and give rise to electromagnetic waves. Dozmorov and Zadonskii[160] proposed to receive gravitational waves by means of an ordinary electromagnetic antenna of the quadrupole type. Forward[36] designed a gravitational antenna of the spherical type capable of selectively receiving tensor and scalar waves. The possibility of reception of gravitational waves in the infrared band with the aid of a liquid binary solution at the critical stratification point was analyzed in Ref. 161. The gravitational radiation causes the solution to break up into its original components. A cylindrical aluminum detector for the reception of gravitational waves in the frequency band up to 31 kHz was proposed in Ref. 162.

Winterberg[163] and Bergmann[164] considered a method of detecting gravitational waves by means of the stellar flicker they produce. However, as shown by detailed calculations in Ref. 165, the effect of this phenomenon is negligibly small. To search for scalar gravitational waves, Weber[166] has constructed a disk antenna that resonated at 1661 Hz and could measure an energy flux density on the order of 10^6 erg/cm^2 sec.

7. SEARCH FOR GRAVITATIONAL WAVES FROM ASTROPHYSICAL SOURCES

For some five years, Weber[166–171] has been systematically recording excitation pulses with the aid of two gravitational antennas operating in coincidence. He interprets these pulses as bursts of gravitational radiation, coming from the center of the galaxy. The receiving apparatus contains the two cylindrical antennas described in the preceding section, spaced 1000 km apart, one at the University of Maryland and the other at the Argonne National Laboratory near Chicago. Piezoelectric pickups glued to the antennas transform the elastic oscillations into electric signals. The outputs of the pickups are connected to coincidence circuits with resolution time 0.1 sec. The antennas are suspended on elastic wires in special vacuum chambers, whose walls serve simultaneously as electromagnetic shields. Protection against external seismic action is provided by special acoustic filters. The sensitivity of the instrument is limited only by the Brownian normal oscillations of the cylinders themselves, corresponding to an energy on the order of kT for each resonant oscillation. The relative change in the cylinder length under the influence of the gravitational radiation is

$$\Delta l/l = \frac{2c^2 Q}{\pi\omega^2} R_{i0j0}.$$

The overall sensitivity of the apparatus is such that the experiments record a displacement of 1.5×10^{-15} cm of the ends of the cylindrical antennas. This corresponds to a gravitational-radiation flux density 2×10^4 erg/cm² sec. However, Braginskiĭ and Rudenko[120] estimate this flux at 3×10^6 erg/cm² sec. The measurement is carried out at the frequency of the fundamental mode of the cylinders, namely 1661 Hz. The antenna has $Q = 10^5$. The system therefore operates in a band $\Delta\nu = \nu_0/Q$ and its cross section for the absorption of gravitational radiation is

$$\sigma = 20\frac{2Gm}{c^2\lambda} QL^2 = 3\cdot10^{-19}\ \text{cm}^2.$$

The detectors were calibrated against a standard noise source and with the aid of a dynamic gravitational field.[146] In the latter case, the field was produced by oscillations of a somewhat smaller second aluminum cylinder located two meters away from the tested detectors. The output power of the detectors agreed approximately with the calculated value.

The experimental procedure consists of recording continuously the total number of coinciding signals from the two antennas, without introducing time delays in the coincidence channels. A delay in one of the channels is then introduced and the background events are measured. The difference between the counts produced in these two measurements is taken to be the true number of coincidences due to the gravitational signals. In the latest experiment[170] the average count of the gravitational signals was about 7 events daily. This is the highest frequency of coincidences, at which the cylinder axes are perpendicular to the direction to the galactic center. The experimental results have led Weber to conclude that the sources of gravitational radiation are located at the center of the galaxy. Weber has also performed an experiment with a disk antenna, aimed at detecting scalar gravitational waves,[168] but with negative results.

The results of Weber's experiments have been widely criticized of late. The basis for the criticism is mainly the fact that the gravitational-radiation power flux measured in the experiment turns out to be too large, 10^6 erg/cm² sec. If this value is real, then the gravitational radiation should carry away from the galactic center approximately 150 M annually[40] (M is the mass of the sun), whereas the total annual electromagnetic luminosity of the galaxy is 10^{-2} M. The large loss of mass from the center of the galaxy to gravitational radiation yields for its lifetime a value 10^6–10^7 years,[126] which is short and differs strongly from the universally accepted cosmological and geological age 15–20 billion years.[172–174]

Some workers suggest that Weber registers in his experiments external disturbances that cannot be controlled in the experiment. Thus, it is suggested by Braginskiĭ *et al.*[126] that the possible source of the signals in the experiment may be a change in g, occurring simultaneously in both detectors, as a result of acoustic waves on earth. The level of the received signals corresponds to $\Delta g/g \approx 10^{-12}$, which may be ineffectively monitored by the gravimeter used in Weber's experiments.

Adamyants *et al.*[172] carried out a correlation analysis of the connection between Weber's data and solar and geomagnetic activity, and also the intensity of cosmic rays. They found that the correlation coefficient for these phenomena is not at all small. This makes highly probable an interpretation of Weber's results as due to the action of the magnetic field of the earth and of cosmic-ray fluxes on the oscillation detectors.

A similar conclusion is arrived at by the authors of Ref. 175. They found that a perturbation of the earth's magnetic field on the order of 10^{-5} A/m is capable of exciting Weber's detectors to the level of the received signals. By way of answer to the criticism in Refs. 172 and 175, Weber and co-workers have performed control experiments with cosmic-ray particles[176] and with alternating magnetic fields.[171] He established that these phenomena are utterly unrelated to the observed signals. A similar conclusion was reached by Beron *et al.*,[177] who tested an antenna of the type used by Weber with a beam of particles from an accelerator.

