Electromagnetic form factors and electrodisintegration of deuterons

A. I. Kirillov, V. E. Troitskiĭ, S. V. Trubnikov, and Yu. M. Shirokov

V. A. Steklov Mathematics Institute, Academy of Sciences of the USSR, Moscow and Nuclear Physics Institute, Moscow State University

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A review is presented of the experimental and, mainly, the theoretical questions involved in elastic electron–deuteron scattering and electrodisintegration, based principally on papers published since 1965. A new method, proposed by Yu. M. Shirikov, is formulated for calculation of the electromagnetic structure of the deuteron. The results of the calculation of the deuteron charge form factor and of the deuteron electrodisintegration matrix element by the new method are presented. The prospects of the method and the unsolved problems are briefly summarized.

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INTRODUCTION

R. Wilson[1] has noted that "deuteron theory is in a confused and obscure state, and there are no fully verified numerical calculations. Even if the experimental technique is greatly improved, the theory will not be in a position to make full use of this fact."

The study of elastic and inelastic scattering of electrons by deuterons is the meeting point of elementary-particle physics and nuclear physics. For elementary-particle physics, the *ed*-scattering data are the main source of experimental information on the electromagnetic structure of the deuteron. For nuclear physics, the study of *ed* scattering yields information on the deuteron wave function and on the nucleon–nucleon interaction potential.

Finally, for both elementary-particle physics and nuclear physics the deuteron is an ideal proving ground for the testing of various methods (including relativistic ones) of describing composite systems.

In spite of the obvious urgency of this problem, a relative abatement of its experimental and theoretical study has been observed recently. The progress in experiment is hindered by a certain lag of the theory. On the other hand, for further development of the theory it is desirable to perform new experiments (including polarization experiments). A comparison of the existing theoretical predictions with the results of such experiments makes it possible to choose the most correct approach to *ed* scattering. Polarization experiments have presently become technically realizable. One can therefore expect in the nearest future an increased interest in the problem of *ed* scattering as a whole.

So far, the principal methods used for deuteron calculations are those of the phenomenological Hamiltonians of nuclear physics. However, methods of nuclear physics are limited because of their nonrelativistic character and because of the appreciable model dependence on the assumptions concerning the dynamics of the *np* system. By now, the possibilities of this method have been practically exhausted, although there are still many unsolved problems in the theory of *ed* scattering. Numerous attempts at introducing relativistic corrections do not improve the situation, since the error due to the incompleteness of relativization cannot be estimated.

To make further progress in the theory of *ed* scattering it is necessary to develop a consistent relativistic approach. Certain hopes of developing such an approach were pinned at one time on the dispersion-relation method, which gave a good account of itself in elementary-particle physics. However, as will be shown below, this method is still not suitable for real quantitative calculations.

We have developed in our papers a new approach to calculation of the electromagnetic structure of composite hadron systems, in which we used the ideas of quantum field theory. This approach is consistently relativistic, takes correct account of the dynamics of the *np* system, and is suitable in practice for numerical calculations.

A review of the electromagnetic form factors, including that of the deuteron, was presented by Griffy and Schiff.[2] Our review covers those changes that have occurred in the experimental and theoretical situation concerning *ed* scattering during the time since 1967. From among the papers published prior to 1966, we mention only the minimum information needed to be able to read the present paper independently of Ref. 2.

1. KINEMATICS OF ONE-PHOTON EXCHANGE

We present, for reference purposes, some kinematic formulas.

The cross section for elastic *ed* scattering depends on the energy E_e and the scattering angle θ_e. The inelastic *ed*-scattering cross section depends also on the energy E'_e of the final electron and on the directions of motion of the proton and neutron in the c. m. s. In experiments on elastic and inelastic scattering one measures the angular and energy distributions of the scattered electrons. In the electrodisintegration of the deuteron, to obtain information on the final states of all the particles, it is necessary to consider, in addition to the aforementioned characteristics of the electron, also the emission direction of one of the nucleons, i. e., to measure the cross section $d^3\sigma/d\Omega_e d\Omega_N dE'_e$.

From the conditions of relativistic invariance alone it follows that in single-photon exchange the elastic-scattering cross section of unpolarized particles takes the form

$$d\sigma/d\Omega_e = (d\sigma/d\Omega_e)_{\text{Mott}}\{A(q^2) + B(q^2)\,\mathrm{tg}^2\,\theta_e/2\}. \quad (1)$$

Here $(d\sigma/d\Omega_e)_{\text{Mott}}$ is the cross section obtained by Mott for the scattering by a spinless and structureless charged particle, and is equal to

$$\left(\frac{d\sigma}{d\Omega_e}\right)_{\text{Mott}} = \frac{e^2}{4E_e^2}\cdot\frac{E_e'}{E_e}\cdot\frac{\cos^2\theta_e/2}{\sin^4\theta_e/2};$$

$A(q^2)$, $B(q^2)$ are functions of the squared momentum transfer and their form is determined by the electromagnetic structure of the deuteron. Equation (1) was derived by Rosenbluth[3] for the scattering of an electron by a spin-$\frac{1}{2}$ particle. The physical meaning of this equation is discussed in Ref. 4.

Gourdin[5] has established that the cross section for the scattering of an electron by a charged unpolarized particle depends on two invariant functions, and in the particular case of electrodisintegration of a deuteron it takes the form

$$\frac{d^2\sigma}{d\Omega_e\,dE_e'} = \left(\frac{d\sigma}{d\Omega_e}\right)_{\text{Mott}}\{C(t, s) + D(t, s)\,\mathrm{tg}^2\,\theta_e/2\}, \quad (2)$$

$$t = -q^2.$$

Drell and Walecka[6] have shown that exactly the same dependence on $\mathrm{tg}^2\theta_e/2$ holds also for the inclusive processes of the type $e + X \to e' + anything$.

According to (1) and (2), the dependence of the ratio $(d\sigma/d\Omega_e)/(d\sigma/d\Omega_e)_{\text{Mott}}$ on $\mathrm{tg}^2\theta_e/2$ should be linear. Deviation from this linearity can be due to the contribution of the non-single-photon mechanism. In full accord with the theoretical predictions, no such deviations were observed in any of the experiments performed. An analogous situation takes place also for inelastic scattering. We shall therefore henceforth assume throughout a single-photon *ed*-collision mechanism, without specially stipulating so. (We do not stop to discuss the effects of multiple production of infrared photons. The theory of these effects is described in Ref. 7.)

The single-photon character of the *ed* interaction causes the elastic-scattering cross section (1) to be expressed in terms of the square of the amplitude of the virtual process $\gamma d \to d$. This amplitude is proportional to the current matrix element $\langle d|j_\mu|d\rangle$. From the relativistic invariance conditions it follows that this matrix element can be parametrized, i.e., represented in the form of a linear combination of three invariant functions $G_C^d(q^2)$, $G_{\text{mag}}^d(q^2)$ and $G_Q^d(q^3)$, called respectively the electric (charge), magnetic, and quadrupole form factors of the deuteron.[8,9]

The directly measured functions $A(q^2)$, $B(q^2)$ from (1) are expressed in terms of form factors by formulas of the type[9]

$$A(q^2) = [G_C^d(q^2)]^2 + \frac{8}{9}\eta^2[G_Q^d(q^2)]^2 + \frac{2}{3}\eta[G_{\text{mag}}^d(q^2)]^2(1+\eta), \quad \eta = \frac{q^2}{4M_d^2}; \quad (3)$$

$$B(q^2) = \frac{4}{3}\eta(1+\eta)^2[G_{\text{mag}}^d(q^2)]^2. \quad (4)$$

The kinematics of the single-photon electrodisintegration of the deuteron can be treated in analogy with the elastic-scattering kinematics just considered. Here, too, the cross section and the matrix element $\langle np|j_\mu|d\rangle$ of the current are expressed in the form of a linear combination of invariant functions, which can be appropriately called the inelastic form factors. The difference lies in the fact that these new form factors are more numerous and depend on a larger number of invariant variables. In contrast to the case just considered of elastic *ed* scattering, in deuteron electrodisintegration there is no universally accepted standard parametrization in terms of the invariant form factors $G(s, t)$.

We present the parametrization results given in Ref. 10 as applied to electrodisintegration of the deuteron.

Each form factor is a function of the variables s and t and pertains to definite values of J, l, and S. Here J, S, and l are respectively the total, spin, and orbital angular momenta in the c.m.s. of the np system. All the form factors are subdivided[1)] into charge (G_Q), magnetic (G_M), and toroidal (G_T) form factors. The multipolarity of the form factor will be designated by a subscript, e.g., $G_{M1}^{JSl}(s, t)$. At small values of the energy variable s, the main contribution to the cross section for electrodisintegration of the deuteron is made by the $l=0$ state of the emitted nucleons. In this case the cross section for electrodisintegration of the S-wave deuteron $d^2\sigma/d\Omega_e\,dE_e'$ is expressed in terms of the invariant inelastic form factors in the following manner:

$$\frac{d^2\sigma}{d\Omega_e\,dE_e'} = \frac{\sigma_{\text{Mott}}}{256\pi^2 M_d M^2}\left(\frac{|\mathbf{p}|}{\omega_p}\right)_{\text{c.m.s}}\left\{\frac{2(s+M_d^2)+t}{8M_d\pi}|G_{Q0}^{110}|^2 + \frac{2}{5\cos^2\theta_e/2}\left(\frac{k_\perp}{E_e}\cdot\frac{k_\perp'}{E_e'} - 2\sin^2\theta_e/2\right)[|G_{M1}^{000}|^2 + |G_{M1}^{110}|^2]\right\},$$

where $k_\perp^{(\prime)}$ are the components of the vectors $\mathbf{k}$ and $\mathbf{k}'$ perpendicular to the vector $\mathbf{q}$.

At $\theta_e = \pi$ we have

$$\frac{d^2\sigma}{d\Omega_e\,dE_e'} = \frac{\alpha^2 M_d}{12\pi(2M)^2}\cdot\frac{|\mathbf{p}_{\text{c.m.s}}|}{E_e^2\sqrt{s}}\{|G_{M1}^{000}|^2 + |G_{M1}^{110}|^2\}.$$

2. EXPERIMENTAL DATA

Much experimental material on the measurement of the differential cross section for elastic $e + d \to e + d$ and inelastic $e + d \to e + n + p$ electron—deuteron scattering has been accumulated since 1955. The motivation for these experiments is described, e.g., in Ref.

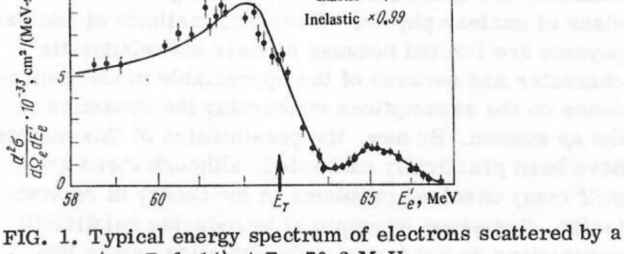


FIG. 1. Typical energy spectrum of electrons scattered by a deuteron (see Ref. 14) at $E_e = 70.2$ MeV.

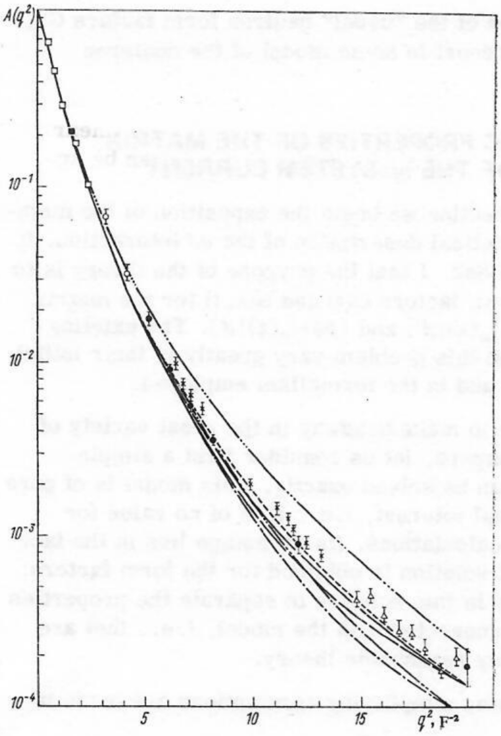


FIG. 2. Experimental value of $A(q^2)$, taken from Ref. 20. The lines show the results of calculations of $A(q^2)$ for different potential models.

2, and the experimental technique is described in Ref. 13.

