Nonlocal quantum electrodynamics

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Fiz. El. Chast. Atom. Yad., 5, 223-256 (1974)

Nonlocal quantum electrodynamics is examined. The axioms for the quantum theory of interacting electromagnetic and electronic fields are formulated. An S matrix is constructed which satisfies unitarity, causality, covariance, gauge invariance, and which is finite to all orders of perturbation theory. This nonlocal theory describes the interactions of a local electromagnetic field with an extended electron which has the shape of a sphere of radius l.

1. INTRODUCTION

The greatest success achieved in quantum field theory has been in quantum electrodynamics which describes the interactions of photons and fermions (electrons, μ mesons, protons, and so on). It has been possible to construct in consistent fashion an S matrix which satisfies all the general requirements of a local quantum field theory, and we have the mathematical apparatus to calculate any of the effects of photons interacting with charged fermions.

Quantum electrodynamics, moreover, agrees brilliantly with experiment. Not a single effect has been found to date which cannot be included within the scope of local quantum electrodynamics.

In recent years, there has been growing interest in the question of the limits to the validity of quantum electrodynamics and the limits on its applicability (see the review by Brodsky and Drell,[1] the paper by Petrunykin[2] at the Second International Conference on Nonlocal Quantum Field Theory, and the review by Solov'ev[3] presented to the 15th International Conference on Elementary Particle Physics in Kiev).

In order to evaluate the experimental consequences to be expected due to nonlocal effects in quantum electrodynamics, it will be necessary to have a good, consistent method for constructing the nonlocal theory which satisfies all the general quantum field theory requirements: covariance, unitary, causality, gauge invariance, and finiteness.

The usual method of inserting nonlocal modifications into the theory is to introduce form factors into the individual structural elements of the Feynman diagrams: that is, the propagators, vertices, and so on. Usually the form factor is a function like the Pauli–Villars regularization. For example, the photon propagator is changed to the following;

$$D_{\mu\nu}(k)=\delta_{\mu\nu}\left[\frac{1}{-k^2-i\varepsilon}-\frac{1}{M^2-k^2-i\varepsilon}\right], \qquad (1.1)$$

where M is the momentum cutoff which is related to the "elementary" length l by the equation $l = 1/M$.

There is another, extremely simple method of introducing nonlocal features. In the local field theory the difference between an experimental cross section for some process and the calculated value is parametrized in the following manner:

$$\frac{\sigma_{exp}}{\sigma_{theor}}=1+\frac{m^4}{M^4}.$$

One then determines the limits over which the new parameter M may vary.

It seems that a similar parametrization may be very remotely connected with a consistent description of a nonlocal interaction. In general the parameter M must be different in each different experiment. Therefore, the results of one experiment are in no way related to the results of another.

The available methods of constructing an internally consistent nonlocal electrodynamics are based on assumptions borrowed from the regularization method of Pauli and Villars (see refs. 4-6 and the earlier works cited therein).

The main source of the difficulties encountered in constructing a theory with the Pauli–Villars regularization from factor [as in Eq. (1.1)] is that additional singularities appear in the amplitudes of certain physical processes. As a result the unitarity and causality of the S matrix are destroyed, and there is difficulty in fulfilling the gauge invariance of the theory. In order to rescue the situation one must turn to theories having an indefinite metric;[6,7] further, one must formulate rather clumsy rules for calculating matrix elements, and these can be used only in the lowest orders of perturbation theory.

It seems to us that the class of meromorphic functions to which the Pauli–Villars regularization belongs is not acceptable from the point of view of the requirements imposed on the S matrix in quantum field theory.

In the present paper we present a variant of nonlocal quantum electrodynamics which is free from the objections stated above. A covariant description will be presented of the interactions of photons with an extended electron, i.e., an electron which is a sphere of radius l, with some distribution of charge density inside the sphere. The S matrix describing this interaction satisfies all the requirements of quantum field theory: covariance, unitarity, causality, gauge invariance, and finiteness.

In order to describe the experiment, we propose, within the realm of a self-consistent quantum field theory, a phenomenological scheme in which the parameters are an elementary length l (which is in fact the electron's size) and a function which gives the charge distribution within a sphere of radius l. Since l is quite small, the equations describing various physical processes will contain moments of the charge distribution function. The advantage of this scheme is that both l and the parameters describing the electron charge distribution will be unique for all experiments in quantum electrodynamics.

In the present paper we survey the ideas and mathematical methods for constructing a nonlocal quantum electrodynamics. The article does not analyze the experimental situation from the new point of view; this subject will be treated in a subsequent paper.

2. INTRODUCTION ON NONLOCALITY INTO ELECTRODYNAMICS

It is well known (see refs. 8, 9) that the interactions of charge particle fields with an electromagnetic field are determined by the condition of gauge invariance. This means that in the description of the electromagnetic field by the potentials A_μ the physical content of the theory is not altered by a gauge transformation of the form

$$A_\mu(x) \to A_\mu(x) + \partial_\mu f(x), \tag{2.1}$$

because the electromagnetic field tensor

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x)$$

is invariant with respect to such a gauge transformation.

It is usually required that the Lagrangian for the system of charged fields Φ_j interacting with the electromagnetic field A_μ be invariant relative to the following gauge transformations:

$$\left.\begin{aligned} &A_\mu(x) \to A_\mu(x) + \partial_\mu f(x);\\ &\Phi_j(x) \to \Phi_j(x)\exp\{iq_j f(x)\};\\ &\Phi_j^*(x) \to \Phi_j^*(x)\exp\{-iq_j f(x)\}, \end{aligned}\right\} \tag{2.2}$$

where q_j denotes the charge of the field Φ_j. The invariance of the total Lagrangian $\mathcal{L}(\Phi_j, \Phi_j^*, A_\mu)$ relative to the transformations of Eq. (2.2) leads to current conservation:

$$\partial_\mu J_\mu(x) = 0, \tag{2.3}$$

where

$$J_\mu(x) = i\sum_j q_j\left\{\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Phi_j^*(x))}\Phi_j^*(x) - \frac{\delta\mathcal{L}}{\delta(\partial_\mu\Phi_j(x))}\Phi_j(x)\right\}. \tag{2.4}$$

It must be emphasized that the transformations in (2.2) already imply that the interaction of the electromagnetic field with the charged fields is local. The only electromagnetic characteristic of the field Φ_j is its charge which enters into the transformations in (2.2). The explicit form of the Lagrangian for the interactions of an electromagnetic field with charged fields is usually selected according to the principle of "minimality" which states that when the variable field Φ_j is acted on by the operator ∂_μ the result takes the form

$$\left.\begin{aligned} &\partial_\mu\Phi_j(x) \to \{\partial_\mu - iq_j A_\mu(x)\}\Phi_j(x)\\ &\partial_\mu\Phi_j^*(x) \to \{\partial_\mu + iq_j A_\mu(x)\}\Phi_j^*(x). \end{aligned}\right\} \tag{2.5}$$

The generally accepted procedure just outlined is the factor which causes all the difficulty in a local quantum field theory (see refs. 8, 9).

Our nonlocal generalization of the theory for electromagnetic interactions consists of the following. We assume that instead of the gauge transformations given in Eq. (2.2) we have these:

$$\left.\begin{aligned} &A_\mu(x) \to A_\mu(x) + \partial_\mu f(x);\\ &\Phi_j(x) \to \Phi_j(x)\exp\left\{iq_j\int dx' K_j\left(\frac{x-x'}{l_j}\right)f(x')\right\};\\ &\Phi_j^*(x) \to \Phi_j^*(x)\exp\left\{-iq_j\int dx' K_j\left(\frac{x-x'}{l_j}\right)f(x')\right\}, \end{aligned}\right\} \tag{2.6}$$

where $K_j(x/l_j)$ is some real function, a generalized form factor, normalized according to the condition

$$\int dx K_j(x/l_j) = 1. \tag{2.7}$$

The parameter l_j (an elementary length) characterizes the extent of the region of the nonlocal interaction.

The transformations of Eq. (2.6) differ from those in Eq. (2.2) in that the phases of the charged field Φ_j transformations are altered. For gauge transformations of constant phase (f = const) the transformations in (2.6) coincide with the more common choices of Eq. (2.2) since the functions K_j are normalized according to Eq. (2.7).

The transformations of Eq. (2.6), with the arbitrary function $f(x)$, formally ensure conservation of the electromagnetic current of the charged fields (2.3), where we now have

$$J_\mu(x) = i\sum_j q_j\int dx' K_j\left(\frac{x-x'}{l_j}\right) \times\left\{\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Phi_j^*(x'))}\Phi_j^*(x') - \frac{\delta\mathcal{L}}{\delta(\partial_\mu\Phi_j(x'))}\Phi_j(x')\right\}. \tag{2.8}$$

The local variant is obtained when $K_j[(x-x')/l_j] = \delta^4(x-x')$.

In place of Eq. (2.5) the principle of minimality of the electromagnetic interaction in the nonlocal case will appear in this form:

$$\left.\begin{aligned} &\partial_\mu\Phi_j(x) \to \left\{\partial_\mu - iq_j\int dy K_j((x-y)/l_j)A_\mu(y)\right\}\Phi_j(x);\\ &\partial_\mu\Phi_j^*(x) \to \left\{\partial_\mu + iq_j\int dy K_j((x-y)/l_j)A_\mu(y)\right\}\Phi_j^*(x). \end{aligned}\right\} \tag{2.9}$$

What is the physical content of the gauge transformation for the charged fields in Eq. (2.6)? The constant q_j determines the charge of the field Φ_j. The nonlocal function $K_j(x/l_j)$ characterizes the charge distribution of the field $\Phi_j(x)$ in x space. This question will be considered in greater detail below (see Sec. 4).

