Quasielastic electron scattering and nuclear structure

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Inelastic electron–nucleus collisions are considered on the basis of many-body theory without using any model approximations. The general properties of the average nuclear field for quasiparticles are considered. Owing to correlative interactions, this field turns out to be complex and energy-dependent. The character of the spectrum of the residual nucleus in (e, e'p) reactions is explained qualitatively and a phenomenological approach to the description of hole excitations is proposed. Rescattering effects in (e, e'n) reactions are considered briefly.

INTRODUCTION

One of the important methods of investigating nuclear structure is the study of electron–nucleus collisions. Although the advantages of this procedure in nuclear physics have been known for a long time,[1,2] it is only in recent years that very subtle measurements have become possible in reactions with electrons, owing to progress in the experimental techniques. Even the first experiments[3] on quasielastic (e, e'p) scattering have pointed to an unexpectedly complicated nature of the deep "hole" states of the nucleus. It turns out that the observed levels have an appreciable width and lie much lower than those predicted by the shell model.[4–6] To be sure, individual disparities with the shell model have been known also from other experiments, but they did not seem very important prior to the experiments on quasielastic knock-out of protons from the lower levels in (e, e'p) and p, 2p) reactions.[7] It was precisely the data on quasielastic knock-out of protons that made it urgent to develop for nuclear theory more rigorous methods than the known model approximations. The present review is devoted to an exposition of a new approach,[8,9] in which an explanation is proposed for the deviations from the shell model observed in quasielastic (e, e'p) processes. This approach is based on field-theoretical methods of many-body theory and its advantage lies in the fact that model representations of the nuclear structure are dispensed with from the very outset. Only simplifications of nonfundamental character, aimed mainly at abbreviating the expositions, are introduced. Thus, we confine ourselves to Coulomb forces between the electron and the nucleus and neglect the interaction with the radiation field.

The use of field-theoretical methods makes it possible to establish general relations between different observable characteristics of a nuclear system. In our problem this leads to a qualitative explanation of the singularities in the hole-excitation spectrum. As to the quantitative description, we are left as before with certain difficulties inherent in the many-body problem as a whole. Therefore the analysis of the experimental data and the comparison with the results of the exact theory can yield valuable information concerning the role of various interactions in the interior of the nucleus.

In Sec. 1, which is introductory, the electron–nucleus scattering problem is formulated in the language of many-body field theory. The relations connecting different observable quantities with the field functions are presented. In Sec. 2, the reaction of quasielastic knock-out of protons is considered for the case of high energy transfers. For a qualitative explanation of the positions and widths of the hole levels we use the projection-operator method in a form somewhat different from the one customarily employed in the theory of nuclear reactions.[10] This approach demonstrates clearly the differences from the shell model, although the final results are not quite suitable for practical calculations. In Sec. 3 we develop a method of describing the same process, but not on the basis of the Green's-function theory, which is most convenient when we arrive (after a number of simplifications) at the phenomenological treatment. In Sec. 4 it is shown that even though the average field is energy dependent and complex for quasiparticles, we can still use a formalism that is close to the shell model. In Sec. 5 we consider briefly an approximation used to take into account the effects of rescattering of the excited proton and the ensuing neutron knock-out reaction.

We intentionally do not dwell on exposition details connected with the kinematics of the electron-scattering processes, since they are dealt with in rather comprehensive reviews.[2,11] Our purpose is to call attention to that new information concerning the nuclear structure, which can be extracted from new experimental data on nucleon knock-out reactions.[12,13]

1. GENERAL RELATIONS

To establish general relations between the observable cross sections and the field quantities that characterize a nuclear system, we represent the total Hamiltonian of the electron–nucleus collision problem in the form $H_T = H + H_e + V$, (H is the Hamiltonian of the internal motion of the nucleons and of the nucleus as a whole; H_e is the Hamiltonian of the free motion of the electron; V is the Hamiltonian of its interaction with the nucleus). Owing to the relatively small role of the radiative effects in the processes considered, we need retain in the Hamiltonian V only the Coulomb interaction, and we can write, in the second-quantization representation,

$$V = \int dx\, dy v_c(x, y)\, \hat{\rho}(x)\, \hat{\rho}_e(y),$$

where x and y are the sets of the spin and spatial coordinates of the proton and electrons, respectively ($x = (\sigma, \mathbf{r})$; $\int dx \equiv \sum_\sigma \int d\mathbf{r}$); v_C is the Coulomb potential; $\hat{\rho}(x)$ and $\hat{\rho}_e(x)$ are the proton and electron density operators [$\hat{\rho}(x) = \overline{\psi}(x)\psi(x)$; $\overline{\psi}$ and ψ are the proton creation and annihilation operators).

We separate the principal effects proportional to the

nuclear charge and determine the interaction in the elastic channel

$$V_0 = \int dx\, dy v_c(x, y)\, \rho_0(x)\, \hat{\rho}_e(y),$$

where $\rho_0(x) = \langle | \hat{\rho}(x) | \rangle$ is the nuclear-charge distribution density in the ground state $|\rangle$. The scattering-problem equation can then be written in the form[1)]

$$\Psi = \Phi_{0k} + \frac{1}{E_k - H - H_e + i\alpha}(V - V_0)\Psi. \quad (1)$$

Here Φ_{0k} is the "initial" state vector corresponding to an elastically scattered electron: $(E_k - H_e - V_0)\Phi_{0k} = 0$; $\Phi_{0k} = \alpha_k^+ |\rangle$; E_k is the energy of the incident electron (the energy is reckoned from the energy of the ground state of the target nucleus, i.e., $H|\rangle = 0$); α_k^+ is the operator of production of an electron with momentum k in the field of the nonexcited nucleus.

It follows from (1) that the inelastic processes can be described with the aid of perturbation theory in the interaction V, but with electron wave functions that are "distorted" by the Coulomb field. Thus, a transition with excitation of the nucleus to the state $|n\rangle$ ($H|n\rangle = \omega_n |n\rangle$) is determined in first order in V by the matrix element $V_{k,nk'} = \langle \Phi_{nk'} | V | \Phi_{0k} \rangle$. The state vector $\Phi_{nk'}$ satisfies the equation

$$(E_{k'} - \omega_n - H_e - V_0)\Phi_{nk'} = 0,$$

where $E_{k'}$ and k' are the energy and momentum of the electron after the collision. This yields $\Phi_{nk'} = \alpha^+_{k'} |n\rangle$ and consequently, the amplitude $V_{k, nk'}$ is expressed in the form

$$V_{k, nk'} = \int dx\, dy v_c(x, y)\, \rho_n(x)\, \varphi^+_{k'}(y)\, \varphi_k(y); \quad (\rho_n(x) = \langle n | \hat{\rho}(x) | \rangle), \quad (2)$$

where $\varphi_k(y)$ is the wave function of the electron with momentum k moving in the Coulomb field of the target nucleus.

Since we are interested in processes in which the electrons have high energies (on the order of several hundred MeV) before and after the collision, the wave functions $\varphi_k(y)$ can be adequately approximated by plane waves. Taking this into account and changing over to the Fourier representation of v_C in (2), we obtain

$$V_{k, nk'} = \frac{4\pi^2 e^2}{q^2} \int dx \rho_n(x) \exp(iqr) \quad (x = (\sigma, \mathbf{r})),$$

where e is the electron charge; $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ is the momentum transfer. The differential cross section of the process with excitation of a nucleus in the state $|n\rangle$ and emission of an electron of energy $E_{k'}$ into a solid angle $\Omega_{k'}$ is given by $V_{k, nk'}$, and takes the form[2]

$$\frac{d^2\sigma_n}{dE_{k'}\, d\Omega_{k'}} = K(\mathbf{k}, \mathbf{q}, \omega) R_{k, nk'}; \quad R_{k, nk'} = \left| \int dx \rho_n(x) \exp(-iqr) \right|^2 \delta(\omega - \omega_n). \quad (3)$$

Here $\omega = E_k - E_{k'}$ is the transferred energy; K is a known kinematic factor, $R_{k, nk'}$ is a quantity that determines the nuclear transition.

For values of ω exceeding the nucleon separation energy, and under conditions when only the final momentum and energy of the electron are determined in the experiment, the total inelastic-scattering cross section contains the quantity

$$R_{kk'} = \sum_n \left| \int dx \rho_n(x) \exp(-iqr) \right|^2 \delta(\omega - \omega_n) = \int dx\, dx' \langle | \hat{\rho}(x) \delta(\omega - H) \hat{\rho}(x') | \rangle \exp[iq(r - r')]. \quad (4)$$

In this equation, the summation sign denotes summation over the discrete states of the nucleus and integration over the continuum.

