Two-nucleon interaction in nuclei near the doubly magic nucleus Z = 82, N = 126

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The problem of nucleon–nucleon interaction in nuclei near ^{208}Pb is analyzed on the basis of the structure of their excited levels. Experimental data are presented on the excited states of these nuclei, the theoretical papers dealing with the application of phenomenological potentials for the description of these nuclei are reviewed, and the results of investigations in which the free-nucleon potentials serve as the starting point are also considered.

INTRODUCTION

The determination of the relation between the interaction of free nucleons and the effective interaction between nucleons in a nucleus is one of the most fundamental problems of modern nuclear physics. Recent progress in both theoretical and experimental physics has made it possible to determine phenomenological potentials that describe adequately nucleon–nucleon scattering and polarization effects in a wide range of energies. On the other hand, many features of the structure of excited levels of definite groups of nuclei can be explained with the aid of the shell model, with an average potential and a residual interaction between the "valence" nucleons. The residual interaction of nucleons above the closed shells can be represented in the form of a linear combination of two-particle interactions.

We are thus able to correlate directly the residual two-nucleon potential with physical quantities that characterize the nucleus, and to choose its parameters on this basis. The results of many recent papers show that the two-nucleon potential of the residual interaction differs in general from the potential of the free nucleons. Two reasons are advanced, viz., the faults inherent in the model describing the nucleus, and physical effects which we shall arbitrarily call effects of the nuclear-matter density.

The influence of the model cannot be eliminated completely, but can be decreased. As is well known, the shell model works well in the closed-shell region Z = 82, N = 126. Focusing the attention on nuclei in this (A, Z) region creates conditions for the investigation of the renormalization of the potential of the free nucleons, when this potential acts in nuclear matter. The problem becomes much simpler if the experimental data and the theoretical calculation results are compared for nuclei in which the number of nucleons above the filled shells (holes, or nucleons and holes) is equal to 2.

This group of eight nuclei will be symbolically designated ^{208}Pb ± 2. Recognizing that the structure of the residual-interaction potential should not differ from a free-nucleon potential that consist of many parts, we can expect different terms of the potential to reflect different nuclear characteristics to different degrees. Refinement of the problem by determining the correlation of different nuclear properties with the corresponding terms of the potential enables us to set up a program for constructing a complete two-nucleon potential of the residual interaction on the basis of the set of experimental data.

This program is the principal topic of the present article. The first part is a survey of the states of one nucleon or of the states of one hole near ^{208}Pb, with allowance for their connection with the collective excitations of the core. The next two parts are devoted to a discussion of theoretical aspects of the interaction of two nucleons (holes, or a nucleon and a hole) in the system ^{208}Pb ± 2, and also to experimental data concerning these nuclei.

1. EXCITED STATES OF THE SYSTEM ^{208}Pb ± 1 NUCLEON

Introduction. Data on single-particle states near the closed shells Z = 82 and N = 126 were obtained from investigations of the excited states of ^{207}Pb and ^{209}Pb as well as ^{209}Bi and ^{207}Tl. Many studies, made mainly in the 1950's, concern the decay of such isotopes via electron capture by excited states and inelastic scattering of neutrons and protons by these nuclei. Studies made in the 1960's with the aid of tandem generators yielded an extremely large amount of new information: It is known that reactions of the transfer type, i.e., transfer or pickup of one nucleon, are particularly useful for the determination and analysis of single-particle states.

The characteristics of the single-particle states near ^{208}Pb are represented in Figs. 1 and 2, which show the level schemes of nuclei with one nucleon or with one hole outside the closed shells. In addition to the energy, spins, and parities of the individual excited states, these schemes contain also data on the spectroscopic coefficients, which were gathered from different papers and critically analyzed by Stein.[1]

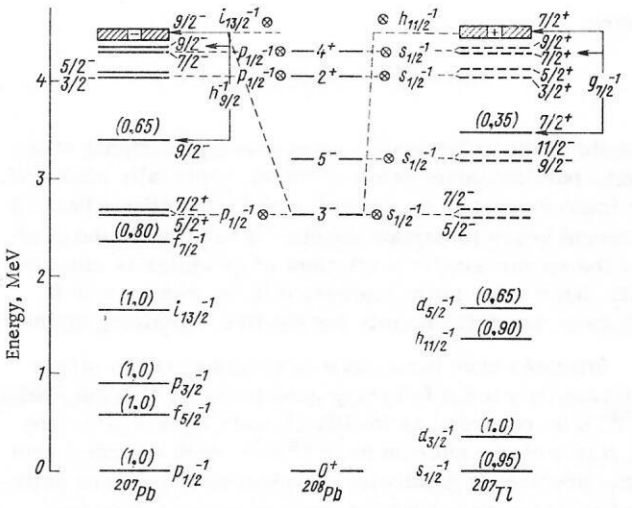

Fig. 1. Scheme of excited single-particle hole states of ^{207}Pb and ^{207}Tl (ref. 1). The numbers in the brackets denote spectroscopic coefficients.

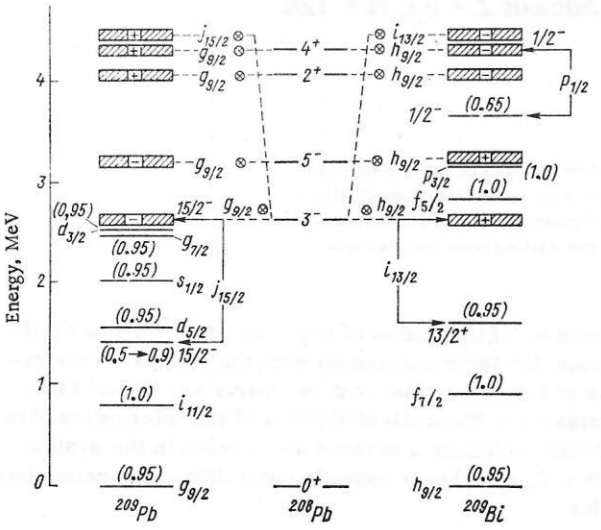

Fig. 2. Diagram of excited single-particle states of ^{209}Pb and ^{209}Bi (ref. 1).

Comparison of the experimental differential cross sections of single-nucleon transfer (pickup) reactions with calculations by the DWBA method makes it possible to determine the spectroscopic coefficient, which can be regarded as a measure of the purity of the single-particle states. For 19 out of the 24 states shown in the figures, the spectroscopic coefficient exceeds 0.8, and for each of these states one observes in the single-nucleon transfer reaction only one strong line. This proves that admixtures of more complicated configurations are not very significant, so that the shell model can be regarded as a good approximation. An analysis of the excited states of nuclei with one nucleon (or hole) above closed shells makes it possible to determine the parameters of the average potential of the entire nucleus (core), in which a single valence nucleon moves. By means of such an analysis, L. A. Sliv and V. A. Volchok[2] were able to refine the parameters V_0, α, and λ of the potential:

$$V_c(r, \mathbf{s}\cdot\mathbf{l}) = V(r) - \lambda\left(\frac{\hbar}{2mc}\right)^2 \frac{\mathbf{s}\cdot\mathbf{l}}{r}\cdot\frac{dV(r)}{dr},$$

where

$$V(r) = V_0[1+\exp\{\alpha(r-r_0)\}]^{-1}.$$

In spite of the undisputably great accomplishments of the single-particle description of nuclei, especially near lead, the improvement in the experimental capabilities has led in recent years to certain doubts. It is all well and good that the spectroscopic coefficient of 19 states is close to unity, but we are more interested in the reason why it differs so much from unity for the five remaining states.

Attempts have been made in recent years to obtain exact answers to the following questions: a) Can the shells of ^{208}Pb be regarded as ideally closed? b) How pure are the states of one nucleon near ^{208}Pb? Such a formulation of the problem is particularly typical of two papers delivered at international conferences on nuclear structure. At the 1967 Tokyo conference, Mottelson[3] called attention in his paper to the particular importance of investigating the interaction of single-particle states and the vibrational states of the core, and analyzed the multiplet identified in ^{209}Bi as an interesting example of such an interaction.

At the 1969 Montreal conference, Stein[1] represented an analysis of the purity of the states of one nucleon near ^{208}Pb and attributed certain deviations from the predictions of the shell model to the inclusion of collective degrees of freedom generated by the core.

Ground state of ^{208}Pb. The first question can be formulated more accurately: Does the ground state of ^{208}Pb contain an admixture of a two-particle–two-hole configuration? If this were so, the wave function of the ground state of ^{208}Pb could be represented as a superposition of the state of a core with closed shells and the two-particle–two-hole (2p–2h) states

$$^{208}Pb_{gs} = \alpha\,\square + \beta\,\square\!\!\square + \gamma\,\square\!\!\square$$

in the proton and neutron shells, plus possible terms corresponding to four-particle–four-hole (4p–4h) states.

Presently available experimental data yield direct information on the wave function of the ^{208}Pb ground state. A Los Alamos group[4], using a tandem generator, measured the differential cross section of the reaction ^{207}Pb(t, p)^{209}Pb and the reaction ^{210}Pb(p, d)^{209}Pb. They investigated the levels in ^{209}Pb in both cases. Among the excited states produced when the target is bombarded with tritium ions or with protons, they identified some belonging to the two-particle–one-hole (2p–1h) neutron configuration. In the case of the first reaction, such configurations should be realized very simply: The state of the $3p_{1/2}$ neutron hole should remain unchanged, and the two neutrons transferred from the tritium should occupy the $2g_{9/2}$ level or a higher level. Calculation of even the lowest level gives grounds for expecting a very complicated level spectrum produced when the $3p_{1/2}$ hole state interacts with the states $(2g_{9/2})^2_{0,2,4,6,8}$. The energies of the remaining levels of this class, as well as their excitation cross sections, can be obtained by starting from the wave function of ^{210}Pb and using fractional-parentage coefficients. If the two-neutron–hole configuration is excited in the reaction ^{210}Pb(p, d)^{209}Pb one neutron should be released from the $3p_{1/2}$ inner shell or from a lower one. A complicated spectrum is produced also when two neutrons in a state corresponding to the ^{210}Pb ground state, i.e., $(g_{9/2})^2$, interact with a neutron–hole state $p_{1/2}, f_{5/2}, \ldots$. These two reactions supplement each other from the point of view of the analysis of the 2p–1h states.

The Los Alamos group has observed spectra corresponding to both types of excitations. The angular distributions were measured, the differential cross sections were determined, and the experimental results were compared with the DWBA calculations, so that the spectroscopic coefficients could be obtained. These results led to the following conclusions: The 2.152-MeV state is the "ground state" for both types of multiplets and is excited in the ^{207}Pb(t, p) reaction with a cross section equal (within 5%) to the cross section of the reaction ^{208}Pb(t, p)^{210}Pb, followed by a transition to the ground state. Thus, an analysis of the multiplets and the equality of the cross sections of the aforementioned reactions confirm the hypoth-

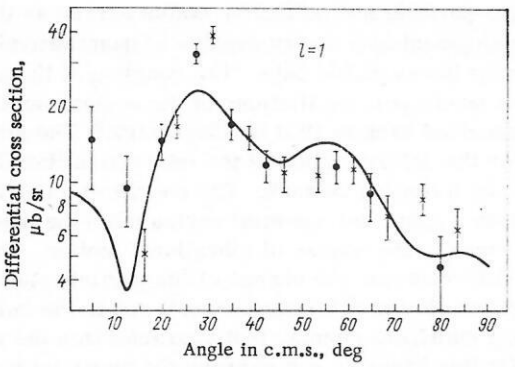

Fig. 3. Differential cross section for excitation of the 2.152-MeV state in the reaction ^{208}Pb(d, p)^{209}Pb.

esis that the configuration of the 2.152-MeV state in ^{209}Pb is analogous to the configuration of the ground state of ^{210}Pb, and that the neutron hole $3p_{1/2}$ does not cause significant changes in the neutron-pair wave function.

Additional information on the purity of the ground state of ^{208}Pb were obtained by a group at Yale,[5] who measured the differential cross section for excitation of the 2.152-MeV state in the reaction ^{208}Pb(d, p)^{209}Pb. The measurement results are represented by the points of Fig. 3, where the smooth curve is the differential cross section calculated by the DWBA for neutron transfer to the $3p_{1/2}$ level.

If the neutron shell of ^{208}Pb is regarded as closed, then a pair of neutrons should first go over in this reaction to the $g_{9/2}$ level, after which the transferred neutron can occupy one of the freed places. The result presented shows that pure transfer with $l = 1$ takes place, and this leads to the conclusion that the 2.152-MeV state is excited when the nucleus is in the (2p−2h) state. The cross section of this reaction is very small. Calculation of the spectroscopic factor, which can be regarded as the square of the modulus of the amplitude of the (2p−2h) state in ^{208}Pb, yields $\beta^2 = 0.015$–0.020. This means that the fraction of the configuration $(^{210}\mathrm{Pb}_{gs} \times p_{1/2}^{-2})$ in the wave function of ^{208}Pb is 12–14%. As already mentioned, in addition to the ground state (2p−1h), certain other states of this class can be excited in the transfer reaction. Although the population of such state is very small in the transfer reaction, their sum can yield a discernible contribution, and this can increase the amplitude β^2. This problem is of great importance for the understanding of nuclear structure, and one should therefore expect further studies aimed at obtaining a more accurate value of the amplitude.