For interactions of an electromagnetic field with the electron–positron field the total Lagrangian of the classical fields is[1)]

$$\mathcal{L}(x) = \mathcal{L}_{em,0}(x) + \mathcal{L}_{e,0}(x) + \mathcal{L}_I(x), \tag{2.10}$$

where

$$\left.\begin{aligned} &\mathcal{L}_{em,0}(x) = -\tfrac{1}{2}\partial_\nu A_\mu(x)\,\partial_\nu A_\mu(x);\\ &\mathcal{L}_{e,0}(x) = -\bar\psi(x)(i\hat\partial + m)\psi(x); \end{aligned}\right\} \tag{2.11}$$

$$\mathscr{L}_I(x) = e\bar{\psi}(x)\gamma_\mu\psi(x)\mathfrak{A}_\mu(l, x). \tag{2.12}$$

Here

$$\mathfrak{A}_\mu(l, x) = \int dy K((x-y)/l) A_\mu(y). \tag{2.13}$$

The Lagrangian $\mathscr{L}_{em,0}(x)$ for the free electromagnetic field takes into account the Lorentz condition $\partial_\mu A_\mu(x) = 0$.

In this case the equations of (2.6) can be written as

$$\left.\begin{aligned} &A_\mu(x) \to A_\mu(x) + \partial_\mu f(x);\\ &\psi(x) \to \psi(x)\exp\left\{ie\int dy K((x-y)/l) f(y)\right\};\\ &\bar{\psi}(x) \to \bar{\psi}(x)\exp\left\{-ie\int dy K((x-y)/l) f(y)\right\}. \end{aligned}\right\} \tag{2.14}$$

Here e is the electron charge and $\alpha = e^2/4\pi = 1/137$ ($\hbar = c = 1$).

In the following we will consider only the quantum electrodynamics of electrons and positrons.

3. STATEMENT OF THE PROBLEM

The following problems arise when studying the Lagrangian of (2.10) with the gauge transformations of (2.14).

1. How does one quantize Eq. (2.10)? Is it possible to construct a canonical formalism and carry out the complete dynamical description of the quantized fields in Eq. (2.10)?

2. How is the S matrix constructed? What conditions must it satisfy? Can it be made finite?

3. What is the physical meaning of the function $K(x/l)$? What is the origin of this function? Is it unique?

At present there are no answers to the first group of questions. They will be the subject of subsequent investigation.

The S matrix, which we propose to construct in quantum electrodynamics, is not the result of solving the field-quantized Schrödinger equation. We shall pursue an axiomatic construction of the theory, as suggested by Bogolyubov, Shirkov, Medvedev and Polivanov.[8,10] This approach to the theory comes from a program suggested by Heisenberg[11] in which one considers only those elements of the S matrix which correspond to transitions between asymptotically stable states. In quantum electrodynamics these matrix elements can be represented by a functional expansion in terms of normal products of the asymptotically noninteracting electromagnetic field $A_\mu(x)$ and the electron field $\psi(x)$:

$$\begin{aligned} S = &\sum_{n,m,l}\frac{1}{n!\,m!\,l!}\int dx_1\ldots\int dx_n\\ &\times\int dy_1\ldots\int dy_m\int dz_1\ldots\int dz_l\\ &\times F^{\mu_1\ldots\mu_n}_{\alpha_1\ldots\alpha_m;\,\beta_1\ldots\beta_l}(x_1\ldots x_n;\ y_1\ldots y_m;\ z_1\ldots z_l)\\ &\times{:}A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\psi_{\alpha_1}(y_1)\ldots\psi_{\alpha_m}(y_m)\\ &\times\bar{\psi}_{\beta_1}(z_1)\ldots\bar{\psi}_{\beta_l}(z_l){:}\\ =&\sum_{n,m,l}\frac{1}{n!\,m!\,l!}\int dk_1\ldots\int dk_n\\ &\times\int dp_1\ldots\int dp_m\int dq_1\ldots\int dq_l\\ &\times\tilde{F}^{\mu_1\ldots\mu_n}_{\alpha_1\ldots\alpha_n;\,\beta_1\ldots\beta_l}(k_1\ldots k_n;\ p_1\ldots p_m;\ q_1\ldots q_l)\\ &\times{:}A_{\mu_1}(k_1)\ldots A_{\mu_n}(k_n)\psi_{\alpha_1}(p_1)\ldots\psi_{\alpha_n}(p_n)\\ &\times\bar{\psi}_{\beta_1}(q_1)\ldots\bar{\psi}_{\beta_l}(q_l){:}, \end{aligned} \tag{3.1}$$

where the operators $A_\mu(x)$ and $\psi(x)$ satisfy these free equations

$$(i\hat{\partial} - m)\psi(x) = 0, \tag{3.2}$$

$$\Box A_\mu(x) = 0 \tag{3.3}$$

supplemented by the Lorentz condition

$$\partial_\mu A^{(-)}_\mu(x)|\ldots\rangle = 0. \tag{3.4}$$

This version of the theory is based on the following physical assumptions.

I. Physical States

1. The asymptotic states of the system contain noninteracting particles which are infinitely far apart, and are described by the amplitudes $|\ldots\rangle$ which are elements in a Hilbert space.

2. Relativistic covariance. Under the action of the Lorentz group L the state amplitudes are transformed with some help from its unitary representation U_L.

3. There exists a vacuum state for which $U_L|0\rangle = |0\rangle$, and it is unique.

4. Completeness and spectral character. There exists a system of proper amplitudes for the 4-momentum states $|n, k_n\rangle$ which corresponds to negative energy values, and together with the amplitude $|0\rangle$ they are complete.

5. The asymptotic states satisfy the Lorentz condition

$$\partial_\mu A^{(-)}_\mu(x)|\ldots\rangle = 0,$$

i.e., longitudinal and time-like photons are not included.

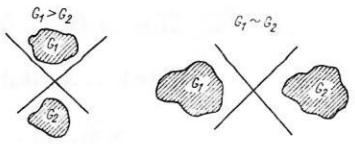
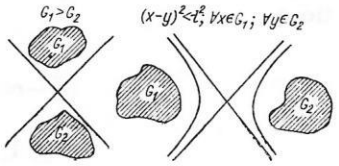
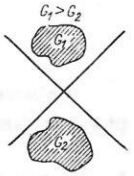
II. S Matrix

1. Unitarity $SS^+ = S^+S = 1$.

2. Causality. We introduce the operations of "turning on" and "turning off" the interactions. We assume the presence of a function e(x) which ranges in value over the interval (0, e), and which characterizes the degree to which the interaction is turned on. The S matrix is a functional of the turning-on function S[e] where $S[0] \equiv 1$ when $e(x) = 0$. We now assume that $e_1(x)$ is nonzero in the region $G_1 \subset R^4$ and $e_2(x)$ is nonzero in the region $G_2 \subset R^4$. If the S matrix corresponds to a local, microcausality theory, then S[e] satisfies the condition

$$S[e_1 + e_2] = S[e_1]\,S[e_2] \tag{3.5}$$

TABLE 1

$S[e_1+e_2]=S[e_1]S[e_2]$ under the following relationships between the regions	
I. A local, microcausality theory	
II. Microcausal, nonlocal theory. The dimension of the region of nonlocal interaction is l	
III. Microcausal, nonlocal theory	

where $G_1 \gtrless G_2$. This means that all the points in G_1 are either spatially similar to those in G_2 or they lie in the future relative to the points of G_2.

If the theory is nonlocal but is microcausal, then S[e] satisfies Eq. (3.5) when the region G_1 is in the future with respect to all the points of G_2; i.e. $G_1 > G_2$. The dimension of the region of nonlocality will be l if Eq. (3.5) is satisfied when $G_1 \sim G_2$ and $(x-y)^2 < -l^2$ for $\forall x \in G_1$ and $\forall y \in G_2$.

The causality conditions for the various cases can be written in table form. The S matrix which we have constructed will satisfy the microcausality condition in form III of Table 1 below. However, a proof of this statement will not be presented in this paper. The proof will be published separately.

3. Gauge invariance.

$$\frac{\partial}{\partial x_{1\mu_1}} \cdots \frac{\partial}{\partial x_{2\mu_r}} \left(\frac{\delta^r S}{\delta A_{\mu_1}(x_1) \ldots \delta A_{\mu_r}(x_r)} \right) = 0 \qquad (3.6)$$

or, in the expansion of Eq. (2.1),

$$k_{1\mu_1} \ldots k_{r\mu_r} \bar{u}_{\beta_1}(q_1) \ldots \bar{u}_{\beta_l}(q_l) \times \tilde{F}^{\mu_1 \ldots \mu_n}_{\alpha_1 \ldots \alpha_m;\, \beta_1 \ldots \beta_l}(\ldots)\, u_{\alpha_1}(p_1) \ldots u_{\alpha_m}(p_m) = 0, \qquad (3.7)$$

where $u_\alpha(p)$ is a Dirac spinor which satisfies the equation

$$(\hat{p} - m)\, u(p) = 0. \qquad (3.8)$$

4. Integrability. The coefficients of the function $F^{\mu_1 \ldots \mu_n}_{\alpha_1 \ldots \alpha_m;\, \beta_1 \ldots \beta_l}(\ldots)$ are integrable on some suitable space of basis functions.

5. Stability. If the vacuum state $|\alpha\rangle$ is a single-particle state or some other stable state, then

$$S|\alpha\rangle = |\alpha\rangle.$$

6. Correspondence principle. For infinitely small e(x) the S matrix must have the form

$$S[e] = 1 + i \int dx e(x) \bar{\psi}(x) \hat{\mathfrak{A}}(l, x) \psi(x), \qquad (3.9)$$

where

$$\hat{\mathfrak{A}}(l, x) = \int dy K((x-y)/l)\, \hat{A}(y).$$

In this approach to the construction of physically acceptable theory it is sufficient to indicate a method of constructing an S matrix which will provide a scattering matrix which satisfies all the requirements enumerated above.

Our problem is to obtain an S matrix in the form of a perturbation series. In formal analogy with the local theory the S matrix can be written as

$$S[e] = T \exp \left\{ i \int dx e(x) \bar{\psi}(x) \hat{\mathfrak{A}}(l, x) \psi(x) \right\}. \qquad (3.10)$$

To obtain the perturbation series we must expand the exponent in Eq. (3.10) in a series in e(x) and then convert to products of N operators for the fields $A_\mu(x)$ and $\psi(x)$, as provided by the Wick theorem.