The quantity $R_{kk'}$ can be expressed in terms of the two-particle Green's function, defined as

$$G(1, 2, 3, 4) = \langle | T(\psi(1)\psi(2)\bar{\psi}(3)\bar{\psi}(4)) | \rangle,$$

where the numbers denote the sets of the coordinates x and the time t ($1 \equiv (x_1, t_1)$), and the t-product sign contains the operators ψ in the Heisenberg representation [$\psi(1) = \exp(iHt_1)\psi(x_1)\exp(-iHt_1)$]. Equating here $(x_1, t_1) = (x_3, t_3)$ and $(x_2, t_2) = (x_4, t_4)$, we get from the definition of the t-product

$$G(1, 2, 3, 4)^{3\to 1}_{4\to 2} = -\theta(\tau_{12}) \langle | \hat{\rho}(x_1) \exp(-iH\tau_{12}) \hat{\rho}(x_2) | \rangle - \theta(-\tau_{12}) \langle | \hat{\rho}(x_2) \exp(iH\tau_{12}) \hat{\rho}(x_1) | \rangle;$$
$$\tau_{12} = t_1 - t_2; \quad \theta(\tau) = \begin{cases} 1 & (\tau > 0); \\ 0 & (\tau < 0). \end{cases}$$

This function coincides, apart from a numerical factor, with the expression for the polarization operator Π (1, 2) (ref. 14). Changing over to the Fourier representation with respect to the time difference τ_{12}, we obtain

$$\Pi(x_1, x_2, \omega) = i \int d\tau_{12} G(1, \ldots, 4) \exp(i\omega\tau_{12})^{1\to 3}_{2\to 4} = \left\langle \left| \hat{\rho}(x_1) \frac{1}{\omega - H + i\alpha} \hat{\rho}(x_2) \right| \right\rangle - \left\langle \left| \hat{\rho}(x_2) \frac{1}{\omega + H - i\alpha} \hat{\rho}(x_1) \right| \right\rangle$$

We can therefore express Eq. (4) in terms of the proton polarization operator Π:

$$R_{kk'} = -\frac{1}{\pi} \operatorname{Im} \int dx\, dx' \Pi(x, x', \omega) \exp[iq(r - r')]. \quad (5)$$

We also establish a connection between the quantity $R_{kk'}$, the vertex function Γ, and the single-particle Green's function $G(1, 2) = -i \langle | T(\psi(1)\psi(2)) | \rangle$. As is well known,[15] Γ is defined by the relation

$$\int d2'\, d3' G(1, 2') \Gamma(2, 2', 3') G(3', 3) = G(1, 3) G(2, 2) - G(1, 2, 3, 2) \quad \left(\int d1 \equiv \int_{-\infty}^{+\infty} dt_1 \int dx_1 \right). \quad (6)$$

Changing over to the Fourier representation with respect

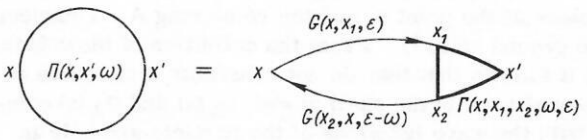

Fig. 1. Diagram representation of the polarization operator in terms of the vertex Γ and the propagator G.

to the times, we can now easily obtain an expression for R:

$$R_{kk'} = -\frac{1}{\pi}\,\mathrm{Im}\int\frac{d\varepsilon}{2\pi i}\int dx\,dx'\,dx_1\,dx_2 \times \Gamma(x', x_1, x_2, \omega, \varepsilon)\,G(x_2, x, \varepsilon-\omega)\exp[iq(r-r')], \quad (7)$$

where G and Γ are the proton single-particle and vertex functions. The diagram relation between the polarization operator Π and the functions G and Γ is shown in Fig. 1.

It follows from (4) that at values of ω below the boundary of the continuum of the eigenstates of the nucleus Eqs. (5) and (7) define transitions with excitation of individual discrete levels n. The investigation of such processes is of interest for nuclear spectroscopy of low-lying levels.[2] At values of ω above this limit, $R_{kk'}$ contains an infinite number of continuum states. In this case the total inelastic-scattering cross section can yield information on groups of states that are distinguished by certain attributes.

To describe a particular concrete process we can use the diagram-cutting method known from field theory.[16] In our problem, this method is applied in the following manner: To find the probability of the partial transition it is necessary to cut the polarization-operator diagram into two parts at places corresponding to the sought process, and then go over to the mass shell. Thus, for a process with excitation of a particle–hole state it is necessary to draw the cut in diagrams whose two parts are joined by particle and hole lines. These can be either proton or neutron lines. It is easy to show if the function Γ in the polarization operator Π is expanded formally in a series in the nucleon interaction and one cut is made in each of the diagrams, then the result is an expression for the probability of a transition with excitation of a particle–hole state:

$$R^{(i)}_{k,\lambda\lambda'k'} = \Big|\int dx\,dx'dx''\psi^{+}_{i\lambda}(x') \times \Gamma_i(x, x', x'', \omega, \varepsilon^{(+)}_{i\lambda})\,\varphi_{i\lambda'}(x')\exp(iqr)\Big|^2\delta(\omega-\varepsilon^{(+)}_{i\lambda}+\varepsilon_{i\lambda'}), \quad (8)$$

where i = n or p; $\varphi_{i\lambda}$ and $\psi_{i\lambda}$ are the proton (i = p) or neutron (i = n) wave functions of the holes and particles, respectively:

$$\varphi_{i\lambda}(x) = \langle\Phi_{i\lambda}|\,\psi_i(x)\,|\rangle;\ H\Phi_{i\lambda} = -\varepsilon_{i\lambda}\Phi_{i\lambda};$$
$$\psi_{i\lambda}(x) = \langle|\,\psi_i(x)\,|\,\Psi_{i\lambda}\rangle;\ H\Psi_{i\lambda} = \varepsilon^{(+)}_{i\lambda}\Psi_{i\lambda}.$$

If the target nucleus contains A nucleons, then $\Phi_{i\lambda}$ and $\Psi_{i\lambda}$ pertain to systems of A − 1 and A + 1 nucleons, respectively. The functions $\varphi_\lambda(x)$ and $\psi_\lambda(x)$ have a simple physical meaning: $\varphi_\lambda(x) = \langle\Phi_\lambda|\psi(x)|\rangle$ is the probability amplitude in the ground state of a nucleus consisting of A nucleons of finding one nucleon at the point x and the remaining A − 1 nucleons in the state Φ_λ; $\psi_\lambda(x) = \langle|\psi(x)|\Psi_\lambda\rangle$ is the probability amplitude of finding in the state Ψ_λ a nucleon at the point x, and the remaining A − 1 nucleons in the ground state $|\rangle$. From the definition of these functions it follows that they do not constitute a complete orthonormal set. In the shell model, $\psi_\lambda(x)$ and $\varphi_\lambda(x)$ coincide with the wave functions of the particle and hole in the average field. In real nuclei, however, the ground state $|\rangle$ as well as the excited states Φ_λ and Ψ_λ are complex superpositions of the shell basis of vectors, and $\varphi_\lambda(x)$ can be regarded as the wave function of a quasiparticle in the average field only for excitations near the Fermi surface.[17] With increasing excitation energy, the quasiparticle concept becomes meaningless because of the increasing role of the correlative interactions.

From the definition of the functions $\varphi_\lambda(x)$ and $\psi_\lambda(x)$ it follows that they differ from zero for all states Φ_λ and Ψ_λ that cannot be classified as single- or many-particle states without resorting to additional physical considerations.

The vertex function Γ determines the response of the system to an external field and includes all possible quasiparticle-interaction effects. In the absence of correlative forces, for example in the shell model without residual interactions, we have $\Gamma_p(y, x, x', \omega, \varepsilon) = \delta(y-x)\,\delta(x-x')$ and $\Gamma_n = 0$. It is precisely the presence of such forces in real nuclei that makes the vertex Γ_n different from zero. Since Γ_n is a quantity on the order of the characteristic nuclear constants, the cross sections for the scattering of electrons with excitation of neutron states via primary proton states will be much larger than the cross sections due to direct electron–neutron interaction.