The present status of experiments can be summarized in the following manner. One measures most frequently the angular and energy distributions of the scattered electrons. This yields the inclusive cross section $d^2\sigma/d\Omega_e\, dE'_e$ of the process $e+d\to e+anything$. This cross section depends on the three variables E_e, E'_e, θ_e.

Figure 1 shows a typical energy spectrum at $E_e = 70.2$ MeV and $\theta_e=\pi$, taken from Ref. 14. The scattering cross section shows a characteristic large smeared-out quasielastic peak corresponding to the scattering of an electron by a free nucleon. Another characteristic feature of all the spectra of this kind is the presence of a distinct small peak at $E'_e \gtrsim E_T$, corresponding to elastic *ed* scattering. The rest of the spectrum (at $E'_e \lesssim E_T$) pertains to different inelastic processes, principal among which (at not too high values of the energy E_e) is the electrodisintegration of the deuteron.

Experiments on elastic *ed* scattering were performed for the squared momentum-transfer values 0.05 F^{-2} $\leqslant q^2 \leqslant 35.4$ F^{-2} (Refs. 15–21). The errors in the measurements of the cross section increase with increasing q^2, from 1% at $q^2=0.05$ F^{-2} to 50% at $q^2=35.4$ F^{-2}. The functions $A(q^2)$ and $B(q^2)$ in (3) and (4) can be determined in two ways. The first consists of measuring, at each value of q^2, the electron forward scattering cross section (1) ($\theta_e \sim 0°$) and backward cross section ($\theta_e \sim 180°$). It follows from (1) that the first of these cross sections receives a contribution only from the function $A(q^2)$, and the second from $B(q^2)$. Another standard method of determining $A(q^2)$ and $B(q^2)$ consists of measuring at each value of q^2 the cross section at several arbitrary scattering angles. A and B are then determined as the coefficients of the linear function that has the smallest rms deviation from the measured cross section.

Most of the recent experimental data on the functions $A(q^2)$ and $B(q^2)$ are shown in Figs. 2 and 3, which are taken from Refs. 14 and 20. (We recall that the deuteron magnetic form factor $G^d_{\text{mag}}(q^2)$ can be expressed directly in terms of the function $B(q^2)$ [see Eq. (4)].)

The squares of the charge form factor $G^d_C(q^2)$ and of the quadrupole form factor $G^d_Q(q^2)$ of the deuteron enter in the cross section in the form of the linear combination $[G^d_C(q^2)]^2 + 8\eta^2[G^d_Q(q^2)]^2/9$ [see Eq. (3)], so that the experiments described above are incapable of determining each form factor separately. Separation of the charge and quadrupole form factors is possible only in polarization experiments, which have not been performed so far. To extract the contributions of G^d_C and G^d_Q from $A(q^2)$ it is therefore necessary at present to resort to theoretical results.

A single experiment[22] was reported in 1968 on the measurement of the polarization vectors of the recoil deuterons in elastic *ed* scattering. This experiment is of interest because, as shown earlier in Refs. 23–25, if T-invariance is violated in electromagnetic interactions then the polarization vector $|\mathbf{P}|$, directed normal to the polarization plane, is proportional only to the new (T-violating) form factor of the deuteron. The near-zero experimental result $|\mathbf{P}| = 0.075 \pm 0.088$ points to the absence of strong violation of T-invariance in hadron electrodynamics.[2)]

Great interest attaches to measurements of the polarization of the recoil deuterons in elastic *ed* scattering, or to experiments on the scattering of electrons by an aligned deuteron target. Such experiments make it possible to measure separately the charge and quadrupole form factors and at the same time to solve certain problems in deuteron theory.[27] Polarization effects in *ed* scattering were calculated in Refs. 28–31. Unfortunately, the literature known to us contains no def-

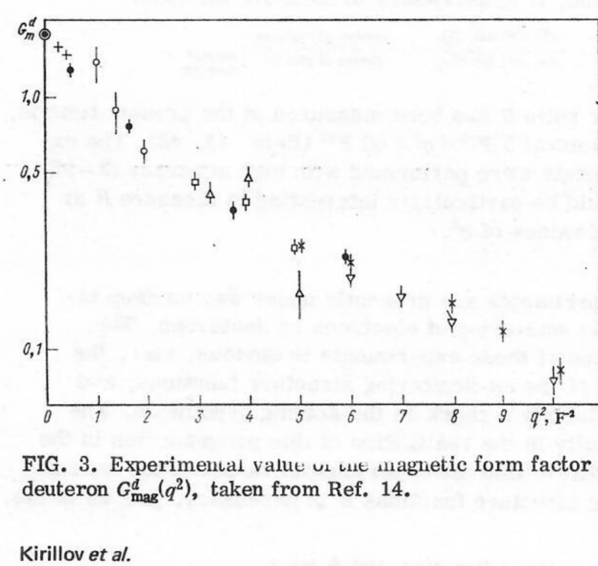


FIG. 3. Experimental value of the magnetic form factor of the deuteron $G^d_{\text{mag}}(q^2)$, taken from Ref. 14.

inite mentions of any plans of performing such experiments in the nearest future. Nevertheless, there is hope that the polarizations in *ed* scattering will be measured.

In most experiments on deuteron electrodisintegration a spectrum of the type shown in Fig. 1 is measured. By now, at an incident-electron energy of several GeV, the cross section $d^2\sigma/d\Omega_e dE'_e$ has been measured in the region $0 < q^2 \leqslant 175$ F^{-2} (Refs. 32–36). Just as in elastic *ed* scattering, the errors in the measurement of the cross section increase with increasing q^2 and amount to 5–10% at small q^2 and to 50% at $q^2 \geqslant 100$ F^{-2}. Most of the measurements were made in the region of the quasielastic peak at small and medium electron-scattering angles in the laboratory frame ($\theta_e \leqslant 90°$), since it is precisely in this case that the cross section is largest and consequently easiest to measure. In Sec. 6 below we shall show that in the region of the quasielastic peak the experimental data lend themselves more readily to theoretical treatment. We emphasize that the main purpose of these experiments is to obtain information on the charge form factor of the neutron.

The published data on large-angle electron scattering and backward scattering ($\theta_e = 180°$) are much scantier.[37–41] Backward scattering is of interest because it separates directly the values of the magnetic multipole *MJ* transitions, which are sensitive to relativistic effects, to exchange meson currents, and to admixtures of nucleonic isobars[27] (electric multipole *QJ* transitions make the dominant contribution to the cross section at $\theta_e \lesssim 90°$). By now, the cross section $d^2\sigma/dE'_e d\Omega_e$ for electron backward scattering ($\theta_e = 180°$) has been measured in the region 0.15 F$^{-2} \leqslant q^2 \leqslant 10.1$ F^{-2} with accuracy 5–10%. However, as indicated in Ref. 42, these experiments will be repeated, inasmuch as only one experiment was performed at $q^2 \geqslant 6$ F^{-2}, and it is desirable to duplicate its results.

Coincidence experiments, i.e., experiments in which the recoil nucleon is registered in addition to the scattered electron, yield much more information than experiments in which only one of the scattered particles is registered. In particular, measurement of the cross section $d^3\sigma/dE'_e d\Omega_e d\Omega_N$ makes it possible to determine with high accuracy the neutron form factors $G_{En\,Mn}$. To this end, it is necessary to measure the ratio

$$R = \frac{d^3\sigma/dE'_e\, d\Omega_e\, d\Omega_n}{d^3\sigma/dE'_e\, d\Omega_e\, d\Omega_p} = \frac{\text{number of neutrons}}{\text{number of protons}}\bigg|_{\substack{\text{emitted}\\ \text{forward}}} .$$

The ratio R has been measured at the present time in the interval 5 F$^{-2} \leqslant q^2 \leqslant 40$ F^{-2} (Refs. 43–48). The experiments were performed with high accuracy (3–5%). It would be particularly interesting to measure R at small values of q^2.

Experiments are presently under way on deep inelastic scattering of electrons by deuterons. The purpose of these experiments is obvious, viz., the study of the *en*-scattering structure functions, and particularly a check on the scaling hypothesis. The difficulty in the realization of this program lies in the fact that to find the experimental values of the *en*-scattering structure functions it is necessary, just as in the determination of the "usual" neutron form factors G_{En} and G_{Mn}, to resort to some model of the deuteron structure.[49]

3. ANALYTIC PROPERTIES OF THE MATRIX ELEMENT OF THE *np*-SYSTEM CURRENT

With this section we begin the exposition of the methods for theoretical description of the *ed* interaction. It follows from Sec. 1 that the purpose of the theory is to obtain the form factors $G(t)$ and $G(s, t)$ for the matrix elements $\langle d | j_\mu(x) | d' \rangle$ and $\langle pn | j_\mu(x) | d \rangle$. The existing approaches to this problem vary greatly in their initial assumptions and in the formalism employed.

To be able to make headway in the great variety of theoretical papers, let us consider first a simple model that can be solved exactly. This model is of pure methodological interest, i.e., it is of no value for quantitative calculations. Its advantage lies in the fact that an exact solution is obtained for the form factors. It is possible in this solution to separate the properties that are not connected with the model, i.e., that are needed for any reasonable theory.

The following simplifying assumptions are made in the model:

a) The nucleons are treated nonrelativistically. The contribution of the inelastic channels is discarded.

b) The nucleons are assumed to have no spin.

c) One of the nucleons is assumed to be an infinitely heavy scattering center having no electromagnetic structure.

d) The interaction potential is separable, i.e., it is of the form[3)]

$$(\mathbf{p} | H_{\mathrm{int}} | \mathbf{p}') = -f(\mathbf{p}^2) f(\mathbf{p}'^2)^* \tag{5}$$

(we set $p = |\mathbf{p}|$ throughout this section).

Thus, in the spinless nonrelativistic model the proton moves in a scattering-center field with potential (5). This proton has an electric structure which is not deformed by the interaction and which is described by the matrix element of the free-particle current:

$$\left.\begin{aligned}\langle \mathbf{p}, t | j_\mu(\mathbf{x}) | \mathbf{p}', t \rangle &= \exp[\mathrm{i}(E - E')t]\, (\mathbf{p} | j_\mu(\mathbf{x}) | \mathbf{p}');\\ (\mathbf{p} | j_0(\mathbf{x}) | \mathbf{p}') &= (2\pi)^{-3} \exp[-\mathrm{i}\mathbf{q}\mathbf{x}]\, F(-\mathbf{q}^2);\\ (\mathbf{p} | \mathbf{j}(\mathbf{x}) | \mathbf{p}') &= (2\pi)^{-3} \exp[-\mathrm{i}\mathbf{q}\mathbf{x}]\, [F(-\mathbf{q}^2)/2m]\, (\mathbf{p} + \mathbf{p}').\end{aligned}\right\} \tag{6}$$

where $F(-q^2)$ is a nonrelativistic form factor; $\mathbf{q} = \mathbf{p} - \mathbf{p}'$; $E = p^2/2m$; $E' = p'^2/2m$. We note that limitations (b) and (c) play no role in the model. An exact solution exists also when the spins are taken into account and when both particles have finite mass.

The solutions of the Schrödinger equation with potential (5) are simple and well known (see e.g., Ref. 50). We are interested in solutions in which the wave function $\Psi^{(+)}(k, p)$ describes the scattering of a particle that has a momentum $\mathbf{k}$ in the initial state (as $t \to -\infty$). As is well known, this solution (for the S state) is given by

$$\Psi^{(+)}(k, p) = \delta(k - p) - \frac{2m}{k^2 - p^2 + \mathrm{i}0} \frac{f(p) f(k)^*}{B(E + \mathrm{i}0)}, \tag{7}$$

where $E = k^2/2m$,

$$B(E \pm i0) = 1 + \int dk' \frac{|f(k'^2)|^2}{E \pm i0 - k'^2/2m}.$$

It is easy to verify directly that the function (7) satisfies the Schrödinger equation with potential (5). The solution (7) enables us to find the desired current matrix element:

$$\langle k, t | j_\mu(\mathbf{x}) | k', t\rangle = \exp[i(E - E')t] \times \int d\mathbf{p} \int d\mathbf{p}' \Psi^{(+)}(k, p)^* (\mathbf{p} | j_\mu(\mathbf{x}) | \mathbf{p}') \Psi^{(+)}(k', p'), \tag{8}$$

where $E' = k'^2/2m$. Expression (8) yields the complete solution of the problem of the electromagnetic properties of the scattering states. Indeed, using (8) we can calculate the average current $\bar{j}_\mu(t, x)$ at any 4-point for an arbitrary initial state.