We assume, by definition, that the "chronological" convolution of operators for the fermion electron field has the usual form:

$$S(x-y) = \langle 0 | T(\psi(x) \bar{\psi}(y)) | 0 \rangle = \frac{1}{(2\pi)^4 i} \int \frac{dp\, e^{-ip(x-y)}}{m - \hat{p} - i\varepsilon}, \qquad (3.11)$$

while the "causal" functions of the "smeared out" electromagnetic field are written as

$$D_{\mu\nu}(x-y) = \delta_{\mu\nu} D(x-y) = \overline{\hat{\mathfrak{A}}_\mu(l, x)\, \hat{\mathfrak{A}}_\nu(l, y)} = \int dx' \int dy' K\left(\frac{x-x'}{l}\right) K\left(\frac{y-y'}{l}\right) \langle 0 | T(A_\mu(x') A_\nu(y')) | 0 \rangle = \frac{\delta_{\mu\nu}}{(2\pi)^4 i} \int \frac{d^4k\, |\tilde{K}(l^2k^2)|^2\, e^{-ik(x-y)}}{-k^2 - i\varepsilon}, \qquad (3.12)$$

where

$$\tilde{K}(l^2k^2) = \int dx\, e^{ikx} K\left(\frac{x}{l}\right). \qquad (3.13)$$

Thus, we obtain the usual series of perturbation theory, with the exception that the causal functions of the photon field,

$$\Delta_{\mu\nu}(x-y)=\delta_{\mu\nu}\Delta(x-y)=\frac{\delta_{\mu\nu}}{(2\pi)^4 i}\int\frac{dke^{-ik(x-y)}}{-k^2-i\varepsilon} \qquad (3.14)$$

are replaced by the functions in Eq. (3.13).

Let us now discuss the formal definition of the S matrix, as presented in Eq. (3.10). We must point out that in the local quantum field theory (and even more so in the nonlocal version) the T product does not really represent the strict time ordering of the field operators because there are ultraviolet divergences present in the perturbation series.

Symbolically, the standard method for constructing a finite S matrix from perturbation theory can be written in this form:

$$S=\lim_{\Lambda\to\infty}T_\Lambda\exp\left\{i\int dx\mathcal{L}_I(x,\Lambda)\right\}, \qquad (3.15)$$

Here we have used the following notation. The symbol T_Λ denotes the chronological product of field operators plus some regularization procedure (this regularization procedure is usually the Pauli–Villars method combined with various possible modifications) which makes all the matrix elements of the perturbation series finite. Absurd diverging expressions arise when the regularization parameter Λ goes to infinity. To compensate for these expressions, a number of counterterms (which depend on Λ as parameter) are introduced into the interaction Lagrangian. The operator structure of the counterterms and their explicit dependence on Λ are selected such that they completely compensate all expressions which diverge in the amplitude calculations when $\Lambda\to\infty$. Therefore, the limit in Eq. (3.15) exists.

What then is wrong with this approach to the S matrix? The nonphysical parameter Λ enters both into the T_Λ of the regularization procedure and into the interaction Lagrangian $\mathcal{L}_I(x,\Lambda)$. This results in a physically meaningless interaction Lagrangian because the parameter Λ has no physical meaning. We find that the Lagrangian for the interacting fields, which determines the physics of the process, thus depends on the mathematical apparatus used to calculate the S matrix elements, which then loses it original physical meaning. Therefore $\mathcal{L}_I(x,\Lambda)$ must be considered as some artificial object which is selected to ensure the reality of the limit in Eq. (3.15). Thus, in this procedure only the S matrix has physical sense (see ref. 8).

We therefore think that it would be advantageous to select a regularization procedure dependent on Λ in such a manner that the meaningless counterterms do not appear in the interaction Lagrangian. Therefore, there must be a function

$$S=\lim_{\Lambda\to\infty}T_\Lambda\exp\left\{i\int dx\mathcal{L}_I(x)\right\} \qquad (3.16)$$

in which $\mathcal{L}_I(x)$ does not depend on Λ. This idea was first put forward by Slavnov[12] who studied the quantum theory of self-acting scalar fields.

We will proceed in this manner in the case at hand. A subsidiary regularization procedure is formulated such that the limit in Eq. (3.16) exists and that the S matrix will satisfy all the necesary requirements in every order of perturbation theory.

The interaction Lagrangian $\mathcal{L}_I(x)$ remains finite. If it becomes necessary to carry out some renormalization in the theory, the procedure will not be for removal of divergences, but rather it will constitute a transformation from less useful to more useful physical parameters. All renormalization constants will be finite.

Thus, in nonlocal quantum electrodynamics the interaction Lagrangian takes the form[8]

$$\begin{aligned}\mathcal{L}_I(x)=e:\bar\psi(x)\,\hat{\mathfrak{A}}(l,x)\,\psi(x):\\ +e(Z_1-1):\bar\psi(x)\,\hat{\mathfrak{A}}(l,x)\,\psi(x):\ -\delta m:\bar\psi(x)\,\psi(x):\\ +(Z_2-1):\bar\psi(x)(i\hat\partial+m)\psi(x):\ -(Z_3-1)\frac{1}{4}:F_{\mu\nu}(x)F_{\mu\nu}(x):.\end{aligned} \qquad (3.17)$$

In the nonlocal theory the renormalization constants Z_1, Z_2, Z_3 and δm will be finite. Because of the Ward identity, $Z_1 = Z_2$.

4. THE NONLOCAL FORM FACTOR K(x − y)

We now turn to the fundamental problem in the nonlocal theory: What is the class of functions which must be examined to produce the form factor $K[(x-y)/l]$ used to construct the S matrix which will satisfy all the requirements stipulated above?

Study of the nonlocal theory of a scalar quantum field has shown[13] that the most important factor in the choice of a class of allowable form factors is that the S matrix be unitary. It was found that a unitary S matrix can be constructed if the form factor $K[(x-y)/l]$ is relativistically invariant and if the function

$$\tilde K(l^2k^2)=\int dxe^{ikx}K\left(\frac{x}{l}\right) \qquad (4.1)$$

is, first, a completely analytic function in the k^2 plane and, second, if it decreases rapidly enough in the space-like direction, i.e., when $k^2\to-\infty$.

For the quantum electrodynamics case we shall assume that the form factor $V(-l^2k^2)=|\tilde K(l^2k^2)|^2$ satisfies the following conditions:

1. V(z) is an integral function in the complex z plane having a finite order of growth $1/2\le\rho<\infty$;

2. $[V(z)]^* = V(z^*)$;

3. $V(x)\ge 0$ for real x;

4. $V(0) = 1$;

5. $|V(u)| = 0(1/u^\alpha)$ for some $a > 0$ when $u\to+\infty$.

These conditions are sufficient to ensure that all the matrix elements in the perturbation theory are finite (see below).

Before discussing the physical meaning of the form factor, we wish to present a number of equations which prove useful in the subsequent calculations. It will be shown below that it is possible to transform to a Eucli-

dean metric in the proposed theory. In Euclidean x space the photon propagator (3.12) can be written as

$$D(x_E^2) = \frac{1}{(2\pi)^4} \int \frac{d^4k_E V(l^2k_E^2)}{k_E^2} e^{ik_E x_E}, \quad (4.2)$$

where k_E and x_E are Euclidean vectors ($x_E^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 \geq 0$). We now introduce the propagator for a "point" photon:

$$\Delta(x_E^2) = \frac{1}{(2\pi)^4} \int \frac{d^4k_E e^{ik_E x_E}}{k_{13}^2} = \frac{1}{(2\pi)^2} \cdot \frac{1}{x_E^2}. \quad (4.3)$$

We write the function $D(x_E^2)$ in the form

$$D(x_E^2) = \frac{1}{(2\pi)^2} \cdot \frac{1}{x_E^2} U\left(\frac{x_E^2}{l^2}\right). \quad (4.4)$$

The function $U(u_E^2/l^2)$ is then related to the form factor $V(l^2k_E^2)$ by the equation

$$U\left(\frac{x_E^2}{l^2}\right) = V(0) - \frac{1}{\pi^2} \int d^4k_E e^{ik_E x_E} V''(k_E^2 l^2)$$
$$= 1 + \int_0^\infty du f_0\left(\sqrt{u \frac{x_E^2}{l^2}}\right) V'(u). \quad (4.5)$$

By using the conditions 1-5 we obtain

$$U(u) = \begin{cases} O(u^a), & u \to 0, \\ 1 + O(\exp\{-u^\gamma\}), & u \to \infty, \end{cases} \quad (4.6)$$

where $\gamma = \rho/(2\rho - 1) > 1/2$ when $\rho > 1/2$ and $U(u) = 1$ when $u > 1$ for $\rho = 1/2$. Then, for $a > 1$ we have, for all $x_E^2 > 0$,

$$|D(x_E^2)| = \left| \frac{1}{(2\pi)^2} \cdot \frac{U\left(\frac{x_E^2}{l^2}\right)}{x_E^2} \right| \leq \left. \frac{U\left(\frac{x_E^2}{l^2}\right)}{(2\pi)^2 x_E^2} \right|_{x^2=0}$$
$$= \frac{1}{(4\pi)^2 l^2} \int_0^\infty dt V(t) < \infty,$$

i.e., the function $D(x_E^2)$ has its maximum value at zero, where $D(0) < \infty$.

The form factor $V(l^2k_E^2)$ is related to $U(x_E^2/l^2)$ through the equation

$$V(l^2k_E^2) = \int_0^\infty du J_0\left(\sqrt{uk_E^2 l^2}\right) U'(u)$$
$$= \frac{1}{2i} \int_{-\beta+i\infty}^{-\beta-i\infty} \frac{d\zeta v(\zeta) (l^2k_E^2)^\zeta}{\sin \pi\zeta}, \quad (4.7)$$

where $0 < \beta < a$.