2. QUASIELASTIC PROTON KNOCK-OUT REACTION (e, e'p)

The differential cross section of this reaction, with a proton momentum **p** and energy ε_p, is expressed in the Born approximation in the Coulomb interaction in the form[2,11]

$$\frac{d^4\sigma}{dE'_k\,d\Omega_k\,d\varepsilon_p\,d\Omega_p} = \sigma_M\left(\frac{q^4_\mu}{q^4}\right)p\varepsilon_p R(\mathbf{p}, \mathbf{q}, \omega),$$

where σ_M is the Mott cross section and q_μ is the 4-momentum transfer: $q_\mu = (\mathbf{q}, i\omega)$. The factor R, which determined the nuclear transitions, can be represented on the basis of (8) in the form

$$R(\mathbf{p}, \mathbf{q}, \omega) = \pi\sum_\lambda\Big|\int dx\,dx'\,dx''\psi^{+}_{\mathbf{p}}(x') \times \Gamma(x, x', x'', \omega, \varepsilon_p)\,\varphi_\lambda(x'')\exp(iqr)\Big|^2\delta(\varepsilon-\varepsilon_\lambda) \quad (9)$$

($\varepsilon = \varepsilon_p - \omega < 0$). The summation sign means here summation over the discrete levels and integration over the continuum of the excitations of a residual nucleus made up of A − 1 nucleons. At outgoing-proton energies on the order of hundreds of MeV, $\psi_{\mathbf{p}}(x)$ can be assumed with a sufficient degree of accuracy to be the wave function of the proton in the optical potential (see below).

It follows from physical considerations that at large energy transfers the value of Γ depends little on ω, and consequently in the region where $\omega - \omega_{\mathbf{p}}$ lies below the boundary of the continuum of the residual nucleus the expression (9) has only δ-function singularities corresponding to the excitation of the discrete levels λ ($H\Phi_\lambda = -\varepsilon_\lambda\Phi_\lambda$). Above this boundary, expression (9) contains an integration over the continuum of the states $\Phi_{\varepsilon\beta}$. We recall that Φ_λ are the state vectors of a system of A − 1 particles and belong to the continuum when $|\varepsilon_\lambda|$ exceeds double the nucleon binding energy. The fact that the ε_λ are negative in this case should not lead to confusion, since the energy

is reckoned from the ground-state energy $\mathcal{E}_0(A)$ of a nucleus consisting of A nucleons: $\varepsilon_\lambda = \mathcal{E}_0(A) - \mathcal{E}_\lambda(A-1) = \mu - \xi$ [μ is the chemical potential and ξ is the excitation energy ($\xi > 0$)].

Introducing the notation

$$F(x, \varepsilon) = \int dx'\, dx'' \psi_p^+(x'')\, \Gamma(x', x'', x, \omega, \varepsilon) \exp(i\mathbf{qr}), \qquad (10)$$

we rewrite (9) in the form

$$R(\mathbf{p}, \mathbf{q}, \omega) = \pi \sum_\lambda \left| \int dx F(x, \varepsilon)\, \varphi_\lambda(x) \right|^2 \delta(\varepsilon - \varepsilon_\lambda)$$
$$= \pi \sum_\lambda \int dx\, dx' F^+(x, \varepsilon)\, \varphi_\lambda^+(x)\, \varphi_\lambda(x')\, F(x', \varepsilon)\, \delta(\varepsilon - \varepsilon_\lambda).$$

We use the spectral representation of the single-particle Green's function[13]

$$G(x, x', \varepsilon) = \sum_\lambda \left\{ \frac{\varphi_\lambda(x)\, \varphi_\lambda^+(x')}{\varepsilon - \varepsilon_\lambda - i\alpha} + \frac{\psi_\lambda(x)\, \psi_\lambda^+(x')}{\varepsilon - \varepsilon_\lambda^+ + i\alpha} \right\} \qquad (11)$$

and represent R in the form

$$R(\mathbf{p}, \mathbf{q}, \omega) = \mathrm{Im} \int dx\, dx' F(x, \varepsilon)\, G(x, x'\, \varepsilon)\, F^+(x', \varepsilon). \qquad (12)$$

In the shell model, R has δ-function singularities in ε, corresponding to the hole levels; R = 0 at all other points. In real nuclei, R has a resonance shape, the resonance widths increasing with increasing excitation energy. To prove this statement, we use the method of projection operators.[10]

We introduce the projection operators P and Q, which single out respectively the continuous and discrete spectra of a certain model problem ($P + Q = 1$; $PQ = 0$). We assume that the model Hamiltonian satisfies the following requirements: It reflects the basic symmetry properties and the character of the spectrum of the exact problem, and the boundaries of the continuous and discrete spectra of the model and real systems coincide. One can use as such a basis the complete set of shell state vectors.

We now consider Eq. (12) with the aid of the operators P and Q. Since R contains only negative ε, it suffices to investigate the expression

$$G_-(x, x', \varepsilon) = \sum_\lambda \frac{\varphi_\lambda(x)\, \varphi_\lambda^+(x')}{\varepsilon - \varepsilon_\lambda - i\alpha} = \left\langle \left| \bar{\psi}(x') \frac{1}{\varepsilon + H - i\alpha} \psi(x) \right| \right\rangle.$$

It is obvious that the functions $\varphi_\lambda(x) = \langle \Phi_\lambda | \psi(x) | \rangle$ differ from zero in a region of the order of the nuclear dimensions. This means that the $\varphi_\lambda(x)$ describe a finite motion of the nucleon; consequently, if the model Hamiltonian is sufficiently well chosen the following approximate equations hold:[2)]

$$Q\psi(x)|\rangle \approx \psi(x)|\rangle; \quad P\psi(x)|\rangle \approx 0.$$

Taking this into account and representing the total Hamiltonian H in the form of the sum

$$H = H_{PP} + H_{QQ} + H_{QP} + H_{PQ};$$
$$(H_{PP} = PHP; \quad H_{PQ} = PHQ; \ldots),$$

we can express the function G_- in the form

$$\left.\begin{aligned} G_-(x, x', \varepsilon) &= \left\langle \left| \bar{\psi}(x') \frac{1}{\varepsilon + H_{QQ} - i\alpha} \sum_{n=0}^{\infty} \left(\Delta_Q(\varepsilon) \frac{1}{\varepsilon + H_{QQ} - i\alpha} \right)^n \right. \right. \\ &\times \psi(x) \left| \right\rangle = \left\langle \left| \bar{\psi}(x') \frac{1}{\varepsilon + H_{QQ} - \Delta_Q(\varepsilon)} \psi(x) \right| \right\rangle; \\ \Delta_Q(\varepsilon) &= H_{QP} \frac{1}{\varepsilon + H_{PP} - i\alpha} H_{PQ}. \end{aligned}\right\} \qquad (13)$$

The complex energy-shift operator $\Delta_Q(\varepsilon)$ is the result of the interaction with the continuum. The small positive imaginary increment $i\alpha$ in the denominator of $\Delta_Q(\varepsilon)$ corresponds to the boundary condition of the system-decay problem. Indeed, let us assume that we know the solutions of the complex-eigenvalue problem with non-Hermitian Hamiltonian $H_{QQ} + \Delta_Q(\varepsilon)$:

$$[H_{QQ} + \Delta_Q(\varepsilon)]\, \chi_\nu(\varepsilon) = \left[\varepsilon_\nu(\varepsilon) + \frac{i}{2} \Gamma_\nu(\varepsilon) \right] \chi_\nu(\varepsilon);$$
$$\mathrm{Re}\, \Delta_Q(\varepsilon) = H_{QP} \frac{\mathcal{P}}{\varepsilon + H_{PP}} H_{PQ}; \quad \mathrm{Im}\, \Delta_Q(\varepsilon) = \pi H_{QP} \delta(\varepsilon + H_{PP}) H_{PQ}, \qquad (14)$$

where $\mathcal{P}$ denotes the principal value of the integral over a continuous spectrum of the eigenvalues of the Hamiltonian H_{PP}. The imaginary part $(i/2)\Gamma_\nu(\varepsilon)$ determines the half-width of the nonstationary level:

$$\Gamma_\nu(\varepsilon) = 2\, \mathrm{Im} \langle \chi_\nu(\varepsilon) | \Delta_Q(\varepsilon) | \chi_\nu(\varepsilon) \rangle$$

(it is assumed that the functions $\chi_\nu(\varepsilon)$ form a complete orthonormal system). It is easily seen that $\Gamma_\nu(\varepsilon)$ differs from zero in the region of values of $|\varepsilon|$ exceeding double the nucleon-separation energy (the hole should "float up" to the Fermi surface with enough energy to knock out a nucleon into the continuum).

