Diffraction dissociation processes at high energies

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The review presents experimental data on diffraction dissociation processes. Identification methods, principal characteristics, and the possible mechanism of these processes are discussed.

INTRODUCTION

This article presents a review of data concerning the diffraction dissociation of elementary particles at high energies.

It is rather difficult to give an exact definition of diffraction dissociation processes. In what follows we will consider this term to mean classes of inelastic interactions in which the secondary particles can be separated kinematically into two definite groups. Each of these groups has the same quantum numbers as the primary particles except perhaps for spin and parity. In addition, the cross section for formation of these groups of particles is a weak function of the primary energy and the slope of the differential cross section is of the same order as for elastic scattering.

The possibility of diffraction dissociation (or diffraction production of particles) was first pointed out by I. Ya. Pomeranchuk and E. L. Feinberg[1] and later discussed in more detail by Good and Walker[2] and Bialas.[3] This possibility is due to the fact that the diffracted wave arising as the result of absorption of the incident wave by the target leads not only to elastic scattering but also under certain conditions to production of new particles, i.e., to inelastic interactions. It is reasonable to suppose that this process should be similar in its characteristics to elastic scattering.

As in the case of elastic scattering, the most widely used model describing diffraction dissociation processes is the Regge pole model. In terms of this model it is often suggested that diffraction dissociation is due to exchange of Pomeranchuk poles. A detailed analysis of the problems arising in such a description is contained in the article by Zachariasen.[4]

The present review considers the possibilities of identifying diffraction dissociation processes, the properties of systems arising in such processes, and the characteristics of the mechanism of formation of the dissociated systems. Data are presented only on exclusive reactions. Discussion of inclusive processes is beyond the scope of this review.

1. ENERGY DEPENDENCE OF THE CROSS SECTION FOR DIFFRACTION PROCESSES

One of the characteristic features of diffraction processes is the weak dependence of their cross sections on the primary energy. For description of the dependence of the cross sections for various reactions on the primary momentum Morrison[5] proposed use of the simple expression

$$\sigma \sim p^{-n}. \tag{1}$$

Here it turned out that diffraction dissociation processes have an exponent n = 0-0.8, in contrast to other reactions where n > 1. As an example we will present several of these reactions in which, in the effective-mass spectra of the systems written in parentheses, there are peaks produced with a cross section which depends weakly on the primary energy. Such peaks can be associated with the resonances whose designations are written in square brackets:

$$\pi^+p \to p(\pi^\pm\pi^+\pi^-) \quad [A_1,\ A_3]; \tag{2}$$

$$K^\pm p \to p(K^\pm\pi^+\pi^-) \quad [Q,\ L]; \tag{3}$$

$$XN \to X(N\pi) \text{ or } X(N\pi\pi) \text{ [various } N^*_{1/2}], \tag{4}$$

$$X - \pi, K, p, \tilde{p};$$

$$\gamma p \to p\begin{pmatrix}\pi^+\pi^-\\ K^+K^-\end{pmatrix} \text{ or } p(\pi^+\pi^-\pi^0)\,[\rho,\ \omega,\ \varphi]; \tag{5}$$

$$ep \to ep(\pi^+\pi^-) \text{ or } ep(\pi^+\pi^-\pi^0)\,[p,\ \omega]^*. \tag{6}$$

The nature of the singularities in the effective-mass distribution for reactions (2) and (3) is shown in Fig. 1.

In study of the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ the energy dependence of the cross section was studied[6] for various effective-mass intervals of the $(\pi^+\pi^-\pi^-)$ system. These data are shown in Fig. 2. As can be seen from the figure, for momenta above 5 GeV/c the cross section dσ/dM in the A-meson region [$0.8 \le M(3\pi) \le 1.4$ GeV] becomes practically constant, while for the region $M(\pi^+\pi^-\pi^-) >$ 1.4 GeV, dσ/dM changes rather substantially with energy. Constancy of the cross sections is observed also for the Q region in the reaction $K^+p \to pK^+\pi^+\pi^-$ (Fig. 3) and in photoproduction of the ρ meson (Fig. 4).

At the present time there are contradictory data on the behavior of the cross section in the L-meson region.

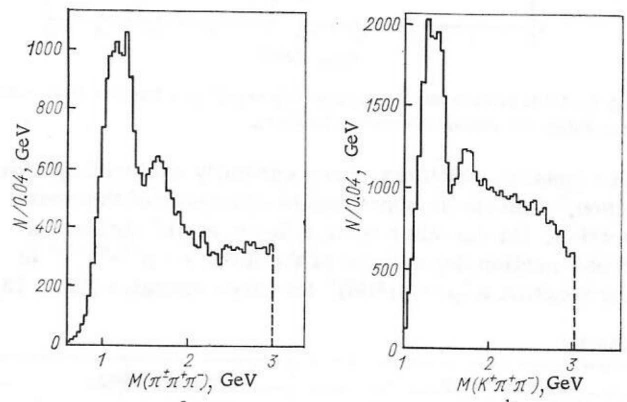

Fig. 1. Combined distribution of effective mass of the (3π) system for the reaction $\pi^\pm p \to p\pi^\pm\pi^+\pi^-$ in the interval from 11 to 20 GeV/c (a) and of the (Kππ) system for the reaction $K^+p \to pK^+\pi^+\pi^-$ in the interval from 7 to 13 GeV/c (b).

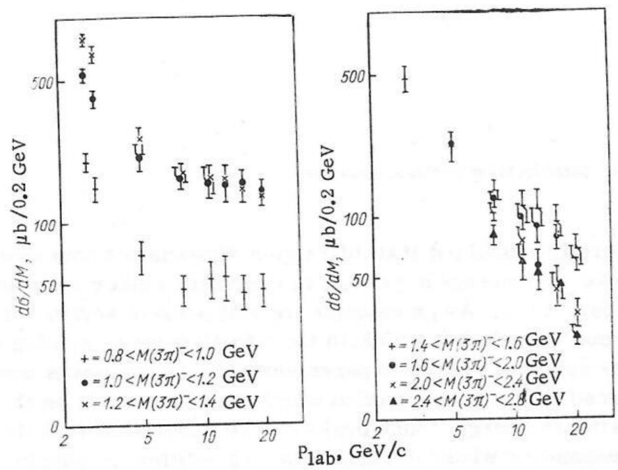

Fig. 2. Cross section for production of the (3π) system in various effective-mass intervals as a function of primary momentum for the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$.

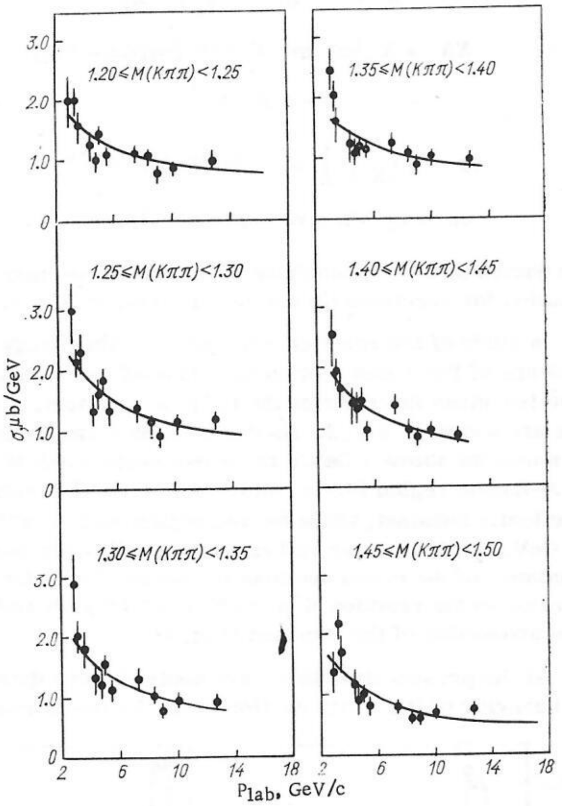

Fig. 3. Cross section for the reaction $K^+p \to pQ^+$ as a function of K^+-meson momentum for various intervals of $M(K\pi\pi)$.