The function

$$v(\zeta) = \frac{1}{2^{2\zeta}\Gamma^2(1+\zeta)} \int_0^\infty du u^\zeta U'(u) \quad (4.8)$$

is

1) regular in the half plane $\mathrm{Re}\,\zeta > -a$, and in this region the following inequality is valid ($\zeta = \xi + i\eta$)

$$|v(\xi + i\eta)| \leq C \frac{e^{\pi|\eta|}}{(1+|\eta|)^M \Gamma\left(\frac{|\xi|}{\rho} + 1\right)}$$

for any $M > 0$ and some $C > 0$;

2) $v(0) = 1$; $v(-m) = 0$, $m = 1, \ldots, [a]$;

3) $[v(\xi)]^* = v(\xi^*)$.

We will now turn to the physical meaning behind the form factor used here. According to Eq. (3.12), any change in the free-photon propagator implies a change in Coulomb's law at small distances. The interaction potential for two electrons due to the exchange of a virtual photon will be calculated in an approximation which neglects the recoil of the charged particles. The interaction potential is related to the photon propagator via the equation

$$W(r) = \frac{e}{(2\pi)^3} \int d\mathbf{k} e^{i\mathbf{kr}} D(\mathbf{k}^2). \quad (4.9)$$

Upon substituting for $D(\mathbf{k}^2)$ according to Eq. (4.4) and using the equations presented above, we obtain

$$W(r) = \frac{e}{2\pi^2} \int_r^\infty \frac{dr'}{r' \sqrt{r'^2 - r^2}} \cdot U\left(\frac{r'^2}{l^2}\right). \quad (4.10)$$

If $U(u) \equiv 1$, then $W(r) = e/4\pi r$ and we obtain the usual Coulomb interaction. For $U(u)$ which satisfy the condition in Eq. (4.6) we easily find that when $\rho > 1/2$

$$W(r) = \begin{cases} \frac{e}{2\pi^2} \cdot \frac{1}{l} \int_0^\infty duV(u^2) + O\left(\frac{r^2}{l^2}\right), & r \ll l, \\ \frac{e}{4\pi} \cdot \frac{1}{r}\left[1 + O\left(\exp\left\{-\left(\frac{r^2}{l^2}\right)^\gamma\right\}\right)\right], & r \gg l. \end{cases} \quad (4.11)$$

When $\rho = 1/2$ we have

$$W(r) = \frac{e}{4\pi} \cdot \frac{1}{r}, \quad r > l. \quad (4.12)$$

Thus the potential $W(r)$ is finite when $r = 0$ and decreases as $r \to \infty$; when $r \gg l$ the deviation from the Coulomb potential is of order $O[\exp\{-(r^2/l^2)^\gamma\}]$ for $\rho > 1/2$, which means that the additional term decreases more rapidly than a linear exponential. When $\rho = 1/2$ the potential $W(r)$ is precisely the Coulomb potential for $r > l$. This suggests that the form factors, which are integral functions with $\rho = 1/2$, describe an electron in which the charge is distributed inside a sphere of radius l and for which the potential $W(r)$ is simply that appropriate to the interaction of two charged spheres.

The charge density $\rho(r)$ may be found from the equation

$$\rho(r) = \frac{1}{r^2} \frac{d}{dr}(rW(r)) \text{ when } r > 0.$$

When $\rho > 1/2$ we find that the electron's charge is distributed throughout all space in a manner such that when $r \to \infty$, $\rho(r) \sim \exp\{-(r^2/l^2)^\gamma\}$; that is, it is a rapidly decreasing function.

From a physical point of view the most interesting case is $\rho = 1/2$ when the stationary electron can be interpreted as a charged sphere of radius l.

Are there any principles, either physical or mathematical in nature, which would permit one to select a unique form factor $V(k^2l^2)$, and therefore a unique potential W(r)? It appears that such principles do exist.

Physically, it is natural to pose the following problem. Find those form factors $V(-l^2k^2)$ for which the potential W(r) for two electrons at rest has the smallest possible value when r = 0. Mathematically the following problem has a solution. Among the integral functions V(z) with $\rho = 1/2$ of type σ satisfying these conditions:

a) V(0) = 1,

b) $V(x) = |f(x)|^2 \geq 0$ along the positive real axis $0 \leq x < \infty$,

c) $\int_0^\infty (du/\sqrt{u})V(u) < \infty$,

find the function which minimizes the functional

$$\mu(V) = \int_0^\infty \frac{du}{\sqrt{u}} V(u) = \text{const}\, W(0). \tag{4.13}$$

This problem belongs to the class of extremal problems in the theory of integral functions (see ref. 14 for more detail) and has unique solution.

Let us derive the solution to this problem.[15] We write Eq. (4.13) as

$$\mu(V) = \int_0^\infty \frac{du}{\sqrt{u}} V(u) = 2\int_0^\infty dt V(t^2) = 2\int_0^\infty dt\, |f(t^2)|^2. \tag{4.14}$$

The function $f(t^2)$ is a first order integral function of type $\sigma/2$. Thus, according to the Paley–Wiener theorem (see Ref. 16) the following representation is valid:

$$f(t^2) = \frac{1}{\sqrt{2\pi}} \int_{-\sigma/2}^{\sigma/2} du e^{iut} \psi(u),$$

where $\psi(u) \in L_2(-\sigma/2, \sigma/2)$. Since $f(t^2)$ depends on t^2, the function $\psi(u)$ is even, so that

$$f(t^2) = \sqrt{2/\pi} \int_0^{\sigma/2} \psi(u) \cos ut\, du. \tag{4.15}$$

Using the Parseval equality we find from Eq. (4.14) that

$$\mu(V) = \int_0^\infty dt\, |f(t^2)|^2 = \int_0^{\sigma/2} |\psi(u)|^2 du. \tag{4.16}$$

Moreover

$$f(0) = \sqrt{2/\pi} \int_0^{\sigma/2} du\psi(u) = 1. \tag{4.17}$$

Let us now assume that we have at our disposal a system of polynomials $\{P_n(x)\}$ which are orthonormal on the interval $[0, \sigma/2]$ with respect to the weighting function l, i.e.

$$\int_0^{\sigma/2} du P_n(u) P_{n'}(u) = \delta_{nn'}$$

and

$$P_0(u) = \sqrt{2/\sigma}.$$

Then the function $\psi(u)$ can be expanded in terms of these polynomials:

$$\psi(u) = \sum_{n=0}^\infty c_n P_n(u).$$

The conditions in Eq. (4.16) and Eq. (4.17) can be written as

$$\sqrt{2/\pi} \int_0^{\sigma/2} du\psi(u) = \sqrt{2/\pi}\, c_0 \sqrt{\sigma/2} = 1;$$

$$\mu(V) = \int_0^{\sigma/2} du\, |\psi(u)| = \sum_{n=0}^\infty |c_n|^2.$$

It is clear that

$$\min_{\{\psi\}} \mu(V) = \min_{\{c_n\}} \sum_{n=0}^\infty |c_n|^2 = |c_0|^2 = \pi/\sigma.$$

This means that

$$\psi(u) = \sqrt{\pi/\sigma}$$

and

$$f(t^2) = \frac{2}{\sigma} \int_0^{\sigma/2} du \cos ut = \sin \sigma t/2/(\sigma t/2).$$

Finally,

$$V(z) = \left[\frac{\sin \frac{\sigma}{2}\sqrt{z}}{\frac{\sigma}{2}\sqrt{z}}\right]^2. \tag{4.18}$$

Using Eq. (4.9) to compute the potential of two electrons at rest we find

$$W(r) = \begin{cases} \frac{e}{4\pi}\cdot\frac{1}{r}, & r > \sigma, \\ \frac{e}{4\pi}\cdot\frac{1}{\sigma}\left(2 - \frac{r}{\sigma}\right), & r < \sigma. \end{cases} \tag{4.19}$$

This potential is that of two charged spheres, of radius $l = \sigma/2$, interacting with one another. In this case, in the gauge transformation of Eq. (2.6) the Fourier transform of the form factor $K[(x - y)/l]$ is

$$\tilde{K}(l^2k^2) = \int dx e^{ikx} K(x/l) = \sin\sqrt{-k^2l^2}/\sqrt{-k^2l^2}. \tag{4.20}$$

In the present interpretation the form factor $K(x/l)$ describes the electron as charged sphere of radius $l = \sigma/2$.

The potential for this case is then

$$V(-l^2k^2) = \left[\frac{\sin\sqrt{-k^2l^2}}{\sqrt{-k^2l^2}}\right]^2. \tag{4.21}$$

In the representation of Eq. (4.7) we easily find the function $v(\xi)$ to be

$$v(\zeta) = 2^{1+2\zeta}/\Gamma(2\zeta+3). \qquad (4.22)$$

It is of interest to calculate the function $D(x_E^2)$ in the Euclidean metric. According to Eq. (4.2) we have

$$D(x_E^2) = \begin{cases} \frac{1}{(2\pi)^2}\cdot\frac{1}{x_E^2}, & x_E^2 > 4l^2, \\ \frac{1}{(2\pi)^2}\cdot\frac{1}{x_E^2}\left\{1-\sqrt{1-\frac{x_E^2}{4l^2}}+\right. & \\ \left.+\frac{x_E^2}{4l^2}\ln\frac{2l}{\sqrt{x_E^2}}\left(1+\sqrt{1-\frac{x_E^2}{4l^2}}\right)\right\}, & x_E^2 < 4l^2. \end{cases} \qquad (4.23)$$

Note that when $x_E^2 \to 0$,

$$D(x_E^2) \to \frac{1}{(2\pi)^2}\ln\frac{4l}{\sqrt{x_E^2}},$$

i.e., the function $D(x_E^2)$ increases logarithmically, while in the local theory when $x_E^2 \to 0$

$$\Delta(x_E^2) \to \frac{1}{(2\pi)^2}\cdot\frac{1}{x_E^2}.$$

If the electron is a uniformly charged sphere of radius l, it is then not difficult to show that

$$V(-l^2k^2) = \frac{9}{(-k^2l^2)^2}\left[\frac{\sin\sqrt{-k^2l^2}}{\sqrt{-k^2l^2}} - \cos\sqrt{-k^2l^2}\right]^2 \qquad (4.24)$$

and

$$v(\zeta) = \frac{9\cdot 2^{4+2\zeta}[2\zeta^2+7\zeta+5]}{\Gamma(2\zeta+7)}. \qquad (4.25)$$

Thus the nonlocal generalized functions

$$K\left(\frac{x-y}{l}\right) = \frac{1}{(2\pi)^4}\int dk e^{ik(x-y)}\tilde{K}(k^2l^2)$$
$$= \sum_{n=0}^{\infty} c_n (l^2\Box)^n \delta^{(4)}(x-y),$$

for which the Fourier transform $\tilde{K}(l^2k^2)$ is an integral function of order 1/2 give us a basis for a relativistically invariant description of extended particles.