Another explanation of Weber's results reduces to the assumption that either the gravitational-radiation sources are located not at the galatic center but much closer to the earth, or else that the radiation from the sources takes the form of a narrow beam.[178] It was also suggested that Weber had received gravitational radiation produced not in our galaxy but which was focused on passing through our galaxy.[179,180] A new model of the galactic center, consisting of black holes, is proposed in Ref. 181 and accounts well for Weber's results.

Weber's experiments have greatly stimulated new gravitational experiments by various scientific groups. One such experiment was performed by Braginskiĭ's group in 1971. In the experiments they used two detectors tuned to 1640 Hz and placed 20 km apart. The apparatus had a sensitivity somewhat lower than that of Weber's apparatus. In the experiment they registered up to several dozen coincidences daily, but a thorough analysis of the amplitude and waveform of the signals coming from the detectors has demonstrated with great reliability the absence of gravitational signals. Nor were Weber's signals observed in the experiments of Tyson[182] and of Levine *et al.*,[183] who used a single aluminum detector at 710 Hz, with a sensitivity higher than Weber's detector. Nor did a Frascati group[184] obtain affirmative results so far. They used two aluminum cylinders operating in coincidence and located in Frascati and in Munich.

In connection with Weber's experiments, the hypothesis was advanced that the gravitational radiation from astrophysical objects should be accompanied by co-moving flashes of radio waves and neutrinos. To verify this hypothesis, experiments were organized[185–187] aimed at observing radio waves that would travel from the center of the galaxy and would be correlated with Weber's signal. These experiments, however, yielded a negative result. Nor were any flashes of neutrino fluxes observed.[188] However, the authors of the papers cited do not conclude categorically from these negative results that there is no gravitational radiation. They assume that the gravitational radiation need not necessarily be accompanied by intense neutrino or radio emission. At the same time, radio waves and neutrinos interact with matter much more strongly than with gravitational waves, and can therefore be strongly screened by the interstellar medium. Fluxes of this radiation can be strongly attenuated by the time they arrive at the earth and can be lower than the registration threshold in the apparatus employed.

Weber believes[189] that although his experiments have not yet convincingly proved the existence of gravitational waves, the method selected by him is correct. For further investigations, it is necessary to increase appreciably the sensitivity of the apparatus.

At the present time, about twenty laboratories are planning or performing experiments aimed at independently verifying Weber's results (these experiments are treated in detail in review articles,[187,190,191]). Most new installations duplicate Weber's experimental setup, but the apparatus has higher sensitivity resulting from partial refinement and from cooling the antennas to cryogenic temperatures. Thus, V.B. Braginskiĭ and co-workers[182] have developed a highly sensitive quadrupole rod-type antenna based on single-crystal sapphire, which has an unprecedented $Q \approx 10^8$. In some experiments, it is proposed to use apparatus with a different operating principle. Thus, Forward *et al.*[150] have developed a detector for the frequency range 0.5—24 kHz based on a Michelson interferometer with an arm 1 km long. At Stanislaus State College in California,[187] they constructed a highly sensitive gravitometer based on a superconducting sphere hovering in a magnetic field. It is planned to use this instrument to investigate quadrupole types of oscillations of the earth, which can be due to gravitational waves. Similar investigations have already been performed with the automatic systems on the moon.[188]

It appears that the entire assembly of new experiments will provide a conclusive answer to the question of Weber's signals in the next three or four years.

CONCLUSION

The present status of the problem of emission and reception of gravitational waves still remains theoretically and experimentally complicated. For example, with respect to the theoretical studies, the problem lies, as already noted, in the strong nonlinearity of the equations of general relativity, for which there are still no exact wave solutions with clear-cut physical meaning. There are therefore still tremendous difficulties even in questions of correctly formulating the problem of determining the energy of the gravitational field and choosing the "correct" coordinate system for the description of the wave processes. This problem is of course being solved, but the directly opposite conclusions frequently arrived at by many theoreticians when answering the fundamental question of the possible existence of gravitational waves in nature shows clearly how far we are from a deep understanding of the essence of gravitation.

However, if the assumed linearization of the solutions of the general relativity equations in the approximation of a weak gravitational field is correct, then the tremendous efforts made by the experimenters in the search for gravitational radiation will ultimately be justified. The question of the discovery of gravitational waves will in this case be only a matter of time.

What are the prospects here? The path taken by Weber several years ago towards the search for gravitational waves of astrophysical origin was, from the point of view of the level of technique of the past years, undoubtedly the most optimal. In the problem as it then existed, the experimenters had to solve only one half of the problem, namely to ensure reliable reception of gravitational radiation. The solution of the second half of the problem, the emission of waves, was taken care of for us by nature. However, by virtue of the exceptional complexity of the experiments and the very high sensitivity of the apparatus employed, which responded to many very weak and almost uncontrollable forms of noise, all the results obtained in these experiments aimed at recording gravitational radiation of astrophysical origin will inevitably contain an appreciable element of uncertainty. This circumstance will also provide a great leeway in the interpretation of the experimental data. The author of the present review therefore believes that an exhaustive solution of the problem of gravitational radiation must be sought by organizing a complete laboratory experiment in the sense of Hertz's well known electrodynamic experiment, in which both a transmitter and a receiver should be used simultaneously. The complexity of the apparatus will be offset in this case by the greater possibilities of "flexibly" performing the experiment. The present status of the technology already allows us to proceed to realization of this program.

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[1)]In some papers G is replaced by the quantity $\kappa = 8\pi G/c^2 = 1.86 \cdot 10^{-27}$ cm^{-1}, called the Einstein gravitational constant.

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