The results, which have been carefully analyzed by Morrison,[7] indicate an approximate constancy of the cross section. On the other hand, Blieden et al.[8] obtained a cross-section dependence of the form $\sigma \sim p^{-1.90 \pm 0.8}$ in the reaction $K^-p \to p(MM)^-$ for three energies (10.9, 13.4,

TABLE 1

Elastic processes		Inelastic processes	
process	n	process	n
K^+p	0.09 ± 0.03	$K^0 \to Q^0$	0.59 ± 0.16
K^-p	0.39 ± 0.04	$K^+ \to Q^+$	0.60 ± 0.05
πN	0.2	$\pi^- \to A_1$	0.41 ± 0.11
NN	0.2	$\pi^- \to A_3$	0.8 ± 0.3

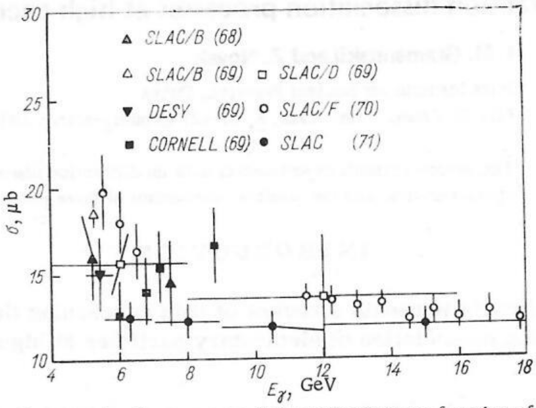

Fig. 4. Cross section for ρ-meson photoproduction as a function of energy in the region of E_γ above 5 GeV.

and 15.9 GeV) in the momentum-transfer interval $0.12 \le |t| \le 0.40\ (\text{GeV/c})^2$ for the L meson.

The results of approximation of the cross-section dependence on the primary momentum by Eq. (1) for various reactions are listed in Table 1.

It should be noted that a zero value of n would indicate that in terms of the theory of complex angular momenta the main contribution to diffraction processes is from the pomeron trajectory $\alpha = 1 + \alpha'_p t$. However, analysis of elastic scattering shows the necessity of introducing secondary degenerate trajectories of the the type $\alpha_R = 1/2 + t$. In this case the energy dependence is expressed as follows:

$$d\sigma/dt \approx A + Bs^{-1/2} + Cs^{-1/4}, \tag{7}$$

where the last term of this expression is unimportant in most cases. A similar dependence can be expected also for diffraction dissociation processes.

In systems with a baryon number B = 1 existence of peaks is also observed in the mass spectrum M_B of these systems in the region of comparatively low M_B values. A characteristic illustration is the missing-mass spectra in the reaction $pp \to p(MM)^+$ at energies[9] of 9.9, 15.1, and 29.7 GeV. Approximation of the spectra by expressions containing a Breit–Wigner formula for description of the resonances and a polynomial for the nonresonance background gave the following parameters:

$$M = 1411 \pm 10\ \text{MeV} \quad \Gamma = 188 \pm 38\ \text{MeV}$$
$$M = 1501 \pm 6\ \text{MeV} \quad \Gamma = 140 \pm 43\ \text{MeV}$$
$$M = 1690 \pm 5\ \text{MeV} \quad \Gamma = 133 \pm 26\ \text{MeV}.$$

Using the data of that work on the mass spectra $d\sigma/dM$ for a fixed value $t = -0.044\ (\text{GeV/c})^2$ and the relation

$$\sigma = \sigma_\infty + \sigma_1 s^{-1/2}, \tag{8}$$

Frampton and Ruuskanen[10] obtained the asymptotic behavior of $d\sigma/dM$ for $s \to \infty$, which is shown in Fig. 5. The smooth curve in this figure shows the combined data obtained after subtraction of the background. On the basis of the agreement of this curve in shape and magnitude with $(d\sigma/dM)_{s \to \infty}$ we can conclude that the cross section

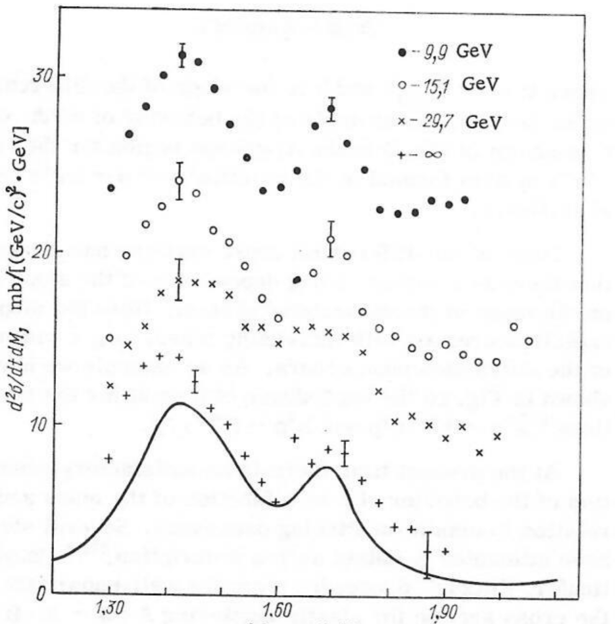

Fig. 5. Dependence of $d^2\sigma/dtdM$ on M for the reaction $pp \to p + (MM)^+$ at 9.9, 15.1, and 29.7 GeV/c and for a fixed value $t = -0.044$ (GeV/c)^2: +) asymptotic value of cross section for $s \to \infty$ obtained from Eq. (8); smooth curve) contribution of resonances after subtraction of background.

of the background processes for $s \to \infty$ becomes negligible, while the cross section of the resonances being discussed remains practically constant.

Considerable interest is presented by study of the effective-mass spectra of (Nπ), (Nππ), and similar states with isospin I = 1/2. Separation of the isospin components of the (Nπ) systems I = 1/2 and I = 3/2 is easily accomplished on the basis of an isospin analysis of the reactions $\pi^+p \to (p\pi^+)\pi^0$, $\pi^+p \to (p\pi^0)\pi^+$, and $\pi^+p \to (n\pi^+)\pi^+$. Such an analysis, carried out in the interval from 4 to 16 GeV/c by Boesebeck et al.[11], shows that the cross-section dependence of the state with I = 1/2 is given by the expression $\sigma(N^*_{1/2}) \sim p^{-0.63}$, in contrast to the cross section for the state with I = 3/2, for which $\sigma(N^*_{3/2}) \sim p^{-1.55}$. Since we have included in the isospin analysis the production of single

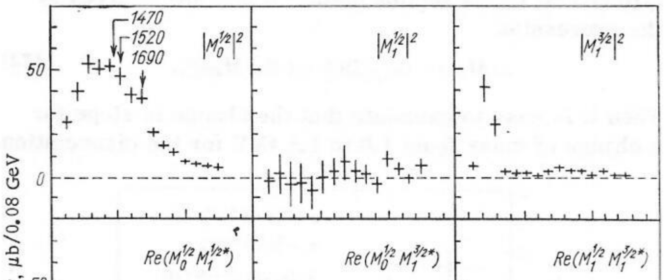

Fig. 6. Square of the modulus of the amplitudes $|M^{I(N\pi)}_{I_{ex}}|^2$ and the corresponding interference terms as a function of the mass of the (Nπ) system for the reaction $\pi^\pm p \to \pi\pi N$ at 16 GeV/c.