5. A REGULARIZATION PROCEDURE

As mentioned above, it is possible to construct an S matrix from perturbation theory only within the confines of a specific mathematical apparatus. We shall now formulate the regularization procedure to be used in calculating the matrix elements of the perturbation theory.

In the perturbation series used in quantum electrodynamics one regularizes the nonlocal photon propagators and the closed spinor bubble diagrams formed by electron propagators.

For the nonlocal form factor $V(-l^2k^2)$ in the region $k^2 < 0$ this Mellin transform holds:

$$V(-l^2k^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} \frac{d\zeta v(\zeta)(-l^2k^2)^\zeta}{\sin\pi\zeta}, \qquad (5.1)$$

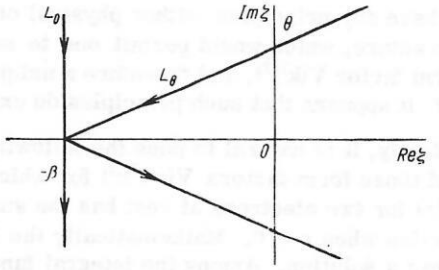


Fig. 1. Integration contours L_0 and L_θ in the complex ζ plane.

where $0 < \beta < a$ and the function $v(\zeta)$ satisfies the conditions enumerated in the previous section. For a transition in the region where $k^2 > 0$ we must integrate in the ζ plane along the contour L_0 to the contour L_θ ($0 < \theta < \pi/2$) where

$$L_\theta = [\zeta : \zeta = -\beta + ue^{\pm i(\pi/2-\theta)},$$
$$0 < \beta < a, \quad \forall u \geqslant 0, \quad 0 \leqslant \theta \leqslant \pi/2],$$

as shown in Fig. 1.

We shall now introduce the regularization function

$$\tilde{D}^\delta(k^2) = \frac{l^2}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} \frac{d\zeta v(\zeta) e^{\delta\zeta^2}[l^2(-k^2-i\varepsilon)]^\zeta}{\sin\pi\zeta}. \qquad (5.2)$$

The function $\tilde{D}^\delta(k^2)$ for $\delta > 0$ is:

1) defined over the entire complex k^2 plane and is everywhere regular except along the cut $[0, +\infty]$;

2) $\tilde{D}^\delta(k^2) = O(1/|k^2|^{1+\beta})$ when $|k^2| \to \infty$;

3) $\lim_{\delta\to\infty}\tilde{D}^\delta(k^2) = D(k^2) = V(-k^2l^2)/(-k^2-i\varepsilon)$.

The Fourier transformation of the function $\tilde{D}^\delta(k^2)$ exists:

$$D^\delta(x) = \frac{1}{(2\pi)^4 i}\int dk e^{-ikx}\tilde{D}^\delta(k^2)$$
$$= \frac{l^2}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} \frac{d\zeta v(\zeta) e^{\delta\zeta^2}(l^2)^\zeta}{\sin\pi\zeta} D(x,\zeta), \qquad (5.3)$$

where

$$D(x,\zeta) = \frac{1}{(2\pi)^4 i}\int dk e^{-ikx}(-l^2-i\varepsilon)^{\zeta-1}$$
$$= \frac{2^{2\zeta}}{(2\pi)^2}\cdot\frac{\Gamma(1+\zeta)}{\Gamma(1-\zeta)}\cdot\frac{e^{-i\pi(1+\zeta)}}{(x^2-i\varepsilon)^{1+\zeta}}.$$

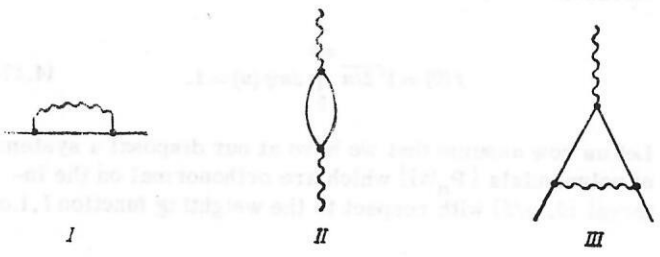


Fig. 2. Divergent primitive irreducible diagrams.

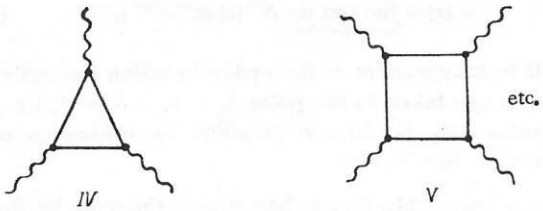


Fig. 3. Convergent primitive irreducible diagrams.

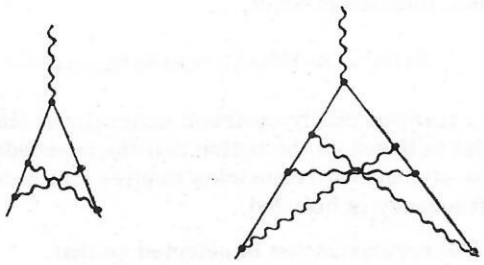


Fig. 4. Reducible vertex diagrams.

It is easily seen that $D^{\delta}(x)$ satisfies this condition:

$$\lim_{|x^2|\to 0} |x^2|^{\varepsilon} |D^{\delta}(x)| = 0, \qquad \forall \varepsilon > 0.$$

We note that we have used[13] the regularization R^{δ} which enables us to regularize form factors for which $1/2 \leq \rho < 1$. The proposed regularization can be used to regularize form factors having arbitrary but finite rates of increase.

To regularize the spinor propagators we shall use the so-called gauge invariant procedure of Pauli and Villars. This means that the causal spinor functions for an electron are not regularized individually; rather, the closed spinor loops

$$\sum_j c_j \operatorname{Sp} [\gamma S_{M_j}(x_1 - x_2)\, \gamma S_{M_j}(x_2 - x_3) \ldots] \tag{5.4}$$

are regularized. Since the fastest divergence in quantum electrodynamics is quadratic, it will be sufficient to use two auxiliary masses M_1 and M_2. The coefficients c_j then satisfy the conditions

$$\left.\begin{aligned} 1 + c_1 + c_2 &= 0; \\ 1 + c_1\Lambda_1 + c_2\Lambda_2 &= 0, \end{aligned}\right\} \tag{5.5}$$

where $M_j^2 = m^2\Lambda_j$; $\Lambda_j (j = 1, 2)$ are large dimensionless regularization parameters which we choose to write as

$$\Lambda_j = \Lambda + \varepsilon_j,$$

where $\Lambda \gg 1$, $0 < \varepsilon_j \ll 1$. With this regularization the electron–electron loop representing the vacuum polarization diverges logarithmically when $\Lambda \to \infty$.

In place of Eq. (5.5) we propose to use three auxiliary masses with the coefficients c_1, c_2, and c_3 related as follows:

$$\left.\begin{aligned} c_1 + c_2 + c_3 &= -1; \\ c_1\Lambda_1 + c_2\Lambda_2 + c_3\Lambda_3 &= -1; \\ c_1 \log \Lambda_1 + c_2 \log \Lambda_2 + c_3 \log \Lambda_3 &= d, \end{aligned}\right\} \tag{5.6}$$

here d is a finite number which must be selected in accord with the condition that the physical charge of the electron be normalized (see Sec. 9 below).

Thus, with our regularization all the matrix elements in the perturbation theory converge.

To remove the regularization in the S matrix elements, it is sufficient to transform into a Euclidean metric over all internal momenta in the integrals corresponding to any linked Feynman diagram because $\widetilde{D}^{\delta}(k^2)$ is regular in the half-plane $\operatorname{Im} k^2 \geq 0$. One may then take the limit $\delta \to 0$ because $\widetilde{D}(k^2)$ decreases rapidly enough when $k^2 \to \infty$ (see refs. 13 and 17 for more detail). The spinor loops are finite in the limit $\Lambda \to \infty$, according to Eq. (5.6).

In the usual local quantum electrodynamics (see refs. 8,9) the matrix elements corresponding to the primitive irreducible Feynman diagrams shown in Fig. 2 are divergent. We shall not consider here the diagrams shown in Fig. 3 because the contribution from Diagram IV is zero in virtue of Furry's theorem, and the integrals for Diagram V converge due to gauge invariance because the electron propagators correspond to the usual local theory (see refs. 8, 9).

The S matrix elements corresponding to the self-energy (I) and vertex parts (III) diagrams will be convergent because the photon propagator $\widetilde{D}(k^2)$ decreases rapidly in the Euclidean region. This is also true of any of the irreducible diagrams which describe a vertex part (Fig. 4). The diagrams shown in Fig. 5, and all the remaining diagrams in the perturbation theory, are also convergent. By virtue of the modified Pauli–Villars procedure which we have adopted the integral corresponding to diagram II of Fig. 2, the so-called vacuum polarization, will converge.

Thus, within the scope of our regularization, all diagrams encountered in the perturbation theory are finite.

6. PROOF OF THE S MATRIX UNITARITY

To prove that the S matrix is unitary in the nonlocal quantum electrodynamics with perturbation theory, one proceeds in the same manner as in the case of the quantum theory of a scalar field (see ref. 17).