As a result of the transformations (13), taking (14) into account we obtain from (12) the following resonance equation:

$$R(\mathbf{p}, \mathbf{q}, \omega) = \sum_\lambda \frac{\frac{1}{2}\Gamma_\nu(\varepsilon)}{(\varepsilon - \varepsilon_\nu(\varepsilon))^2 + \frac{1}{4}\Gamma_\nu^2(\varepsilon)}$$
$$\times \int dx F(x, \varepsilon) \left\langle \chi_\nu(\varepsilon) \left| \psi(x) \right\rangle \exp(i\mathbf{qr}) \right|^2. \qquad (15)$$

We see therefore that at energies below threshold, when there is no continuum and $\Gamma_\nu(\varepsilon) \to 0$ (the region of stationary levels), R vanishes everywhere on the ε axis, with the exception of individual points ε_s that are solutions of the equation

$$\varepsilon_\nu(\varepsilon_s) = \varepsilon_s \qquad (16)$$

corresponding to the energies of the stationary levels of the nucleons. At these points R has δ-like singularities. In the region of nonstationary levels [$\Gamma_\nu(\varepsilon) \neq 0$], R has a resonance shape with the positions and widths of the levels dependent on the energy ε. The centers of the resonance peaks are located at the points ε_s satisfying Eq. (16).

The function R contains the functions $\langle \chi_\nu(\varepsilon) | \psi(x) | \rangle$, which determine the nature of the resonances: It can be assumed that the maximum values of these functions, depending on the number ν of the state, correspond to single-hole states, while the remainder correspond to more com-

plicated states $\chi_\nu(\varepsilon)$ (in the shell model, many-particle states make no contribution to R). It seems that this classification is the only one possible in the given approach. From this point of view, both single- and many-particle excitations of the residual nucleus contribute to the cross section of the (e, e'p) reaction. It follows therefore that in experiments with good resolution, at large $(\omega - \varepsilon_p)$ there should be observed a fine structure of broad hole resonances, in analogy with the case of the giant single-particle resonances in nucleon–nucleus scattering.

We note in conclusion that the practical solution of (14) is no less complicated a task than the direct solution of the Schrödinger equation for a system of many nucleons. The situation is aggravated also by the fact that for deep hole levels, owing to the absence of small parameters, there is no hope of being able to obtain an approximate solution on the basis of the shell model with allowance for the residual forces and a small number of configurations. Nonetheless, the analysis presented of the structure of the cross section of the (e, e'p) reaction makes it possible to establish the general regularities of the excitation spectrum of the residual nucleus, and the difference between this spectrum and the shell spectrum.

3. MASS OPERATOR AND HOLE-EXCITATION SPECTRUM

We now continue the investigation of the general properties of the excitation spectrum of the residual nucleus by the Green's-function theory methods. We consider Dyson's equation for a single-particle propagator:[14,15,17,18]

$$(\varepsilon - T_x)\, G(x, x', \varepsilon) - \int dx_1 M(x, x_1, \varepsilon)\, G(x_1, x', \varepsilon) = \delta(x - x'), \tag{17}$$

where T_x is the nucleon kinetic-energy operator; $M(x, x', \varepsilon)$ is the mass operator, and determines all the possible interactions between the particle or hole and the nucleus. In the general case, the mass operator is complex, nonlocal in space, and dependent on the energy ε. A particular case of the mass operator is the average local potential V_x customarily used in the shell model:

$$M = (x, x', \varepsilon) = \delta(x - x')\, V_x. \tag{18}$$

In this case the single-particle function takes on the simple form

$$G_{sh}(x, x', \varepsilon) = \sum_\lambda \frac{\varphi_\lambda(x)\, \varphi_\lambda^+(x')}{\varepsilon - \varepsilon_\lambda + i\alpha \operatorname{sign}(\varepsilon_\lambda - \varepsilon_F)}, \tag{19}$$

where $\varphi_\lambda(x)$ and ε_λ are the eigenfunction and the self-energy of the single-particle equation $(T_x + V_x)\varphi_\lambda(x) = \varepsilon_\lambda \varphi_\lambda(x)$; ε_F is the energy of the limiting Fermi level.

In many-body field theory one can write down an expression for the mass operator in terms of the interaction potential of a pair of free particles and the two-particle Green's function.[14] To analyze the general properties of M, however, this expression is not very suitable in the absence of small parameters. It would be much more useful to establish a connection between M and the effect of the interaction of the quasiparticles in the nucleus, since it is precisely this interaction which enters in the different experimentally observed quantities.

To obtain such a connection, we consider first the two-particle Green's function[3] $G(1, 2, 3, 4) = \langle | T [\psi(1) \psi(2) \overline{\psi}(3) \overline{\psi}(4)] | \rangle$ at $x_4 = x_2$ and $t_4 = t_2 + \delta$ (δ is a small positive quantity), write the t-product, and then obtain, taking into account the equality $\int dx \overline{\psi}(x) \psi(x) = \hat{N}$ ($\hat{N}$ is the particle-number operator),

$$\int dx_2 G(1, 2, 3, 2^+) = i\{A + \theta(\tau_{12})\theta(\tau_{23}) - \theta(\tau_{21})\theta(\tau_{32})\} G(1, 3)$$
$$(\tau_{ij} = t_i - t_j).$$

By taking the Fourier transform with respect to the times, we get

$$\int dx_2\, dt_1\, dt_3 G(1, 2, 3, 2^+) \exp(i\varepsilon_1 t_1 - i\varepsilon_3 t_3)$$
$$= 2\pi i A \delta(\varepsilon_1 - \varepsilon_3)\, G(x_1, x_3, \varepsilon_1)$$
$$+ \{G(x_1, x_3, \varepsilon_1) - G(x_1, x_3, \varepsilon_3)\} \exp[i(\varepsilon_1 - \varepsilon_3) t_2]/(\varepsilon_1 - \varepsilon_3). \tag{20}$$

On the other hand, the Bethe–Salpeter equation for the two-particle Green's function in the particle–hole channel is of the form:[17]

$$L(1, 2, 3, 4) = G(1, 4)\, G(2, 3) - i \int d1' \ldots d4'$$
$$\times G(1, 2')\, G(4', 3)\, V(1', \ldots, 4')\, L(3', 2, 1', 4);$$
$$L(1, \ldots, 4) = G(1, 3)\, G(2, 4) - G(1, \ldots, 4), \tag{21}$$

where the effective interaction V is defined as an aggregate of irreducible diagrams, none of which can be represented in the form of two parts joined only by two particle and hole propagation lines. By changing over in (21) to the Fourier transform with respect to time and taking (20) into account we get ($\varepsilon_1 - \varepsilon_3 = \omega$)

$$G(x, x', \varepsilon) - G(x, x', \varepsilon - \omega) = -\omega \int dx_1 G(x, x_1, \varepsilon)\, G(x_1, x', \varepsilon - \omega)$$
$$+ \int \frac{d\varepsilon'}{2\pi i} \int dx_1 \ldots dx_4 G(x, x_1, \varepsilon)\, G(x_3, x', \varepsilon - \omega)$$
$$\times V(x_1, \ldots, x_4, \varepsilon, \varepsilon', \omega)\{G(x_4, x_2, \varepsilon' + \omega) - G(x_4, x_2, \varepsilon')\}. \tag{22}$$

From this and from Dyson's equation (17) we obtain for the mass-operator difference at different energies

$$M(x, x', \varepsilon) - M(x, x', \varepsilon - \omega)$$
$$= \int \frac{d\varepsilon'}{2\pi i} \int dx_1\, dx_2 V(x, x_1, x', x_2, \varepsilon, \varepsilon', \omega)$$
$$\times \{G(x_2, x_1, \varepsilon' + \omega) - G(x_2, x_1, \varepsilon')\}. \tag{23}$$

This equation is one of the formulations of Ward's identity in difference form, which goes over to the well known differential form[15] as $\omega \to 0$:

$$\frac{\partial M(x, x', \varepsilon)}{\partial \varepsilon} = \int \frac{d\varepsilon'}{2\pi i} \int dx_1\, dx_2 V$$
$$\times (x, x_1, x', x_2, \varepsilon, \varepsilon', 0) \frac{\partial G(x_2, x_1, \varepsilon')}{\partial \varepsilon'}.$$

Equation (23) can be regarded as a nonlinear equation for the operator M. It should be noted that an approximation using an effective interaction V that does not depend on the energy variable leads, as is seen from (23), to a mass operator that is likewise independent of the energy (static potential). The latter is equivalent to the shell model with a nonlocal average field.