Single-particle excited states of nuclei near ^{208}Pb. Let us attempt to formulate, within the framework of the presently available information, an answer to the second question: How pure are the known single-particle states near lead? As already mentioned, only one strong line is observed in the transfer reaction for each of the 24 known single-particle states, thus indicating a small contribution made to the single-particle state by the admixture of more complicated configurations. Recent investigations show that the main source of the admixture to the wave function of the single-particle states is the collective states of the core.

The simplest model of interaction between single-particle and collective states was proposed by de Shalit.[6] By way of an example called upon to explain many experimental facts, he considered the excited state of ^{203}Tl. The nucleus ^{202}Hg has a collective state 2^+ with energy 440 keV. In ^{203}Tl there are observed three levels: $1/2, 3/2^+, 5/2^+$. The lifetimes of the states 3/2 and 5/2 confirm the hypothesis that they are the result of the interaction of an $s_{1/2}$ proton with the excited 2^+ state of the ^{202}Hg core. Attention should be called to the fact that the energy of one of the levels is lower, and that of the other is higher than the energy of the 2^+ state, but the center of gravity of the doublet has an energy 479 keV, i.e., it is close to the energy of the collective state.

At the 1967 Tokyo conference on nuclear structure, Mottelson called attention to the particular importance of investigating the particle−core interaction near ^{208}Pb. The interaction of the 83rd proton in the state $h_{9/2}$ with the known 3^- vibrational state should lead to the appearance of a multiplet with levels $I = 3/2, 5/2, \ldots, 15/2$ and positive parity in ^{209}Bi in the region of the excitation energy of the octupole state. The insufficient resolution of the spectrometers did not make it possible to separate individual close-lying lines of such multiplets. However, progress in the experimental techniques, especially the use of tandem generators and precision magnetic spectrometers, has made it possible to investigate the interaction of particles with a core in the excited state.

Several recent papers report data on the multiplets of ^{207}Pb and ^{209}Bi. The results of Ellegard and Vedelsby,[7] obtained in the ^{208}Pb(^{3}He, d)^{209}Bi reaction at 20.3 MeV, are shown in Fig. 4, which is taken from Mottelson's paper.[3] A magnetic spectrometer was used to detect lines corresponding to excitations of known single-particle states in ^{209}Bi, and a number of low-intensity lines belonging to multiplets of the phonon-particle interaction were additionally identified. One can see in Fig. 4 the lines of the proton single-particle states $1h_{9/2}$, $2f_{7/2}$, $1i_{13/2}$, $2f_{5/2}$, $3p_{3/2}$, and $3p_{1/2}$, and also a line with spin 13/2 at an energy close to 2.6 MeV, a line near 3.5 MeV, and a weakly resolved multiplet with energy between 4 and 5 MeV with one strong line at 4.43 MeV. On the basis of his own measurements, as well as results by others, Stein[1] compiled the level

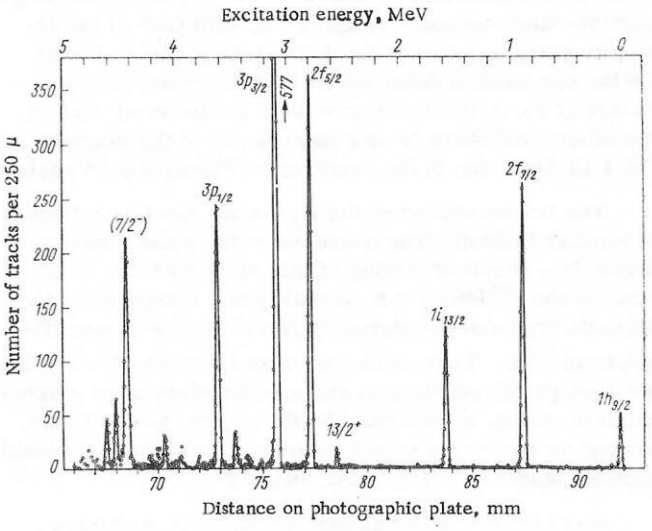

Fig. 4. Spectrum of deuterons from the reaction[7] ^{208}Pb(^{3}He, d)^{209}Bi.

schemes of ^{209}Pb, ^{209}Bi, ^{207}Pb, and ^{207}Tl shown in Figs. 1 and 2.

Comparing the experimental data of Ellegard and Vedelsby with Stein's scheme for ^{209}Bi, we see all of the single-particle states from $h_{9/2}$ to $p_{1/2}$ in normal sequence. However, as seen from the spectroscopic coefficients, some of these states cannot be regarded as pure. The factors for the first five coefficients are close to unity, but the sixth level with energy 3.64 MeV is already a 65% "fragment" of the $3p_{1/2}$ state, while the remaining 35% is at 4.43 MeV.

To explain the weak lines, which we regard to be the lines of a multiplet, we must depart from de Shalit's simple model. In the case of ^{209}Bi, the 83rd proton, the only one outside the closed shell, interacts with the all excited states of the core, and this leads to the following states: $[^{208}\text{Pb}\ (3^-) \times h_{9/2}]$, $[^{208}\text{Pb}(5^-) \times h_{9/2}]$, $[^{208}\text{Pb}\ (2^+) \times h_{9/2}]$, and $[^{208}\text{Pb}\ (4^+) \times h_{9/2}]$. Each of them constitutes a separate multiplet of low intensity with line spacing on the order of several tens of keV. These multiplets are represented by shaded strips in the level schemes (Figs. 1 and 2). For example, the multiplet of the first of the indicated states in ^{209}Bi should be characterized by the spins $3/2^+$, $5/2^+$, ..., $15/2^+$. In ^{209}Bi and ^{209}Pb, the multiplets are quite complicated, this being the consequence of the large spin of the 83rd proton and the 127th neutron. The situation is different in ^{207}Pb and ^{207}Tl, for in this case the multiplets $[^{208}\text{Pb}\ (3^-) \times p_{1/2}^{-1}]$ and $[^{208}\text{Pb}\ (3^-) \times s_{1/2}^{-1}]$ reduce to doublets. Hansen et al.[8] observed doublets in ^{207}Tl, a fact of great significance in determination of the strength of the particle–core interaction.

Let us turn to the question of the purity of individual states and configuration mixing. If the energy of the single-particle state is close to the energy interval occupied by the multiplet, and if the multiplet contains a state with the same spin and parity as the single-particle state, then conditions exist for mixing of the states, so that "fragmentation" of the single-particle state can be expected as a result. The energy of the already mentioned $3p_{1/2}$ state of ^{209}Bi (without interaction with the core) would be very close to the energy of the multiplet $[^{208}\text{Pb}\ (4^+) \times h_{9/2}]$ with components $1/2^-$, $3/2^-$, ..., $17/2^-$. The conditions thus favor a strong mixing of both $1/2^-$ states, so that the single-particle state becomes fragmented, with part of the intensity going to enhance the $1/2^-$ state in the multiplet. On the one hand, a decrease of the spectroscopic coefficient of the 3.54-MeV line to 0.65 is observed, and on the other hand there is an enhancement of the intensity of the 4.43-MeV line in the spectrum of Ellegard and Vedelsby.

The fragmentation of the $i_{13/2}$ state can be explained in similar fashion. The decrease of the spectroscopic factor is a result of mixing of this state with the $13/2^+$ state of the $[^{208}\text{Pb}(3^-) \times h_{9/2}]$ multiplet. Examples of the opposite type are the states $3p_{3/2}$ and $2f_{5/2}$ with negative pairty in ^{209}Bi. They cannot be mixed with the close-lying positive-parity multiplets, and are therefore pure single-particle states, as indicated by the spectroscopic factor. Analogous reasoning explains the fragmentation of the single-particle states in ^{209}Pb, ^{207}Pb, and ^{207}Tl.

Particle – vibration coupling scheme. The qualitative explanation offered above for the mixing of single-particle and collective states served as the basis for development of a theory capable of quantitatively interpreting the available data. The coupling of the nucleon with the quadrupole oscillations of the nuclear surface was described back in 1954 by Choudhury.[9] The starting point for the determination of the interaction Hamiltonian can be the following remark: The energy of a particle moving in a spherical potential varies when the potential is deformed in the course of vibrational motion. For small deformations, the change of the particle energy is proportional to the deformation itself and to the corresponding multipole moment that characterizes the vibration. On this basis we can express the principal term of the particle – core interaction in the form

$$H = k(r)\,(2\lambda+1)^{1/2}\,(\alpha_\lambda Y_\lambda(p))_0,$$

where k(r) is the form factor of the deformation and can be represented in first-order approximation as: $k(r) = -r[dV_0(r)/dr]$; $V_0(r)$ is the single-particle spherical potential; α_λ is the deformation parameter; $Y_\lambda(p)$ is a spherical harmonic connected with the particle coordinate; $(2\lambda+1)^{1/2}$ is a normalization coefficient. This Hamiltonian has been known for quite a long time, since it had been used successfully to describe inelastic neutron or proton scattering leading to excitations of vibrational states. The matrix element of the particle–vibration interaction is

$$\langle (j_2\lambda)\, j_1 | H | j_1\rangle = \langle j_2 | k(r) | j_1\rangle \times \langle j_2 \| Y_\lambda \| j_1\rangle\,(2j_1+1)^{-1/2}\left(\frac{\hbar\omega}{2C}\right)^{1/2},$$

where j_1 is the state of the particle in the field of the undeformed nucleus, λ is the state of the excited core, and j_2 is the state of the particle in the field of the excited core. The last term is the amplitude of the oscillation and can be determined from the excitation cross section of this state or from the matrix element of the decay of this state, ω determines the oscillation frequency, and C is the core rigidity parameter.

We are interested most in the mixing of single-particle states with multiplet states, for in this case experiment yields direct information that can be compared with theory. The condition for the mixing is the equality of the spins and parities of the single-particle state j_1 and one of the multiplet states I: $j_2 = I$.

In this case we can take into account the admixture of the single-particle state in the multiplet state I:

$$|(j_1\lambda)\, I\rangle \to |(j_1\lambda)\, I\rangle + \varepsilon\,|\, j_2 = I\rangle,$$

where

$$\varepsilon = \frac{\langle (j_1\lambda)\, j_2 | H | j_2\rangle}{E_2 - E_1 - \hbar\omega};$$

E_1 is the energy of the state j_1 interacting with the core, and E_2 is the energy of the single-particle state j_2 that mixes with the appropriate multiplet state. The energy shift produced in the state I by mixing with the state j_2 ($j_2 = I$) is determined by the quantity $\delta E = -\varepsilon^2 (E_2 - E_1 - \hbar\omega)$. The accurately identified and well investigated multiplet in ^{209}Bi is most frequently used as the basis for theoretical discussions. We already know that the only

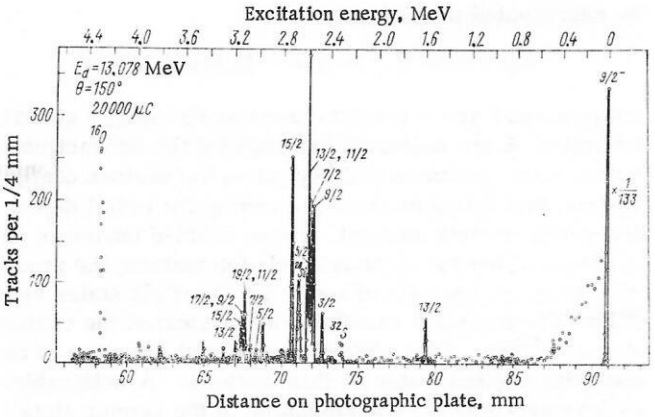

Fig. 5. Spectrum of inelastic scattering of deuterons from the reaction ^{209}Bi (d, d'); the multiplet ($g_{9/2}$, 3$^-$) is distinctly seen.

single-particle state capable of interacting with the ($h_{9/2}$, 3$^-$)multiplet is the state $i_{13/2}$ with energy $E_2 = 1.601$ MeV. The matrix element of the interaction can be obtained from spectroscopic data, namely $B(E3, 3^- \to 0) = 39B_{sp}(E3)$ for ^{208}Pb. This rather well known experimental fact makes it possible in turn to determine the oscillation amplitude: $\hbar\omega/2C = 2 \cdot 10^{-3}$. The radial integral was calculated by Blomquist and Wahlborn,[10] who obtained $\langle i_{13/2}|k(r)|h_{9/2}\rangle = 60$ MeV. Using these data, we can easily obtain the change of the matrix element, namely $\langle(h_{9/2}, 3^-)i_{13/2}|H|i_{13/2}\rangle = 0.22$ MeV. Using the two equations presented above, we obtain the mixing coefficient $\varepsilon^2 = 4.8 \cdot 10^{-2}$ and the energy-level shift $\delta E_{I=13/2} = 48$ keV.