As compared with the scalar field theory, the quantum electrodynamics perturbation theory contains two kinds of propagators, those for photons and for fermions,

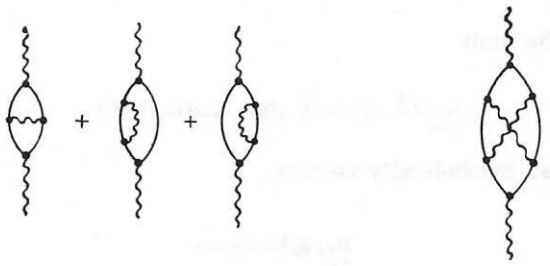


Fig. 5. Reducible vacuum polarization diagrams.

and there are two types of regularization, one for photon propagators and one for fermion bubbles. Therefore, the S matrix is regularized by the parameter δ and a set of parameters $\{\Lambda_m\}$ since each bubble is regularized by itself; thus we have the matrix $S^{\delta\{\Lambda\}}$.

As mentioned earlier, the removal of the regularization on Λ, i.e. the transition to $\Lambda \to \infty$, is in fact nontrivial only in one vacuum polarization diagram (diagram II of Fig. 2). The integrals for any other closed bubble will converge even without regularization. Study of the vacuum polarization diagram shows (see Sec. 9) that the analytic properties of this diagram are in complete accord with the requirements of unitarity. Therefore the operator

$$S^{\delta} = \lim_{\{\Lambda_m \to \infty\}} S^{\delta, \{\Lambda\}} \tag{6.1}$$

exists, and its properties with respect to the operators of the fermion fields are determined by the local theory.

The proof of the unitarity of the scattering matrix

$$S = \lim_{\delta \to 0} S^{\delta} \tag{6.2}$$

now proceeds exactly like the proof used in the nonlocal quantum theory of a scalar field (see ref. 17).

Rather than reproducing the entire proof in detail here, we shall instead formulate it in a general outline. The regularization procedure proposed here satisfies the following conditions:

1. The regularized $S^{\delta}[e]$ matrix is defined and the limit

$$\lim_{\delta \to 0} S^{\delta}[e] = S[e] \tag{6.3}$$

exists. This point was discussed in an earlier section.

2. The positive-frequency photon Green's functions, which define the product operation in

$$S[e]\, S^{+}[e] = \underset{Df}{S[e] \circledast S^{+}[e]},$$

i.e., corresponding to the transition to a normal product according to the Wick theorem, is also regularized

$$\widetilde{D}^{\delta}_{(-)}(k) = \frac{1}{2i} \int_{-\beta+i\alpha}^{-\beta-i\infty} \frac{d\zeta v(\zeta)\, e^{\delta\zeta^2}}{\sin \pi\zeta} \times \frac{1}{i}\left[(-k^2 - i\varepsilon)^{\zeta-1} - (-k^2 - i\varepsilon k_0)^{\zeta-1}\right], \tag{6.4}$$

and the limit

$$\lim_{\delta \to 0} \widetilde{D}^{\delta}_{(-)}(k) = \widetilde{D}_{(-)}(k) = 2\pi\theta(k_0)\,\delta(k^2) \tag{6.5}$$

exists. Symbolically we have

$$\lim_{\delta \to 0} \circledast^{\delta} = \circledast \equiv \cdot \tag{6.6}$$

3. In the equation

$$J[e] = \lim_{\delta_1 \to 0} \lim_{\delta_2 \to 0} \lim_{\delta_3 \to 0} S^{\delta_1}[e] \circledast^{\delta_2} S^{\delta_3 +}[e] \tag{6.7}$$

the limit is independent of the order in which the individual limits are taken to the point $\delta_1 = \delta_2 = \delta_3 = 0$, i.e., the operator $J(\delta_1, \delta_2, \delta_3) = S^{\delta_1} \circledast^{\delta_2} S^{\delta_3 +}$ is continuous at the point $\delta_1 = \delta_2 = \delta_3 = 0$.

The proof of this fact is based on a theorem by Bogolyubov and Shirkov[8] which asserts that if $K_1(x_1 \ldots x_n)$ and $K_2(y_1 \ldots y_m)$ are translationally invariant generalized functions, and the arguments $x_1 \ldots x_n$ and $y_1 \ldots y_n$ are independent, then the product

$$K_1(x_1 \ldots x_n)\, \Pi\Delta_{(-)}(x_s - y_t)\, K_2(y_1 \ldots y_m)$$

is also a translationally invariant generalized function. The proof is based on the notion that the boundedness of the sums of positive frequencies implies that each individual frequency is bounded.

4. The regularization is selected so that

$$S^{\delta}[e] \circledast^{\delta} S^{\delta +}[e] \equiv 1 \quad (\delta > 0).$$

This identity results from the fact that the regularized functions $D^{\delta}(x)$, $D^{\delta}_{(-)}(x)$ and $D^{\delta *}(x)$ are related as follows (when $\delta > 0$)

$$\left.\begin{aligned} D^{\delta}(x) &= \theta(x_0)\, D^{\delta}_{(-)}(x) + \theta(-x_0)\, D^{\delta}_{(+)}(x); \\ D^{\delta *}(x) &= \theta(-x_0)\, D^{\delta}_{(-)}(x) + \theta(x_0)\, D^{\delta}_{(+)}(x); \\ D^{\delta}_{(-)}(x) &= \theta(x_0)\, D^{\delta}(x) + \theta(-x_0)\, D^{\delta *}(x), \end{aligned}\right\} \tag{6.8}$$

equations which are identical to those in the local theory which guarantee the unitary of the S matrix. A a consequence, the following chain of equalities is valid:

$$\begin{aligned} J[e] &= S[e]\, S^{+}[e] = S[e] \circledast S^{+}[e] \\ &= \lim_{\delta_1 \to 0} \lim_{\delta_2 \to 0} \lim_{\delta_3 \to 0} S^{\delta_1}[e] \circledast^{\delta_2} S^{\delta_3 +}[e] \\ &= \lim_{\delta \to 0} S^{\delta} \circledast^{\delta} S^{\delta +} = \lim_{\delta \to 0} 1 = 1. \end{aligned}$$

This indicates that the S matrix is unitary to all orders of perturbation theory.

7. GAUGE INVARIANCE OF THE S MATRIX

The requirement of gauge invariance for the S matrix, which means that the S matrix must be invariant under the transformation

$$A_{\mu}(x) \to A_{\mu}(x) + \partial_{\mu} f(x) \tag{7.1}$$

for an arbitrary function $f(x)$, can be written in the form

$$\frac{\partial}{\partial x_{1\mu_1}} \cdots \frac{\partial}{\partial x_{n\mu_n}} \left(\frac{\delta^n S}{\delta A_{\mu_1}(x_1) \ldots \delta A_{\mu_n}(x_n)} \right) = 0 \tag{7.2}$$

when the fermion operators of the electron field obey the free equations of motion.

It is sufficient for the proof of (7.2) to consider just the case n = 1, i.e.,

$$\partial_{\mu} \frac{\delta S}{\delta A_{\mu}(x)} = 0. \tag{7.3}$$

First we shall present a formal proof based only on the following representation:

$$S = T\exp\left\{i\int dx\mathcal{L}_I(x)\right\}. \tag{7.4}$$

It will be assumed that the representation in Eq. (7.4) ensures the construction of a perturbation series from the causal functions (3.11)-(3.12) and that the S matrix is expanded in a series of normal products of field operators which obey the free equations of motion. We shall not make use of our regularization, but we will show that the proof presented is valid within the realm of our regularizations.

By using Eq. (7.4) we find that

$$\begin{aligned}\frac{\delta S}{\delta A_\mu(x)} &= iT\left\{\left(\frac{\delta}{\delta A_\mu(x)}\int dy\mathcal{L}_I(y)\right)S\right\}\\ &= i\int dyK\left(\frac{x-y}{l}\right)T\left\{\left(\frac{\delta}{\delta\mathfrak{A}_\mu(l,y)}\int dz\mathcal{L}_I(z)\right)S\right\}\\ &= i\int dyK\left(\frac{x-y}{l}\right)T\{e\bar\psi(y)\gamma_\mu\psi(y)S\}.\end{aligned} \tag{7.5}$$

We have used the following equations:

$$\left.\begin{aligned}-i\partial_\mu\gamma_\mu T\{\psi(x)S\} &= T\{[m\psi(x)+ie\hat{\mathfrak{A}}(l,x)\psi(x)]S\};\\ -i\partial_\mu T\{\bar\psi(x)\gamma_\mu S\} &= T\{[-m\bar\psi(x)-ie\bar\psi(x)\hat{\mathfrak{A}}(l,x)]S\}.\end{aligned}\right\} \tag{7.6}$$

These relationships are valid if, in accord with Wick's theorem, the perturbation theory is constructed using a chronological contraction of the fermion operators as in Eq. (3.11), and the S matrix in Eq. (3.1) depends on field operators which satisfy the free equations.

With the aid of these equations we find that

$$\begin{aligned}\partial_\mu\frac{\delta S}{\delta A_\mu(x)} &= \int dyK\left(\frac{x-y}{l}\right)\frac{\partial}{\partial y_\mu}\frac{\delta S}{\delta\mathfrak{A}_\mu(l,y)}\\ &= \int dyK\left(\frac{x-y}{l}\right)T\{[m\bar\psi(y)\psi(y)+ie\bar\psi(y)\hat{\mathfrak{A}}(l,y)\psi(y)\\ &\quad -m\bar\psi(y)\psi(y)-ie\bar\psi(y)\hat{\mathfrak{A}}(l,y)\psi(y)]S\}=0.\end{aligned} \tag{7.7}$$

Thus the S matrix is gauge-invariant within the scope of this formal treatment. We shall now show that these formal transformations are valid within the limits of our regularization procedure.