The Fourier transform of the current

$$\langle k, t | \tilde{j}_\mu(\mathbf{q}) | k', t\rangle = \int d\mathbf{x} \exp(i\mathbf{q}\mathbf{x}) \langle k, t | j_\mu(\mathbf{x}) | k', t\rangle \tag{9}$$

is expressed in terms of the form factor $G(E, E', -q^2)$:

$$\left.\begin{aligned} &\langle k, t | \tilde{j}_0(\mathbf{q}) | k', t\rangle = \exp[i(E - E')t] G(E, E', -q^2);\\ &\langle k, t | \tilde{\mathbf{j}}(\mathbf{q}) | k', t\rangle\\ &= \exp[i(E - E')t] G(E, E', -q^2)(\mathbf{k} + \mathbf{k}')\, 2m. \end{aligned}\right\} \tag{10}$$

For this form factor we obtain from (6)—(9) (for $2m = 1$)

$$\begin{aligned} G(E, E', -q^2) &= F(-q^2)\Big[k^2 k'^2 \tilde{\theta}(k, q, k')\\ &- \frac{k^2 f(k')^*}{B(E' + i0)} \int p'^2 dp' \frac{f(p')\tilde{\theta}(k, q, p')}{k'^2 - p'^2 + i0}\\ &- \frac{k'^2 f(k)}{B(E - i0)} \int dp p^2 \frac{f(p)^* \tilde{\theta}(p, q, k')}{k^2 - p^2 - i0}\\ &+ \frac{f(k) f(k')^*}{B(E - i0) B(E' + i0)} \int \frac{p^2 dp f(p)^*}{k^2 - p^2 - i0}\\ &\times \int \frac{p'^2 dp' f(p')}{k'^2 - p'^2 + i0} \tilde{\theta}(p, q, p')\Big], \end{aligned} \tag{11}$$

where

$$\tilde{\theta}(p, q, p') = \int d\Omega_{\mathbf{p}} \int d\Omega_{\mathbf{p}'} \delta^3(\mathbf{q} - \mathbf{p} + \mathbf{p}'). \tag{12}$$

Denoting the terms in the right-hand side of (11) by $G_{00}, G_{0i}, G_{i0}, G_{ii}$, respectively, we can rewrite the expression for the form factor in the form

$$\begin{aligned} G(E, E', -q^2) &= G_{00}(E, E', -q^2) + G_{0i}(E, E' + i0, -q^2)\\ &+ G_{i0}(E - i0, E', -q^2) + G_{ii}(E - i0, E' + i0, -q^2). \end{aligned} \tag{13}$$

We shall show below that each term has a clear physical meaning.

For the scattering solution to exist, it is sufficient to impose on the function $f(p^2)$, which defines the separable potential (5), certain conditions of smoothness and of rapid decrease at infinity. For a bound state to exist, this function should also satisfy the inequality[50]

$$2m \int dk' |f(k'^2)|^2 \geqslant 1.$$

It is easy to verify that the wave function $\Psi_d(p)$ of the bound state takes the form

$$\Psi_d(p) = \text{const} \frac{f(p^2)}{E_d - p^2/2m}, \tag{14}$$

where $E_d \leqslant 0$ is the binding energy determined from the equation

$$B(E_d) = 1 + \int dk' \frac{|f(k'^2)|^2}{E_d - k'^2/2m} = 0.$$

Using (14), we can obtain solutions for the current matrix elements $\langle k, t | j_\mu(x) | d, t\rangle$ and $\langle d, t | j_\mu(x) | d', t\rangle$, where $|d, t\rangle$ denotes a bound state. Namely,

$$\left.\begin{aligned} &\langle k, t | j_\mu(\mathbf{x}) | d, t\rangle = \exp[i(E - E_d)t] \int d\mathbf{p} \int d\mathbf{p}' \times\\ &\times \Psi^{(+)}(k, p)^* (\mathbf{p} | j_\mu(\mathbf{x}) | \mathbf{p}') \Psi_d(p');\\ &\langle d, t | j_\mu(\mathbf{x}) | d', t\rangle = \int d\mathbf{p} \int d\mathbf{p}' \varphi_d(p) (\mathbf{p} | j_\mu(\mathbf{x}) | \mathbf{p}') \varphi_{d'}(p'). \end{aligned}\right\} \tag{15}$$

The Fourier transforms (9) of these matrix elements are expressed, in analogy with (6) and (10), in terms of the factors $G(E, -q^2)$ and $G(-q^2)$:

$$\left.\begin{aligned} &\langle k, t | \tilde{j}_0(\mathbf{q}) | d, t\rangle = \exp[i(E - E_d)t] G(E, -\mathbf{q}^2);\\ &\langle d, t | \tilde{j}_0(\mathbf{q}) | d', t\rangle = G(-\mathbf{q}^2). \end{aligned}\right\} \tag{16}$$

From (14)—(16) we obtain for these form factors the expressions

$$\begin{aligned} G(E, -q^2) &= F(-q^2) = \text{const}\Big[k^2 \int p'^2 dp' \frac{f(p')\tilde{\theta}(k, q, p')}{E_d - p'^2}\\ &+ \frac{f(k)}{B(E - i0)} \int \frac{p^2 dp f(p)^*}{k^2 - p^2 - i0} \int \frac{p'^2 dp' f(p')}{E_d - p'^2} \tilde{\theta}(p, q, p')\Big];\\ G(-q^2) &= F(-q^2)(\text{const})^2 \int \frac{p^2 dp f(p)^*}{E_d - p^2/2m}\\ &\times \int \frac{p'^2 dp' f(p')}{E_d - p'^2/2m} \tilde{\theta}(p, q, p'). \end{aligned} \tag{17}$$

The form factors $G(E, -q^2)$, $G(-q^2)$ are not independent, but are expressed in terms of $G(E, E', -q^2)$. It is also seen from (11) that the terms G_{0i}, G_{ii} have poles with respect to the variable E' at the bound-state energy $E' = E_d$. The residues at these poles are proportional respectively to G_0 and G_1:

$$\left.\begin{aligned} &G_0(E, -q^2) = \text{const} \lim_{E' \to E_d} (E' - E_d) G_{0i}(E, E' + i0, -q^2);\\ &G_1(E - i0, -q^2) = \text{const} \lim_{E' \to E_d} (E' - E_d)\\ &\times G_{ii}(E - i0, E' + i0, -q^2);\\ &G(E, -q^2) = G_0(E, -q^2) + G_1(E - i0, -q^2). \end{aligned}\right\} \tag{18}$$

Analogously, the form factors G_{i0}, G_{ii} have poles in E at the point $E = E_d$. The residues at these poles yield the form factors $G_0(E', -q^2)^*$ and $G_1(E' + i0, -q^2)^*$.

Finally, the form factor $G_1(E - i0, -q^2)$ has in turn a pole in the variable E at the same point $E = E_d$. The residue at this pole is proportional to the bound-state form factor $E(-q^2)$:

$$G(-q^2) = \text{const} \lim_{E \to E_d} G_1(E - i0, q^2)(E - E_d). \tag{19}$$

The proportionality factor can be expressed in terms of the proton—neutron—deuteron binding constant. To de-

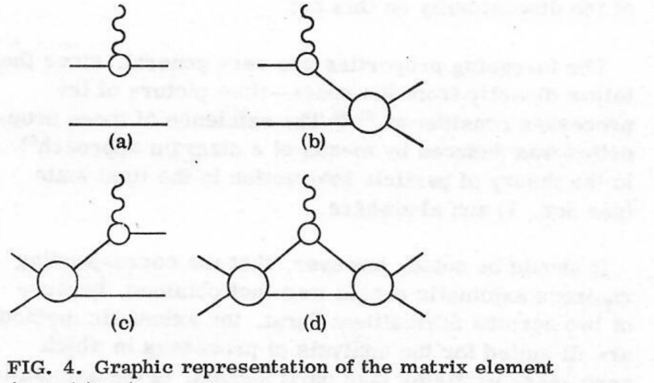


FIG. 4. Graphic representation of the matrix element $\langle np | j_\mu(x) | np\rangle$.

termine this constant it is necessary to continue the npd vertex analytically to the pole. We do not wish to consider matrix elements in the unphysical region, and shall therefore determine the proportionality factor from the condition that the charge form factor be normalized to unity at $q^2=0$.

The form factor $G(-q^2)$ can be regarded as an analytic function of the variable $t=-q^2$. In terms of this variable, the form factor has a well-known logarithmic singularity, called the anomalous threshold, at the point $t=16\,M\epsilon$ ($\epsilon=-E_d$) on the real axis.

Let us summarize now those properties of our model which are general in character and by the same token should be reflected in any realistic theory. The first general property is the separation of the form factor $G(E, E', t)$ into the four terms (13). This separation corresponds to a representation of the general diagram of the matrix element $\langle pn|j_\mu(x)|pn\rangle$ by the graphic sum shown in Fig. 4, where we have omitted, for the sake of brevity, the obvious-type terms reflecting the electromagnetic structure of the neutron. It should be noted that the diagrams in Fig. 4 have a symbolic character and are not interpreted with the aid of the Feynman rules. Rather, these diagrams are close to those used in dispersion theory to represent imaginary parts of amplitudes.

The second general property of the model is the presence of single-particle poles in the form factors. $G_{0i}(s, s', t)$ and $G_{ii}(s, s', t)$ have poles in s' at the point M_d^2, with residues proportional to $G_0(s, t)$ and $G_1(s, t)$, respectively; the form factors $G_{i0}(s, s', t)$ and $G_{ii}(s, s', t)$ have at the same point M_d^2 poles in s, with residues proportional to $G_0(s', t)^*$ and $G_1(s', t)^*$; the form factor $G_1(s, t)$ has a pole at $s=M_d^2$, with a residue proportional to the deuteron form factor $G_d(t)$. Thus

$$G_d(t)=\text{const}\lim_{s\to M_d^2} G_1(s, t)(s-M_d^2) \tag{20}$$

etc. It follows therefore that for a unified calculation of elastic ed scattering and the electrodisintegration process it suffices to determine the current matrix element of the np system.

A third common property is that in a real deuteron the form factor $G(t)$ has an anomalous threshold, from which an anomalous cut is drawn; this cut is dominant in the sense that the form factor is expressed quantitatively, with high accuracy (in integral form), in terms of the discontinuity on this cut.

The foregoing properties are very general, since they follow directly from the space–time picture of the processes considered.[51,52] The existence of these properties was deduced by means of a diagram approach[53] in the theory of particle interaction in the final state (see Sec. 7) and elsewhere.

It should be noted, however, that the corresponding rigorous axiomatic proofs were not obtained, because of two serious difficulties: First, the axiomatic methods are ill suited for the analysis of processes in which zero-mass particles take part; second, in the axiomatic theory there is still no rigorous definition of the in-basis or out-basis (i.e., there is no construction of the Haag–Ruell type) for composite states of the scattering of two particles with fixed relative orbital angular momentum. In spite of this, the existence of the above-noted general properties is hardly in doubt.

4. METHODS OF CALCULATING THE ELECTROMAGNETIC FORM FACTORS OF THE DEUTERON

The first calculations of the electromagnetic form factors of the deuteron were carried out using nonrelativistic wave functions.[54–56] In spite of the individual successes of other approaches, this method still remains the only one that gives quantitative results that are acceptable to some degree for all the experimentally attainable momentum transfers. However, the analysis of the electromagnetic properties of the deuteron with the aid of a nonrelativistic wave function encounters many basic difficulties. Therefore the use of this method at large q^2, when the relativistic corrections are large, is dictated only by the lack of other quantitative methods.

The nonrelativistic deuteron wave functions used to calculate the form factors are obtained as solutions of the Schrödinger equation with various nucleon–nucleon potentials. Realistic potentials, as a rule, contain many parameters, which fit the theoretical predictions to the experimental results on nucleon–nucleon scattering. For calculation of the form factors, the most frequently employed are the wave functions of Hamada–Johnston,[57] Feshbach–Lomon,[58] McGee,[59] and Hulthén.[60]

The serious difficulty of this approach lies in the fact that it is impossible to describe quantitatively the magnetic moment of the deuteron. To illustrate this, we present a table[21] of the results of the calculation of the magnetic and quadrupole moments with different models of potentials. We see that it is impossible to obtain agreement with the experimental values of the magnetic moment for good values of the quadrupole moment. It is customarily assumed that a way out of this situation is to take into account the relativistic corrections and the exchange meson currents. To the indicated difficulties of the nonrelativistic approach it is necessary to add also the strong dependence on the choice of the potential, especially at large q^2 (see, e.g., Fig. 2), and the complications that arise when the same potential model is used to describe two closed processes, elastic and inelastic ed scattering, the close connection between which was demonstrated in Sec. 3.