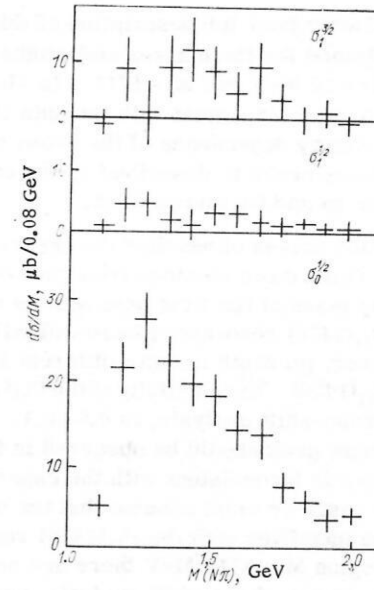

Fig. 7. Cross section as a function of the mass of the (Nπ) system in the reaction $K^-p \to \tilde{K}\pi N$ at 10 GeV/c for various isospin states. The superscript indicates the isospin of the (Nπ) system, and the subscript the isospin of the exchange particle.

pions in π^-p interactions, we can obtain, in addition to the isospin of the (Nπ) system, information on the isospin of the exchange particle. The results of such an analysis, carried out by Boesebeck et al.[12] for $\pi^\pm p$ interactions at 16 GeV/c, are shown in Fig. 6. It can be seen that the contribution of amplitudes with $I_{ex} = 1$ is negligible. For amplitudes with $I_{ex} = 0$ in the low-mass region a broad peak is observed without visible structure. Similar data on analysis of the (Nπ) system for the reaction $K^-p \to \tilde{K}(N\pi)$ are shown[13] in Fig. 7.

Let us turn to discussion of the spectra of the (Nππ) system. Many workers have observed two peaks in the distribution in $M(p\pi^+\pi^-)$ in reactions $Xp \to Xp\pi^+\pi^-$. Figure 8 shows the distribution in $M(p\pi^+\pi^-)$ from data obtained[14] in π^+p interactions at 8 and 16 GeV/c, π^-p interactions at 16 GeV/c, and K^-p interactions at 10 GeV/c. Two Breit–Wigner functions and various expressions for

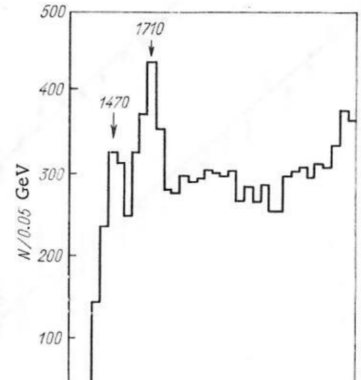

Fig. 8. Distribution in the effective mass $M(p\pi^+\pi^-)$ in reactions $X + p \to X + p\pi^+\pi^-$, where X is π^+ at 8 and 16 GeV, π^- at 16 GeV, and K^- at 10 GeV.

the background were used for description of this spectrum. The values obtained for the masses and widths ($M = 1462 \pm 10$ MeV, $\Gamma = 60 \pm 20$ MeV and $M = 1711 \pm 10$ MeV, $\Gamma = 57 \pm 15$ MeV) are in good agreement with the data of other studies. The energy dependence of the cross section for formation of these peaks is described by the expression $\sigma \sim p^{-0.2\pm0.1}$ for πp and Kp interactions.

The question arises of whether the observed peaks represent the resonances obtained from phase-shift analysis data. The mass of the first peak agrees with the mass of the $P_{11}(1470)$ resonance (the so-called Roper resonance). However, its width is quite different from the width of the $P_{11}(1470)$. The elasticity of the $P_{11}(1470)$, obtained from phase-shift analysis, is 0.6 ± 0.1. Consequently, a narrow peak should be observed in the $M(N\pi)$ spectrum, which is inconsistent with the experimental data. On this basis we must assume that the observed peak cannot be identified with the $P_{11}(1470)$ resonance. In the mass region $M = 1710$ MeV there are several resonances obtained from phase-shift analysis, and identification of the observed peak with any definite resonance is practically hopeless.

Thus, the data presented suggest that the cause of the appearance of the peaks discussed may be diffraction dissociation processes.

2. ANALYSIS OF DIFFERENTIAL CROSS SECTIONS

An important characteristic of diffraction processes is the dependence of the differential cross section on t. As in the case of elastic scattering, this dependence can be represented for small scattering angles in the form of an exponential

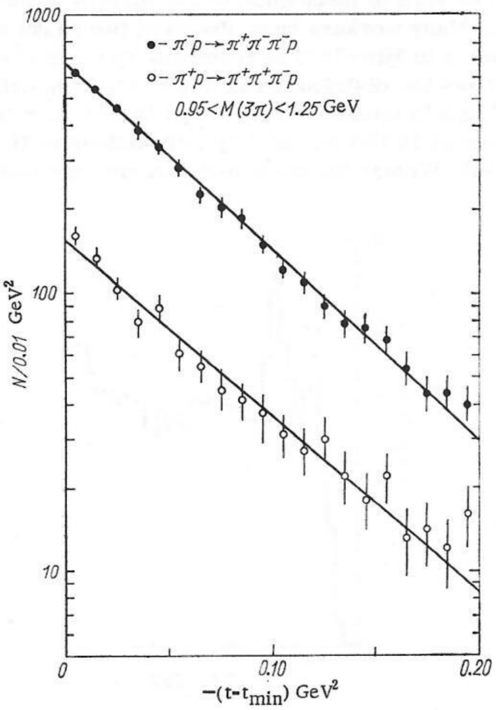

Fig. 9. Distribution in momentum transfer for the reactions $\pi^\pm p \to A_1^\pm$. Approximation of the experimental data by Eq. (9) gives values $b^{(+)} = 14.50 \pm 0.73$ (GeV)$^{-2}$ and $b^{(-)} = 15.59 \pm 0.53$ (GeV)$^{-2}$.

$$d\sigma/dt = A\exp(bt'), \tag{9}$$

where $t' = |t - t_{min}|$ and b is the slope of the differential cross section. An example of the behavior of $d\sigma/dt'$ with t' is shown in Fig. 9 in the A_1-meson region for the ($\pi^+\pi^+\pi^-$) system formed in the reaction[15] $\pi^\pm p \to p\pi^\pm\pi^+\pi^-$ at 15 GeV/c.

Study of the differential cross sections has shown that there is a rather strong dependence of the slope b on the mass of the dissociated system. Here the slope b rapidly decreases with increasing mass, i.e., a widening of the diffraction peak occurs. As an example we have shown in Fig. 10 the dependence of b on M for the reactions[16] $\bar{K}^0p \to K^{*-}\pi^+p$ and $K^0p \to K^{*+}\pi^-p$.