The perturbation series for the S matrix can be represented by a set of Feynman diagrams in which the electromagnetic field operator $A_\mu(x)$ is always connected to an electron line. Two cases are possible: Either this electron line is not closed, or it forms a closed loop. Mathematically, the proof of the gauge invariance reduces to determining the effect of the operator $\partial_\mu\delta/\delta A_\mu(x)$ on the S matrix. Since this operator contains just the first variational derivative with respect to the electromagnetic field $A_\mu(x)$, one may consider transformations of the kind shown in Eq. (7.5)-(7.7) for each closed line and each open line completely independently.

Since the electron mass is unchanged by a gauge transformation, and the transformations in Eqs. (7.5)-(7.7) affect only operators and propagators of charged fields, the calculations are independently valid for each individual open line and each individual loop found in any diagram.

8. SELF-ENERGY DIAGRAM

We now wish to consider the self-energy diagram shown in Fig. 2 (I). This diagram corresponds to the matrix element

$$-i:\bar\psi(x)\Sigma(x-y)\psi(y):, \tag{8.1}$$

where

$$\Sigma(x-y) = -ie^2\gamma_\mu S(x-y)\gamma_\mu D(x-y). \tag{8.2}$$

Going to a momentum representation and using our regularization procedure, which enables us to change to a Euclidean metric via the rotation $k_0 \to \exp[i(\pi/2)]k_4$, we find in the limit $\delta \to 0$ that

$$\begin{aligned}\tilde\Sigma(p) &= \lim_{\delta\to0}(-ie^2)\int dxe^{ipx}\gamma_\mu S(x)\gamma_\mu D^\delta(x)\\ &= \lim_{\delta\to0}\frac{ie^2}{(2\pi)^4}\int dk\tilde D^\delta(k)\gamma_\mu\tilde S(p-k)\gamma_\mu\\ &= \frac{e^2}{(2\pi)^4}\int dk_E\frac{V(k_E^2l^2)}{k_E^2}\gamma_\mu\frac{m+\hat p_E-\hat k_E}{m^2+(p_E-k_E)^2}\gamma_\mu,\end{aligned} \tag{8.3}$$

where $P_E = (ip_0, \mathbf{p})$, $\hat k_E = \gamma_0 ik_4 - \boldsymbol{\gamma}\mathbf{k}$, $p_E^2 = -p^2$. By using the representation in Eq. (4.7) for the form factor $V(k_E^2l^2)$ we obtain after some effort

$$\begin{aligned}\tilde\Sigma(p) &= \frac{e^2}{(2\pi)^4}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}\frac{d\zeta v(\zeta)(l^2)^\zeta}{\sin\pi\zeta}\int\frac{dk_E[4m-2(\hat p_E-\hat k_E)]}{(k_E^2)^{1-\zeta}[m^2+(k_E-p_E)^2]}\\ &= -\frac{e^2}{(2\pi)^4}\cdot\frac{2\pi^3}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}\frac{d\zeta v(\zeta)(m^2l^2)^\zeta}{(\sin\pi\zeta)^2\Gamma(1+\zeta)}F(\zeta,p);\end{aligned} \tag{8.4}$$

$$F(\zeta,p) = \frac{1}{\Gamma(1-\zeta)}\int_0^1 du\left(\frac{1-u}{u}\right)^\zeta\left(1-\frac{p^2}{m^2}u\right)^\zeta(2m-\hat pu). \tag{8.5}$$

The function $F(\zeta, p)$ is regular in the half plane $\mathrm{Re}\,\zeta > -1$. Finally, we obtain

$$\begin{aligned}\tilde\Sigma(p) &= -\frac{e^2}{8\pi^2}\sum_{n=0}^{\infty}\frac{v(n)}{n!}(m^2l^2)^n\\ &\times\left\{F(n,p)\left[\ln m^2l^2+\frac{v'(n)}{v(n)}-\psi(n+1)\right]+\frac{\partial}{\partial n}F(n,p)\right\}.\end{aligned} \tag{8.6}$$

Here $\psi(u) = (d/du)\ln\Gamma(u)$.

We assume that the elementary length l is small, i.e.

$$m^2l^2 \ll 1.$$

Then it is easy to show that up to terms of order $(m^2l^2)^2$,

$$\begin{aligned}\tilde\Sigma(p) &= \frac{e^2}{8\pi^2}\int_0^1 du(2m-u\hat p)\ln\frac{m^2}{m^2-up^2}\\ &+\frac{e^2}{16\pi^2}\left[\left(3\ln\frac{1}{m^2l^2}+3v'(0)+3\psi(1)+1\right)\right.\\ &\left.+m^2l^24v(1)\left(\ln\frac{1}{m^2l^2}-\frac{v'(1)}{v(1)}-\frac{5}{12}\cdot\frac{p^2}{m^2}\right)\right]\\ &+\frac{e^2}{16\pi^2}(m-\hat p)\left[\left(\ln\frac{1}{m^2l^2}-v'(0)+1\right)-m^2l^2v(1)\frac{1}{3}\cdot\frac{p^2}{m^2}\right]\\ &+O((m^2l^2)^2).\end{aligned} \tag{8.7}$$

This expansion is valid for momenta p^2 in the region $p^2l^2 \ll 1$.

It is of interest to compute the correction to the electron mass:

$$\delta m = m - m_0 = \tilde{\sum}(m) = \frac{3e^2}{16\pi^2}\left\{\ln\frac{1}{m^2l^2} + O(1)\right\}. \quad (8.8)$$

If it is assumed that the electron mass is due entirely to electromagnetic causes (i.e., $m_0 = 0$), then

$$ml \simeq \exp\left\{-\frac{8\pi^2}{3e^2}\right\} \sim 10^{-10^2} \quad (8.9)$$

and

$$l \leqslant 10^{-10^2}\ \text{cm}. \quad (8.10)$$

In other words, we have obtained a ridiculously small value for the elementary length.

Since $(e^2/16\pi^2)\ln(1/m^2l^2) \sim 1$ in this case, we do not, strictly speaking, have the right to use perturbation theory. Therefore the numbers obtained must be treated as very rough estimates. However, if it is assumed that the elementary length is somewhere in the range

$$l \sim 10^{-16} \div 10^{-17}\ \text{cm} \quad \text{or} \quad \frac{1}{l} \sim 100 \div 1000\ \text{GeV}, \quad (8.11)$$

then

$$\ln\frac{1}{m^2l^2} \sim 50 \div 60$$

and

$$\delta m = m - m_0 \sim 0{,}1m \sim 0.05\ \text{MeV}. \quad (8.12)$$

This indicates that in this case the electron mass is of nonelectromagnetic origin, and the electromagnetic corrections are small.

9. VACUUM POLARIZATION DIAGRAM

Consider now the vacuum polarization diagram shown in Fig. 2 (II). The term in the scattering matrix which corresponds to this diagram can be written as

$$-i : A_\mu(x)\,\Pi_{\mu\nu}(x-y)\,A_\nu(y) :, \quad (9.1)$$

where

$$\Pi_{\mu\nu}(x-y) = -ie^2\,\mathrm{Tr}\{\gamma_\mu S(x-y)\,\gamma_\nu S(y-x)\}. \quad (9.2)$$

Using our regularization procedure, we find in the momentum representation

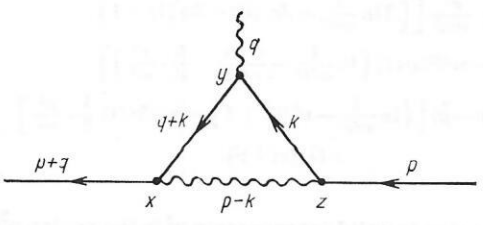


Fig. 6. Vertex diagram.

$$\mathrm{reg}\,\Pi_{\mu\nu}(x-y) = \frac{1}{(2\pi)^4}\int dp e^{-ip(x-y)}\,\mathrm{reg}\,\tilde{\Pi}_{\mu\nu}(p), \quad (9.3)$$

where

$$\mathrm{reg}\,\tilde{\Pi}_{\mu\nu}(p) = \frac{e^2}{(2\pi)^4 i}\int dk \sum_{j=0}^{3} c_j\,\mathrm{Tr}\left\{\gamma_\mu \frac{1}{M_j - \hat{k} - i\varepsilon}\gamma_\nu \frac{1}{M_j - (\hat{k}-\hat{p}) - i\varepsilon}\right\}$$

$$= (p_\mu p_\nu - \delta_{\mu\nu}p^2)\,\frac{e^2}{2\pi^2}\int_0^1 d\alpha\,(1-\alpha)\,\alpha$$

$$\times\left\{\ln\frac{m^2-\alpha(1-\alpha)p^2}{m^2} + \sum_{j=1}^{3} c_j \ln\left(\Lambda_j - \alpha(1-\alpha)\frac{p^2}{m^2}\right)\right\}. \quad (9.4)$$

In view of our additional condition in Eq. (5.6) we find in the limit $\Lambda \to \infty$ that

$$\tilde{\Pi}_{\mu\nu}(p) = \lim_{\Lambda\to\infty}\lim_{\varepsilon_j\to 0}\mathrm{reg}\,\tilde{\Pi}_{\mu\nu}(p) = (\delta_{\mu\nu}p^2 - p_\mu p_\nu)\,\tilde{\Pi}(p^2); \quad (9.5)$$

$$\tilde{\Pi}(p^2) = \frac{e^2}{12\pi^2}\left\{d + p^2\int_{4m^2}^{\infty}\frac{d\varkappa^2}{\varkappa^2(\varkappa^2 - p^2 - i\varepsilon)}\sqrt{1-\frac{4m^2}{\varkappa^2}}\left(1+\frac{2m^2}{\varkappa^2}\right)\right\}. \quad (9.6)$$

Therefore, with our regularization procedure the polarization operator is finite when the regularization is removed and it agrees with the renormalized expression in the usual local electrodynamics if we set the arbitrary regularization constant

$$d = 0. \quad (9.7)$$

In this case $\tilde{\Pi}(p^2)$ is normalized by the condition

$$\tilde{\Pi}(0) = 0. \quad (9.8)$$

This indicates that the constant d must determine the renormalized electron charge, and the choice d = 0 means that, at least up to second-order perturbation theory, the charge e is not renormalized, i.e., the physical electron charge coincides with the bare charge.