TABLE 1. Properties of different nucleon–nucleon potentials.

Potential	Percentage of D state	Magnetic moment	Quadrupole moment, F^{-2}	Radius of core or of the boundary condition, F^{-2}
Hamada–Johnston	6.96	0.840	0.281	0.485
Bressel	6.49	0.842	0.281	0.686
Feshbach–Lomon	4.31	0.854	0.268	0.734
Bethe–Reid (hard core)	6.50	0.842	0.277	0.548
Bethe–Reid (soft core)	6,47	0.842	0.280	0.057
Hulthén (without core)	4.00	0.856	0,271	0.000
Hulthén	4.00	0.856	0.271	0.432
Hulthén	4.00	0.856	0.271	0.561
Experiment	...	$0.85741\pm$ ± 0.00008	$0{,}282\pm$ ± 0.002	...

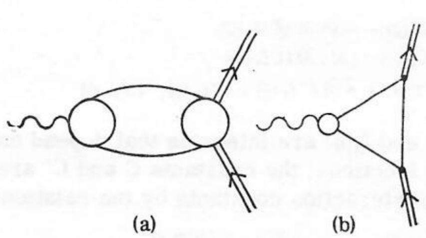


FIG. 5. Diagrams of $\langle d|j_\mu|d\rangle$ containing a nucleon—nucleon intermediate state. The double line stands for the deuteron.

The difficulties of calculations with nonrelativistic wave functions have stimulated the development of other methods of describing the electromagnetic structure of the deuteron. One of them is the dispersion approach. It is based on the assumption that the electromagnetic form factors of the deuteron satisfy a dispersion relation in terms of the squared momentum transfer t, which takes the form, e.g., for the charge form factor of the deuteron[61]

$$G_C^d = \frac{1}{\pi}\int\limits_{T^2}^{\infty}\frac{\operatorname{Im} G_C^d(t')}{t'-t}\,dt, \tag{21}$$

where T^2 is the threshold value. The main contribution to (21) is made by diagrams having the smallest T^2. As already noted in Sec. 3, the nearest singularity in the deuteron form factor is the anomalous threshold, which determines the value of T^2. This singularity stems from the diagrams containing two-nucleon intermediate states (Fig. 5a). A detailed discussion of its properties can be found, e.g., in Refs. 62 and 63. We note here only that the position of this threshold depends on definite combinations of variables describing the diagram with the anomalous singularities, in contrast to the normal threshold, which is determined only by the mass of the channel that is opened. For the diagram of Fig. 5a, in the limit of low deuteron binding energies ϵ, the threshold is $T^2 = 16M\epsilon = 16\alpha^2 < 4m_\pi^2$.

The simplest diagram having an anomalous threshold and contributing to the imaginary part of (21) is shown in Fig. 5b. Replacing the npd vertex by the constant Γ_0, we obtain for the imaginary part $\operatorname{Im} G_C^d$ the expression

$$\operatorname{Im} G_C^{d(\Delta)}(t) = \frac{\Gamma_0^2 f_N(t)}{\sqrt{t\,(1-t/4M_d^2)}}. \tag{22}$$

Here $f_N(t)$ denotes the charge form factor of the nucleon, and the symbol Δ stands for the calculation in the approximation of the triangular diagram of Fig. 5b. (We shall henceforth omit the inessential numerical factors.) We recall that in the physical scattering region we have $t < 0$. It is clear from (22) that the integration in (21) must be cut off at $t' \leqslant 4M_d^2$, lest the current become non-Hermitian; G_C^d depends little on the choice of the cutoff parameter. This is clear from the following reasoning. If we write down the dispersion relation with one substraction (as is usually done for the charge form factor), then as $\epsilon \to 0$ the integral (21) with $\operatorname{Im} G_C^d(t')$ from (22) will diverge at the lower limit, i.e., at small ϵ the predominant role is played by the contribution from a small integration region near the lower limit. From considerations of correspondence to the Schrödinger approach, we obtain for the cutoff parameter precisely the maximum value $t' = 4M_d^2$. In this case we have

$$G_C^{d(\Delta)} = \int\limits_{16\alpha^2}^{4M_d^2}\frac{dt'}{(t'-t)\sqrt{t'(1-t'/4M_d^2)}}$$
$$= \frac{1}{(1-t/4M_d^2)}\left[\frac{\sqrt{1-t/4M_d^2}}{\sqrt{-t}/4\alpha}\operatorname{arctg}\frac{\sqrt{-t}/4\alpha}{\sqrt{1-t/4M_d^2}}\right]. \tag{23}$$

In the nonrelativistic limit ($t/4M_d^2 \approx 0$, $\sqrt{-t} \to |\mathbf{q}|$) expression (23) duplicates[64] the results of the Bethe—Peierls theory

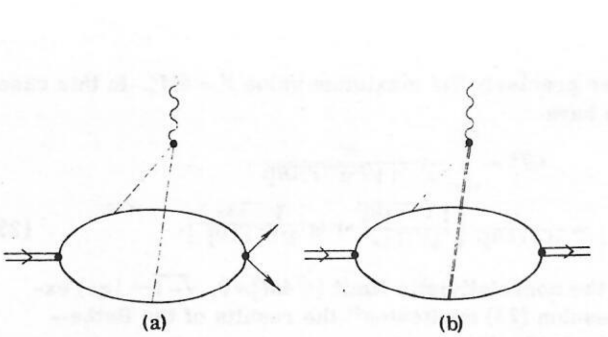
$$[G_C^{d(\Delta)}(q^2)]_{\text{nonrel}} = (4\alpha/|q|)\operatorname{arctg}(|q|/4\alpha). \tag{24}$$

At small t, there is hardly any difference between (23) and (24).

So far we have disregarded the spins of the nucleons and of the deuteron. Unfortunately, once spin is considered, the simplicity and clarity of the picture are spoiled. In this case (22) is replaced by $\operatorname{Im} G_C^{d(\Delta)}(t) = t^2\Gamma_0^2 f_N(t)/\sqrt{t(1-t/4M_d^2)}$. Now, when calculating the dispersion integral (21), the contribution of the region at small t' tends to zero. Thus, the arguments presented above are no longer valid, and the integral depends essentially on the cutoff parameter. In addition, it can be shown that when a more consistent account is taken of the npd vertex function, the factor $(1-t/4M_d^2)^{-1/2}$ vanishes and the integral diverges at the upper limit. Thus, in the spin case we cannot confine ourselves to calculation of the contribution of only one diagram that is closest in mass and contains an anomalous threshold (the triangular diagram shown in Fig. 5b). It is necessary to take into consideration diagrams with higher thresholds.

Thus, the dispersion method as applied to the description of the electromagnetic structure of the deuteron is quite complicated, and furthermore it results in quantitative agreement only at small momentum transfers. It demonstrates clearly the importance of the singular structure of the Feynman diagrams. However, the properties of the dispersion method are such that it tends to combine the internal structure of the interacting particles, i.e., the deuteron wave functions and the nucleon form factors, into a common deuteron current, and this complicates the matter greatly. We must agree with the assessment of this approach given in Ref. 2: "Many workers (Jones, [65] Gross, [66,67] and Nuttall[68]) have attempted to improve these calculations [i.e., calculations with the aid of the nonrelativistic wave function (author's remark)], using dispersion relations to describe the deuteron structure. At the present time, the results of these authors are suitable only for an approximate estimate of the errors inherent in the nonrelativistic approach, and cannot be used for exact numerical calculations."

Many attempts to improve the description of the electromagnetic structure of the deuteron involve a relativistic generalization of the concept of the wave function. Being unable to analyze in detail all these approaches, we confine ourselves only to a brief summary of the results. The starting point is the use of an equation of the Bethe—Salpeter type to find the wave function. One seeks as a rule relativistic corrections to the impulse approximation. Some simplifying assumptions



(a) (b)

FIG. 6. Diagrams with smallest intermediate-particle mass, due to exchange meson currents: a) contribution to the electrodisintegration of the deuteron; b) to elastic ed scattering; ---) pion; ===) ρ meson.

must be made with respect to the relativistic wave function. Thus, in Ref. 69 they are obtained from a simultaneous solution of the Bethe–Salpeter equation with a kernel of separable form of the type of the Yamaguchi nonrelativistic potential. Naturally, it is difficult to establish here the reliability of the quantitative estimates of the relativistic corrections. Quasipotential equations were used to calculate the relativistic corrections to the magnetic moment of the S-wave deuteron. [70] (A discussion of the possibility of using this approach to describe bound states, primarily hydrogen-like atoms, is presented in Ref. 71.)

The best known method of calculating relativistic corrections to the impulse approximation is that developed in the papers of Gross, [72,73] principally because of its successful use to determine G_{En} at small momentum transfers[72] (see Sec. 6). Gross, starting from a Bethe–Salpeter relativistic wave function, has shown that, accurate to $(v/c)^2 = q^2/4M^2$, the wave function of the deuteron in the spinless case[73] is

$$\Psi_d(p) = (1 - d^2/16M^2)\,\Psi_0\,[p^2 - \Delta(p, d)],$$

where $\Delta(p, d) = M^{-2}[(p\cdot d)^2 - 2(p\cdot d)(p^2+\alpha^2)]/4$; p is the relative-motion momentum; d is the total three-momentum of the deuteron; $\Psi_0(p)$ is the usual nonrelativistic wave function in the rest system of the deuteron. Hence, expanding in a Taylor series in $(v/c)^2$, one can obtain the corrections to the wave function. The details of the description in the case with spin are given in Refs. 67 and 72.

So far we took into account only the two-particle np and eN interactions, i.e., we confined ourselves to the impulse approximation. We now dwell on the corrections that go beyond the scope of this approximation, namely, we attempt to take into account the contribution of ternary forces to ed scattering. A typical example of these forces is the so-called exchange meson currents.

An attempt to estimate the contribution of the exchange meson currents to the electromagnetic form factors of the deuteron was made in Refs. 74–77. In Ref. 74, they calculated the simplest diagram, shown in Fig. 6b (the contribution of the diagram of Fig. 6a is equal to zero, since the deuteron is isoscalar), and the two pions were replaced by a ρ meson. The results can be represented in the form of additive increments to the deuteron form factors; they take the form (see, e.g., Ref. 21):

$$\Delta G_C^d(q) = -(8/3)\,\eta C' I_1(q);$$
$$\Delta G_Q^d(q) = (M_d/M)\,C I_1(q);$$
$$\Delta G_{\mathrm{mag}}^d(q) = (3/2\sqrt{2})\,[(2+\sqrt{8/9})\,C' I_1(q) + C' I_2(q) - C I_3(q)].$$

where $I_1(q)$, $I_2(q)$, and $I_3(q)$ are integrals that depend on the deuteron wave functions; the constants C and C' are connected with the interaction constants by the relations

$$C = -3G g_{\rho\pi\gamma} a/(16 m_\rho e); \quad C' = -3G g_{\rho\pi\gamma} a/(16 m_\rho e)$$

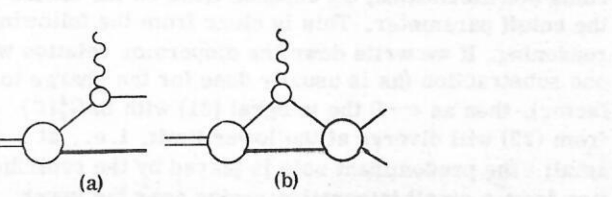
Here $G^2/4\pi = 14$ is the pion–nucleon interaction constant; e is the electron charge; m_ρ is the mass of the ρ meson; $a = G_{Ev}(0) = \frac{1}{2}$ and $b = G_{Mv}(0) = 2.35$ are the constants of the ρ-nucleon interaction. For electric scattering at $q^2 < 10\ \mathrm{F}^{-2}$, the corrections due to the meson current make a small contribution. Principal interest attaches to ΔG_{mag}, inasmuch as this increment makes it possible to correct the theoretical value of the deuteron magnetic moment, retaining at the same time the correct value of the D-wave admixture, namely $P_D \approx 7\%$.

A recent paper[77] dealt with the correction to the deuteron form factors necessitated by processes of the type of the Glauber double-scattering model, namely, a virtual photon is converted into an ω meson, which is scattered by one nucleon and is absorbed by another nucleon. The scattered ω meson can also be transformed into a ρ meson. The magnetic moment is fitted to this model by choosing the corresponding value of one of the parameters that enter in the theory.