Information on the mixing coefficients ε can be obtained from two other reactions, one-nucleon transfer and inelastic scattering. Comparison of the cross section for excitation of the single-particle state $i_{13/2}$ with the excitation cross section of the 13/2 state of the multiplet has made it possible to determine the magnitude of the mixing. This estimate yields $\varepsilon^2 = 5.4 \cdot 10^{-2}$, which agrees within the limits of errors with the preceding result. The cross section for excitation of the multiplet states has one more feature that is very valuable in analysis of the experimental data. The small distances between the individual lines of the multiplet are evidence that the interaction between the particle and the oscillation is weak. In this case we are justified, with good approximation, in representing the wave function of the state in the form of a product of the proton state j and the core state λ. From this assumption we obtain an equation for the inelastic-scattering cross section:

$$d\sigma[(j\lambda)I] = d\sigma(\lambda)\frac{2I+1}{(2j+1)(2\lambda+1)}.$$

It follows from this relation that the intensity of the multiplet line with spin 15/2 excited in inelastic scattering is four times larger than that of the line with spin 3/2. Figure 5 shows the spectrum of deuterons inelastically scattered by ^{209}Bi, obtained by Diamond et al.[11] We see a complete multiplet ($g_{9/2}$, 3$^-$), in which the line-intensity ratios remain in quantitative agreement with the predictions of the foregoing equation.

The information presented on the particle–oscillation interaction in the nuclei ^{208}Pb ± 1 nucleons is necessary for the understanding of the spectra of the excited levels of nuclei of the type ^{208}Pb ± 2 nucleons.

2. INTERACTION OF TWO NUCLEONS OUTSIDE CLOSED SHELLS

Introduction. Two approaches are noted in papers dealing with nucleon–nucleon interactions in nuclei. The first is the tendency to choose a simple potential capable of describing the available experimental data such as the energies, spins, and parities of the states, as well as the probabilities of the transitions between the states, the nuclear decay probabilities, etc. The second approach reflects the tendency to describe the nucleon–nucleon interactions in nuclei by starting from a potential for free nucleons. This problem is considered to be one of the most basic problems in nuclear physics. In this survey we review papers in which both tendencies are reflected.

Before we proceed to the main topic of this section, we recall briefly the experimental data that serve as basic guidelines for the choice of potentials describing nucleon–nucleon interactions in nuclei.

Figure 6 represents a group of nuclei near ^{208}Pb, which differ from the doubly magic lead in that the number of particles and holes above the closed shells is equal to 2. These eight nuclei are the most convenient for the investigation of the residual interaction. By investigating the structure of their excited levels we can study the interaction of two protons, two neutrons, neutron–neutron holes, and proton–proton holes in closed shells, and also interactions of all particle–hole combinations.

Figure 7 shows the sequence with which the neutron and protons are occupied, and helps predict the main features of the level structures of individual nuclei in this group. We see that the 83rd proton can occupy in succession the levels $1h_{9/2}$, $2f_{7/2}$, $1i_{13/2}$, etc. Then, for example, the structure of ^{210}Po should be characterized by the configurations $(h_{9/2})^2$, $(h_{9/2}, f_{7/2})$, $(h_{9/2}, i_{13/2})$, ..., $(f_{7/2})^2$, $(f_{7/2}, i_{13/2})$, ... Since we know from the level scheme of ^{209}Bi the energies of the single-particle states of the proton, viz., $f_{7/2}$ (0.90 MeV), $i_{13/2}$ (1.56 MeV) ..., we can determine in the zeroth approximation the energy difference between the states of two protons, namely, between $(h_{9/2})^2$ and $(h_{9/2}, f_{7/2})$, and between $(h_{9/2})^2$ and $(h_{9/2}, i_{13/2})$. These are precisely 0.9 and 1.56 MeV. We now need to turn on the interaction between the protons. As a result, the states of the two protons are transformed into multiplets.

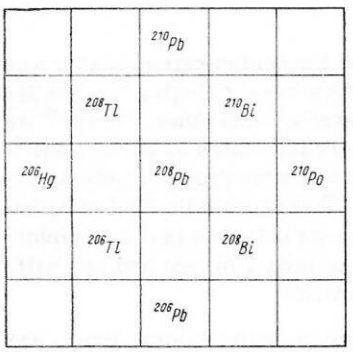

Fig. 6. Group of nuclei near ^{208}Pb that differ from ^{208}Pb in having a total of two particles and holes above the closed shells. This group is symbolically designated ^{208}Pb ± 2.

Protons		Neutrons	
hole states	83rd particle	hole states	127th particle
$3s_{1/2}$	$1h_{9/2}$	$3p_{1/2}$	$2g_{9/2}$
$2d_{3/2}$	$2f_{7/2}$	$2f_{5/2}$	$1i_{11/2}$
$1h_{11/2}$	$1i_{13/2}$	$3p_{3/2}$	$1j_{15/2}$
$2d_{5/2}$	$2f_{5/2}$	$1i_{13/2}$	$3d_{5/2}$
$1g_{7/2}$	$3p_{3/2}$	$2f_{7/2}$	$4s_{1/2}$
	$3p_{1/2}$	$1h_{9/2}$	$2g_{7/2}$
			$3d_{3/2}$

Fig. 7. Sequence in which the neutron and proton levels (particle and hole) are filled near ^{208}Pb.

Two interacting $h_{9/2}$ protons yield the multiplets 0^+, 2^+, 4^+, 6^+, 8^+; two $h_{9/2}$ and $f_{7/2}$ protons yield 1^+, 2^+, . . ., 7^+, 8^+; and the interaction of $h_{9/2}$ and $i_{13/2}$ protons leads to the negative-parity states $2^-, 3^-, \ldots, 10^-, 11^-, \ldots$.

In the next section we shall gather the experimental data on the excited levels of the nuclei of interest to us. The level scheme of ^{210}Po (see Fig. 23) shows distinctly the level groups that form multiplets, namely $(h_{9/2})^2$, $(h_{9/2}, f_{7/2})$, and $(h_{9/2}, i_{13/2})$.

The connection of the character of the interaction of the valence nucleons with the structure of the multiplet, i.e., with the sequence of its individual levels and the distances between them, is the main topic of the present review.

The spectrum of the excited states of two nucleons in the field of an even–even core made up of the remaining nucleons of this nucleus carries information concerning the extent to which the interaction of the free nucleons is modified in the nucleus and, on the other hand, can serve as the key to the understanding of spectra with more complicated configurations.

First phenomenological potentials for nuclei near ^{208}Pb. The first papers in which residual interactions were taken into account within the framework of the shell model were published in the early fifties. Flowers and Edmonds,[12] Pryce,[13] and Talmi[14] published the calculated energies of different configurations of individual nuclei, using very simple two-particle potentials for the residual interaction. It was assumed at that time that a definite state is realized with the aid of a single configuration.

Allowance for configuration mixing was the next step forward, and was made by Inglis,[15] de Shalit and Goldhaber,[16] Elliot and Flowers,[17] and other. Pryce[18] was the first to calculate the excited states of nuclei near ^{208}Pb. Very little was known concerning interactions of nucleons in the nucleus. All that could be said on the basis of nucleon–nucleon scattering data was that the triplet interaction of free nucleons is larger by one and one half times than the singlet interaction.

To simplify the calculations, Pryce had assumed that the radial dependence of the interaction could be represented in the form of a δ function. Neglecting configuration mixing, the proton–neutron interaction energy can be represented in the form

$$E_{np} = A_s \cdot a_s \cdot R + A_t \cdot a_t \cdot R = A_s \cdot R \cdot (a_s + 1.5a_s),$$

where s and t are subscripts denoting the singlet and triplet states, A are constants determining the interaction force, a are constants that depend on the nucleon configuration, and R is a function describing the radial dependence of the matrix element. Pryce avoided laborious calculations of the radial function by determining the product RA_S from an analysis of known two-particle states near ^{208}Pb. The principal calculations[19] concerned the excited states of ^{206}Pb. They yielded the correct sequence of the low-lying excited states of this nucleus. A noticeable achievement was the determination of the isomer state 7^- in ^{206}Pb and of its decay characteristics.

True and Ford[20] showed that inclusion of configuration-mixing effects is a necessary condition for an appreciable improvement of the agreement between calculations and experiment for ^{206}Pb. They took into account in their calculations not only configuration mixing, but also interaction with the quadrupole oscillations of the core, and this led to agreement between the calculated and experimental values of the probability of an E2 transition from the first excited state.

Further improvements obtained by True and Ford, in comparison with the Pryce method, were the use of a finite-radius potential (instead of a δ function) and an exact calculation of the radial integrals. Their results[20] offer evidence of both the correctness of the approach itself and of the correctness of the choice of the parameters for the description of the low-lying levels. True and Ford drew the following conclusions:

a) Calculations of the excited-state energies, using the interaction of the free nucleons and including configuration mixing, yield results for low-lying levels that agree with experiment.

b) The choice of the singlet interaction and of the strength of interaction with the core oscillations is essential.

c) The choice of the particle–oscillation interaction on the basis of B(E2) for ^{206}Pb gives simultaneously the correct value of the probability of the electric quadrupole interaction in ^{207}Pb.

d) Agreement between the calculated and experimental levels is obtained in a wide range of energies for a fixed interaction with the collective motion if the strength of the interaction between two neutrons in the nucleus is assumed to be 75% of the interaction between free neutrons. Similar calculations were successfully performed[3] for ^{205}Pb using the same interaction parameters. It is difficult to apply Pryce's method, and all the more the method of True and Ford, to isotopes with more than three valence nucleons or holes, in view of the large volume of the computations. The problem becomes much simpler, however, if the superfluid-nucleus model is used.

Mention should be made here of the classic paper by Kisslinger and Sorensen,[22] who presented an exhaustive analysis of nuclear structure in the region Ni–Pb on the basis of this model. The residual interaction is broken up into two parts: short-range forces binding pairs of

particles with identical spin in a state with zero spin, and long-range forces that generate collective effects.

The success of the work by Kisslinger and Sorensen lies mainly in the description of the principal features of the nuclear structure in a very wide interval of atomic numbers with the same nuclear-interaction parameters, but the description of the excited states, say of ^{206}Pb, is inferior in accuracy to the results of True and Ford.

The interaction of two odd nucleons in the field of a spherical core was investigated by Kim and Rasmussen.[23] Starting from the properties of the low-lying levels of ^{90}Y, ^{210}Bi, and ^{210}Po, they concluded that tensor forces play an important role in the residual interaction of the proton and neutron.

The excited levels of ^{208}Tl and ^{208}Bi, which have respectively one hole in the proton shell and an extra neutron, and one extra proton and a neutron hole, were analyzed in ref. 24.

The problem of calculating the matrix element with a potential consisting of singlet even and odd parts, a triplet even part, and tensor triplet even and odd parts is quite complicated. The results of the calculations and the experimental data are shown in Fig. 8. We see that good agreement was obtained. Inclusion of the tensor forces is a necesary condition for the explanation of the main features of low-lying levels of odd–odd spherical nuclei (if the remaining components of the potential are to be kept within reasonable limits). It is particularly difficult to obtain the ground state of ^{210}Bi with spin 1 without tensor forces.

Theoretical method of multiplet analysis. To analyze the multiplets of two-particle states, it is necessary to understand the connection between the parameters of the potential and the characteristics of the multiplet, namely the energies of its levels and the sequence of the spins in the multiplet. We describe here briefly the main idea that has led to definite expressions for the matrix elements of different potentials. The complete theoretical formalism is presented in a monograph by de Shalit and Talmi,[25] and in a monograph by Sliv and Kharitonov.[26]

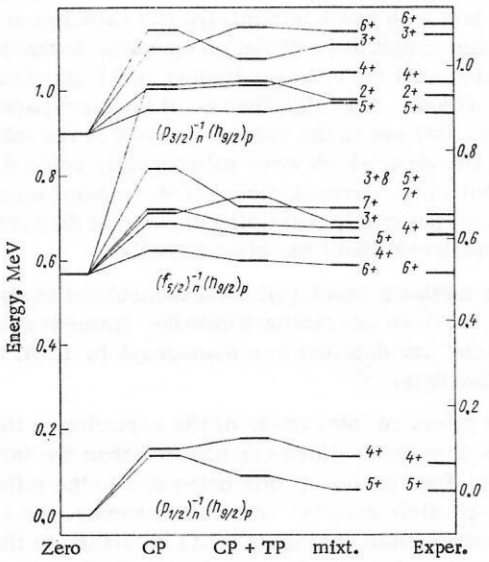

Fig. 8. Comparison of experimental and calculated excited states of ^{208}Bi: CP) central potential; TP) tensor potential.

It must be emphasized that there are also other approaches to the calculation of the state energies. One of the best known is Brown's method, in which the reaction matrices are used.[27]

The theoretical calculations of the energies of a two-particle system in the field of a core are most frequently based on perturbation theory. The depth of the potential well, which is replaced in the shell model by the averaged action of all the nucleons on a given nucleon, amounts to 35-40 MeV for medium Z, while the interaction energy of the two-nucleon system usually does not exceed 1 MeV. Thus, perturbation theory can be used for this problem.