10. THE VERTEX DIAGRAM

Now consider the diagram shown in Fig. 2 (III). This diagram corresponds to the matrix element

$$ie : \bar{\psi}(x)\,\Gamma_\mu(x, z\,|\,y)\,\psi(z)\,A_\mu(y) :, \quad (10.1)$$

where we have introduced the third-order vertex function

$$\Gamma_\mu(x, z\,|\,y) = ie^2\gamma_\nu S(x-y)\,\gamma_\mu S(y-z)\,\gamma_\nu D(x-z). \quad (10.2)$$

We have selected the momentum variables as shown in Fig. 6. Going to the momentum representation, we find in the Euclidean metric

$$\tilde{\Gamma}_\mu(p, q) = \lim_{\delta\to 0} ie^2\int dy\int dz e^{ipz+iqy}\gamma_\nu S(y)\,\gamma_\mu S(z-y)\,\gamma_\nu D^\delta(z)$$

$$= -\frac{e^2}{(2\pi)^4}\int\frac{dk_E V(l^2(p_E-k_E)^2)\,\gamma_\nu(m+\hat{k}_E+\hat{q}_E)\,\gamma_\mu(m+\hat{k}_E)\,\gamma_\nu}{(p_E\ \ k_E)^2(m^2+(k_E+q_E)^2)(m^2+k_E^2)}$$

$$= -\frac{e^2}{8\pi}\cdot\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}\frac{d\zeta v(\zeta)(m^2l^2)^\zeta}{(\sin\pi\zeta)^2\,\Gamma(1+\zeta)}$$

$$\times[\gamma_\mu F_1(\zeta;\ p, q) + F_{2\mu}(\zeta;\ p, q)], \quad (10.3)$$

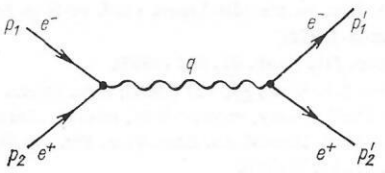


Fig. 7. Diagram for elastic scattering $e^+ + e^- \to e^+ + e^-$.

where

$$F_1(\zeta; p, q) = \frac{1}{\Gamma(1-\zeta)} \int_0^1\int_0^1\int \frac{d\alpha\, d\beta\, d\gamma \alpha^{-\zeta}\delta(1-\alpha-\beta-\gamma)}{\left[\beta+\gamma+\alpha\gamma\frac{p^2}{m^2}+\beta\gamma\frac{q^2}{m^2}+\alpha\beta\frac{(p+q)^2}{m^2}\right]^{-\zeta}}; \quad (10.4)$$

$$F_{2\mu}(\zeta; p, q) = \frac{1}{\Gamma(-\zeta)} \int_0^1\int_0^1\int \frac{d\alpha\, d\beta\, d\gamma \alpha^{-\zeta}\delta(1-\alpha-\beta-\gamma)}{\left[\beta+\gamma+\alpha\gamma\frac{p^2}{m^2}+\beta\gamma\frac{q^2}{m^2}+\alpha\beta\frac{(p+q)^2}{m^2}\right]^{1-\zeta}} \times \frac{1}{m^2}[m^2\gamma_\mu - 2mq_\mu + 4m(\beta q_\mu - \alpha p_\mu) + (\alpha\hat{p}-\beta\hat{q})\gamma_\mu\hat{q} + (\alpha\hat{p}-\beta\hat{q})\gamma_\mu(\alpha\hat{p}-\beta\hat{q})]. \quad (10.5)$$

It is easily shown that the vertex part satisfies the gauge-invariance condition

$$q_\mu \bar{u}(\mathbf{p}+\mathbf{q})\tilde{\Gamma}_\mu(p, q)\, u(\mathbf{p}) = 0, \quad (10.6)$$

where u(**p**) and u(**p** + **q**) are Dirac spinors describing the electron in its initial and final states.

We now want to calculate the correction to the electron's anomalous magnetic moment induced by the nonlocal effects. To this end we find the vertex part in the approximation where the momenta p and (p + q) are on the mass shell, or $p^2 = (p+q)^2 = m^2$, and terms of order q^2 and (m^2l^2) are omitted. Since an infrared divergence arises in this approximation, it will be necessary to introduce a small photon mass λ_Φ ($\lambda_\Phi \ll m$) into Eqs. (10.2) and (10.3). After some calculation we find

$$\tilde{\Gamma}_\mu(q) = \frac{\alpha}{4\pi}\gamma_\mu\left[\ln\frac{1}{m^2l^2} - 2\ln\frac{m^2}{\lambda_\Phi^2} + O(1)\right] - \frac{\alpha}{2\pi}\cdot\frac{1}{2m}\sigma_{\mu\nu}q_\nu\left(1-\frac{2v(1)}{3}m^2l^2\right), \quad (10.7)$$

where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$.

The first term in (10.7) contains terms subject to renormalization. The second term contributes to the anomalous magnetic moment. We have

$$\Delta\mu = \frac{\alpha}{2\pi}(1 - 2v(1)\, m^2l^2/3). \quad (10.8)$$

If the electron is a charged sphere, we find from Eq. (4.22) that

$$v(1) = 4\cdot 2/\Gamma(5) = 1/3.$$

If, however, the electron is a uniformly charged sphere, then $v(1) = 9\cdot 2^6\cdot 13/\Gamma(9) = 13/70$.

The present experimental value for the electron's anomalous magnetic moment is

$$\Delta\mu_{\text{exp}} = \frac{\alpha}{2\pi} - 0{,}32846\left(\frac{\alpha}{\pi}\right)^2 + (0.54 \pm 0.58)\left(\frac{\alpha}{\pi}\right)^3 \quad (10.9)$$

a value which is completely explained by the local quantum electrodynamics.[1-3] This can be used to calculate an upper limit on the elementary length:

$$\left.\begin{aligned} ml &\leqslant \frac{3}{\sqrt{2}}\cdot\frac{\alpha}{\pi} \quad \text{charged sphere,} \\ ml &\leqslant \sqrt{\frac{70}{13}}\cdot\frac{\alpha}{\pi} \quad \text{charged ball} \end{aligned}\right\} \quad (10.10)$$

or

$$\left.\begin{aligned} l &\leqslant 1.9\cdot 10^{-13}\,\text{cm charged sphere,} \\ l &\leqslant 2.1\cdot 10^{-13}\,\text{cm charged ball.} \end{aligned}\right\} \quad (10.11)$$

It can thus be seen that, in spite of the very high precision of the measurements of the anomalous magnetic moment, this experiment still allows the existence of an electron having rather large dimensions.

We point out once again that the elementary lengths cited in refs. 1 and 2 are not the actual values for the electron's dimensions or for the region of nonlocal interaction; they are only indirectly related to these items. In our model of the nonlocal interaction, however, l does have the connotation of an exact dimension of the electron, which is the radius of the sphere.

11. HIGH ENERGY BEHAVIOR

An important feature of the present variant of nonlocal quantum electrodynamics is an increase in the amplitude for physical processes as the energy increases. Consider, for example, the process $e^+ + e^- \to e^+ + e^-$ which is described by the diagram shown in Fig. 7.

For a given order of perturbation theory, the ratio of the cross sections calculated in the local and nonlocal theories will be given by the equation

$$\frac{\sigma_{\text{nonloc}(e^+e^-\to e^+e^-)}}{\sigma_{\text{loc}(e^+e^-\to e^+e^-)}} = [V(-sl^2)]^2. \quad (11.1)$$

It follows from Eq. (4.21) that as $s \to \infty$ this ratio increases as $\exp\{2sl^2\}$ where $s = q^2 = (p_1 + p_2)^2$.

We also note that there is a rapid increase in the radiative corrections to the scattering process $e^+e^- \to e^+e^-$.

Thus, in this model of nonlocal interactions it turns out that for energies $s \gtrsim 1/l^2$ the entire perturbation series now becomes important. The electromagnetic interactions become strong. In the usual local electrodynamics this situation arises at very high energies, roughly defined by the condition

$$\frac{e^2}{4\pi}\ln\frac{s}{m^2} \sim 1.$$

This indicates that when the energy is sufficiently great one cannot use expansions of the amplitudes in terms of the electron charge. One must then develop methods which go beyond the usual perturbation theory.

Equations like (11.1) can be used to provide estimates of the deviations expected from the predictions of local

quantum electrodynamics. For example, study of the process $e^+ + e^- \to e^+ + e^-$ provides a more restrictive limit on the elementary length as compared with the measurements of the electron's anomalous magnetic moment. Analysis of the experimental data[18] obtained for a total energy $E_{e^+} + E_{e^-} \sim 2$ GeV indicates that

$$\left.\begin{aligned} l &\leqslant 0{,}9\cdot 10^{-14} \text{ cm charged sphere,} \\ l &\leqslant 1{,}5\cdot 10^{-14} \text{ cm charged ball.} \end{aligned}\right\} \qquad (11.2)$$

We have used the form factors of Eqs. (4.21) and (4.24) to obtain the above estimates.

We propose to conduct a complete analysis of the available experimental data from the point of view of this nonlocal quantum electrodynamics.

[1)]We have employed the following notation system:

$$\partial_\mu = \partial/\partial x_\mu;\ \gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha = 2g_{\alpha\beta};$$
$$g_{\alpha\beta} = 0,\ \alpha \neq \beta;\ g_{00} = -g_{11} = -g_{22} = -g_{33} = 1;$$
$$\hat{p} = p_\alpha\gamma_\alpha = p_0\gamma_0 - \mathbf{p}\boldsymbol{\gamma},\ pk = p_0k_0 - \mathbf{pk};$$
$$\hat{\partial} = \gamma_\alpha\partial_\alpha = \gamma_\alpha\partial/\partial x_\alpha.$$

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