On the whole, the question of the corrections introduced by the exchange meson currents into the deuteron form factors is in a somewhat confused state. The calculation results depend strongly on the interaction constants, and even the sign of this correction is not certain, nor is the size of the contribution of the unaccounted-for diagrams. The problem of the exchange meson currents calls for further study.

5. THEORY OF DEUTERON ELECTRODISINTEGRATION

The theoretical description of the electrodisintegration of the deuteron cannot be regarded as complete, in spite of the number of papers devoted to it. In fact, the uncertainties in the calculations of the cross section for $e+d \to e+n+p$ are even larger than the uncertainties in the calculations of the process $e+d \to e+d$. It follows from Sec. 3 that to describe the electrodisintegration of the deuteron it is necessary to determine the contribution of the diagrams shown in Fig. 7, i.e., to determine the form factors G_{10} and G_{11}. This means that it is necessary to determine the amplitude of the electrodisintegration in the Born approximation and to take into account the contribution of the final-state interaction. At the present time the Born amplitude is



(a) (b)

FIG. 7. Graphic representation of the matrix element $\langle np|j_\mu|d\rangle$.

calculated by the wave-function method or the dispersion-relation method. The contribution of the interaction in the final state is taken into account by the distorted-wave method or with the aid of the Muskhelishvili–Omnes equation. Let us discuss first the nonrelativistic approach, since it is precisely in this formalism that the greatest number of studies were made and the principal practical results were obtained. In principle it is possible, by solving the Schrödinger equation for both the deuteron and for the uncoupled np system with the same neutron–proton potential, to describe the electrodisintegration of the deuteron under the same assumptions as elastic electron–deuteron scattering (see, e.g., Ref. 78 and 79). However, the Schrödinger equation with any kind of realistic np potential can be solved only numerically, and a realization of this program would greatly complicate the problem. Therefore, as a rule, one determines only the deuteron wave function, and the interaction in the final state is taken into account only approximately and under additional assumptions.[80] We note here also a technical difficulty that is common to all the calculations and is connected with the need for taking into account a large number of partial waves with respect to the relative motion of the emitted nucleons. This circumstance makes the calculations of the electrodisintegration much more complicated.

In most theoretical papers, the cross section of the electrodisintegration of the deuteron is calculated in the region of the quasielastic peak. This circumstance is not accidental and is connected with the fact that in this region it is possible to introduce many simplifications in the calculations.

In view of the "looseness" of the deuteron, it can be assumed as a first approximation that in the reaction $e+d\to e+n+p$ the electron is scattered independently by the proton and by the neutron[80]:

$$(d\sigma/d\Omega)_{\rm inel} = (d\sigma/d\Omega)_p + (d\sigma/d\Omega)_n. \qquad (25)$$

This leads to the appearance of a large quasielastic peak in the spectrum $d^2\sigma/d\Omega_e dE'_e$ at a final-electron energy corresponding to scattering by a free nucleon[80]:

$$(E'_e)_{\rm reac} = \frac{E_e-\epsilon}{1+(2E_e/M)\sin^2\theta_e/2}.$$

Allowance for the interaction of the nucleons in the deuteron, i.e., for their relative motion, leads to a "smearing" of the peak. In the region of the quasielastic peak, the cross section is larger by approximately one order of magnitude than outside this region. In this region, the relative energy of the nucleons ω in their c.m.s. is large (the ratio ω/M is not small), and the final-state interaction plays a minor role (it changes the cross section by approximately 10%). This greatly simplifies many theoretical calculations.

Taking these remarks into account, let us consider first methods of calculating the Born amplitude, and postpone the discussion of the final-state interaction. The subject of our discussion will be the attempt at a pure dispersion calculation of the Born amplitude (see Fig. 7a).

The most consistent attempt to develop a dispersion theory of the electrodisintegration of a deuteron is contained in Ref. 81. The method of Ref. 81 is based on a postulated Mandelstam representation of the transition current matrix $\langle np|j_\mu|d\rangle$. However, as noted by the authors of Ref. 81, the complete calculation of the double spectral function is "presently beyond human capabilities." It is therefore necessary to use the nearest-singularity approach. In Ref. 81 they took into account the pole terms in three Mandelstam variables, single dispersion terms, and in part the contribution of the double dispersion representation for gauge invariance (Fig. 8).

In the calculation it is necessary to known the npd vertex off the mass shell.[82] This vertex is determined from the condition of best agreement between the theoretical and experimental deuteron photodisintegration cross sections.[81] It can also be determined[83] from the condition that the dispersion theory in the nonrelativistic limit agree with the Schrödinger theory.

There are interesting papers[84] in which the npd vertex is calculated by the dispersion method with account taken of the lowest-order (in perturbation theory) diagrams of the np interaction (account is taken of exchange of 1, 2, and 3 pions). Insofar as we know, these procedures were not used for a detailed calculation of the process $e+d\to e+n+p$.

To take the final-state interaction into account, it is necessary to carry out a partial-wave expansion of the amplitude. In the region of the quasielastic peak, where the parameter ω/M is not small, this expansion converges slowly, so that the calculation becomes very cumbersome. The fact that the final-state interaction is strong only at small l leads to an appreciable simplification. Therefore the final-state interaction was taken into account in Ref. 81 only for the lower partial waves

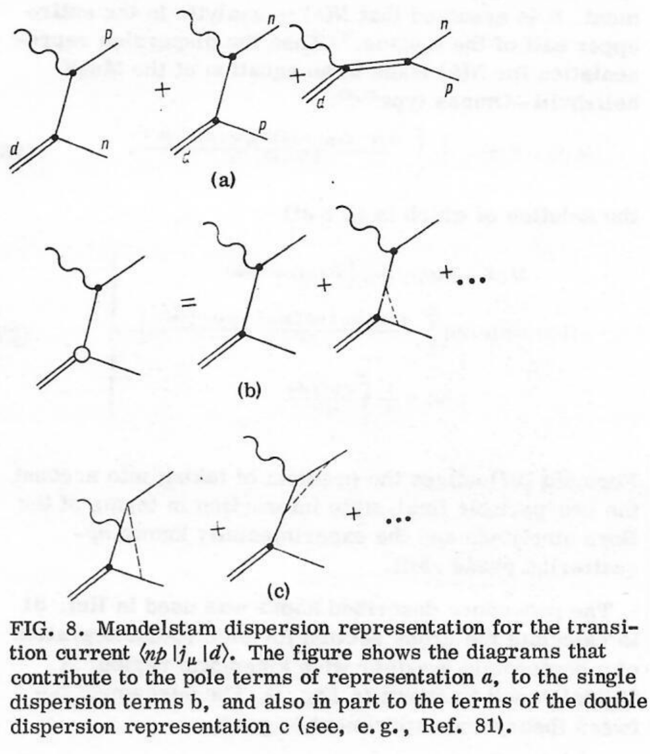


FIG. 8. Mandelstam dispersion representation for the transition current $\langle np|j_\mu|d\rangle$. The figure shows the diagrams that contribute to the pole terms of representation a, to the single dispersion terms b, and also in part to the terms of the double dispersion representation c (see, e.g., Ref. 81).

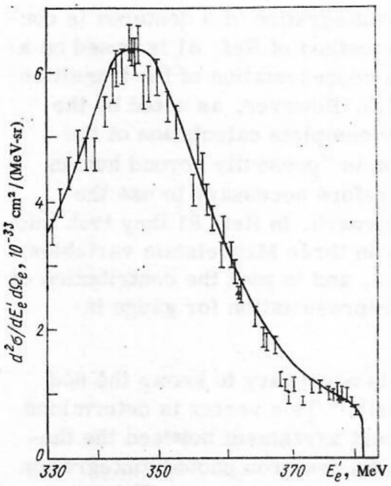


FIG. 9. Deuteron electrodisintegration cross section in accordance with the theory of Renard *et al.*[81]

(in the noninteracting-channel approximation), and the remaining amplitudes were taken in the Born approximation.

It is best to take the final-state interaction into account by the method based on the Muskhelishvili–Omnes equation.[85] The advantages of this approach reduce to the following: 1) consistent relativism; 2) no model assumptions are used. We describe the simplest variant of this method, which is not cluttered up with technical details.

Let k be the relative momentum of the nucleon in the c. m. s. The transition matrix element $\langle np|j|d\rangle = M(k) = B(k) + N(k)$ to a state with fixed orbital angular momentum in the final state takes, according to Watson's theorem, the form $M_l = |M_l| \exp(i\delta_l)$, where $\delta_l(k)$ is the np-scattering phase shift; $B(k)$ is the Born matrix element. It is assumed that $N(k)$ is analytic in the entire upper half of the k plane.[4)] Then the dispersion representation for $N(k)$ leads to an equation of the Muskhelishvili–Omnes type[81,85]:

$$M(k) = B(k) + \frac{1}{\pi}\int_0^\infty \frac{M(k')\exp[-i\delta(k')]\sin\delta(k')\,dk'^2}{k'^2 - k^2 - i\varepsilon}, \tag{26}$$

the solution of which is $(\omega = k^2)$

$$\left.\begin{aligned} &M(\omega) = \exp[i\rho(\omega)]\Big\{B(\omega)\cos\delta(\omega) \\ &+ (1/\pi)\exp[\rho(\omega)]\int_0^\infty \frac{B(\omega')\sin\delta(\omega')\exp[-\rho(\omega')]\,d\omega'}{\omega'-\omega}\Big\}, \\ &\rho(\omega) = \frac{1}{\pi}\int_0^\infty \frac{\delta(\omega')\,d\omega'}{\omega'-\omega}. \end{aligned}\right\} \tag{27}$$

Formula (27) solves the problem of taking into account the two-particle final-state interaction in terms of the Born amplitude and the experimentally known np-scattering phase shift.

The procedure described above was used in Ref. 81 to calculate the cross section for electrodisintegration of a deuteron in a rather wide kinematic region. A typical result is shown in Fig. 9. The agreement between theory and experiment is good.

In nuclear physics there is a proved Siegert theorem[81,87] according to which the contribution of the exchange meson currents to the charge multipole QJ transitions is strongly suppressed. Therefore the problem of the contribution of the exchange meson currents to the structure of the deuteron is best investigated in magnetic multipole MJ transitions, for which it is necessary to obtain the cross section of $e+d \to e+n+p$ at $\theta_e = 180°$.

The question of the experimental determination of the exact contribution of the exchange meson currents at low energies and momentum transfers was succinctly formulated in Ref. 88. The idea of the solution is that the deviation of the cross section observed in experiment at $\theta_e = \pi$ from that calculated from the formulas of the relativistic impulse approximation yields precisely the value of this contribution.

We turn now to attempts at a theoretical calculation of the contribution of the exchange meson currents. The results obtained since 1966 are the following. Since the deuteron is a weakly bound system, it suffices to take into account only the lower Feynman diagrams to calculate the contribution of the exchange meson currents to the electrodisintegration of the deuteron (as in the case of elastic ed scattering). Such a calculation was carried out in Refs. 78 and 79. Unlike elastic ed scattering, the main contribution is made here by the diagram of the two-pion exchange current (see Fig. 6a). In the calculation of Ref. 78, the deuteron and the final np system were described with the aid of the wave functions of the Hamada-Johnston potential. In addition to the contribution of the $(\pi\pi)$ current, they calculated also the contributions of the diagrams of the $(\omega\pi)$, $(\rho\eta)$, and $(\rho\rho)$ currents. As expected, these contributions turn out to be smaller by one order of magnitude than the contribution of the $(\pi\pi)$ current. However, errors crept into the addition of the partial S and D states of the deuteron in Ref. 78. Therefore the conclusions drawn in Ref. 78, that the contribution of the exchange meson currents and of the deuteron D state to the cross section is small at large q^2 (5–10 F^{-2}) and that there is an appreciable discrepancy between theory and experiment, are incorrect.

Recent calculations[79] yielded practically the same partial amplitudes as in Ref. 78. In Ref. 79, however, the errors of Ref. 78 in the summation of the amplitudes were corrected. The results of these calculations are the following: a) at $q^2 > 5$ F^{-2} the contribution of the two-pion diagram of the exchange meson currents dominates in the cross section $d^2\sigma/d\Omega_e dE'_e|_{\theta_e=\pi}$; b) at $q^2 = 5$–10 F^{-2}, the calculated cross section agrees well with the experimental one.