The correction to the energy of a definite state, needed to account for the interaction of a nucleon pair, is obtained by calculating the matrix element with the corresponding potential. It is convenient here to use the Slater expansion, which has the following advantages: It makes it possible to separate the angular and radial variables, it permits, through the use of tensor algebra, an appreciable simplification of the interaction matrix element, and also permits the use of wave functions without specifying their form in detail. The expansion of the interaction with the aid of Legendre polynomials yields

$$V(|\mathbf{r}_1-\mathbf{r}_2|)=\sum_{k=0}^{\infty} v_k(r_1, r_2)\,P_k(\cos\omega_{12}),$$

where ω_{12} is the angle between the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. Expressing the Legendre polynomial in terms of spherical harmonics of the angles of two particles in a polar coordinate system, we change over to tensor operators:

$$V(|\mathbf{r}_1-\mathbf{r}_2|)=\sum_{k,\lambda}\frac{4\pi}{2k+1}v_k(r_1, r_2)\,Y^*_{k,\lambda}(\Omega_1)\,Y_{k,\lambda}(\Omega_2),$$

where $\Omega \equiv (\theta, \varphi)$ (there is additional summation over the index λ, which varies in the range $-k \le \lambda \le k$).

The wave function can be represented as a product of functions that depend separately on the radial and angular variables:

$$\Psi(l_1l_2LM)=(1/r_1r_2)\,R(r_1)\,R(r_2)\cdot\varphi(l_1l_2LM).$$

The angular part $\varphi(l_1l_2\mathrm{LM})$ of the wave function is expressed as a product of spherical harmonics of the angles of the first and second particles. Using the Slater integral

$$F^k=\int|R(r_1)\,R(r_2)|^2\,v_k(r_1, r_2)\,dr_1\,dr_2$$

and applying the Wigner–Eckart theorem we represent the two-particle interaction in the form

$$\langle l_1l_2LM|\,V(|\mathbf{r}_1-\mathbf{r}_2|)\,|l_1l_2LM\rangle=\sum_k f_kF^k.$$

Here f_k now depends only on the angle variables of the first and second nucleons:

$$f_k=(-1)^{l_1+l_2+L}(l_1\|C^{(k)}\|l_1)(l_2\|C^{(k)}\|l_2)\begin{Bmatrix} l_1l_2L \\ l_2l_1L\end{Bmatrix},$$

where

$$C^{(k)}(\Omega) = \sqrt{\frac{4\pi}{2k+1}}\, Y_{k,\lambda}(\Omega).$$

Only even k take part in the summation, since $(l \| C^{(k)} \| l)$ vanished for odd k.

Thus, the matrix element of the interaction of two nucleons contains two factors: f_k, which depends only on quantum numbers and on the spatial configurations of the angular-momentum vector, and F^k, which contains the dependence on the radial variables of the potential. Such a representation of the interaction is convenient when it comes to choosing the parameters of the potentials in order to obtain the experimental value of the energy.

An expression was thus obtained for the nucleon-interaction matrix element, under the assumption that the potential is independent of the spin. This expression can, however, be generalized in rather simple fashion. In the j–j coupling model, F^k remains unchanged, and f_k is defined with the aid of a tensor comprising the product of the spin operator σ with the above-defined quantity $C^{(k)}$, namely: $T_{\rho}^{(1k)r} = [\sigma \times C^{(k)}](\frac{r}{\rho})$. From the point of view of the tensor classification, this is an irreducible tensor operator of rank r, obtained from a tensor of rank 1 (spin vector) and a tensor of rank k (angle variables).

We thus have a new expression

$$f_k' = (-1)^{j_1+j_2+J}(j_1 \| T^{(k)} \| j_1)(j_2 \| T^{(k)} \| j_2) \cdot \begin{Bmatrix} j_1 j_1 J \\ j_2 j_1 k \end{Bmatrix}$$

and accordingly the interaction energy is given by the matrix element

$$\Delta E_J = \langle l_1 l_2 SLJM \,|\, V_{12} \,|\, l_1 l_2 SLJM \rangle = \sum_k f_k' F^k.$$

We call attention to the fact that the parameters of the potential in the expression for the matrix element of the two-particle interaction are chosen on the basis of an analysis of the energies of $2j_2 + 1$ levels of the entire multiplet $(j_1 j_2)$, $j_2 \leq j_1$, since ΔE_J describes all the states of the multiplet. In our expression we have $j_2 + 1$ parameters of the function F^k (for $0 \leq k \leq 2j_2$), which characterizes the dependence of the potential on the radial variables.

The foregoing discussion is only a sketch that demonstrates primarily the logic of the papers in which analysis of multiplets of two-nucleon interaction is used to obtain information on the form of the potential.

Research by the Leningrad scientists. Among the studies devoted to a theoretical analysis of two-particle interaction in nuclei near lead, the work of L. A. Sliv and his co-workers is worthy of special attention. In view of the scientific importance of this work and of the consistency in the choice of research methods, we shall discuss their results separately. The present author, being an experimentalist, does not regard himself sufficiently qualified to comment authoritatively on the results of all the work done by Sliv et al. He will therefore describe only briefly the principal ideas of this work, and focus attention on the connection between the individual components of the nucleon–nucleon interaction and various experimental data. The connection between theory and experiment is a characteristic feature of the work by Sliv et al.

In the first papers, attention was concentrated on effects due to an averaged potential acting on an individual nucleon. The parameters of the central potential V_c (r, s, l) and of the quadrupole component V_s (r, θ, φ) were fitted on the basis of the spectrum of the excited states of the individual nucleon above the closed shell. It is shown in a paper by L. A. Sliv and V. A. Volchok[2] that the parameters of the central potential can be regarded as constant in a wide interval of atomic numbers, from ^{17}O to ^{209}Bi. They have also established that excitation of the core is manifested to a small degree in the energies of the single-particle levels, but significantly influences the transition probability.

This was followed by a study of the two-particle interaction, which naturally plays a more important role when there are two nucleons above the filled shell. In this case the valence nucleons move under the influence of the potential $V = V_c + V_s + V_p$ (1, 2). In the calculation[28] of the states of, say, ^{206}Pb, the two-particle interaction was chosen in the form $V_p = -[v_t \pi_t + v_s \pi_s] f(r)$, where v_t and v_s are the parameters of the triplet and singlet interactions, and π_t and π_s are the corresponding projection operators: $\pi_t = (3 + \sigma_1\sigma_2)/4$; $\pi_s = (1 - \sigma_1\sigma_2)/4$.

The radial dependence of the interaction was chosen most frequently in Gaussian form (the details are of no significance in the integral effect; only the depth and the effective radius of the forces are important). The task of the first studies by Sliv et al. consisted of fitting the parameters v_t and v_s as well as the effective radius of the two-particle interaction ρ to the available experimental data. In ref. 29, published in 1961, the level spectra of the even–even nuclei ^{206}Pb, ^{210}Pb, and ^{210}Po, and also of the odd–odd nuclei ^{206}Tl and ^{210}Bi were used to estimate the principal parameters of the two-particle interaction, namely $\rho = 2$ F and an approximate potential depth 35 MeV. The quadrupole interaction parameters were the phonon energy $\hbar\omega = 3$ MeV and the effective surface tension C = 1000–2000 MeV.

The difficulties that were gradually overcome by the physicists engaged in the problem of residual interaction of nucleons in the nucleus were the lack of exact experimental data with which to compare the calculation results, the limited capabilities of the computation techniques of that period, and the need for finding new theoretical-analysis methods. The significance of the first papers of Sliv et al. lies not in the very derivation of the interaction parameters, which were subsequently refined many times, but in the correct choice of the method used to obtain a total potential, consisting of several different parts, for the nucleons above the closed shells.

The methods developed for a theoretical analysis of nucleon–nucleon interaction within the framework of the shell model are detailed in a monograph by L. A. Sliv and Yu. I. Kharitonov.[26]

The effective interaction of the particles in the nucleus is apparently more complicated than the interaction of the free nucleons, this being due to the influence of many-particle effects. One can, however, start out with an expression analogous in its structure to that for the interaction between free nucleons. In this case the effective interaction consists principally of central, spin–

orbit, tensor, and centrifugal terms:[30]

$$V(1,2) = V_c + (\mathbf{L}\cdot\mathbf{S})\,V_{LS} + S_{12}V_T + L_{12}V_{LL}.$$

It should be borne in mind that V_C, V_{LS}, V_T, and V_{LL} are in the general case different arbitrary functions of r, and depend also on the spin and on the isospin. We recall also that the phenomenological nucleon–nucleon potential is described by a large number of parameters; for example, the Hamada–Johnston potential[31] is determined with the aid of 29 parameters. The inclusion of terms that depend on the velocities of the interacting particles becomes important, since data on the nucleon–nucleon interaction show that their contribution is significant at short distances. The operator L_{12} in the Hamada–Johnston potential can be represented in the form

$$L_{12} = [\delta_{LJ} + (\sigma_1\cdot\sigma_2)]\cdot\mathbf{L}^2 - (\mathbf{L}\cdot\mathbf{S})^2.$$

Sliv et al. take into account only matrix elements of velocity-independent forces, so that the potential energy can be represented in the simplified form

$$V(1,2) = [V_w + V_\sigma(\sigma_1\cdot\sigma_2) + V_T\cdot S_{12}]\,f(r).$$

If effects of tensor forces, which will be shown below to influence only the positions of levels with spins I = 0 and 1, are disregarded in the first-order approximation, then V(1, 2) can be expressed as a linear combination of spin-independent Wigner forces v_0 and singlet forces. Retaining the previous notation, we have $V(1,2) = (v_0 + v_1\pi_S)f(r)$, where $v_0 = v_t$; $v_1 = v_S - v_t$. A study of the behavior of multiplets as a function of the configuration of the two nucleons and of the spin of the two-nucleon state was the principal topic of a paper by L. A. Sliv and Yu. I. Kharitonov.[32] They calculated the dependence of the diagonal matrix element on the spin for Wigner and singlet forces. They used a two-particle interaction with a finite force radius and harmonic-oscillator wave functions, and this enabled them to express the radial part of the matrix element F^k in the form of a linear combination of Talmi integrals, which in turn depend on only one parameter $\lambda = r_0/\rho$, where r_0 is the effective radius of the nuclear forces and $\rho = \sqrt{2\hbar/M\omega}$ is the radius of the oscillator well. An example of such a calculation is shown in Fig. 9. This is a plot of the diagonal matrix element as a function of the spin I of the state, which determines the splitting of the configuration $(1h_{9/2}, 2f_{7/2})$ under the influence of Wigner and singlet forces for $\lambda = 0.45$. The results show that for the singlet forces the course of the curve depends not only on the spin of the two-particle system, but also on the parity of the Nordheim number N ($N = j_1 + j_2 + l_1 + l_2$). The dependence of the matrix elements on l is described, for both the Wigner and singlet forces, by two smooth curves. Consequently, the curves connect separately even and odd values of I.

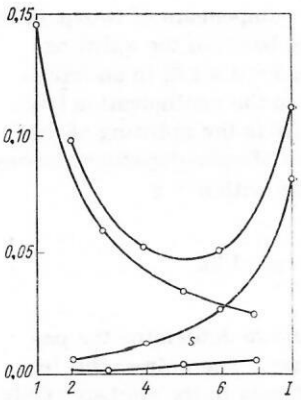

Fig. 9. Dependence of the diagonal matrix element on the spin for Wigner and singlet forces, in the case of the configuration $(1h_{9/2}, 2f_{7/9})$ for $\lambda = 0.45$.

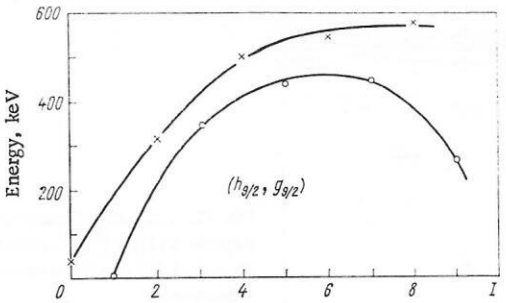

Fig. 10. Splitting of the multiplet of the configuration $(1h_{9/2}, 2g_{9/2})$ in ^{210}Bi.

The arguments advanced above were used to estimate the parameters of the residual forces in ^{210}Po and ^{210}Pb (ref. 33). The best agreement for the ^{210}Po multiplets $(1h_{9/2}, 2f_{7/2})$ and $(1h_{9/2}, 1i_{13/2})$ is obtained when the ratio of the Wigner to the singlet forces is approximately 3:2 and $\lambda = 0.5$. These parameters describe also the multiplet $(1h_{9/2})^2$.

Figure 10 shows by way of example the experimental splitting of the multiplet of the configuration $(1h_{9/2}, 2g_{9/2})$ in ^{210}Bi.