At the present time there is no satisfactory description of the behavior of b as a function of the mass and its relation to elastic-scattering processes. Several studies have attempted to obtain such a description.[17,18] In particular, Elitzur[17] proceeded from the well-known fact that the cross section for elastic scattering $A+B \to A+B$ can be described by means of the electromagnetic form factors of the interacting particles

$$d\sigma/dt = \text{const}\,[F_A(t)\cdot F_B(t)]^2. \tag{10}$$

He further suggested that Eq. (10) is valid also for creation of excited states: $A+B \to A+B^*$. Here the dependence of the form factor on t is the same function with an argument multiplied by the scale factor[2)] $(M_B/M_{B^*})^2$, i.e., $F_{B^*}(t, M_{B^*}^2) = F_B[t(M_B/M_{B^*})^2]$. Then the cross section for production of the excited state B^* will be

$$(d\sigma/dt)_{AB\to AB^*} = \text{const}\left[\frac{d\sigma_{AB}(t)}{dt} \times \frac{d\sigma_{AB}(t(M_B/M_{B^*})^2)}{dt}\right]. \tag{11}$$

Equation (11) has been used to calculate the cross sections for production of the isobars formed in pp interactions[17] at 24 GeV. The results of the calculation agree quite well for a number of isobars with the experimental data given in Fig. 11. According to this scheme the dependence of b on M_B for an exponential behavior of the differential cross section $d\sigma/dt = A\exp(bt)$ is given by the expression

$$b(M_{B^*}) = (b_{AB}/2)[1 + (M_B/M_{B^*})^2]. \tag{12}$$

Then it is easy to calculate that the change in slope for a change of mass from 1.0 to 1.5 GeV for the dissociation

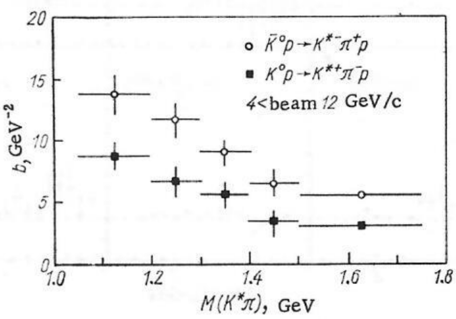

Fig. 10. Differential cross-section slope b as a function of $M(K^*_{890}\pi)$ in the reactions $K^0p \to p(K^{*+}_{890}\pi^-)$ and $K^0p \to p(K^{*-}_{890}\pi^+)$. Averaging was carried out over the interval $4 \leq p_{beam} \leq 12$ GeV/c.

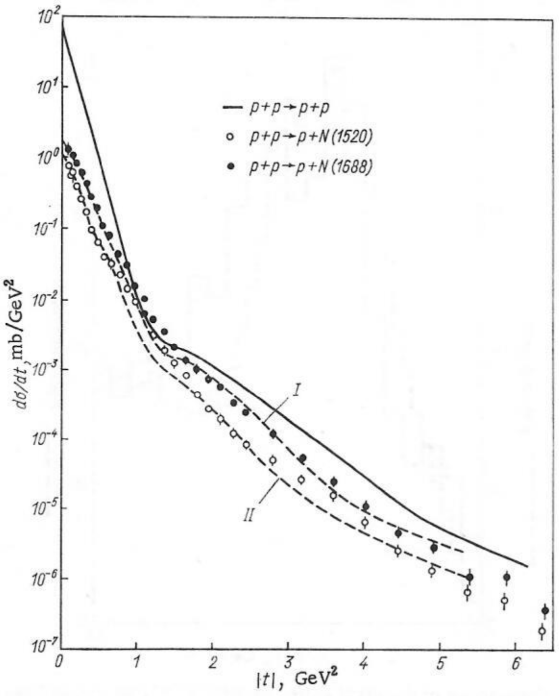

Fig. 11. Comparison of results calculated according to Elitzur's model on production of resonances with I = 1/2 with experimental data on the reactions pp → pN*⁻(1520) (curve II) and pp → pN*(1688) (curve I) at 24 GeV.

$\pi \to A_1$ will amount to about 1% and for the dissociation $K \to Q$ about 15%. The observation of a strong dependence of b on M (see, for example, Fig. 10) indicates that this scheme is apparently not applicable[3)] for boson-dissociation processes.[20]

It is well known that in elastic scattering the behavior of the diffraction peak with increasing energy is different for different particles. For example, the diffraction peak narrows for pp scattering, remains constant for πp scattering, and widens for $\bar{p}p$ scattering.

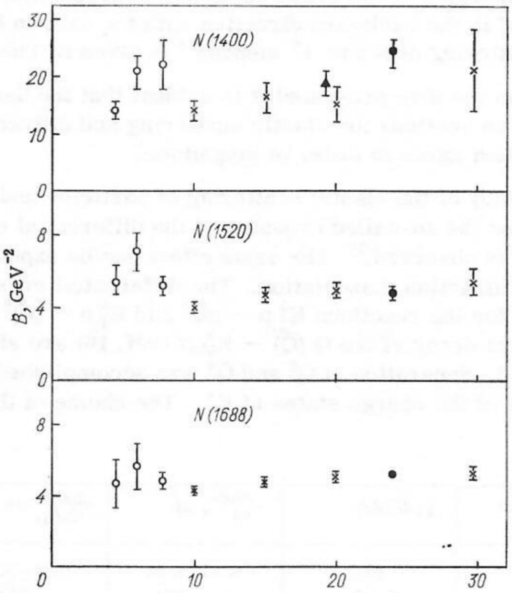

Fig. 12. Dependence of the parameter b defined in Eq. (9) as a function of primary momentum, for production of isobars with I = 1/2.

TABLE 2

Reaction	b, GeV^{-2}	
	p = 8 GeV/c	p = 16 GeV/c
$\pi^- p \to \pi^- p$	8.2±0.3	7.8±0.3
$\pi^- p \to \pi^- N^*$ (1400)	13.3±1.3	15.9±1.3
$\pi^- p \to \pi^- N^*$ (1520)	4.4±0.2	5.1±0.15
$\pi^- p \to \pi^- N^*$ (1690)	4.1±0.1	4.6±0.1

The experimental data existing at the present time on diffraction dissociation processes do not permit a detailed analysis to be made of the dependence of b on s. The difficulties of the analysis are due mainly to the fact that different intervals in t have been used for determination of b in different studies. In Fig. 12 we have shown the dependence of b on primary momentum for production of isobars in pp interactions according to the data of Allaby et al.[21] These data strongly suggest that the slope for production of isobars apparently increases with energy. The values of b in reactions with formation of πN^*-meson isobars at 8 and 16 GeV are given[22] in Table 2 and also indicate some rise in b with increasing s, while the slope for elastic $\pi^- p$ scattering remains constant or even decreases.

It is interesting to note that in the reaction $K^+ p \to pQ^+$ the narrowing of the diffraction peak is observed[23] for the decay mode $Q^+ \to K^+\pi^+\pi^-$ and is absent for the

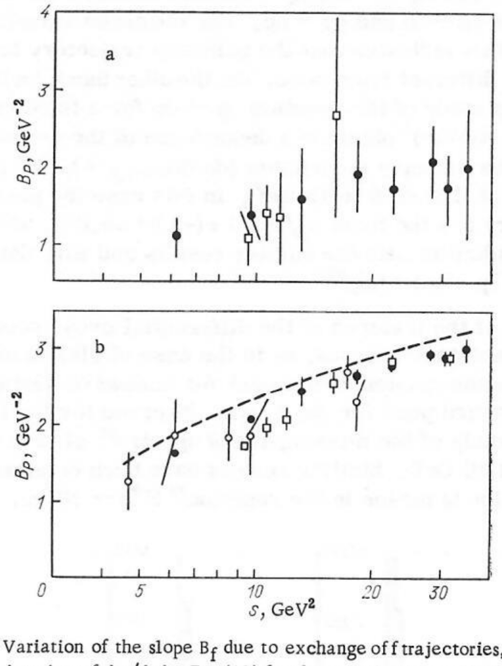

Fig. 13. Variation of the slope B_f due to exchange of f trajectories, obtained by approximation of $d\sigma/dt$ by Eq. (13) for the reaction $\gamma p \to \gamma p$ and $\gamma p \to pp$ (a) and of the slope B_P due to exchange of a pomeron for the reaction $\gamma p \to \gamma p$ (hollow squares), $\gamma p \to \rho p$ (solid circles), and $\gamma p \to \varphi p$ (hollow circles) (b). For the last reaction the approximation was made by the expression $d\sigma/dt \sim \exp[2B_P t]$.