From the definition of the vertex Γ in terms of the two-particle propagator we can obtain relations (22) and (23) in different form:

$$G(x, x', \varepsilon) - G(x, x', \varepsilon - \omega) = -\omega \int dy\, dx_1\, dx_2 G(x, x_1, \varepsilon) \times \Gamma(y, x_1, x_2, \omega, \varepsilon) G(x_2, x', \varepsilon - \omega); \quad (24)$$

$$M(x, x', \varepsilon) - M(x, x', \varepsilon - \omega) = \omega \left\{ \delta(x - x') - \int dy \Gamma(y, x, x', \omega, \varepsilon) \right\}. \quad (25)$$

Obviously, the difference between the vertex Γ and the free vertex Γ_f (y, x, x', ω, ε) = δ (y − x) δ (x − x'), and consequently also the dependence of the mass operator on the energy, is due to the presence of correlative interactions.

Equations (24) and (25) enable us to establish a number of useful relations. Thus, from (24) and the spectral representation (11) of the propagator G(x, x', ε) we obtain as $\varepsilon \to \varepsilon_\lambda$ and $\omega \to \varepsilon_\lambda - \varepsilon_{\lambda'}$ (λ and λ' are discrete levels) a condition for the normalization of the function $\varphi_\lambda(x)$ introduced in Sec. 1:

$$\int dy\, dx\, dx' \varphi^+_\lambda(x) \Gamma(y, x, x', \varepsilon_\lambda - \varepsilon_{\lambda'}, \varepsilon_\lambda) \varphi_{\lambda'}(x') = \delta_{\lambda\lambda'}. \quad (26)$$

A similar condition applies also to the functions $\psi_\lambda(x)$. To obtain a similar equation for the states of the continuum we use the scattering-problem equation $\Psi_p = [i\alpha/(\varepsilon - H + i\alpha)]\Psi_{0p}$.

If we take the initial-state vector in this case to be $\Psi_{0p} = \int dx \bar{\psi}(x) |\rangle \varphi_p(x)$, where $\varphi_p(x)$ is the wave function of a nucleon with momentum p moving in a certain average field of the nucleus, then we easily obtain from (24) a normalization condition for the function[4] $\psi_p(x) = \langle |\psi(x)| \Psi_p \rangle$. The situation is somewhat more complicated with the continuum of the system consisting of A − 1 particles, but we can use the same formal procedure and obtain an equation similar to (26).

We now consider the equation of motion of a certain fictitious particle in a nonlocal potential M(x, x', ε) that depends on the energy ε as a parameter:

$$(T_x + M(x, \varepsilon)) \varphi_\lambda(x, \varepsilon) = \tilde{\varepsilon}_\lambda(\varepsilon) \varphi_\lambda(x, \varepsilon). \quad (27)$$

For simplicity, we introduce the notation M(x, ε) φ(x) = $\int$ dx' M (x, x', ε) φ(x'). We assume that this equation admits of a solution with eigenvalues $\tilde{\varepsilon}_\lambda(\varepsilon)$ (they can be complex) and with a complete orthonormal system of eigenfunctions φ_λ (x, ε). The single-particle propagator can then be written in diagonal form:

$$G(x, x', \varepsilon) = \sum_\lambda \frac{\varphi_\lambda(x, \varepsilon) \varphi^+_\lambda(x', \varepsilon)}{\varepsilon - \tilde{\varepsilon}_\lambda(\varepsilon) + i\alpha \operatorname{sign}(\varepsilon - \mu)} \quad (28)$$

(μ is the chemical potential). Comparing this expression with the representation (11), we can conclude that for the region of the discrete states the energy levels are determined from the equation

$$\tilde{\varepsilon}_\lambda(\varepsilon_\nu) = \varepsilon_\nu \quad (29)$$

and consequently

$$\int dx \varphi^+_\nu(x) \varphi_\nu(x) = (1 - \partial\varepsilon_\lambda(\varepsilon)/\partial\varepsilon)^{-1}_{\varepsilon \to \varepsilon_\nu}. \quad (30)$$

Since the normalization integral $\int dx |\varphi_\nu(x)|^2$ is always smaller than unity, as follows from the definition of $\varphi_\lambda(x)$ (see Sec. 1), we get, taking (24) and (25) into account,

$$\frac{\partial\varepsilon_\lambda(\varepsilon)}{\partial\varepsilon}\Big|_{\varepsilon \to \varepsilon_\nu} = \int dx\, dx' \varphi^+_\lambda(x, \varepsilon) \frac{\partial M(x, x', \varepsilon)}{\partial\varepsilon} \varphi_\lambda(x', \varepsilon)\Big|_{\varepsilon \to \varepsilon_\nu} < 0. \quad (31)$$

Here, as above, the subscript λ denotes the set of quantum numbers characterizing the state of the particle in the potential M(x, ε). Equation (29), however, can have also several real roots ε_ν, corresponding to different states Φ_ν (or Ψ_ν) with the same set λ. In this case, just as in Sec. 2, we can assume an approximate classification of the levels: The maximum values of the normalization integral (or of the derivative $\partial\varepsilon_\lambda(\varepsilon_\nu)/\partial\varepsilon_\nu$), depending on ε_ν, correspond to single-particle states, and all others correspond to many-particle states.

We consider now the qualitative picture of the spectrum of Eq. (27) on the basis of the reasoning presented above. In the region $2\mu < \varepsilon < 0$ the mass operator is real, and Eqs. (27) and (29) determine the spectrum of the discrete stationary levels of nuclei with A − 1 ($\varepsilon < \mu$) and A + 1) ($\varepsilon > \mu$) particles. For $\varepsilon < 2\mu$, a continuum is produced for the nucleus with A − 1 nucleons, and the mass operator becomes complex. Equation (27) defines in this case[5] the complex energy eigenvalues $\tilde{\varepsilon}_\lambda(\varepsilon)$:

$$\left.\begin{aligned} &\tilde{\varepsilon}_\lambda(\varepsilon) = \varepsilon_\lambda(\varepsilon) + i\Gamma_\lambda(\varepsilon)/2; \\ &\varepsilon_\lambda(\varepsilon) = \operatorname{Re} \int dx \varphi^+_\lambda(x, \varepsilon)(T_x + M(x, \varepsilon)) \varphi_\lambda(x, \varepsilon); \\ &\Gamma_\lambda(\varepsilon) = 2\operatorname{Im} \int \partial x \varphi^+_\lambda(x, \varepsilon) M(x, \varepsilon) \varphi_\lambda(x, \varepsilon). \end{aligned}\right\} \quad (32)$$

It is easily seen that the centers of these resonances are determined by the real roots of an equation similar to (29):

$$\varepsilon_\lambda(\varepsilon_\nu) = \varepsilon_\nu; \ (\varepsilon_\lambda(\varepsilon) = \operatorname{Re} \tilde{\varepsilon}_\lambda(\varepsilon)). \quad (33)$$

In addition to quasistationary levels, the solutions of (27) with $\varepsilon < 2\mu$ contain also a continuum. However, it cannot have a real physical meaning, for the imaginary part of M is then positive. Analogously, the mass operator becomes complex also in the region $\varepsilon > 0$, but with a negative imaginary part. Here, to the contrary, the solutions with physical meaning are those in the continuum, since they correspond to the problem of scattering with absorption.

We discuss now the qualitative differences between the real-system spectrum defined by Eqs. (27), (29), and (33), and the shell-model (or statistical-approximation) spectrum. We assume that the normalization integrals (30) corresponding to the many-particle levels are small and will compare only the single-particle excitation branches.

We equate $\varepsilon = \mu$ in (25) and express the mass operator in the form

$$M(x, x', \mu - \omega) = V(x, x') - \omega\gamma(x, x', \omega), \quad (34)$$

where ω is the excitation energy of the particles ($\omega < 0$)

or holes ($\omega > 0$); V is the value of the mass operator on the Fermi surface:

$$\left.\begin{aligned} V(x, x') &\equiv M(x, x', \mu)\ (\operatorname{Im} V(x, x') = 0; \\ \gamma(x, x', \omega) &= \delta(x-x') - \int dy \Gamma(y, x, x', \omega, \mu). \end{aligned}\right\} \quad (35)$$

It can be shown, on the basis of relations (24) and the spectral representation (11), that the function γ has no poles on the real ω axis.