The first attempts of Sliv et al. to explain the spectra of the excited levels of the odd–odd nuclei with the aid of only Wigner and singlet forces encountered great difficulties.[32] The role of the tensor forces in the residual interaction of the nucleons in the nucleus was the subject of separate studies. In one of them[34] they calculated the matrix element of the tensor forces $V_T = S_{12}\cdot V(r)$:

$$\langle j_1 j_2 J \,|\, V_T \,|\, j_1 j_2 J\rangle = \sum_k \Phi(j_1, j_2, k)\,(-1)^J W[j_1 j_2 j_1 j_2; Jk],$$

where $\Phi(j_1, j_2, k)$ is a function that does not depend on the total spin J; the summation index k is determined by the rank of the tensor operator in the expansion of V_T (ref. 25). From the properties of the Racah coefficients, which determine the relation

$$\sum_k (-1)^J (2J+1)\cdot W[j_1 j_2 j_1 j_2; Jk] = 0,$$

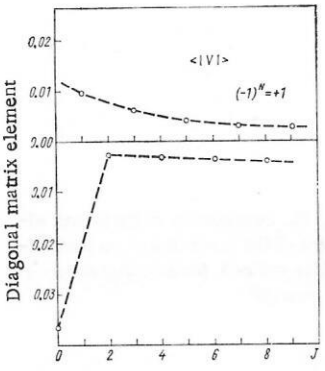

Fig. 11. Dependence of the diagonal matrix element of the tensor forces on the total spin J for even and odd Nordheim numbers.

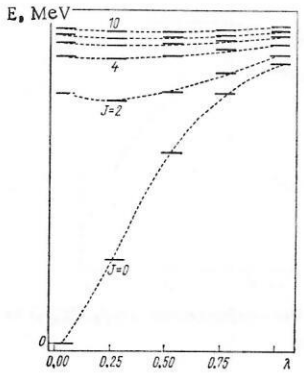

Fig. 12. Splitting of multiplet of configuration $(1i_{11/2})^2$ under the influence of central forces with Gaussian radial dependence.

it follows that the center of gravity ρ of the levels belonging to the configuration split by the tensor forces is given by

$$\rho = \sum_k (2J+1)\langle j_1 j_2 J \mid V_T \mid j_1 j_2 J\rangle / \sum_k (2J+1) = 0.$$

As seen from Fig. 11, the dependence of the diagonal matrix element on the total spin J is of alternating sign. The result depends also on the parity of the Nordheim number N. For $(-1)^N = -1$ we have the situation obtained by rotating Fig. 11 through 180° around the J axis.

The methods developed for calculating the matrix elements of the tensor forces were used to calculate the spectrum of the excited states of ^{210}Bi. It was shown that it is impossible to obtain the correct sequence of levels in the multiplet $(2g_{9/2}, 1h_{9/2})$ ^{210}Bi by using only central forces. Inclusion of the tensor forces raises the $J = 0^-$ level by 200 keV and lowers the $J = 1^-$ level by 50 keV, and this leads to a correct sequence in this multiplet. The splitting of the entire $(2g_{9/2}, 1h_{9/2})$ multiplet amounts to 600 keV.

As already stated, the work by Sliv et al. is characterized by attempts to obtain empirical quantities that would be separately critical to each term of the interaction V(1, 2). An analysis of these possibilities is given in ref. 30.

It is known that the effective radius r_0 of the central forces comes into play in the splitting of a multiplet of the type $(j)^2$ of identical particles; Bang et al.[30] carried out a systematic analysis of the dependence of the splitting on $\lambda = r_0/\rho$ with oscillator wave functions and of the radial dependence of forces of Gaussian type, $\exp(-r^2/r_0^2)$, and of the Yukawa type $r_0/r\,[\exp(-r/r_0)]$. Figure 12 shows

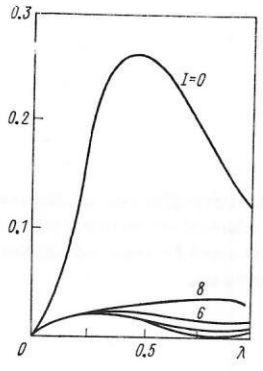

Fig. 13. Dependence of the matrix element of the tensor forces on their effective radius λ for a configuration of the type $(j)^2$.

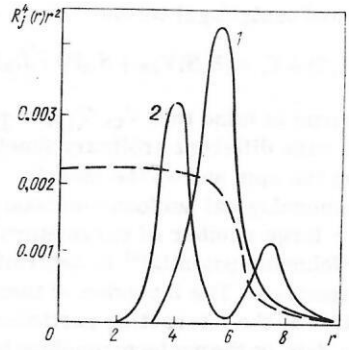

Fig. 14. Dependence of the integrand $R_j^4(r)\,r^2$ on r for the states $1h_{9/2}$ (1) and $2g_{9/2}$ (2). The density distribution for nuclei near ^{208}Pb is also shown.

by way of example the splitting of the configuration $(1i_{11/2})^2$ under the influence of central forces with a Gaussian radial dependence. Since the position of the 0^+ level is most sensitive to the parameter λ, the measure of the splitting was taken to be the quantity $\alpha = (E_0 - E_2)/(E_2 - E_{Jmax})$, where E_J is the energy of the level with spin J. It turned out that the form of the radial dependence of the central forces has in itself little effect on the value of α, but the dependence of α on λ is equally strong for all the considered configurations, so that conclusions can be drawn concerning the effective radius r_0 of the forces. It should be recalled here that other components of the potential can also affect the splitting, and that collective effects must be excluded. In the $(j)^2$ configuration, the character of the splitting is the same for singlet and Wigner forces.[33] Therefore, the two components cannot be separated on the basis of the splitting alone.

Bang et al.[30] investigated also the influence of tensor forces on the splitting of a multiplet. They have shown that the splitting of a two-nucleon configuration under the influence of tensor forces is of alternating sign as a function of the spin (see Fig. 11).

To assess the influence of tensor forces as a function of their effective radius, a number of calculations were performed of the matrix elements of configurations of the type $(j)^2$ and (j_1, j_2) for a potential in Gaussian form and in Yukawa form. The dependence of the matrix element on λ, as seen from Fig. 13, is quite weak for all I, with the exception of I = 0 (ref. 33), and the matrix element vanished identically for λ = 0. This means that all the calculations with δ-function forces exclude a possible manifestation of tensor forces. A very interesting fact is that the matrix element has a maximum precisely at λ = 0.5.

The spin dependence of the components of two-particle forces can be investigated on the basis of the splitting ΔE of each level J into a doublet $I = J \pm 1/2$ in an interaction with the nucleon $J_1 = 1/2$ in the configuration $\{j^n J;\ j_1 = 1/2;\ I\}$, n = 2, 3, 4, ... If Δε is the splitting of the $j \pm 1/2$ levels under the influence of spin-dependent forces, then we have for the configuration with $n \geq 2$

$$\Delta E = \frac{2J+1}{2j+1}\,\Delta\varepsilon\,(j, j_1 = 1/2).$$

The quantity ΔE makes it possible to determine the parameters of spin-dependent forces. The spin–orbit interaction plays a very importnat role in the nucleus. Only

after these forces were taken into account did it become possible to obtain the correct level sequence in the shell model, and in particular to obtain the magic numbers.

The splitting of the spectrum of the excited states under the influence of spin–orbit interaction is described with the aid of the potential

$$V_{ls} = -\xi(\mathbf{l}\cdot\mathbf{s})\frac{1}{r}\cdot\frac{\partial V}{\partial r},$$

where l and s are the orbital and spin angular momenta of the nucleon. The parameter ξ, which determines the intensity of the interaction, is approximately the same for all states of one particle above the filled shells,[2] and for hole states it is approximately 1.5 times larger than for single-particle states. It is much easier to separate the spin–orbit interaction from the total potential and to determine its parameters if data are available on the configuration of interest to us in several neighboring nuclei.

The nonlocality of the residual interaction can be taken into account by introducing a dependence of the forces on the density ρ. The significant role of these forces was established in the theory of nuclear matter.[35] For example, the relation $V_\rho(1, 2) = [1-(\rho/\rho_0)^{2/3}]\cdot V(1,2)$, where $\rho = \rho_0\{1 + \exp[(r - c)/a]\}^{-1}$, leads to an enhancement of the interaction in the outer layer of the nucleus. Such a potential was used[33] to analyze the multiplets of two configurations, namely $(g_{9/2})^2$ in ^{210}Pb and $(h_{2/3})^2$ in ^{210}Po.

A decisive role in the splitting of multiplets is played by the radial integral $\int R_j^4(r)\,\varphi\,[\rho(r)]r^2\,dr$. Figure 14 shows a plot of the function $R_j^4(r)\,r^2$ for the states $2g_{9/2}$ and $1h_{9/2}$, as well as the density distribution, which is similar for both nuclei. A qualitative estimate of these radial integrals explains the difference in the character of the splitting of the configurations $(g_{9/2})^2$ and $(h_{9/2})^2$: The former correspond to a function located in the high-density region, whereas the function of the second configuration lies on the surface of the nucleus.

The latest experimental data indicate the need for including in the potential a term that describes the density dependence. If the latest results of calculations of the residual interaction by the Brueckner–Hartree–Fock method are taken into account, then the dependence of the potential on ρ should be of the type $\rho^{2/3}$. L. A. Sliv[36] has reached the conclusion that if the potential $V(1, 2) = -V(r_{12}) + \alpha\rho(r)^{2/3}\delta(\mathbf{r}_{12})$ is used, the parameters of the nn and pp interaction can remain the same for all the nuclei near lead.

In conclusion, it can be stated that a possibility exists of determining the total residual interaction by establishing the relation between the physical quantities and individual terms of the effective potential. The appearance of a large amount of experimental data has made it possible to broaden greatly our understanding of the role of any given term of the residual interaction. In particular, it was ascertained that the ratio of the Wigner to the singlet forces is approximately 3:2, i.e., the use of $v_0 = -(27\text{-}30)$ and $v_1 = -(18\text{-}20)$ MeV makes it possible to describe many multiplets of nuclei near lead. In addition, it is becoming clear that the use of contact δ forces, like the use of surface forces, does not yield correct results, and finite-radius forces with a parameter $\lambda = 0.5$ describe the experimental data well. The role of the dependence of the forces on the density has also become clearer recently: The results of calculations in the theory of nuclear matter ($\rho^{2/3}$) yield identical parameters for nuclear nn and pp interactions for all nuclei near lead. The need for including tensor forces to determine the correct multiplets of odd–odd nuclei is perfectly obvious. However, their participation in nn and pp interactions is masked by other parts of the potential.

Allowance for the interaction with phonons leads to a certain modification of the radial integrals. This effect does not influence the structure of the spectrum, but shifts the entire multiplet by a certain amount.

Interaction of free nucleons, and residual interaction in a nucleus. The connection between the interaction of free nucleons and that of nucleons in a nucleus is the subject of an extensive literature. We attempt to describe here the principal trends and achievements in this field. The amount of exact experimental data has greatly increased of late, both on the spectra of excited states of nuclei, which are of interest from the point of view of comparison with theory, and on the interaction of free nucleons,[37] so that a discussion of the relation between the interaction of nucleons that are free and of those situated in a nucleus has become possible.

In shell-model calculations one uses a bounded configuration space. It is precisely this limitation which makes it impossible to obtain correct results in the case when realistic potentials are used. Consequently, within the framework of this model, it is impossible to use the effective interaction, which consists of the free-nucleon interaction plus a correction connected with the bounded character of the configuration space. The best potentials are presently regarded to be the Hamada–Johnston potential[31] and the Yale potential,[38] which are characterized by an infinite repulsive core.

Both potentials adequately describe nucleon–nucleon scattering up to 350 MeV and the main properties of the deuteron, and both potentials go over at large distances into the one-meson-exchange potential. The latter property is particularly valuable, since the principal task of nuclear physics is to describe the interaction of free nucleons with the aid of a mesic potential, and then to describe the physical phenomena in the nucleus with the aid of the nucleon–nucleon interaction potential. The third potential considered for possible applications to nuclear theory is the Tabakin potential,[39] with the so-called soft core, which is a nonlocal potential. All three potentials have been used rather successfully in the theory of nuclear matter, so that there are grounds for assessing their suitability for calculations of the properties of concrete nuclei as well. Kuo and Brown[27] have investigated the Hamada–Johnston potential from this point of view. The method presented by them consists of a transition from a nucleon-interaction potential V to a reaction matrix G.

The calculation technique developed was used to investigate particle–hole multiplets in ^{208}Bi (ref. 40). It was shown that the wave function for most low-lying levels

contains insignificant admixtures of other configurations, and that inclusion of the core polarization does not alter the level energy appreciably. Very good agreement was obtained for the 36 calculated and experimental levels in ^{208}Bi.

The study by Freed and Rhodes[41] belongs to the group of studies in which direct connections were sought between the interactions of free nucleons and interactions of nucleons in a nucleus. Of the three potentials used for calculations in the nucleus, they chose the Tabakin potential, which has a soft core and takes into account the nonlocality of the interaction. They call attention to the fact that agreement with experiment can be obtained for realistic potentials only by including mixing of simple single-particle configurations as well as complicated configurations that account for collective effects.

Polarization of closed shells by valence nucleons may mean that the core is excited by one nucleon and the excitation is removed by another. In this case, the interaction between the valence nucleons is appreciably modified in comparison with the interaction of free nucleons.

The Feynman diagrams of Fig. 15 correspond to different particle-interaction means that lead to renormalization of the potential. Particle interaction via production of a pair (particle and hole) from the core, shown in Fig. 15a, makes the largest contribution to renormalization.