TABLE 3

Reaction	$\gamma p \to \rho p$	$\pi N \to \pi N$
A_p, mb$^{1/2}$ GeV^{-1}	4.6±0.3	4.82±0.14
A_f, mb$^{1/2}$	6.4±0.6	5.4±0.5

mode $Q^+ \to K^0\pi^0\pi^+$. The differential cross sections for photoproduction of resonances and their dependence on s have been investigated in terms of the so-called dual model with absorption.[24] According to this model the differential cross section for the reactions $\gamma p \to \gamma p$ and $\gamma p \to \rho p$ is approximated by the expression

$$d\sigma/dt = A_P^2 \exp(2B_P t) + 2A_P A_f \exp[(B_P + B_f)t] J_0(R\sqrt{t})\, s^{-1/2}, \quad (13)$$

where the subscripts P and f refer to the pomeron and f trajectories. The function B(s) obtained as a result of the approximation is shown in Fig. 13. The values of A_p and A_F, multiplied by the scale factor $\gamma_{\gamma p}/\sqrt{\pi\alpha}$ which follows from the model of vector dominance, are listed in Table 3. Also listed in Table 3 are the similar data obtained in analysis of elastic πN scattering. The good agreement of these values may mean that the role of pomeron exchange and f-trajectory exchange in the two processes is identical.

Freund[25] and Barger and Cline[25] suggested that, in contrast to the reactions discussed, only the vacuum trajectory is important in photoproduction of the φ meson. Study of polarization effects has confirmed this suggestion. Therefore the differential cross section for the reaction $\gamma p \to \varphi p$ should be described by the expression $d\sigma/dt \sim \exp(2B_p t)$. Values of B_p are also given in Fig. 13 for various s and agree roughly with data obtained for the reactions $\gamma p \to \gamma p$ and $\gamma p \to \rho p$. The observed variation of B_p with s indicates that the pomeron trajectory has a slope α' different from zero. On the other hand, Anderson et al.[26] in study of the reaction $\gamma p \to \varphi p$ for a fixed value $|t| = 0.6\ (\mathrm{GeV}/c)^2$ obtained a dependence of the cross section on the primary momentum $(d\sigma/dt)_{t=0.6} = E^{-0.02}$ in the interval of E from 6 to 19 GeV. In this case the pomeron trajectory has the form $\alpha_p(t) = 1 \pm (-0.03 \pm 0.13)t$, which is in contradiction with the earlier results and with data on pp and K^+p scattering.[27]

One of the features of the differential cross section for diffraction processes, as in the case of elastic scattering, is the existence of a peak for backward scattering. The backward peak for the A_1 was observed for the first time in study of the missing-mass spectra[28] of $\pi^-p \to p(MM)$ at 16 GeV. Similar results have been obtained in study of the Q meson in the reaction[29] $K^+p \to NK\pi\pi$. In

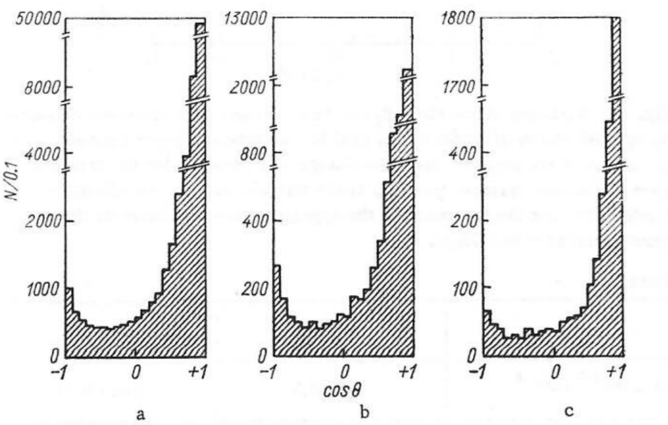

Fig. 14. Angular distribution of the $(K\pi\pi)$ system in the center of mass for the reactions: a) $K^+p \to K^+\pi^+\pi^-p$; b) $K^+p \to K^0\pi^+\pi^0p$; c) $K^+p \to K^0\pi^+\pi^+n$.

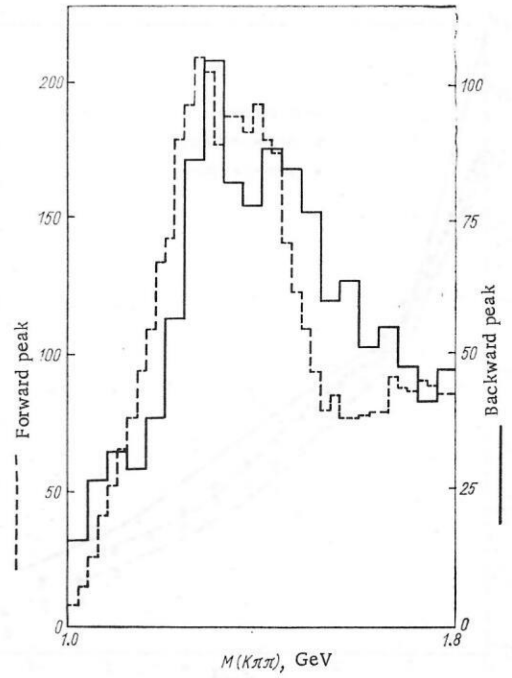

Fig. 15. Comparison of the distributions of $M(K^+\pi^+\pi^-)$ for events from the backward and forward hemispheres in the center of mass for the reaction $K^+p \to pK^+\pi^+\pi^-$.

Fig. 14 we have shown the angular distribution of the $(K\pi\pi)$ system in the center of mass. The mass spectrum of the $(K\pi\pi)$ system for events from the backward peak in the reaction $K^+p \to pK^+\pi^+\pi^-$ is shown in Fig. 15, where for comparison we have also shown the mass spectrum of this same system emitted in the forward direction. On separation of the mass interval $1.16 \le M(K^+\pi^+\pi^-) \le 1.52$ GeV, i.e., the Q-meson region, the energy dependence for the backward cross section turned out to be $\sigma(s) \sim s^{-4.2\pm0.2}$. This leads to an averaged Regge trajectory of the form $\alpha(0) = -1.1 \pm 0.1$, which is in good agreement with the Λ trajectory.

Comparison of the cross sections for production of Q^+ and A_1^- in the backward direction with the data on backward scattering of K and π^+ mesons[30] is given in Table 4.

From the data presented it is evident that the backward cross sections for elastic scattering and diffraction dissociation agree in order of magnitude.