6. ELECTROMAGNETIC FORM FACTORS OF THE NEUTRON

The neutron is a fundamental particle, and therefore an investigation of all its properties, including the electromagnetic ones, is of undoubted interest. Unfortunately, an experimental study of the electromagnetic structure of the neutron is hindered greatly by the lack of neutron targets. One of the ways of getting around this difficulty is to interchange the roles of the electron and neutron in the *en* collisions, i.e., to scatter the neutrons by electrons of atoms. Another way is to attempt to determine the *en* scattering from its contribution to the *ed* scattering. This approach has a mixed experimental-theoretical character, inasmuch as it is necessary to "subtract" the presence of the proton from the results of the measurement of the *ed*-scattering cross sections.

The interaction of thermal neutrons with electrons of atoms is discussed in Refs. 89 and 90, and we shall not concern ourselves with these questions further. Let us dwell on the problem of extracting the neutron form factors from data on *ed* scattering.

Information on neutron form factors in the region $0 < q^2 \lesssim 25\ \mathrm{F}^{-2}$ can be obtained from the data on elastic *ed* scattering. The elastic *ed*-scattering cross section receives contribution only from the isoscalar form factors of the nucleon, i.e., from the quantities $(G_{Ep} + G_{En})$, $(G_{Mp} + G_{Mn})$. The form factors G_{Ep} and G_{Mp} are known from experiments on *ep* scattering. When G_{En} is determined from the difference $[(G_{Ep} + G_{En}) - G_{Ep}]$, the absolute errors in the measurement of $(G_{Ep} + G_{En})$ and G_{Ep} combine. Since the form factor G_{En} is close to zero, this leads to large relative errors in its value. Thus, to determine G_{En} it is necessary to determine very accurately the values of G_{ES} and G_{Ep}. It is obvious that we do not encounter this problem when determining G_{Mn}, since this form factor is close to G_{Mn} in absolute value. Therefore the determination of G_{Mn} from G^d_{mag} is more reliable than that of G_{En} from $A(q^2)$.[5]

The determination of G_{En} at small q^2 is facilitated by the fact that in this case (e.g., at $q^2 \lesssim 1\ \mathrm{F}^{-2}$) the contribution of $G^d_G(q^2)$ and $G^d_{\mathrm{mag}}(q^2)$ to $A(q^2)$ is small. Therefore measurement of the ratio of the cross sections of *ep* and *ed* scattering in this case means measurement of the ratios $A(q^2)/G^2_{Ep}(q^2) = [G^d_C(q^2)]^2/G^2_{Ep}(q^2)$, i.e., the measurement of

$$\frac{G^d_C}{G_{Ep}} = \frac{G_{En} + G_{Ep}}{G_{Ep}} \left(1 - \frac{q^2}{8M^2}\right) C_E(q^2).$$

where C_E is the charge structure function of the deuteron (see, e.g., Ref. 21), and factor $(1 - q^2/8M^2)$ takes into account the relativistic corrections in accordance with Ref. 73.

For a long time, a contradiction existed between the results of the direct measurements of the slope $dG_{En}(q^2)/dq^2$ at $q^2 = 0$ and experiments on *en* scattering, which yielded $G'_{En}(0) = (0.0189 \pm 0.0004)\ \mathrm{F}^{+2}$ (Ref. 90), and the fact that the value of G_{En} extracted from the deuteron form factors is zero on the average. The reason for this discrepancy is that to extract G_{En} it is necessary to determine exactly the structure function, which takes into account, in particular, the relativistic corrections. In Ref. 73 (see also Ref. 91) it was shown that allowance for the relativistic corrections to C_E in the description of the deuteron by the Feshbach-Lomon wave function eliminates this contradiction.

The determination of G_{En} from the form factors of the deuteron at $1\ \mathrm{F}^{-2} \leqslant q^2 \leqslant 20\ \mathrm{F}^{-2}$ can be found, e.g., in Ref. 20, where citations to earlier work can be found. In the analysis of the experimental values of $A(q^2)$ it was assumed that $G_{Mn} = \mu_n G_{Ep}$. Therefore the unknown quantities in $A(q^2)$ are (besides G_{En}) the structure functions, which were calculated by using various potentials. The relativistic corrections were taken into account in accordance with Ref. 72. The value of G_{En} depends on the choice of the *np*-interaction potential. Thus, it was found that G_{En} agrees well with the relation $G_{En} = -\mu_n \tau G_{Ep}/(1 + p\tau)$; $\tau = q^2/4M^2$, where $p = 5.6$ for the Feshbach-Lomon potential and $p = 10.7$ for the Hamada-Johnston potential (Fig. 10). Figure 11 shows that the choice of the different values of G_{En} from (-0.05) to $(+0.05)$ yields the same scatter in the value of $A(q^2)$ as the choice of different potentials (see Fig. 2).

At $q^2 \gtrsim 30\ \mathrm{F}^{-2}$, the measurement of the cross sections of elastic *ed* scattering becomes very complicated. The main source of information on the neutron form factors for these values of q^2 is experiments on deuteron electrodisintegration. The methods of extracting neutron form factors from data on deuteron electrodisintegration are described in detail in Ref. 81. We shall describe three such methods: 1) the area method, 2) the quasielastic peak method, 3) the method of simultaneous registration of an electron and a nucleon.

The first method consists of calculating $(d\sigma/d\Omega_e)_n$ from formula (25). It is clear that this method is essentially approximate, since it does not take the *np* interactions into account. It is attractive, however, because of its simplicity, and also because the experiment does not call for high energy resolution of the final electron, i.e., results of not very accurate experiments can be used for reduction by the area method. The area method permits a calculation of the neutron form factors with 20% accuracy.

In the second method, the deuteron electrodisintegration cross section $d^2\sigma/d\Omega_e dE'_e$ is measured in the region of the quasielastic peak (see Sec. 5). The experimentally determined structure functions C and D of the cross section [see (2)] are connected with the form factors of the nucleons by the following typical formulas[81]:

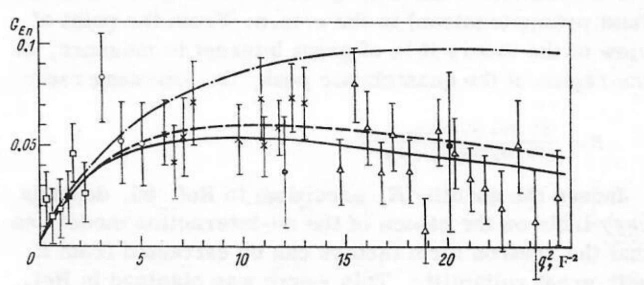


FIG. 10. Experimental value of the charge form factor of the neutron G_{En}, obtained from $A(q^2)$ with the aid of the Feshbach-Lomon wave function (cited from Ref. 20): dashed and dash-dot curves—values G_{En} calculated from the formulas $G_{En} = -\mu_n \tau G_{Ep}$ and $G_{En} = -\mu_n \tau G_{Ep}/(1 + \tau)$ respectively; solid curve—result of calculation of G_{En} from the formula $G_{En} = -\mu_n \tau G_{En}/(1 + p\tau)$ with $p = 5.6$.

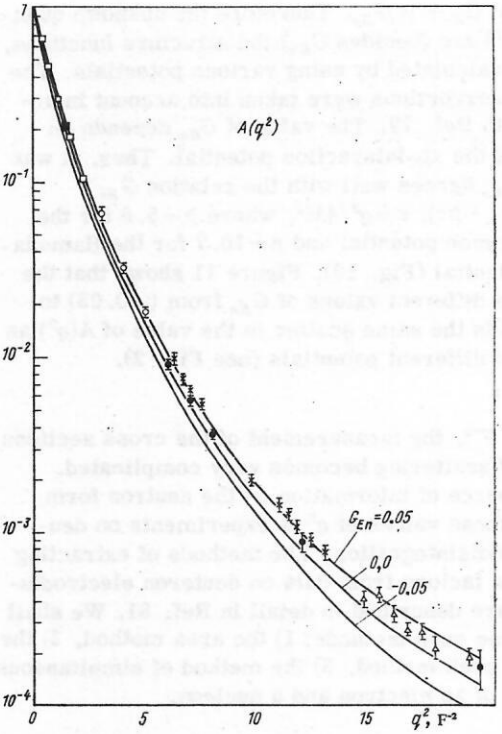


FIG. 11. Dependence of $A(q^2)$ on the choice of the neutron charge form factor.[20]

$$\left.\begin{array}{c} C=\sigma_T+\sigma_L; \quad D=\sigma_T; \\ \sigma_T=A_T\left(G_{Mp}^2+G_{Mn}^2\right)+2B_T G_{Mp}G_{Mn}+C_T\left(G_{Ep}^2+G_{En}^2\right) \\ +2D_T G_{Ep}G_{En}+2E_T\left(G_{Ep}G_{Mp}+G_{En}G_{Mn}\right) \\ +2F_T\left(G_{Ep}G_{Mn}+G_{En}G_{Mp}\right); \\ \sigma_L=A_L\left(G_{Ep}^2+G_{En}^2\right)+2B_L G_{En}G_{Ep}+C_L\left(G_{Mp}^2+G_{Mn}^2\right) \\ +2D_L G_{Mp}G_{Mn}+2E_L\left(G_{Ep}G_{Mp}+G_{En}G_{Mn}\right) \\ +2F_L\left(G_{Ep}G_{Mn}+G_{En}G_{Mp}\right). \end{array}\right\} \tag{28}$$

The quantities $A_{T,L}, \ldots, F_{T,L}$ are calculated theoretically with the aid of the methods described schematically in Sec. 5, and are tabulated in Ref. 81. The neutron form factors G_{En} and G_{Mn} are calculated from formulas (28).

We consider now the method of joint registration of the electron and nucleon. Let $\theta_{p(n)}$ be the angle between the 3-momentum transfer **q** and the 3-momentum of the final proton (neutron) in the c. m. s. From the point of view of the theory it is of great interest to measure, in the region of the quasielastic peak, the following ratio:

$$R=\frac{(d^3\sigma/dE'_e\, d\Omega_e\, d\Omega_n)_{\theta_n=\pi}}{(d^3\sigma/dE'_e\, d\Omega_e\, d\Omega_p)_{\theta_p=\pi}}.$$

Indeed the quantity R, according to Ref. 80, depends very little on the choice of the np-interaction model, so that the neutron form factors can be extracted from R with great reliability. This result was obtained in Ref. 80 in the wave-function formalism without allowance for the final-state interaction, but in the region of the quasielastic peak the contribution of the final-state interaction is negligible.[80,81] According to estimates given in Ref. 81, allowance for the final-state interaction changes R by 1%. Since this uncertainty is much less than the experimental errors on the measurements of R, which as a rule amount to 3–5%, it is possible to disregard the final-state interaction in determination of the neutron form factor from R, as is done, e.g., in Ref. 47. Summarizing the foregoing, we conclude that in the third method the form factors of the neutron in the region of the quasielastic peak can be calculated from the formula

$$R=\frac{G_{Mn}^2\{[(G_{En}/G_{Mn})^2+\tau]/(1+\tau)+2\tau+\mathrm{tg}^2\,\theta/2\}}{G_{Mp}^2\{[(G_{Ep}/G_{Mp})^2+\tau]/(1+\tau)+2\tau+\mathrm{tg}^2\,\theta/2\}}.$$

The corrections to this formula are analyzed in Ref. 81.

The extraction of the neutron form factors from the experimental data on deuteron electrodisintegration is the subject of extensive studies,[36,45] in which the values of G_{En} and G_{Mn} are calculated from formulas analogous to those given above (mainly with the aid of the first and third methods).

By now, the study of the electrodisintegration of the deuteron has made it possible to determine the magnetic form factor of the neutron $G_{Mn}(q^2)$ with 10% accuracy in the interval $q^2 \lesssim 50\ \mathrm{F}^{-2}$ (and with lower accuracy at larger q^2). For the charge form factor $G_{En}(q^2)$, however, only an upper bound could be obtained for its absolute values, $|G_{En}|^2 < 0.1$ for all q^2.

7. INTEGRAL REPRESENTATION FOR ELECTROMAGNETIC FORM FACTORS OF THE *np* SYSTEM

In this section we describe a new method of calculating the electromagnetic properties of the np system, a method having important advantages over those discussed in Secs. 4 and 5. In this approach, the np pair interaction is taken into account consistently and relativistically, while the form factors of the np system are expressed directly in terms of the np-scattering phase shifts and the nucleon form factors.

The principal basis of the method was set forth in Ref. 51 in connection with the general problem of the relation between fields and particles off the mass shell. A relativistic treatment is given in Ref. 52. The concrete case of ed interaction is considered on this basis in Refs. 92–96. The initial premises of the method of Ref. 51 are refined in Ref. 97, where this method is generalized in such a way that inelastic interaction in the np system can be taken into account in principle.