We consider now an equation for the vertex Γ [it can be obtained from (21) for the two-particle Green's function and from the definition (6)]:

$$\begin{aligned} \Gamma(y, x, x', \omega, \varepsilon) &= \delta(y-x)\delta(x-x') \\ &+ \int \frac{d\varepsilon'}{2\pi i} \int dx_1 \ldots dx_4 V(x, x_1, x', x_2, \varepsilon, \varepsilon', \omega) \\ &\times G(x_2, x_3, \varepsilon'-\omega) G(x_4, x_1, \varepsilon') \Gamma(y, x_3, x_4, \omega, \varepsilon'-\omega) \end{aligned} \quad (36)$$

or in symbolic form

$$\Gamma = 1 + VGG\Gamma.$$

The vertex in (35) corresponds to an unphysical transition, induced by the external field, from a hole state to a hole state ($\omega > 0$) or from a particle state to a particle state ($\omega < 0$). It follows from general considerations that the effective interaction should be attracting for such processes.[6] This is confirmed also by perturbation theory, when allowance for the higher-order approximations in the bare attracting interaction increases the particle binding energy. We can therefore conclude that the quantity Re γ should be positive at all ω. This causes the effective depth of the "single-particle potential" $M(\mu - \omega)$ to increase with increasing excitation energy for hole states ($\omega > 0$) and to decrease their particle states ($\omega < 0$).

In concluding this section, let us dwell briefly on the energy of the ground state of the nucleus from the point of view of the results obtained above. It can be shown that the energy $\mathcal{E}_0(A)$ is expressed in terms of the single-particle Green's function as follows:

$$\begin{aligned} \mathcal{E}_0(A) &= \frac{1}{2} \sum_\nu \int dx \varphi_\nu^+(x)(T_x + \varepsilon_\nu)\varphi_\nu(x) \\ &= \operatorname{Im} \int_{-\infty}^{\mu} \frac{d\varepsilon}{2\pi} \int dx (T_x + \varepsilon) G(x, x', \varepsilon)_{x' \to x}. \end{aligned}$$

Substituting here the function $G(x, x', \varepsilon)$ in the form (28) and taking (32) into account, we obtain

$$\begin{aligned} \mathcal{E}_0(A) = \sum_\lambda \int_{-\infty}^{\mu} \frac{d\varepsilon}{2\pi} \cdot \frac{\Gamma_\lambda(\varepsilon)/2}{(\varepsilon - \varepsilon_\lambda(\varepsilon))^2 + \Gamma_\lambda^2(\varepsilon)/4} \\ \times \int dx \varphi_\lambda^+(x, \varepsilon)(T_x + \varepsilon)\varphi_\lambda(x, \varepsilon). \end{aligned} \quad (37)$$

In the interval $2\mu < \varepsilon < \mu$ the width $\Gamma_\lambda(\varepsilon) \to 0$ and the resonant factors turn into δ functions at the points $\varepsilon = \varepsilon_\nu$ corresponding to the roots of (29). In the region $\varepsilon < 2\mu$ the $\Gamma_\lambda(\varepsilon)$ differ from zero, and the integral with respect to ε from $-\infty$ to 2μ is apparently a smooth function of the mass number A. We can thus state that the fluctuations of the energies $\mathcal{E}_0(A)$ from nucleus to nucleus (shell corrections) stem from the integration region $(2\mu, \mu)$.

It should be borne in mind, however, that unlike the shell model, contributions are made to (37) also by many-particle states. We shall assume that the normalization factor is determined in the case of the quasidiscrete levels (32) with the expression $(1 - \partial \operatorname{Re} \tilde{\varepsilon}_\lambda(\varepsilon)/\partial\varepsilon)^{-1} \to \varepsilon_\lambda$, where ε_ν are the real roots (33). We note that Eq. (27), like the analogous Eq. (14), has no solutions at complex values of ε.

4. PHENOMENOLOGICAL APPROACH

We consider now the phenomenological description of quasiparticle excitations of nuclei. We must stipulate immediately that the single-particle potential V obtained by reducing the experimental data on the nucleon distribution density and on the level positions near the Fermi surface should not coincide with the mass operator M at $\varepsilon = \mu$. The primary reason is that the functional form of V is usually chosen to make the calculations relatively simple, whereas the mass operator has a complicated nonlocal coordinate dependence also for weak excitations. The frequently employed procedure of changing over from a nonlocal to a local potential by expansion in the momentum operator cannot be regarded as justified in the case of finite systems, owing to the abrupt boundary in the spatial distribution of the nucleons. To be sure, in a rough approximation it can be assumed that the nonlocal potential reduces to a local one by introducing the effective mass of the particle into the kinetic-energy operator. But even in this case it is impossible to establish experimentally the value of this parameter, since the choice of the phenomenological single-particle Hamiltonian again becomes ambiguous. Nonetheless, remaining within the framework of the phenomenological approach, we shall attempt to draw from the experimental data certain qualitative conclusions concerning the properties of the average field at relatively large hole excitations.

To simplify the comparison with the customarily employed phenomenological average field, we shall assume approximately that the mass operator expressed in the form (34) is local:

$$\begin{aligned} M(x, x', \mu - \omega) &\approx \delta(x-x') M(x, \mu - \omega) \\ &\approx \delta(x-x')[V_x - \omega\gamma(x, \omega)]. \end{aligned} \quad (38)$$

This form of M corresponds to the assumption that at small excitations ($\omega \to 0$) the average field coincides with the empirical potential V_x customarily used in the calculations.

We now compare expression (38) with the optical potential obtained by reducing the experimental data on nucleon–nucleus scattering. Indeed, it is known that the elastic scattering of nucleons by different nuclei can be accounted for in a wide energy range[19] from 30 to 200 MeV by introducing an energy dependence into the imaginary and real parts of the optical potential. Thus, for protons this dependence can be described for the real part approximately by the equation

$$\operatorname{Re} V(\varepsilon) \approx V_0 \exp(-0.007\varepsilon);\ V_0 \approx -54\ \text{MeV},$$

where ε is the energy of the incident proton and $V(\varepsilon)$ is

the value of the potential at the center of the nucleus. At energies $\varepsilon \sim 30$–40 MeV, the value of $|\omega|$ reckoned from the Fermi surface is approximately 40–50 MeV, and expression (38) for Re M at the center of the nucleus can be represented in the form

$$\mathrm{Re}\, M(\varepsilon) \approx V_0 - 0{,}4\omega;\ (\varepsilon = \mu - \omega;\ \omega < 0). \qquad (39)$$

We assume in this equation that the energy-dependent increment γ has the same spatial form as the potential V_x (say a Woods–Saxon potential). The potential for the neutrons is similar in form.[19] We can thus state that Re γ at the center of the nucleus is approximately equal to 0.4. It is obvious that with increasing ε the linear dependence on the excitation energy ω becomes unsatisfactory, since at large $|\omega|$ the value of $\omega\gamma(\omega)$ should tend to a certain constant. As to the imaginary part of $\omega\gamma(\omega)$, it can also be determined by comparison with the empirical value of Im V (ε), but it will no longer be a linear function of ω even at small ω.

A similar comparison can be made also for hole excitations of nuclei, by using the data on the (p, 2p) reaction[7] and the (e, e'p) reaction[12,13] of quasielastic knockout of high-energy protons. If we interpreted the observed proton spectra by starting from the simple shell model, then the kinematic analysis of the measurement results leads to the conclusion that the position of the deepest nuclear shells (the 1s and 1p states) does not agree with the calculations with the static average field.[4,5] For example, in the Woods–Saxon potential with the customarily assumed parameter values, the proton energy in the 1s state of ^{27}Al should be approximately 40 MeV; the experimental value, however, is about 55–60 MeV. Obviously, such a discrepancy cannot be eliminated by any reasonable variation of the average field. At the same time, an adequate agreement with the experimental data is obtained from Eq. (39) at positive ω with the same optical-potential parameter for nucleon scattering.

The fact that the values of Re γ turn out to be approximately the same for particles and holes can be understood from the following reasoning. The energy-dependent increment to the optical potential is due to excitation of different degrees of freedom by the incident particle, and to their interaction with one another. If the "external" particle is regarded only as the carrier of the excitation energy, and we neglect the fact that it is identical with the remaining nucleons of the medium, then we can conclude that the values of γ are approximately equal for particles and holes. The energy increments to the mass operators are then symmetrical about the point $\omega = 0$, in view of the chosen level from which the excitation energy is reckoned.