In study of the elastic scattering of particles and antiparticles the so-called crossing of the differential cross sections is observed.[31] The same effect can be expected also in diffraction dissociation. The differential cross sections for the reactions $K_L^0 p \to p\bar{Q}^0$ and $K_L^0 p \to pQ^0$ with subsequent decay of the Q $(\bar{Q}) \to K^*_{890}\pi$ (ref. 16) are shown in Fig. 16. Separation of Q^0 and $\bar{Q}^0$ was accomplished on the basis of the charge states of K^*. The choice of these

TABLE 4

Initial state	p, GeV/c	σ_{el}^{back}, μb	σ_{Q,A_1}^{back}, μb
π^-p	16.0	0.24±0.03	$0.35^{+0.87}_{-0.28}$
K^+p	5.2	5	7
K^+p	6.9	1.5	3

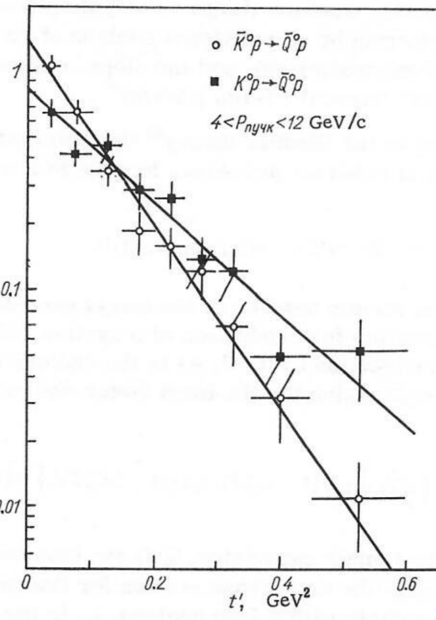

Fig. 16. Differential cross sections for the reactions $K^0p \to Q^0p$ and $\bar{K}^0p \to Q^0p$ in the primary-momentum interval 4-12 GeV/c. The slopes of the differential cross sections are $b_Q = 5.9$ (GeV/c)$^{-2}$ and $b_Q = 9.7$ (GeV/c)$^{-2}$.

reactions is due to the fact that in this case there is no problem in the conditions of normalization. It should be emphasized that, in spite of the difference of the differential cross sections, the total cross sections for these reactions are practically identical: $\sigma(K^0p \to pQ^0)/\sigma(\bar{K}^0p \to \bar{Q}^0p) = 0.99 \pm 0.08$. As can be seen from Fig. 16, the intersection of the curves describing the differential cross sections occurs at $|t'| = 0.13 \pm 0.03$, i.e., for the same value of t as in the case of elastic scattering. The most likely explanation of the crossing effect, as in the case of

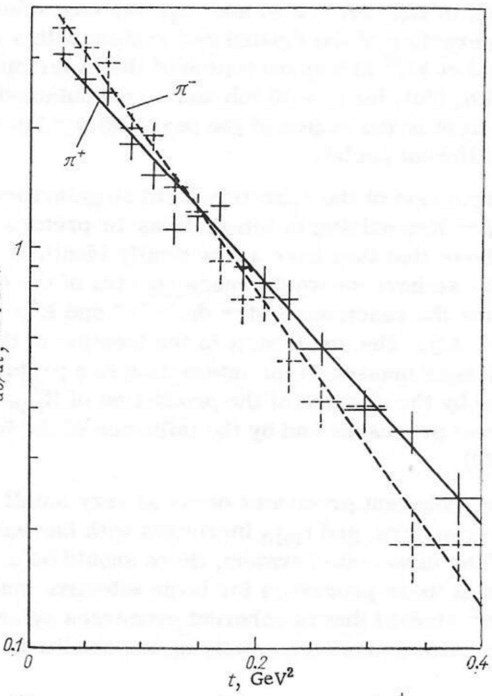

Fig. 17. Differential cross section for dissociation of $\pi^\pm$ mesons in the reactions $\pi^\pm p \to p\pi^\pm\pi^+\pi^-$ at 16 GeV/c. Identification of events corresponding to the dissociation process was accomplished by analysis in longitudinal phase space.

elastic scattering, in terms of Regge theory is the existence of secondary trajectories.

We will designate the pomeron, vector, and tensor parts of the amplitude by A_P, A_V, and A_T, respectively. Then, if the amplitude A can be represented for the reaction $K^0p \to pQ^0$ in the form $A = A_P + A_T - A_V$, then for the reaction $\bar{K}^0p \to p\bar{Q}^0$ the amplitude is $A = A_P + A_T + A_V$. Neglecting quadratic terms and assuming that the pomeron amplitude is pure imaginary, it is easy to obtain the relation

$$d\sigma/dt\,(p\bar{Q}^0) - \frac{d\sigma}{dt}\,(pQ^0) = \sum_{\lambda\mu} A_P^{\lambda\mu}\,\mathrm{Im}\,A_V^{\lambda\mu}, \tag{14}$$

where the superscripts indicate the change in helicity at the two vertices. Since a decrease in $d\sigma/dt$ at $|t| \sim 0$ is not observed in these reactions, then it follows from the reasoning given below that in scattering forward $\lambda = \mu = 0$. Using the value of the ratio $\alpha = \mathrm{Re}\,A/\mathrm{Im}\,A \approx 1$ from $K_L^0p \to K_S^0p$, we can calculate the following quantities:[16]

$$A_P^{00}/A_P^{el} = \sqrt{\left(\frac{d\sigma}{dt'}\right)(pQ^0)\Big/\left(\frac{d\sigma}{dt}\right)(K^+p)}\;\Bigg|_{t'=0} = 0.4 \pm 0.1; \tag{15}$$

$$\mathrm{Im}A_V^{00}/\mathrm{Im}A_V^{el} = \frac{\left(\frac{d\sigma}{dt'}\right)(p\bar{Q}^0) - \left(\frac{d\sigma}{dt'}\right)(pQ^0)}{\left[8\left(\frac{d\sigma}{dt'}\right)(pQ^0)\left(\frac{d\sigma}{dt}\right)(K_S^0p)\right]^{1/2}}\Bigg|_{t'=0} = 0.8 \pm 0.3; \tag{16}$$

$$|\,\mathrm{Im}A_V^{00}/A_P^{00}| = \frac{\left(\frac{d\sigma}{dt'}\right)(p\bar{Q}^0) - \left(\frac{d\sigma}{dt'}\right)(pQ^0)}{4\left(\frac{d\sigma}{dt'}\right)(pQ^0)}\Bigg|_{t'=0} = 0.17. \tag{17}$$

All of the values were obtained by averaging over the energy interval considered from 4 to 12 GeV. It should be noted that the data obtained refer to the entire mass interval of the ($K\pi\pi$) system in the Q region. For elastic scattering according to the data of Davier and Harari[32] the ratio $|\mathrm{Im}A_V^{el}/A_P^{el}| = 0.19$, which is practically identical to the value given by Eq. (17).

The effect of crossing of the cross sections of the (3π) system in the reactions $\pi^\pm p \to p\pi^\pm\pi^+\pi^-$ has been observed[33] at 16 GeV/c (Fig. 17). The values obtained for the crossing points for $|t'| = 0.15$ (GeV/c)^2 correspond to the values given above. Here this effect can be explained as the result of interference of amplitudes due to exchange of pomeron and ρ-meson poles. However, we should then expect the presence of the A_1 peak in reactions with charge exchange. Approximate estimates by Kane[34] employing the difference in the differential cross section, isotopic ratio, and so forth, give a cross section for the reaction $\pi^-p \to nA_1^0$ approximately equal to 0.4 mb, which for 16 GeV/c amounts to $\sim 1/4$ of the cross section for the reaction $\pi^-p \to n\pi^0$. This result is in contradiction with the experimental data.

Beaupre et al.[33] state that the crossing of the cross sections is absent in the reactions $\pi^-p \to \pi^-N^*$ for the ($p\pi^+\pi^-$) system. On the other hand, Cohen-Tannoudji et al.[35] observe this effect in the reactions $pp \to pN^*$ and $\tilde{p}p \to \tilde{p}N^*$.