Comparison of the matrix element of such an interaction (more accurately, of its principal term, describing the particle and hole in the state I = 2, T = 0) with an interaction matrix element of the type P_2 (cos θ_{12}), used by Kisslinger and Sorensen,[22] shows that they are very close.[42] It was therefore concluded that the configuration-mixing component connected with the particle–hole excitation from the core can be taken into account by including quadrupole interaction in the potential. The next correction, which can be particularly important for tensor forces, is the consequence of the excitation of valence nucleons in the intermediate state – a mechanism analyzed by Kuo and Brown.[27]

The need for mixing a tremendous number of single-particle configurations makes the calculation complicated and unrealistic. As already mentioned, if the matrix is limited to dimensions that are reasonable from the point of view of the calculations, the omitted configurations must be compensated for by corrections to the potential. Kuo and Brown note that diagrams 15b and 15c make noticeable contributions only to the pairing interaction P_0. The total potential of the interaction of the nucleons in the nucleus can then be written in the form $V = V_T + P_0 + P_2$. Comparison of the calculated and experimental energies of the states 0^+ and 2^+ in ^{210}Po, ^{206}Pb, and ^{210}Pb has made it possible to select the constants of the pairing and quadrupole interactions.

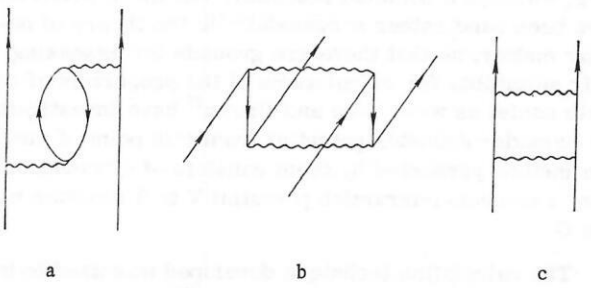

Fig. 15. Feynman diagrams of the main types of interaction of two particles, which lead to renormalization of the potential of free nucleons.

An interesting attempt to select the residual interaction was undertaken by Haderman and Ader.[43] Their aim was to describe the characteristics of excited states with the aid of a "more realistic" potential than, say, the "pairing plus quadrupole" interaction potential, and yet less complicated than the actually realistic nucleon–nucleon interaction potential. They chose a deuterium potential consisting of radial, spin, and tensor parts, with four parameters: The potential depth V_0, the nuclear-force effective radius α, and the spin-and tensor-part intensities a and b. The parameters α, a, and b were chosen on the basis of the low-lying levels of ^{206}Pb, ^{210}Pb, and ^{210}Po.

The best agreement is obtained for an interaction radius 1.72 F, whereas from nucleon–nucleon scattering we have 1.78 F, a = 0.10 (deuterium theory yields 0.13), and b = 0.77 (deuterium theory yields b = 0.77). With these parameters, the depth of the potential differs by 25% from the depth of the free-nucleon potential.

The cited papers far from exhaust the subject of this section, but illustrate well the principal trend, namely, description of nuclear characteristics with the aid of the nucleon–nucleon interaction requires that all possible configurations be taken into account. If the configuration mixing is restricted to manageable size, then an appreciable alteration of the potential is necessary.

3. EXPERIMENTAL DATA FOR NUCLEI OF THE TYPE ^{208}Pb ± 2 NUCLEONS

The amount of data on the excited states of nuclei near lead has greatly increased of late. This is due to the appearance of new experimental possibilities, as well as to

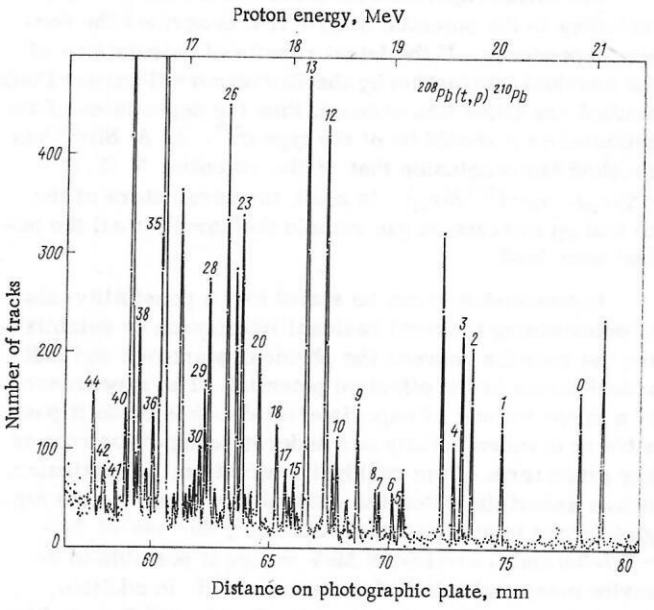

Fig. 16. Spectrum of protons from the reaction ^{208}Pb(t, p), as obtained by Flynn et al.[44]

the increased interest in these nuclei. The principal role was played here by tandem-type accelerators equipped with high-resolution magnetic spectrometers or with spectrometers with semiconductor detectors. Reactions in which one nucleon is transferred, and sometimes two, at particle energies 10-20 MeV, uncover tremendous possibilities for the identification of states of the very simplest configurations of the single-particle type at excitations not attainable in classical spectroscopy.

The spectra of the protons from ^{208}Pb (t, p) reactions, obtained by Flynn et al.,[44] can serve as an example of such measurements. Figure 16 shows clearly the resolved spectrum of the protons from this reaction. For most lines, measurements were made also of the angular distributions, and this helped to determine the spins and parities of the ^{210}Pb states. The level schemes presented in this section are simply working materials gathered in the last few years and containing data that are useful in the analysis of two-nucleon multiplets.

We discuss below the experimental material concerning nuclei of the type ^{208}Pb ± 2 nucleons, shown in Fig. 6.

^{206}Hg. The data on this isotope are very skimpy.[44]

^{206}Tl is a short-lived isotope ($T_{1/2}$ = 4.3 min) with 81 protons and 125 neutrons, i.e., from the point of view of the shell model, an isotope having one hole each in the proton and neutron shells.

The excited levels of this nucleus were investigated by L. I. Rusinov et al.,[46] by Spejewski,[47], and by Walen and Bastin-Scoffier[48] on the basis of the α decay of ^{210m}Bi. To analyze the ^{206}Tl level scheme, Wolf[49] used also the β decay of ^{206}Hg, obtained in the reaction ^{208}Pb(p, 3p) with high-energy protons.

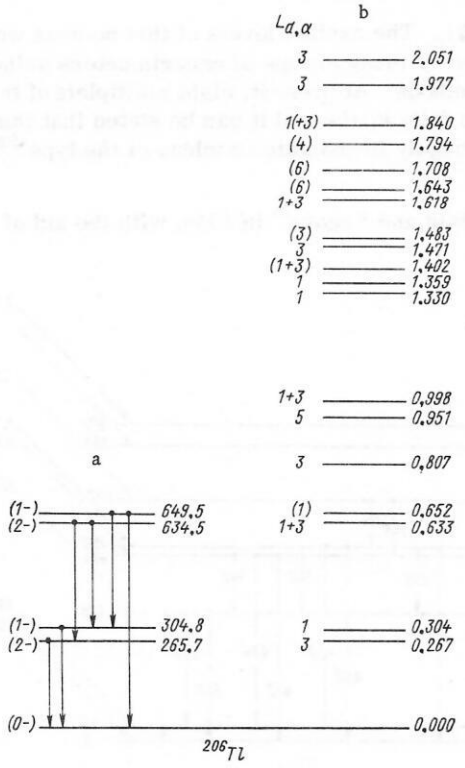

Fig. 17. Excited-level scheme of ^{206}Tl: a) levels identified in decay; b) levels from the two-nucleon-transfer reaction.

Some investigations of ^{206}Tl were made with the aid of nuclear reactions. Erskine[50] and Mukherjee[51] used the one-nucleon-transfer reaction ^{205}Tl(d, p) ^{206}Tl to investigate the single-particle state; Lewis and Daehnik[52] used the reaction ^{208}Pb(d, α) ^{206}Tl.

The first four excited levels of ^{206}Tl are populated in α decay. The most accurate measurements, those of Spejewski,[47] have made it possible to determine their energies within a fraction of a keV, and also their spins and parities. In addition, transitions between levels were observed (Fig. 17).

The β decay of ^{206}Hg proceeds to the ground state of ^{206}Tl and to states with energies 305 and 650 keV. An analysis of the β-decay probabilities and of the multipolarity of the γ transitions between these levels confirms the assumption that the ground and two excited states are characterized by spins and parities 0^-, 1^-, and 1^-.

It can this be assumed that the low-energy part of the level scheme, the part connected with the α decay of ^{210}Bi and β decay of ^{206}Hg, has been established with sufficient accuracy. The lifetime of the 305-keV state has also been measured. The result obtained by Wolf[49] ($T_{1/2} \leq 8 \cdot 10^{-11}$ sec) is of very great significance to the interpretation of the level scheme, since it allows us to assume that this is a state of the doublet ($s^{-1}_{1/2}$, $p^{-1}_{1/2}$) I = 0.1. The first calculations of the splitting of this doublet, performed on the basis of the shell model by Yu. I. Kharitonov et al.[29] and also by Kim (as cited by Erskine[50]), yielded ≤ 10 and 60 keV, respectively, and they suggest that a level 1^- that has not been resolved in the measurements exists near the ground state. An earlier measurement of the lifetime of the 305 keV level, which yields too high a result,[46] also favors this hypothesis.

The higher excited levels were identified with the aid of nuclear reactions. Erskine,[50] using the one-nucleon-transfer reaction ^{205}Tl(d, p) ^{206}Tl, observed the first excited states, and also a state with energy 802 keV, which he identified as 1^-.

The two-nucleon-transfer reaction ^{208}Pb(d, α) ^{206}Tl has made it possible to investigate highly excited states of a neutron- and proton-hole configuration.[52] The results of that study are shown in Fig. 17b.

^{206}Pb. The first data on the structure of the excited states of this nucleon were published by Altburger and Pryce.[54]

The γ spectra, the conversion-electron spectra, and also the eγ-coincidence spectra from the ^{206}Bi decay via electron capture have made it possible to determine the first level scheme of ^{206}Pb, which was supplemented in 1958 with the measurements of True and Ford[20] and the data of a Swedish group.[55]

Data on ^{206}Pb states excited in nuclear reactions were published in the sixties. Vallois et al.[56] determined the orbital angular momentum and the spin of a number of levels on the basis of the angular distributions of inelastic proton scattering.

Tickle et al.[57] determined, on the basis of data from

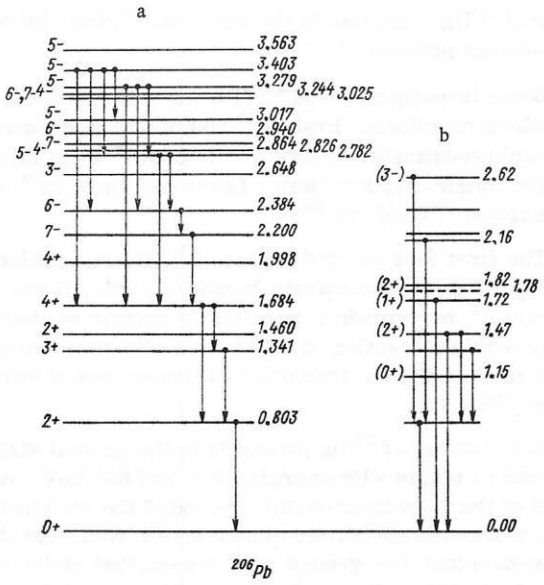

Fig. 18. Excited-level scheme of ^{206}Pb, constructed on the basis of an investigation of the decay of ^{206}Bi (a) and with the aid of data from nuclear reactions (b).

the reaction ^{207}Pb (d, t), the amplitudes of individual single-particle configurations in the wave function of several states, and finally Smith et al.[58] investigated the states of two neutron holes with the aid of the transfer reaction ^{208}Pb(p, t) ^{206}Pb.

The studies of Manthuruthil et al.[59] concerned the decay of ^{206}Bi into ^{206}Pb. Owing to the large intensity of the source obtained in the Pb(p, xn) reaction, and also owing to the high accuracy with which the γ spectrum was measured with a Ge(Li) spectrometer, they were able to identify, in addition to the already known transitions, a number of new γ transitions. The level scheme shown in Fig. 18 is based on the results of this group.

The scheme of Fig. 18a shows only the γ transitions of high intensity. Figure 18b shows a fragment of the level scheme constructed with the aid of data from nuclear reactions.

Taking into account the sequence in which the shells with from 82 to 126 neutrons are filled, we can predict the following configurations for ^{206}Pb: The ground state can be characterized by an empty neutron shell $3p_{1/2}$ and completely filled shells $2f_{5/2}$, $3p_{3/2}$, $1i_{13/2}$, $2f_{7/2}$, and $1h_{9/2}$. In this case, the spin of the ground state can be only 0^+, in agreement with the experimental data.

The pair breaking in the $2f_{5/2}$ shell and the transfer of one neutron to the $3p_{1/2}$ shell lead to a hole configuration $(p_{1/2}, f_{5/2})$ with spins 2^+ and 3^+, and the shift of a neutron pair to the $3p_{1/2}$ shell gives the configuration $(f^{-5}_{5/2})$ with spins 0^+ 2^+, and 4^+. The transfer of one neutron from $2f_{5/2}$ to the $3p_{1/2}$ shell, and of another to $2g_{9/2}$, makes it possible to form the negative-parity 4^- and 5^- states. It is possible to evaluate in this manner also other configurations, possible spins and parities of states, and their energies in the zeroth approximation.