To illustrate the new approach clearly, we turn to the model with separable potential, considered in Sec. 3. From the function $\Psi^{(+)}(k, p)$ we can easily determine the scattering matrix

$$\langle p\,|\,S\,|\,k\rangle=\lim_{t\to\infty}\exp\left[-\mathrm{i}\left(k^2-p^2\right)t\right]\Psi^{(+)}(k,\,p). \tag{29}$$

By virtue of spherical symmetry, only the S-phase $\delta(E)$, defined by the relation

$$\langle p\,|\,S\,|\,k\rangle=\delta\left(E'-E\right)\exp\left[2\mathrm{i}\delta\left(E\right)\right] \tag{30}$$

does not vanish in this matrix. With the aid of $\delta(E)$ we obtain the function

$$B\left(E\right)=\frac{E-E_d}{E}\exp\left\{-\frac{1}{\pi}\int_0^\infty\frac{\delta\left(E'\right)dE'}{E'-E}\right\}. \tag{31}$$

This function is a natural generalization of the Jost function, which is widely used in potential-scattering theory. The Jost function is introduced here with an eye to the following. We continue analytically the form factors into the complex plane, but we take the scattering phase shifts only in the actually measurable physical range of their values. Obviously, this purpose will be accomplished if the form factors can be expressed in terms of the Jost function. Indeed, it is seen from (31) that $B(E)$ is expressed, on the one hand, in terms of the phase shift in the physical region, and on the other hand is defined in the entire complex E plane with a cut along the real semiaxis. The S matrix is obtained from $B(E)$ by taking the limit:

$$S(E)=\exp[2i\delta(E)]=\lim_{\epsilon\to 0}\frac{B(E-i\epsilon)}{B(E+i\epsilon)}. \tag{32}$$

We now represent the form factors G_{i0}, G_{0i}, and G_{ii} in the form[6)]

$$\begin{aligned}&G_{i0}(E-i0,E',-q^2)\\&=\frac{F(-q^2)k'^2}{2\pi iMB(E-i0)}\int\frac{p^3\,dp}{k^2-p^2-i0}\tilde{\theta}(p,q,k')\Delta B\left(\frac{p^2}{2M}\right)\\&-\frac{F(-q^2)k'^2}{B(E-i0)}\int\frac{p^2\,dp}{k^2-p^2}\tilde{\theta}(p,q,k')[f(k)-f(p)];\end{aligned} \tag{33}$$

$$\begin{aligned}&G_{0i}(E,E'-i0,-q^2)\\&=\frac{F(-q^2)k^2}{2\pi iMB(E'+i0)}\int\frac{p'^3\,dp'}{k'^2-p'^2+i0}\tilde{\theta}(k,q,p')\Delta B\left(\frac{p'^2}{2M}\right)\\&-\frac{F(-q^2)k^2}{B(E'+i0)}\int\frac{p'^2\,dp'}{k'^2-p'^2}\tilde{\theta}(k,q,p')[f(k')^*-f(p')^*];\end{aligned} \tag{34}$$

$$\begin{aligned}&G_{ii}(E-i0,E'-i0,-q^2)=-\frac{F(-q^2)}{4\pi^2M^2B(E-i0)B(E'+i0)}\\&\times\int\frac{p^3\,dp\Delta B(p^2/2M)}{k^2-p^2-i0}\int\frac{p'^3\,dp\Delta B(p'^2/2M)}{k'^2-p'^2+i0}\tilde{\theta}(p,q,p')\\&+\frac{F(-q^2)}{B(E-i0)B(E'+i0)}\int\frac{p^2\,dpf(p)^*}{k^2-p^2-i0}\int\frac{p'^2\,dp'f(p')}{k'^2-p'^2-i0}\\&\times\tilde{\theta}(p,q,p')[f(k)f(k')^*-f(p)f(p')^*].\end{aligned} \tag{35}$$

In each of the representations (33)–(35), the investigated form factor breaks up into a sum of two terms. The first term is expressed directly in terms of the scattering phase shift and has an anomalous singularity. The second term has no anomalous singularity. It is precisely this separation which is the basis of the new method.

We consider the four current matrix elements $\langle P^{\text{out}}_{\text{in}}|j_\mu|P'^{\text{out}}_{\text{in}}\rangle$ of a system consisting of a proton and a neutron that are in the S state of relative motion. These matrix elements can be expressed in terms of the form factors in the following manner[7)]:

$$\langle s,P(k)|j_\mu(0)|s',P'(l)\rangle=\frac{F(s,k;t;s',l)}{(2\pi)^3}\left\{K_\mu-Q_\mu\frac{KQ}{Q^2}\right\}, \tag{36}$$

where $K_\mu=P_\mu+P'_\mu$; $Q_\mu=P_\mu-P'_\mu$, $s=P^2$, $s'=P'^2$, $t=Q^2$, while k and l label the out and in bases. The vectors of the out-basis have an index k (or l) $=+1$, and those of the in-basis an index k (or l) -1.

The form factor $F(s,k;s',l)$ can be represented in the form

$$F(s,k;t;s',l)=F_0(s,t,s')+F_C(s,k;t;s',l), \tag{37}$$

where F_0 is the form factor of the system of the free proton and neutron in the S state of relative motion. The contribution of the np interaction is fully taken into account by the form factor $F_C(s,k;t;s',l)$. We emphasize that the breakdown (37) is not connected in any way with perturbation theory or with any model of the interaction of the system particles. This breakdown is due only to the possibility of representing the current matrix of a two-particle system by a sum of connected and unconnected parts.

The form factor $F_0(s,t,s')$ is expressed explicitly in terms of the nucleon form factors with the aid of the methods of relativistic kinematics.[12,52] To determine F_C, we assume that this form factor can be represented in the form (Ref. 97)[8)]:

$$F_C(s,k;t;s',l)=F_{10}(s+ik0,k;t;s',l)+F_{01}(s,k;t;s'-il0,l). \tag{38}$$

The function $F_{10}(s+ik0,k;t;s',l)$ is assumed here to be analytic in $s+i\epsilon$ in a certain upper ($k=1$) or lower ($k=-1$) semi-region of the physical region of variation of the variable s: $4M^2\leqslant s<+\infty$. The function F_{10} as a function of s' cannot be continued from the real axis. To the contrary, F_{01} is analytic in a certain upper ($l=-1$) or lower ($l=+1$) semi-region of the physical region of values of s', and cannot be continued from the real axis as a function of s. We note that the form factor $F_C(s,k;t;s',l)$ on the whole is not assumed to be analytic in either s or s'.

The conditions for finding F_{10} and F_{01} are realizable by the S matrix that connects the in- and out-basis. With the aid of these connections it was shown in Refs. 51 and 97 that Eq. (38) leads to an integral representation for $F_C(s,k;t;s',l)$ in the form

$$\begin{aligned}F_C(s,k;t;s',l)&=\frac{1}{2\pi iB(s+ik0)}\int_{4M^2}^{\infty}\frac{\Delta B(x)F_0(x,t,s')}{s-x+ik0}\,dx\\&+\frac{1}{2\pi iB(s'-il0)}\int_{4M^2}^{\infty}\frac{F_0(s,t,y)\Delta B(y)}{s'-y-il0}\,dy\\&-\frac{1}{4\pi^2B(s+ik0)B(s'-il0)}\int_{4M^2}^{\infty}\frac{dx\Delta B(x)}{s-x+ik0}\\&\times\int_{4M^2}^{\infty}\frac{dy\Delta B(y)}{s'-y-il0}F_0(x,t,y)\\&+\frac{G(s;t;s',l)}{B(s+ik0)}+\frac{G(s';t;s,k)^*}{B(s'-il0)}.\end{aligned} \tag{39}$$

Here

$$B(z)=\frac{z-M_d^2}{z-4M^2}\exp\left\{-\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{\delta(z')\,dz'}{z'-z}\right\}; \tag{40}$$

$\delta(z)$ is the nucleon-nucleon scattering phase shift; $\Delta B(x)\equiv B(x+i0)-B(x-i0)$. The function $G(s;t;s',l)$ is analytic in s in the vicinity of the physical region $4M^2\leqslant s<+\infty$. Expression (39) is obtained from formula (17) of Ref. 97, if we neglect in the latter the contribution of the inelastic interactions, designate

$$C(s;t;s',l)+\frac{1}{2\pi iB(s'-il0)}\int_{4M^2}^{\infty}\frac{dx}{s'-x-il0}$$

$$[B(x+i0)C(s;t;x,-1)-B(x-i0)C(s;t;x,+1)]$$

by $G(s;t;s',l)$, and recognize that $B(s'+il0)=B(s'-il0)^*$, $\Delta B(x)^*=-\Delta B(x)$ because the np-scattering phase shift δ is real. The physical meaning of (39) is illustrated by Fig. 4.

We note that the functions B have zeros at the point M_d^2. Therefore the form factor $F(s,k;t;s',l)$ has poles at $s,s'=M_d^2$. As indicated in Sec. 3, the residues at these poles determine the form factors of the current

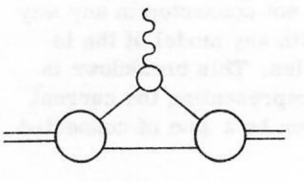


FIG. 12. Graphic representation of the deuteron current $\langle d|j_\mu|d\rangle$.

matrix elements $\langle d|j_\mu|d\rangle$ and $\langle np|j_\mu|d\rangle$. Namely, the form factor of the electrodisintegration current is given by

$$F(t;s'.l)=\frac{\Gamma^{-1}}{2\pi i B'(M_d^2)}\int_{4M^2}^{\infty}\frac{\Delta B(x)\,dx}{M_d^2-x}F_0(x,t,s')$$

$$+\frac{\Gamma^{-1}}{4\pi^2B'(M_d^2)B(s'-il0)}\int_{4M^2}^{\infty}\frac{\Delta B(x)\,dx}{M_d^2-x}\int_{4M^2}^{\infty}\frac{\Delta B(y)\,dy}{y-s'+i l0}F_0(x,t,y). \quad (41)$$

For the deuteron form factor we obtain the expression

$$F(t)=-\frac{\Gamma^{-2}}{4\pi^2B'(M_d^2)^2}\int_{4M^2}^{\infty}\frac{\Delta B(x)\,dx}{M_d^2-x}\int_{4M^2}^{\infty}\frac{\Delta B(y)\,dy}{M_d^2-y}F_0(x,t,y), \quad (42)$$

where B' is the derivative. In (41) and (42) we have neglected the contribution of the functions $G(s;t;s',l)$ and $G(s';t;s;k)^*$. This can be done because G and G^* do not contain a contribution from the anomalous cut. The constant Γ is determined from the normalization condition $F(0)=1$ of the charge form factor. Formulas (41) and (42) are represented graphically in Figs. 7 and 12.

Using relations (26) and (27), we can easily verify that the double integral in (41) is a correction, for the final-state interaction, to the first single integral. Consequently, the single integral in (41) is the analog of the Born amplitude of the electrodisintegration. This amplitude is usually calculated with the aid of wave functions or by perturbation theory (see Sec. 5). In our approach, the Born amplitude turned out to be automatically expressed in terms of the physical np-scattering phase shift.

We shall use the Omnes-Muskhelishvili equation to determine directly the Born amplitude (the first diagram of Fig. 7). To this end we consider the form factor $F_0(s,t,s')$ [see (37)], which describes the electromagnetic properties of the free np system, and take into account the interaction in the initial state. The form factor $F(s,t,s')$, which contains the contribution of the interaction in the initial state, is determined from the equation [cf. (26)]

$$F(s,t,s')=F_0(s.t.s')+\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{h(x)F(x,t,x')\,dx}{x-s-i0}, \quad (43)$$

where $h(x)=\exp[i\delta(x)]\sin\delta(x)$; $\delta(x)$ is the np scattering phase shift. The solution of (43) is

$$F(s,t,s')=F_0(s,t,s')+\frac{1}{2\pi i B(s-i0)}\int_{4M^2}^{\infty}\frac{\Delta B(x)F_0(x,t,s')}{s-x-i0}\,dx. \quad (44)$$

Taking the residue at the deuteron pole in F, we obtain

$$F_B(t,s')=\frac{\Gamma^{-1}}{2\pi i B'(M_d^2)}\int_{4M^2}^{\infty}\frac{\Delta B(x)F_0(x,t,s')}{M_d^2-x}\,dx. \quad (45)$$

This expression coincides fully with the single integral in (41).