3. DIFFRACTION DISSOCIATION IN COHERENT PROCESSES

In study of inelastic interactions with nuclei it is

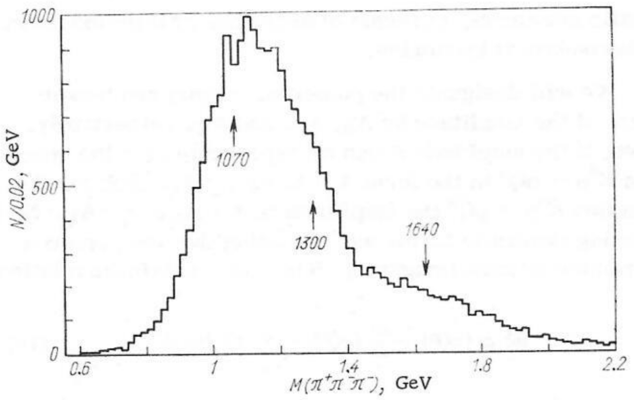

Fig. 18. Combined distribution of effective mass of the (3π) system in coherent processes $\pi^- A \to A(3\pi)$ in the nuclei Be, C, Al, Si, Ti, Ag, Ta, Pb at 15.1 GeV/c.

necessary to distinguish a special class of reactions in which the recoil nucleus remains in the ground state. These processes are given the name coherent. The study of coherent reactions at high energies has been carried out by various methods.[36,37]

Analysis of coherent interactions of nucleons and mesons with nuclei shows that in the secondary-particle effective-mass spectra, in the low-mass region, broad peaks are also observed. Examples of these spectra are shown in Fig. 18 from the work of Bemporad et al.[37] Analysis of coherent reactions is accomplished by studying the distribution in momentum transfer. This distribution has a characteristic form (Fig. 19) which can be crudely described by the sum of two exponentials:[38,39]

$$d\sigma/dt' = A\exp(-at') + B\exp(-bt'). \tag{18}$$

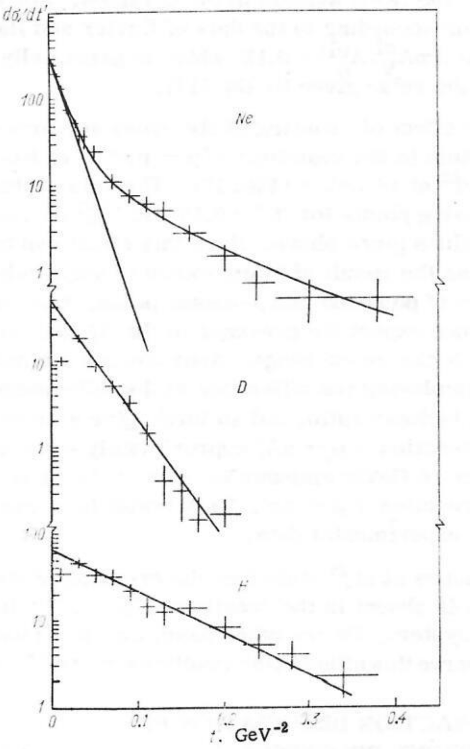

Fig. 19. Distribution in momentum transfer t' to the (3π) system for the nuclei H, D, and Ne.

The slope b in this equation (large values of t') corresponds to scattering by an individual nucleon of the nucleus (incoherent scattering), and the slope a becomes important in the interval 30-100 $(\mathrm{GeV}/c)^{-2}$.

According to the Glauber theory[40] the differential cross section of coherent processes has the following form:

$$d\sigma/dt = A^2 C(t, M)\,|F(t, M, A)|^2, \tag{19}$$

where A is the atomic number of the target nucleus, C is the cross section for production of a system with mass M in a free nucleon, and F(t, M, A) is the nuclear form factor. The expression for the form factor can be written in the form

$$F \approx \int \exp\left\{ i\mathbf{Q}\mathbf{b}\,\frac{1}{2}\left[\sigma_1(1-\alpha_1)T_1 + \sigma_2(1-i\alpha_2)T_2\right]\right\} \rho d^2 b db_z, \tag{20}$$

where $\mathbf{b}$ is the impact parameter, Q is the transverse momentum, σ_1 is the total cross section for interaction of the incident particle with a free nucleon, α_1 is the ratio of the real part of the elastic-scattering amplitude to its imaginary part at t = 0, σ_2 and α_2 are the same quantities for the second (dissociated) system, ρ is the density of nucleons inside the nucleus ($\int \rho d^3 r = 1$); $T_1 = \int_z^\infty A\rho\, dz$; $T_2 = \int_{-\infty}^z A\rho\, dz$. If we assume that the density of nucleons is given by the Woods–Saxon expression[41]

$$\rho(r) = \rho_0 / \{1 + \exp[(r - 1.12A^{1/3})/0.545]\}, \tag{21}$$

and use the known data on scattering of particles by nucleons, i.e., σ_1 and α_1, then the only unknown parameters remaining in Eq. (20) are σ_2 and α_2: the characteristics of the interaction of the dissociated system with a nucleon. Bemporad et al.[37] in approximation of the experimental data by Eq. (20), for σ_2 = 25 mb and α_2 = 0 obtained agreement of dσ/dt in the region of the peak $M(3\pi) \le 1.6$ GeV for the different nuclei.

Comparison of the near-threshold singularities $A_1 \to 3\pi$ and $Q \to K\pi\pi$ arising in interactions in protons and nuclei shows that they have a practically identical shape. In Fig. 20 we have shown the mass spectra of the $(K\pi\pi)$ system for the reactions $K^+d \to dK^+\pi^+\pi^-$ and $K^+p \to pK^+\pi^+\pi^-$ (ref. 42). The small shift in the location of the peak toward larger masses in the interaction in a proton is explained by the absence of the production of $K^*_{1420} \to K\pi\pi$ in coherent processes and by the influence of the form factor (20).

Since coherent processes occur at very small momentum transfers, and t_{min} increases with increasing mass of the dissociated system, there should be a suppression of these processes for large effective masses. Bingham[43] showed that in coherent processes systems can arise with a mass M_b satisfying the condition

$$M_b^2 \leqslant M_a^2 + 0.56A^{1/3}p_a,$$

where p_a is the primary-particle momentum. However,

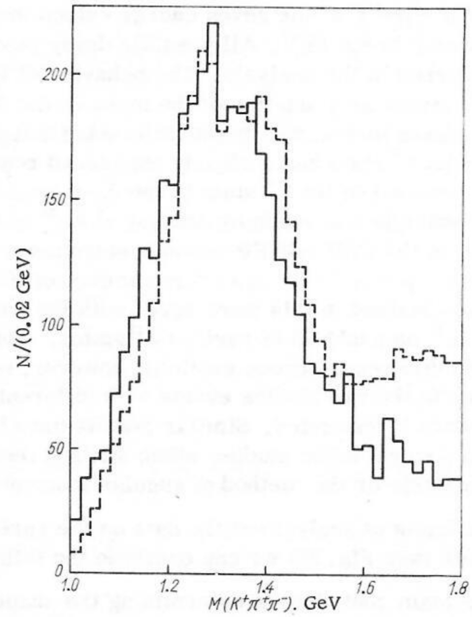

Fig. 20. Distribution in $M(K^+\pi^+\pi^-)$ for the reaction $K^+d \rightarrow dK^+\pi^+\pi^-$ at 12 GeV/c (events corresponding to formation of d^* have been excluded); the dashed line is the distribution in $M(K\pi\pi)$ for the reaction $K^+p \rightarrow pK^+\pi^+\pi^-$, normalized to the number of events in the interval $1.1 \leq M(K\pi\pi) \leq 1.5$ GeV for the first reaction.

this limitation cannot explain the fact that the peaks corresponding to A_3 and L mesons in coherent processes are not observed in any nuclei except deuterium (see Fig. 18). Bemporad et al.[37] observed coherent processes leading to five pions in the final state. The shape of the spectrum, which is shown in Fig. 21, turned out to be extremely similar to the spectrum in the case of three pions, but shifted toward larger masses.