The predicted states correspond in final analysis to the experimentally identified states, with the possible exception of the 0^+ state, the existence of which, as follows from the scheme, cannot be regarded as firmly established.

The discussion of the possible excited states in ^{206}Pb leads to one more conclusion, that at low energies one cannot expect the existence of multiplets with large numbers of levels. The first high-energy multiplet with six levels $2^-, 3^-, \ldots, 7^-$ could be constructed within the framework of the configuration $\{(f^{3}_{5/2})_{5/2}\}$. It is likely that the 7^- state with energy 2200 keV and lifetime 0.13 msec is a member of this multiplet.

Attention should be called to the fact that the first excited state 2^+, the aforementioned state 7^-, and also the 2648-keV 3^- state exhibit strong collective properties.

^{208}Tl is also known as ThC", is a β-unstable isotope, and decays to the states 5^-, 4^-, and 3^- of ^{208}Pb. The excited levels of ^{208}Tl were observed by α decay of ^{212}Bi. By now, eight α lines have been identified and their energies and intensities have been determined. The $\alpha\gamma$ coincidences and correlations were measured by Horton and Sherr[60] and also by Horton.[61]

The configuration $(s_{1/2}, g_{9/2})$, which admits of the possibility of the spins 5^+ and 4^+, is expected for the ground state of ^{208}Tl on the basis of the shell model. An analysis of the β decay (intense decays to the states 5^- and 4^- and weak ones to 3^-) has led Weale[62] to the assumption that the spin of the ground state should be 5^+, and that of the first-excited state 4^+. An M1 multipolarity was established for the transition from the 40-keV level by Graham and Bell.[63] The next four excited levels with spins 5^+, 4^+, 3^+, and 6^+ should be the members of the multiplet $(d_{3/2}, g_{9/2})$.

The previously established level spins and parities were confirmed by Cobb,[64] who performed accurate measurements of the $\alpha\gamma$ correlations. The level scheme of ^{208}Tl is shown in Fig. 19.

^{208}Bi. The excited levels of this nucleus were investigated by many groups of experimenters using different methods. At present, eight multiplets of this nucleus are fully known and it can be stated that this is the most throughly investigated nucleus of the type ^{208}Pb ± 2 nucleons.

Duffield and Vegors[65] in 1958, with the aid of the (d, p)

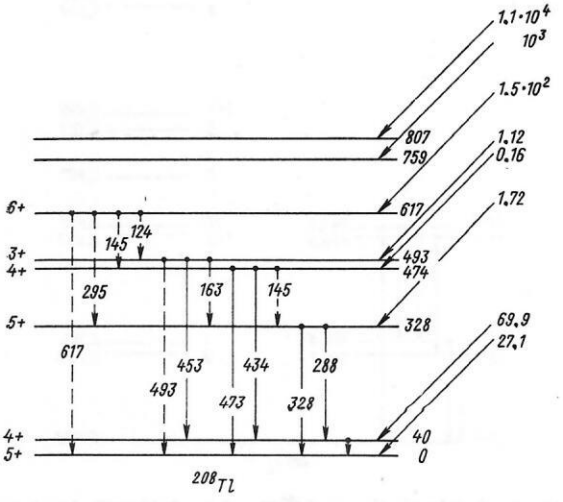

Fig. 19. Excited levels of ^{208}Tl, obtained from α decay of ^{212}Bi.

reaction, identified five excited states and established the existence of an isomeric state with energy 1.43 MeV and lifetime 2.7 msec. Measurement of the γ spectrum has made it possible to obtain the first estimates of spin of definite states. They have also investigated the decay of the 10^- isomeric state. From the coincidences of the 921- and 501-keV γ transitions and their multipolarities, they determined the decay to the ground state as $10^- \rightarrow 7^+ \rightarrow 5^+$.

The reaction ^{209}Bi (d, t) ^{208}Bi was used for analysis of the excited states of the final nucleus by Mukherjee and Cohen,[66] and also by Erskine,[67] who used a high-resolution magnetic spectrometer to identify several excited levels. Measurement of the differential cross section of this reaction has made it possible to determine the spins of the states. It was also possible to separate levels belonging to the multiplets $(h_{9/2}, p_{3/2}^{-1})$ and $(h_{9/2}, f_{5/2}^{-1})$.

Excited states of ^{208}Bi are populated also in α decay. Jones[68] investigated the α spectrum of the decay of ^{212}At, obtained in the reaction ^{209}Bi(α, n)^{212}At. He identified the spins and parities of the ground and first excited states as 4^+ and 5^+. This agrees with the assumption that this is a state of the doublet $(h_{9/2}, p_{1/2}^{-1})$. The same reaction was recently investigated by Reeder,[69] using a cyclotron to accelerate α particles to 23 MeV. The lifetimes of the ground state of At and of the metastable state, which also decays via α decay, are short (0.305 and 0.12 sec, respectively), so that it was possible to measure the α spectrum between the cyclotron pulses. Reeder has observed 20 α transitions and correlated the α-line energies with the energies of the excited states of ^{208}Bi. The decay of ^{208}Po by electron capture also makes it possible to investigate the excited states of the nucleus of interest to us.

Hagee et al.[70] have shown that the levels populated in the decay are 3^+ with energy 1074 keV (3.6%), 2^+ with energy 925.6 keV (77.4%), and 3^+ with energy 634.4 keV (19%). The γ spectrum of the decay of these states to the ground state of ^{208}Bi, measured with high accuracy by a

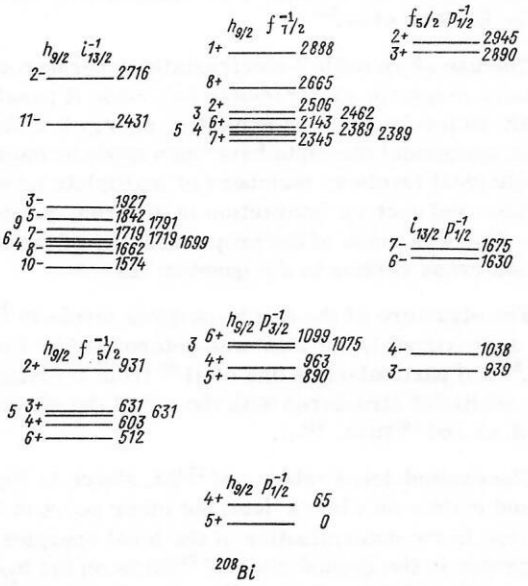

Fig. 20. Excited-level scheme of ^{208}Bi with breakup into individual multiplets, in accordance with the data of Alford et al.[71]

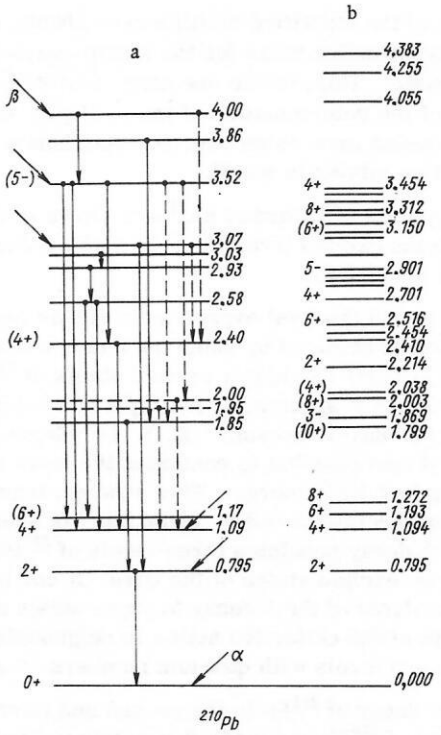

Fig. 21. Excited-level scheme of ^{210}Pb: a) levels obtained from an analysis of the γ spectra (β decay of ^{210}Tl); b) levels established by Flynn et al. in the reaction ^{208}Pb(t, p).

germanium detector, and also the spectrum of the coincidences between several γ lines, have made it possible to obtain more accurate values for the energies of the low-lying levels and for their spins.

The largest amount of data on the structure of excited states of ^{208}Bi was obtained by Alford, Schiffer, and Schwarz.[71] They performed a number of measurements, using a tandem generator and a magnetic spectrometer with high resolution. Four nuclear reactions in which the final nucleus was ^{208}Bi were investigated, namely: ^{207}Pb(^{3}He, d), ^{207}Pb(α, t), ^{209}Bi(d, t), and ^{209}Bi(^{3}He, α). The spectra of the particles emitted in the reactions were measured at 10 angles, and the resultant angular distributions served as a basis for determination of the transferred angular momentum and of the spectroscopic factors.

The reaction (^{3}He, α) has an appreciable cross section, compared with (d, t), for the excitation of states with transfer of large l; this has helped identify high-spin states, for example those belonging to the multiplet $(h_{9/2}, i_{13/2})$.

An analysis of all the experimental data has made it possible to establish a very important fact concerning the purity of individual states, namely, that the configuration mixing in the case of ^{208}Bi is small, even at high excitation energies.

For pure particle–hole states. the cross section of the one-nucleon transfer reaction is proportional to the statistical factor $2J + 1$ (J is the spin of the final state). In the case of bismuth we can therefore use the value of the cross section to identify the spins of the levels of individual multiplets. Certain multiplets were analyzed also from the point of view of the strength function. It turned out that the spectroscopic strength summed over

the states of the identified multiplets is always very close to the corresponding value for the single-particle or single-hole state. This, on the one hand, confirms the correctness of the determination of the multiplet and, on the other, indicates once more that the contribution of the configuration mixing is small.

The results of Alford et al.[71] are shown in Fig. 20. These are the excited levels of ^{208}Bi with breakup into individual multiplets.

^{210}Pb. In the first experiments of this isotope, its levels were determined by using the β decay of the 1.3-minute ^{210}Tl (RaC") to highly excited states of ^{210}Pb. On the basis of the γ spectra, both simple and coincidence, published by Mayer-Kuckuk,[72] B. S. Dzhelepov,[73] and Stetter,[74] it was possible to construct the level scheme of ^{210}Pb up to 4 MeV energy. This scheme, together with the corrections introduced by Weinzirl,[75] is shown in Fig. 21a. The β decay populates three levels of ^{210}Pb, which appear to be excited states of the core. It can be deduced from an analysis of the β decay to these states and from an analysis of the collective states in neighboring nuclei that these are levels with quantum numbers 5^- and 6^-.

The α decay of ^{214}Po to the ground and first two excited states of ^{210}Pb has made it possible to identify the spin and parities of these states as 0^+, 2^+, and 4^+, and gave grounds for identifying the multiplet $(2g_{9/2}^2)$. Extensive information on the levels in ^{210}Pb up to an excitation 5 MeV was obtained by using tandem generators.

Bjergaard et al.[76] have determined with the aid of (t, p) reactions the cross section for the transfer of a pair of neutrons to individual levels of ^{210}Pb (a number of new levels were observed at the same time).

Ellegard et al.[77] used inelastic (p, p') and (t, t') scattering for the study of the part played by collective degrees of freedom in the spectra of excited states. They obtained a pair of 3^- states at 1.85 and 2.81 MeV. Since the excitation cross sections of these states amount to 2/3 and 1/3 of the excitation cross section of the 3^- state in ^{208}Pb (2.62 MeV), there are grounds for stating that a strong mixing of the collective state 3^- with the state $(g_{9/2}, j_{15/2})_3$- takes place.

Recently, Flynn et al.[78] repeated the measurements of the transfer cross sections to the states of ^{210}Pb in the ^{208}Pb (t, p) reaction at 20 MeV. They identified 45 excited states (many of which were known from earlier studies). The proton angular distributions could be measured for most transitions and conclusions could be drawn concerning the spins and parities of a number of excited levels. The excited levels determined by Flynn et al.[78] are shown in Fig. 21b (several levels with energies higher than 4.383 MeV are omitted from the figure). In spite of the extensive experimental material indicated in Fig. 21, it is difficult to establish complete correspondence between the identified levels and the members of individual multiplets of the possible two-neutron configurations. The ground state and the first four excited states undoubtedly form a multiplet $(2g_{9/2}^2)$ with the states 0^+, 2^+, 4^+, 6^+, and 8^+.

On the basis of the data on the excited states of the 127th neutron in ^{209}Pb (see Fig. 7), we can predict the presence of other multiplets of ^{210}Pb. Thus, levels of the multiplets $(1i_{11/2}^2)$, $(1j_{15/2}^2)$, $(3d_{5/2}^2)$, $(2g_{9/2}, 1i_{11/2})$ are to be expected, including three 0^+ levels, as well as a number of other levels not identified in the scheme. What is particularly strange is the absence of 0^+ states among the excited states of ^{210}Pb.

If, for example, we assume for the multiplet $(1i_{11/2}^2)$ the simplest interaction of the δ-force type, then the state 0^+ of this multiplet should appear at an energy near 1.5 MeV, and a calculation with more realistic forces should not shift the 0^+ state above 2 MeV.