We now turn again to Eq. (12) for the nuclear-transition probability in the (e, e'p) reaction. Substituting in (12) the Green's function in the representation (28) and taking (32) into account, we obtain

$$R(\mathbf{p}, \mathbf{q}, \omega) = \sum_{\lambda} \frac{\Gamma_\lambda(\varepsilon)/2}{(\varepsilon - \varepsilon_\lambda(\varepsilon))^2 + \Gamma_\lambda^2(\varepsilon)/4} \left| \int dx F(x, \varepsilon)\, \varphi_\lambda(x, \varepsilon) \right|^2. \qquad (40)$$

We see therefore that the transition to the shell model is effective in the limit as $\omega\gamma \to 0$ and $\Gamma_\lambda \to 0$. Then R has δ-function singularities in the energy ε. From the microscopic point of view, this limit can occur only if the Hartree–Fock approximation is sufficiently accurate for the determination of the average field from the interaction of the pair of free particles. This approximation cannot be regarded as satisfactory for real nuclear forces. It is easily seen that allowance for the next higher approximation already leads to an energy dependence of the average field.[14]

The unique symmetry of the particle and hole states was noted already in the discussion of the phenomenological methods of determining the real part of the mass operator. It can be assumed that a similar symmetry obtains for the imaginary parts of the mass operator, at least in a certain energy region. The following argument can be advanced in favor of this statement: The real part of M is connected with the imaginary part by a dispersion relation.[17] If symmetry about the value of μ exists for the real part, then we can expect the imaginary parts also to be symmetrical. We assume that the imaginary parts in the energy regions $\varepsilon > 0$ for particles and $\varepsilon < 2\mu$ for holes are the same in absolute magnitude but of opposite sign. These assumptions cannot be rigorously proved, and their experimental verification calls for more accurate measurements.

To estimate the positions of the deep hole levels we used a mass operator in the form (39) with a different coordinate dependence.[8] It was assumed that Re γ depends weakly on ω. For the hole states, the imaginary part of M was assumed to be approximately equal to the imaginary part of the optical potential for nucleon scattering, with the sign reversed, at corresponding excitation energies. The width $\Gamma_\lambda(\varepsilon)$ was determined in first order in Im M:

$$\Gamma_\lambda(\varepsilon) \approx 2\,\mathrm{Im} \int dx \varphi_\lambda^+(x, \varepsilon)\, M(x, \varepsilon)\, \varphi_\lambda(x, \varepsilon),$$

where $\varphi_\lambda(x, \varepsilon)$ is the solution of the Schrödinger equation

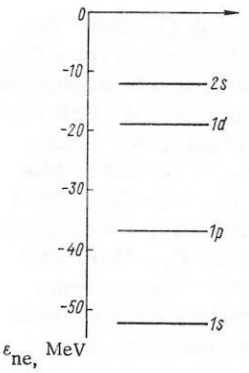

Fig. 2. Positions of hole levels in the rectangular dynamic potential (39) for ^{40}Ca.

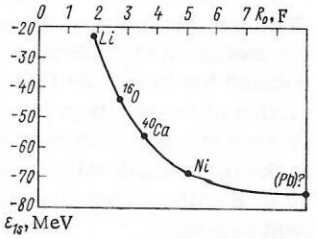

Fig. 3. Position of 1s level in a dynamic well for different nuclei.

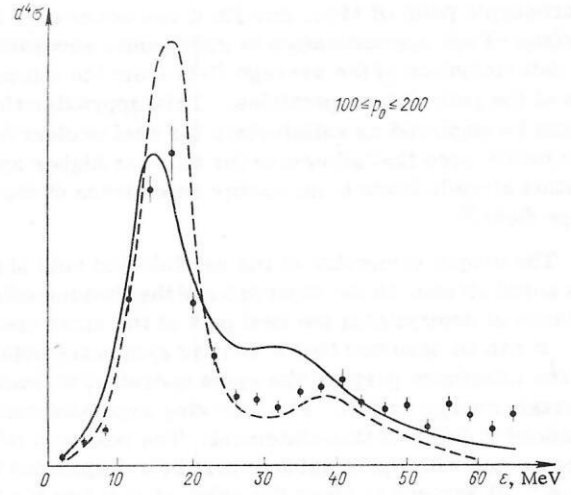

Fig. 4. Cross section of (e, e'p) reaction for ^{12}C as a function of the energy transfer ε, with averaging over the momentum $p_0 = q - p$ in the interval[12] $100 \le p_0 \le 200$ MeV/c: Dashed curve – calculated from ref. 6 with fitting of the well depth and the widths;[12] solid curve – calculation in a square well (39).

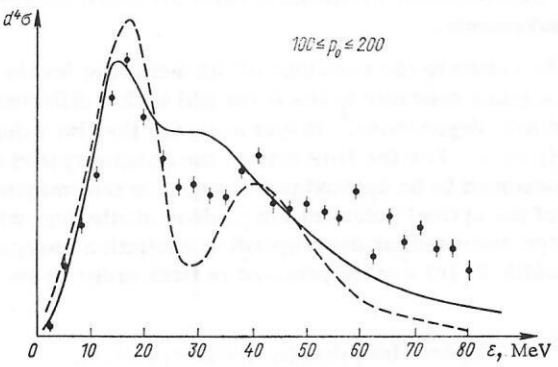

Fig. 5. The same as Fig. 4, but for ^{40}Ca.

with the real part of M in accordance with Eq. (39). The centers of the resonance peaks were determined from (33). The positions and widths of the 1s and 1p levels calculated in this manner for different nuclei duplicate satisfactorily the experimentally observed picture (Figs. 2-6). Thus, for ^{12}C we have $\varepsilon_{1s} \approx -40$ MeV and $\Gamma_{1s} \approx 12$ MeV, while for ^{40}Ca we have $\varepsilon_{1s} \approx -60$ MeV and $\Gamma_{1s} \approx 20$ MeV. With increasing mass number, the hole levels drop lower, leading to an increase in Im M, meaning also Γ_λ. Saturation sets in, however, for nuclei with A > 60: The positions and widths of the levels become practically constant (see Fig. 3). This agrees with the experimental data on the (p, 2p) reaction.[7]

These calculations were performed in the plane-wave approximation for electrons and protons, while the vertex Γ in F [see (9)] was assumed equal to the free vertex. The Coulomb distortion of the electron functions and the proton motion in the optical potential of the nucleus were accounted for in ref. 20 (Fig. 7). It turned out that the distortion of the electron functions does not alter the cross section strongly, whereas allowance for the proton motion in the optical potential strongly affects the absolute value of R without changing the overall picture of the resonant behavior.

It is obvious that with our choice of the dynamic potential in the approximation (39) which is linear in ω, Eq. (40) does not contain many-particle resonances. Only allowance for the dependence of γ on the excitation energy can lead to additional roots in Eq. (33), and it is these roots which will correspond to the many-particle states. At the same time, this enables us to determine the normalization factors $(1 - \partial \varepsilon_\lambda(\varepsilon)/\partial \varepsilon)^{-1}$, which in our approximation do not satisfy the requirement (31). We can nevertheless state that the proposed simple form of the dynamic potential and its symmetry with respect to the particle and hole excitations reflect the main qualitative regularities in the spectrum of the hole levels and explain the averaged picture of the resonant behavior of the (e, ep') cross section in experiments on quasielastic proton knock-out. In particular, calculations with Eq. (40) lead also to the observed resonance-peak asymmetry due to the energy dependence of the potential M. We note

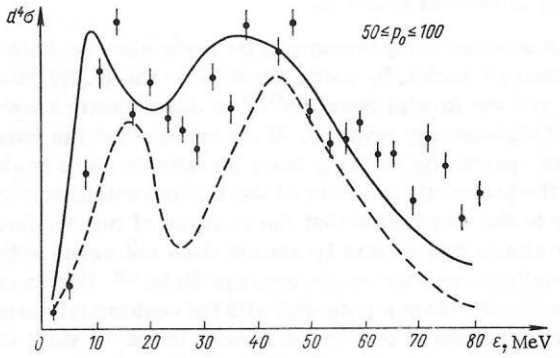

Fig. 6. The same as Fig. 4 for ^{40}Ca, but with averaging of p_0 in the interval $50 \le p_0 \le 100$ MeV/c.