It was shown above that the representations for the form factors in (41) and (42) took consistent account of the inelastic np interaction in the initial and final states. Using (39), we can obtain for the form factor $F=F_0+F_C$ an integral representation in which the contribution of the elastic interaction is expressed in terms of the np-scattering phase shift. It is shown in Ref. 97 that the contribution of the inelastic interaction can also be expressed in terms of observable quantities, namely the multiparticle matrix elements of the electromagnetic current and the scattering amplitudes. The functions G contain the contributions of the exchange currents. Within the framework of our approach, there is no special prescription for the calculation of this contribution. However, since the representation (37)–(39) enables us to take into account the elastic interaction consistently and relativistically, the deviation of the experimentally observed cross section from that calculated by formulas (41) and (42) enables us to estimate the exact contribution of the exchange currents and of the inelastic interaction.

8. CALCULATION OF *ed* SCATTERING

It was shown in Sec. 3 that calculation of the elastic *ed* scattering and of the electrodisintegration can be carried out in a unified manner by starting from the matrix element of the np-system current. In Sec. 7 we have described a method that permits a realization of this possibility. We now describe the main stages of actual calculations based on this method.

For a realistic calculation it is necessary to determine the form factors $F_0(s,t,s')$ of the matrix element of a system consisting of a free proton and a free neutron, and also the Jost function $B(s)$. The form factors of the deuteron and of the matrix element of the electrodisintegration current are then determined from formulas (41) and (42).

The matrix element of the current of a free np system is

$$\begin{aligned}&\langle \mathbf{p}_n, m_n;\mathbf{p}_p, m_p|j_\mu|\mathbf{p}'_n, m'_n;\mathbf{p}'_p, m'_p\rangle\\ &=\langle \mathbf{p}_n, m_n|\mathbf{p}'_n, m'_n\rangle\langle \mathbf{p}_p, m_p|j_\mu|\mathbf{p}'_p, m'_p\rangle\\ &+\langle \mathbf{p}_n, m_n|j_\mu|\mathbf{p}'_n, m'_n\rangle\langle \mathbf{p}_p, m_p|\mathbf{p}'_p, m'_p\rangle,\end{aligned} \quad (46)$$

where $\mathbf{p}$ are the nucleon momenta and m are the projections of their spins on the z axis. This relation corresponds to the unconnected diagram of Fig. 4a. Starting from (46), we can determine the matrix element of the current between the states with definite values of P, s, J, S, l, m_J:

$$\langle \mathbf{P}, s, J, S, l, m_J|j_\mu|\mathbf{P}', s', J', S', l', m'_J\rangle. \quad (47)$$

where $P_\mu=p_{n\mu}+p_{p\mu}$, $s=P^2$; J, l, and S are respectively the total, orbital, and spin angular momenta of the np system; m is the projection of J on the z axis. The matrix element (47) is determined from (46) with the aid of the Clebsch-Gordan coefficient of the Poincaré group (see, e.g., Refs. 98 and 99). The form factors F_0 are obtained by parametrization of the "partial" matrix elements of the current (47) and are given by

$$F_0(s,s',t)=\theta(s,s',t)[A(s,s',t)G_{ES}(t)+B(s,s',t)G_{MS}(t)], \quad (48)$$

where $\theta(s,s',t)$ is the relativistic analog of the function

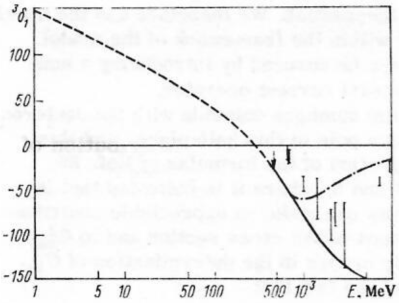


FIG. 13. Experimental set of $^3S_1$ phase shifts with different variants of extrapolation, used in the calculation given in Ref. 94.

$\tilde{\theta}(p,q,p')$ [see (12)]; A and B are matrices in (J,S,l), (J',S',l'), the elements of which are algebraic functions of the variables s, s', and t. To calculate the elastic ed scattering we need A and B with indices $S=S'=1$, $J=J'=1$, $l, l'=0; 2$. In this case, each form factor of the free np system is a 2×2 matrix in l and l'. To calculate the electrodisintegration we need form factors with $S=1$; $J=1$; $l=0$ or 2; $S'=0$ or 1; $J'=0$, 1; etc.; $l'=J+1$, $J-1$ (the latter, of course, only at $J\geqslant 1$). The explicit form factors for certain values of (J,S,l), (J',S',l') are given in Refs. 12, 94, and 96.

In the general case $B(s)$ is a matrix in l and l'. Its elements are determined from the physical scattering matrix $S_{ll'}(s)$ with the aid of the system of equations

$$B_{ll'}(s-i0) = \sum_{l''} S_{ll''}(s) B_{l''l'}(s+i0). \tag{49}$$

This system is the matrix analog of relation (32). If we neglect the admixture of partial waves with l or $l'>0$, then the Jost function $B(s)$ takes the form (40). We confine ourselves here to this simplest case. The integrals in (41) and (42) were calculated with a computer. The experimental set of phase shifts at energies up to 750 MeV was taken from Ref. 100. The phase shifts at higher energies were obtained with the aid of different extrapolations, each of which was reconciled with the results of high-energy phase-shift analyses (see, e.g., Ref. 101). Examples of the extrapolation of $^3S_1$ phase shift are shown in Fig. 13.

A systematic two-channel relativistic numerical calculation of ed scattering, by the method described in Sec. 7, has not yet been completed. We present here the results obtained by neglecting the contribution of the D state of the np system. In this approximation, physical interest attaches only to $G_C^d(q^2)$, inasmuch as for $G_Q^d(q^2)$, and $G_{\text{mag}}^d(q^3)$ a correct allowance for the D-wave admixture is one of the most important problems (see Sec. 4). The results of numerical integration in accordance with (42) are shown in Fig. 14, which is taken from Refs. 94 and 102. A value 2.12 F was obtained for the rms radius, in agreement with the experimental value.[16] At $q^2 \leqslant 6$ F^{-2} we have $A(q^2)\approx [G_C(q^2)]^2$, so that the numerical value of $|G_C^d(q^2)|$ can be compared with the experimental value $A(q^2)$. It is seen from Fig. 14 that the value of $|G_C^d(q^2)|$ obtained in the calculations agrees within 10% with the experimental value (the results of the calculations by dispersion theory agree somewhat worse with experiment). To determine the contribution of the relativistic corrections to the S-wave

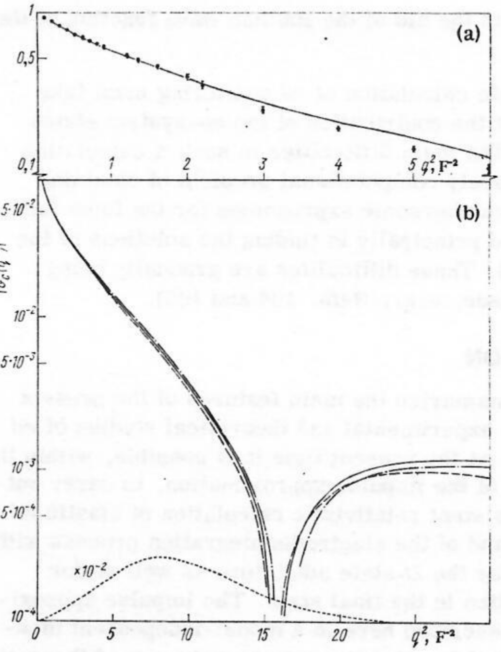


FIG. 14. Charge form factor of deuteron, calculated in Refs. 94 and 102: the dotted line shows the contribution made to $|G_C^d(q^2)|$ by the second term of (48), which is proportional to $G_{MS}(t)$. This term is of pure relativistic origin.[12]

deuteron, the integral (42) was calculated with the form factor F_0 taken both in accordance with the complete relativistic formula[94] and in the nonrelativistic approximation. It is seen from Fig. 14 that the use of the nonrelativistic approach leads to a small decrease of $|G_C^d(q^2)|$ (~10% for all q^2).

Formula (41) was used to calculate the electrodisintegration of a deuteron for backward scattering of the electron near the deuteron disintegration threshold $(s-4M^2)/4M^2 \ll 1$. The results are shown in Fig. 15. The cross section calculated for 0.16 F$^{-2}\leqslant q^2 \leqslant 5$ F^{-2} agrees within the limits of errors with the experimental results [two points of (Ref. 40) do not fall on the curve, but these data seem to be in error[103]]. The relativistic corrections are negative and do not exceed 5%.

Formulas (41) and (43) duplicate the results of the Schrödinger theory. Namely, in the nonrelativistic approximation for F_0, with effective-range phase shifts $G_C^d(q^2)$ (Ref. 92), the form factors of the electrodisintegration[95] agree in the Born approximation with those

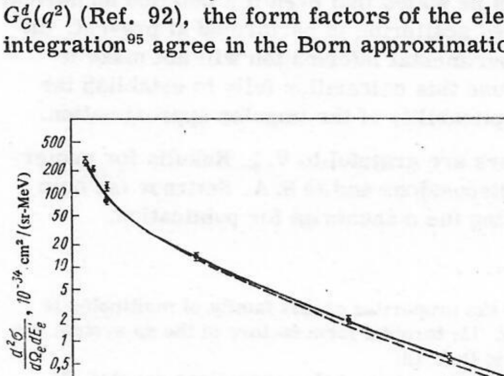


FIG. 15. Cross section for deuteron electrodisintegration in backward scattering of the electron, as calculated in Ref. 96.

obtained with the aid of the Hulthén wave function of the deuteron.

A complete calculation of *ed* scattering must take into account the contribution of the *np*-system states with $\ell > 0$. The main difficulties in such a calculation lie in the purely computational problem of obtaining the rather cumbersome expressions for the form factors F_0, and principally in finding the solutions of the system (49). These difficulties are gradually being overcome (see, e.g., Refs. 104 and 105).

CONCLUSION

Let us summarize the main features of the present state of the experimental and theoretical studies of *ed* scattering. At the present time it is possible, within the framework of the impulse approximation, to carry out a unified consistent relativistic calculation of elastic *ed* scattering and of the electrodisintegration process with allowance for the *D*-state admixture as well as for the interaction in the final state. The impulse approximation is described here in a model-independent manner, so that it is not necessary to make use of theoretical assumptions concerning the character of the *np* interaction.

We have seen above that the *ed*-scattering cross sections are expressed in terms of form factors of the nucleons and the phase shifts of the *np* interaction. To be able to extract from the *ed* scattering reliable data on the neutron form factors (or on the *np*-interaction phase shifts), it is necessary to have reliable estimates of the errors of the impulse approximation, i.e., estimates of the contribution made by the exchange currents and by the inelastic channels. The evaluation of these estimates is the basic problem of *ed* scattering. It is still impossible to do so theoretically, owing to the lack of a consistent theory of strong interactions. Another way is to determine them experimentally, by determining the quantitative discrepancy between the detailed theoretical calculation and the experimental data.

The existing experimental situation makes this procedure impossible, since the accuracy of the data at large q^2 is low, and especially because no polarization experiments have been made.

It can thus be stated that even if a detailed numerical analysis of *ed* scattering is performed at present, the existing experimental information will not make it possible to use this calculation fully to establish the limits of applicability of the impulse approximation.

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1) A review of the properties of this family of multipoles is given in Ref. 11; toroidal form factors of the *np* system are considered in Ref. 12.

2) For a detailed discussion of these questions see Ref. 26.

3) The separable potential is nonlocal, so that no conserved-current local operator can be introduced for the particles whose interaction is described by this potential. For the purposes of the present section, however, the current-conservation condition is inessential. We therefore use the usual local current operator within the framework of the model. Current conservation can be ensured by introducing a nonlocal increment to the usual current operator.

4) If the final-state quantum numbers coincide with the deuteron numbers, then $N(k)$ has a pole in this half-plane, and this leads to a slight modification of the formulas of Ref. 86.

5) See, however, Secs. 4 and 5, where it is indicated that the exchange meson currents can make an appreciable contribution to the transverse part of the cross section and to G^d_{mag}; this leads to systematic errors in the determination of G_{Mn}.

6) We took account here of the fact that

$$B(E+i0)-B(E-i0)=\Delta B(E)=-2\pi i\sqrt{m/2E}\,|f(2mE)|^2.$$

7) The nucleons in (36) are assumed to have zero spin. The case of real nucleons is considered in Sec. 8.

8) In the particular case of the matrix element $\langle P_{\text{in}}|j_\mu|P'_{\text{in}}\rangle$ ($k=1$, $l=-1$), the representation for the form factor is a relativistic generalization of the representation (13).

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