This fact points to the need of further experimental investigation of the structure of the ^{210}Pb levels, particularly the levels of higher multiplets of two-nucleon configurations. On the other hand, the lowest group of levels, namely the ground-state multiplet $(2g_{9/2}^2)$, has been well determined in experiment and can already serve as a basis for checking the theoretical calculations and for deducing important results.

^{210}Bi. The unstable isotope ^{210}Bi decays from the ground state with a lifetime 5.01 days, emitting β or α particles. This isotope, known under the name RaE during the time of research on natural radioactivity, is characterized also by α decay from an isomeric state, with a lifetime $2.6 \cdot 10^6$ years.

Measurements by L. I. Rusinov et al.,[79] G. A. Korolev and G. E. Kokharov,[80] and also by Spejewski,[47] have made it possible to determine the fine structure of the α spectrum and have led to a determination of a number of excited levels in ^{206}Tl.

From the point of view of the shell model, ^{210}Bi is a nucleus with one proton and one neutron above the closed magic shells. The first experimental investigations of the structure of this nucleus have shown convincingly that the excited state of ^{210}Bi can be described within the framework of the shell model. Harvey[81] and also Holm et al.[82] have observed in the reaction ^{209}Bi(d, p)^{210}Bi a proton spectrum in which each of the lines could be related to a definite single-particle state in which a neutron was picked up in the transfer reaction. This reaction was investigated also by Erskine el al.[83]

The use of an 8-MeV electrostatic generator and a precision magnetic spectrometer has made it possible to identify 40 levels (up to an excitation energy 3.2 MeV). Rather successful attempts have been made to interpret the individual levels as members of multiplets produced by proton and neutron interaction in different quantum systems. Use was made of the proportionality of the transfer-reaction cross section to the quantity $(2J + 1)$.

The structure of the single-particle levels in ^{210}Bi up to approximately 3.2 MeV was determined by Kolata,[84] Motz,[85] and particularly Cline et al.[82] from investigations of the multiplet structures with the aid of the reactions ^{209}Bi(d, p) and ^{209}Bi(α, ^{3}He).

The excited-level scheme of ^{210}Bi, shown in Fig. 22, is based mainly on Cline's data, but other sources were also used in the determination of the level energies. The 83rd proton in the ground state of ^{209}Bi is on the $h_{9/2}$ orbital, and the 127th neutron in the ground state of ^{209}Pb is on the $g_{9/2}$ orbitral. We can therefore expect the low-

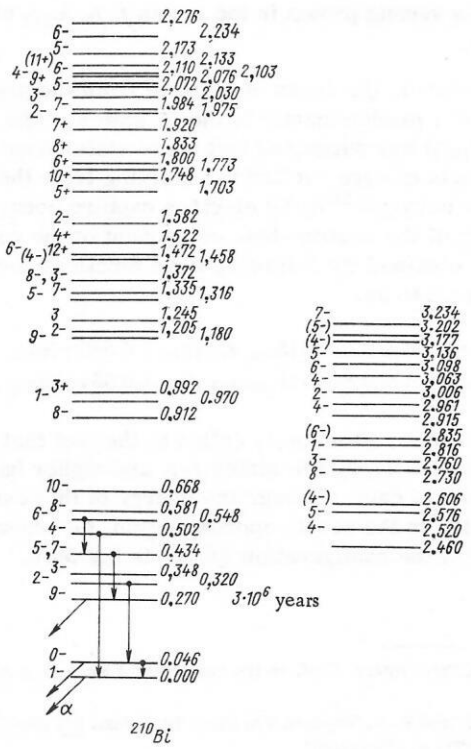

Fig. 22. Excited levels of ^{210}Bi, obtained by Cline et al.[86] with the aid of the reactions ^{209}Bi(d, p) and ^{209}Bi(α, ^{3}He).

lying levels in ^{210}Bi to be the members of the multiplet $(h_{9/2}, g_{9/2})$ with spins $0^-, 1^-, \ldots, 9^-$.

If the energies of the first nine low-lying levels are plotted as functions of the spins (separately for even and odd spins), then two smooth curves typical of particle–particle multiplets[87] are obtained (see Fig. 10). This confirms the hypothesis that the levels belong to one multiplet.

The 270-keV state 9^- of this multiplet is an isomeric state with a lifetime $2.6 \cdot 10^6$ years that decays with emission of an α particle. As seen from the level scheme (Fig. 22) and from the dependence of the energies of the multiplet states on the spin (see Fig. 10), all the high-spin states of the $(h_{9/2}, g_{9/2})$ multiplet have energies higher than the state 9^-, and this is the cause of the higher stability of this multiplet.

At energies 1 MeV and higher, there exist multiplets resulting from the interaction of the $h_{9/2}$ proton with a neutron in the excited states $(h_{9/2}, i_{11/2})$, $(h_{9/2}, j_{15/2}) \ldots$, and also multiplets in which the proton is on higher orbits: $(f_{7/2}, g_{9/2})$, $(i_{13/2}, g_{9/2}) \ldots$.

At the present time, even the lowest-lying multiplets have not yet been fully identified, owing to the lack of complete experimental data (and particularly to the ambiguity in the determination of the spins and parities of the levels at high level density), and to configuration mixing. For example, the multiplet pairs $(h_{9/2}, d_{5/2})$ and $(f_{7/2}, i_{11/2})$, $(h_{9/2}, d_{5/2})$ and $(h_{9/2}, s_{1/2})$, and $(h_{9/2}, g_{7/2})$ and $(h_{9/2}, d_{3/2})$ have levels that are close in energy and have identical spin and parity, leading to configuration mixing and consequently to appreciable shifts of their energies. This effect complicates greatly the identification of levels of individual multiplets.

^{210}Po, which has two protons above the closed shell and a completely closed neutron shell, is an α active isotope ($T_{1/2}$ = 138.4 d), known also as the link radium-F (RaF) in the radioactive uranium–radium (4n + 2) chain. The excited levels of ^{210}Po were investigated on the basis of the decay of ^{210}At by electron capture or with the aid of nuclear reactions. Hollander, Perlman, and Seaborg[88] recorded in 1953 the γ lines 0.25, 1.15, and 1.49 MeV connected with the decay of ^{210}At. The results of more exhaustive investigations carried out with a γ spectrometer and also with a conversion-electron spectrometer were reported in 1954 by Mihelich et al.[89] An analysis of nine γ transitions has led to the conclusion that five excited states exist. Similar investigations spanning several years were carried out by Hoff and Hollander.[90]

The use of a strong ^{210}At source and of the $\gamma\gamma$ coincidence technique has made it possible to confirm the data of Mihelich, and also to observe several new levels. Attempts were made to describe the ^{210}Po levels theoretically, using Pryce's method.[13]

Schima et al.[91] made further contributions to the understanding of the structure of ^{210}Po. Measurements of the $\gamma\gamma$ coincidences and of the angular correlations of the γ transitions has helped refine the level scheme. Measurement of the energy difference between the ground states of ^{210}At and ^{210}Po has made it possible to determine log ft of transitions to three levels with energy $\approx$ 3 MeV. The statement that these are allowed transitions ($\log ft$ = 5.9, 6.4, and 5.9) is of importance for the determination of the character of the excited states populated in the decay. Next, much attention was paid in ref. 91 to the first excited states (2^+, 4^+, 6^+), which we now know to make up the multiplet of configuration $(h_{9/2})^2$ together with the 0^+ ground state and the then not yet identified 8^+ state.

The lifetimes of the 6^+ and 4^+ states were measured ($T_{1/2}$ = 38.4 ± 5.1 and 1.8 ± 0.1 nsec) and the probabilities of the transitions between the states 6^+ and 4^+ and between 4^+ and 2^+ were calculated with the aid of the wave functions of Newby and Konopinski.[92] Comparison of these results with the experimentally established transition intensities has pointed to considerable deviations from the predictions of the single-particle model.

An attempt at further refining the decay scheme of ^{210}At was made by Hollander and Prussin.[93] They measured, with high accuracy, the spectra of the γ quanta and the spectra of the conversion electrons. They identified 50 transitions and established the multipolarity for most of them on the basis of the K-conversion coefficient. The proposed scheme includes 10 levels, and five more are regarded by the authors as probable. The results of their investigations were compared by Prussin and Hollander with the calculations of Kim and Rasmussen.[23]

As already mentioned, a number of investigations of the excited levels of ^{210}Po by the method of nuclear reactions have been reported.

Yamazaki and Ewan[94] measured the γ spectrum of the transition of the final nucleus excited in the reaction ^{208}Pb(α, 2n)^{210}Po.

Bergstrom et al.[95] also used the (α, 2n) reaction to study the levels of ^{210}Po. Tickle and Bardwick[96] used the pickup reaction ^{209}Bi(^{4}He, t)^{210}Po at an α-particle energy

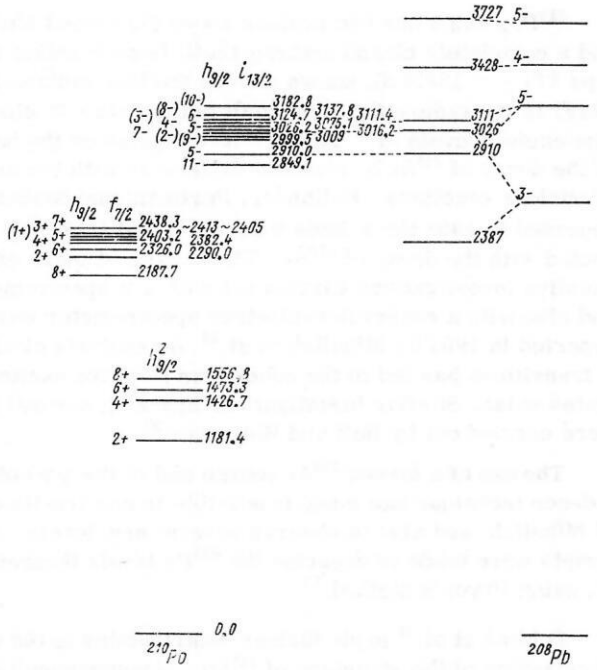

Fig. 23. Excited levels of ^{210}Po with subdivision into multiplets, in accordance with the data of Jardin et al.[97]

45 MeV. They measured the proton spectrum with very good resolution (15 keV), and this made it possible to separate the individual levels of the multiplets $(h_{9/2})^2$, $(h_{9/2}, f_{7/2})$, $(h_{9/2}, i_{13/2})$, and $(h_{9/2}, f_{5/2})$. They observed in the spectrum 21 lines, which they related to corresponding excited states.

The spins of these states were identified by measuring the nucleon-transfer cross section. As is well known, in these reactions if we are dealing with pure states, the multiplet line intensity is proportional to $2J + 1$.

Jardin et al.,[97] using the results of the measurements of the γ spectra, the γ-coincidence spectra, and the conversion-electron spectra for the decay of ^{210}At, analyzed the ^{210}Po scheme anew. They defined a number of data and compiled a scheme containing 23 levels (of which six are doubtful). The scheme shown in Fig. 23 is a result of the reduction, by the authors, of their own data and data by other groups.

The scheme indicates individual multiplets and the connection between certain states and the collective states of the ^{208}Pb core. [Several of the highest levels (> 3.5 MeV), which are difficult to relate to any particular multiplet because of the uncertainty in the spins, have been left out of the scheme.] The excitations of the core are strongly reflected in the level structure of ^{210}Po.

Thus, the pair of 5^- states in the multiplet $(h_{9/2}, i_{13/2})$, which are "fragments" of the 5^- collective state in ^{208}Pb, were the subjects of many discussions ever since it was established that the main part of the ^{210}At decay populates these levels, namely, 71% and 19.2% of the decays go to the levels 2910 and 3026.8 keV, respectively.

Hoff and Hollander[90] have called attention to the fact that the decay of ^{210}At to some level of identified multiplets should be strongly forbidden, since it means replacement of the $h_{9/2}$ proton by a $p_{1/2}$ neutron (with possible excitation of a second proton in the states $f_{7/2}$, $i_{13/2}$ or higher states).

However, the decay of astatine from the ground state (spins 5^+) predominantly to the 5^- states of the multiplet $(h_{9/2}, i_{13/2})$ has suggested that these states have a strong component of core excitation. Starting from the premise that the decay of ^{210}At by electron capture occurs at the expense of the neutron-hole component of the core, Jardin et al.[97] obtained the following wave functions for the states of interest to us:

$$\Psi_{5^-}(2910) \approx 0.534/\pi\,(h_{9/2}, i_{13/2}))_{5^-} + 0.846/\nu\,(g_{9/2}, p^{-1}_{1/2}))_{5^-};$$
$$\Psi_{5^-}(3026) \approx 0.846/\pi\,(h_{9/2}, i_{13/2}))_{5^-} + 0.534/\nu\,(g_{9/2}, p^{-1}_{1/2}))_{5^-}.$$

In conclusion, attention is called to the fact that multiplets with protons excited to states $f_{7/2}$ and higher have not been identified to date, although the energy of this excitation is small. In the zeroth approximation, for example, the energy of the configuration $(f_{7/2})^2$ is 3.2 MeV.

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