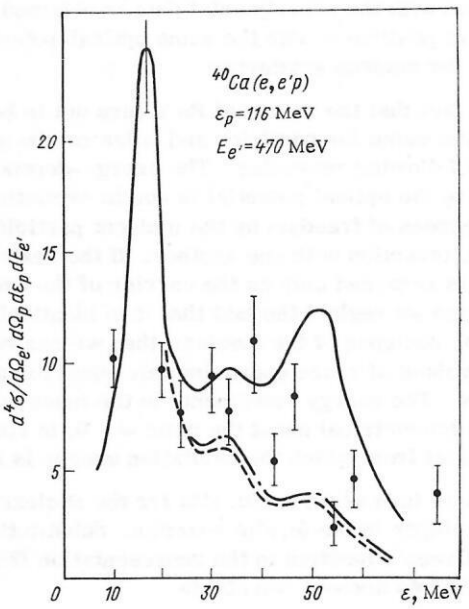

Fig. 7. Comparison of the cross section of the (e, e'p) reaction in ^{40}Ca with experiment: dashed curve – Born approximation; dash–dot curve – allowance for electron distortion; solid curve – allowance for distortions in the proton and electron channels. The hole-state functions were calculated in a Woods–Saxon potential. All the curves were made to coincide at the peak of the 1d resonance. Rescattering was not taken into account.[20] The positions and the widths were calculated in a dynamic potential (see Sec. 4).

here that the nonorthogonality of the function $\varphi_\lambda(x, \varepsilon)$ at different values of ε is also the consequence of such a dependence, which results in the generalized orthonormalization condition (26).

5. RESCATTERING EFFECTS AND (e, e'n) REACTIONS

We consider now the role of the rescattering of an excited proton by a hole, or in other words we take into account their interaction and estimate the value of Γ. At large energy transfer (on the order of hundreds of MeV), it is reasonable to regard the vertex Γ as a smooth function of the energy variables ω and ε contained in it. We can therefore expect rescattering in (e, e'p) reactions with high-energy protons to change only the absolute values of the cross sections, but not their resonant dependence on the excitation energy of the residual nucleus.

In Eq. (36) for the vertex Γ, the effective interaction V is responsible for the transitions of the excited nucleon and the residual nucleus to different states of the decaying compound system. Since the channel with proton emission is not the only open channel at large energy transfers, V should be a complex quantity. It is obvious that the determination of the interaction V, which includes complicated excitation processes, followed by the solution for the equation of the vertex Γ, is a most difficult problem. Let us estimate the value of Γ in the case of large energies ω and ε_p and use the following approximations. First, we regard the excited nucleon as a particle that is not identical with the remaining nucleons of the nucleus. Second, we apply to the description of the nuclear system the Thomas–Fermi approximation[14] and neglect the difference between the occupation numbers of the protons and neutrons. These approximations are justified if $\omega \gg \varepsilon_F$ and if the "external" nucleons have momenta $p \gg p_F$ (ε_F and p_F are the Fermi energy and momentum).

Under these simplifying assumptions, the Green's functions G_{ex} and G of the external and nuclear nucleons, respectively, can be expressed in the form:[14]

$$\left.\begin{aligned} G_{ex}(x, x', \varepsilon) &= \int \frac{d\mathbf{p}}{(2\pi)^3} \\ &\times \frac{\delta_{\sigma\sigma'} \exp[i\mathbf{p}(\mathbf{r}-\mathbf{r}')]}{\varepsilon - \varepsilon_{\mathbf{p}} - V + i\alpha} \quad (\varepsilon_{\mathbf{p}} = p^2/2m); \\ G(x, x', \varepsilon) &= \int \frac{d\mathbf{p}}{(2\pi)^3} \cdot \frac{\delta_{\sigma\sigma'} \exp[i\mathbf{p}(\mathbf{r}-\mathbf{r}')]}{\varepsilon - \varepsilon_{\mathbf{p}} - V + i\alpha \operatorname{sign}(p - p_F)}, \end{aligned}\right\} \quad (41)$$

where V is the single-particle potential in the nucleus. This yields for the momentum representation of Eq. (36)

$$\Gamma_i(Q, P) = \delta_{ip} + 2 \sum_{j=p,\, n} \int \frac{d^4P'}{(2\pi)^4 i} V_{ij}(P, P', Q) G(P'+Q) G(P') \times \Gamma_j(Q, P'+Q). \quad (42)$$

The subscript i takes on the values n and p; P = (p, ε) is the 4-momentum of the excited nucleon, and Q = (q, ω) is the 4-momentum transfer. At large P and Q it can be assumed that the interaction V_{ij} (P, P', Q) depends little on P and P' and is the same for arbitrary i and j, i.e., V_{ij} (P, P', Q) $\approx$ V (Q).

The following arguments can be advanced in favor of this form of V. If the momentum p and the energy ε_p of the excited nucleon greatly exceed the Fermi momentum and energy, then V should depend little on the motion of the nucleons of the medium, i.e., on P'. Consequently, V depends only on Q, inasmuch as for $p \gg p'$ we can assume that $p \approx Q$. In this approximation of V, the vertex Γ_i also depends only on Q and Eq. (42) can be easily solved:

$$\Gamma_p(Q) = 1 + \Gamma_n(Q); \quad \Gamma_p(Q) = \frac{1}{1 - V(Q)\,\Pi(Q)}, \quad (43)$$

where Π (Q) is a polarization operator defined by the relation

$$\Pi(Q) = 2 \int \frac{d^4P}{(2\pi)^4 i} G_{ex}(P+Q) G(P) \approx \frac{2}{(2\pi)^3} \int\limits_{p<p_F} \frac{d\mathbf{p}}{\omega - \varepsilon_{\mathbf{p}+\mathbf{q}} + \varepsilon_{\mathbf{p}} + i\alpha}. \quad (44)$$

Substituting (43) into Eq. (8) for the (e, e'p) and (e, e'n) reactions, we obtain

$$R_i(\mathbf{p}, \mathbf{q}, \omega) = \pi |\Gamma_i(\mathbf{q}, \omega)|^2 \times \sum_\lambda \left| \int dx \psi^+_{i\mathbf{p}}(x) \varphi_{i\lambda}(x) \exp(i\mathbf{q}\mathbf{r}) \right|^2 \delta(\varepsilon - \varepsilon_{i\lambda}).$$

This form of the transition probability is convenient for an experimental determination of the rescattering effects by comparing the cross sections of the neutron and proton yields:

$$\left.\begin{aligned} \frac{d^4\sigma_n}{d^4\sigma_p} &= \left| \frac{\Gamma_n(\mathbf{q}, \omega)}{\Gamma_p(\mathbf{q}, \omega)} \right|^2 = |\Gamma_n(\mathbf{q}, \omega)|^2; \\ \frac{d^4\sigma_p - d^4\sigma_n}{d^4\sigma_n} &= \frac{1 - 2\,\mathrm{Re}\,\Gamma_n}{|\Gamma_n|^2}. \end{aligned}\right\} \quad (45)$$

We have used here relations (43). If we neglect for purposes of estimates all but the pair interactions and set the potential V equal to its value for free nucleons [$V_q = (\pi^2/mp_F) F_q$; $F \approx -2, -3$)], then we find that for transferred energies ω and momenta on the order of several hundred MeV the ratio of the neutron yield in the (e, e'n) reaction to the proton yield in the (e, e'p) reaction can reach 20-30%.

CONCLUSION

From the foregoing data we can conclude the extent to which valuable information can be obtained from experiments on electron scattering. The advantage of electrons over, say, nucleons is obvious: Reactions with electrons can yield directly information on nuclear transitions. This pertains primarily to the (e, e'p) quasielastic nucleon knock-out reactions. It therefore becomes possible to use a new phenomenological approach to the determination of the optical model for hole states, so that processes at high energies can be analyzed on a more rigorous basis with a more accurate average field. At the same time this will undoubtedly add to our knowledge of the structure of the ground state and of the role of correlative interactions. Thus, in the study of different excitations of the particle–hole type it is necessary to take into account the energy dependence of the average field and of the effective interaction, and this will lead to a more complete description of the phenomena, for example, with respect to the lifetimes of such states. Of course, we cannot state that all the refinements of the average field are connected with its energy dependence, for it is important to investigate also other characteristics such as nonlocality, the spin

and isospin characteristics of the field, etc.

[1)] We use the system of units with $\hbar = c = 1$.

[2)] In essence, these equations must be regarded as an additional requirement on the choice of a model basis.

[3)] Unless otherwise stipulated, the coordinates x here and below include also the isospin variables τ [$x \equiv (\sigma, \tau, \mathbf{r})$].

[4)] This is precisely the function that enters in expression (9) for $R(\mathbf{p}, \mathbf{q}, \omega)$.

[5)] It should be borne in mind that in the problem of eigenvalues in a complex potential, the functions $\varphi_\lambda^+(x, \varepsilon)$ satisfy an equation conjugate to (27), in which M^+ must be replaced by M.

[6)] For real particle–hole excitations, the effective interaction V should be repelling because of the condition of self-contraction of the system.[17]

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