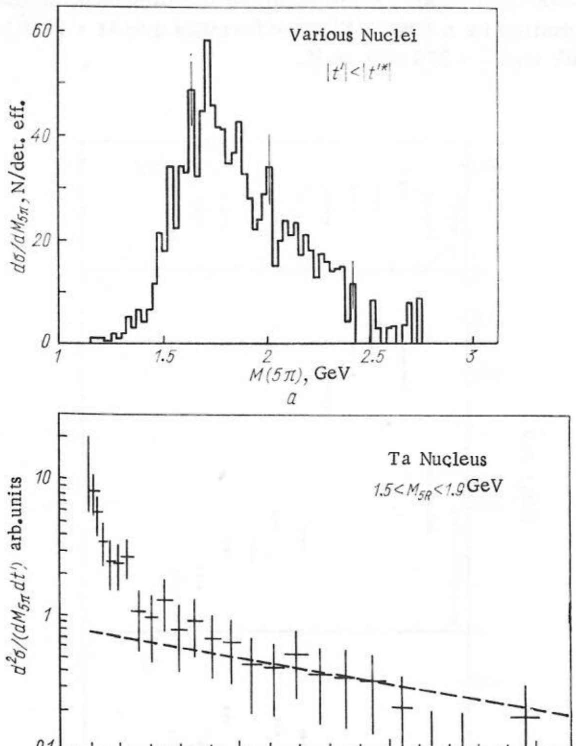

Fig. 21. Distribution in effective mass of the (5π) system for various nuclei with correction for detection efficiency and cutoff in t for various nuclei (a) and distribution in momentum transfer t to the (5π) system for the Ta nucleus (b).

According to some models the rapid drop in the mass distribution in diffraction dissociation processes is due to the appearance of new open channels with increase of the effective mass of the system.[44] Then, if coherent processes are induced by the same mechanism as diffraction processes in protons, they can be used to check this statement. It is particularly important that the background conditions are substantially improved in these processes.

The best check of such models would be an experiment on analysis of the missing mass. However, such an experiment has not yet been set up. In the composition of the effective-mass spectra of systems of three and five charged π mesons[37] it remains unclear what is the contribution of decays containing neutral π mesons and also what should be the normalization of these spectra, since the isospin states of the intermediate systems are unknown. It should be noted that, with a rather large variation of the relative normalization of these spectra, it is not possible to obtain a smooth total spectrum.

Study of coherent processes in nuclei with dissociation of nucleons has been carried out by Longo et al.[45] These authors studied the reaction $nA \rightarrow A + p\pi^-$ in carbon, copper, and lead nuclei. The mass spectra of the $(p\pi^-)$ system, shown in Fig. 22, have a broad peak in the low-mass region, in which separated resonances are not observed. A similar result was obtained[46] in the reaction $dp \rightarrow dp\pi^+\pi^-$ at 25 GeV.

4. ANALYSIS OF SECONDARY-PARTICLE ANGULAR DISTRIBUTIONS

Study of the angular distributions of secondary particles in diffraction processes can give useful information on the following problems:

a) determination of spin and parity J^P of the system appearing;

b) determination of the spin density matrix for the purpose of studying the mechanism of diffraction dissociation and, in particular, the question of helicity conserva-

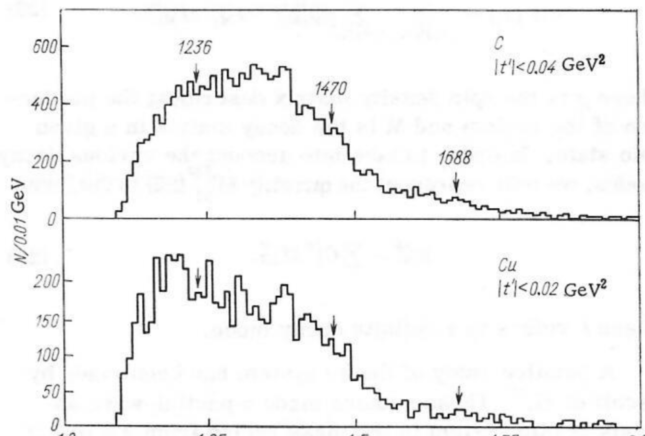

Fig. 22. Effective-mass distribution $M(p\pi^-)$ for reactions $nA \rightarrow (p\pi^-)A$.

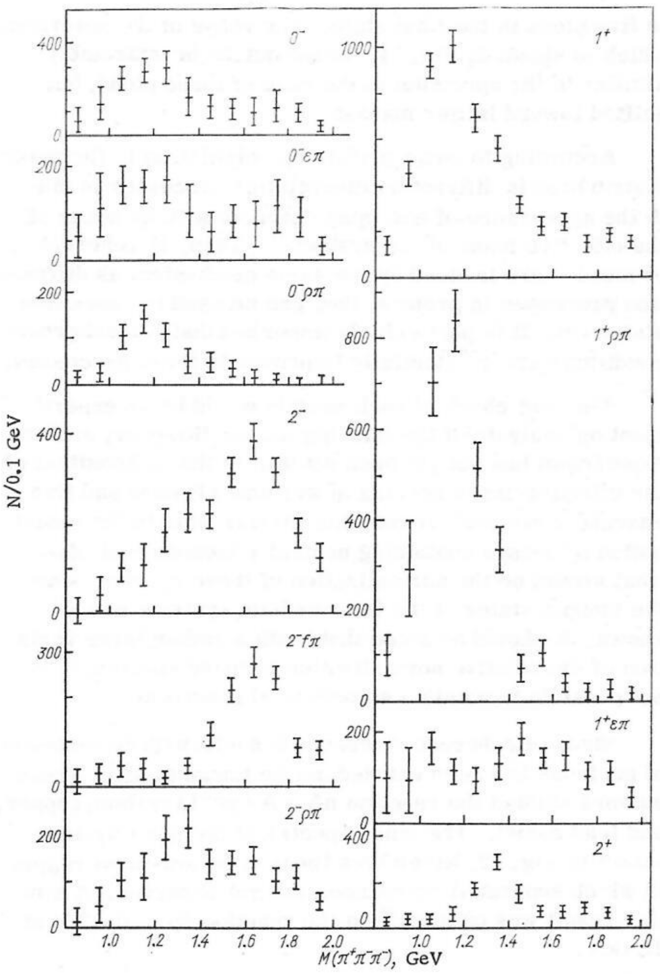

Fig. 23. Partial contributions of various J^P states for different decay modes of the (3π) system as a function of $M(3\pi)$ in the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ in the energy interval 11-25 GeV.

tion and the possibility of determining the quantum numbers of the exchange system;

c) study of the interaction in the intermediate state in three-particle decays.

The most general expression for the distribution of decay angles x_i has the following form:

$$W(x_i) = \sum_{J_1P_1M_1} \sum_{J_2P_2M_2} \rho_{M_1M_2}^{J_1P_1J_2P_2} M_{M_1}^{J_1P_1} M_{M_2}^{J_2P_2}, \tag{22}$$

where ρ is the spin density matrix describing the production of the system and M is the decay matrix in a given spin state. In order to take into account the various decay modes, we will represent the quantity M_M^{JP} (22) in the form:

$$M_M^{JP} = \sum_l C_l^{JP} M_{Ml}^{JP}, \tag{23}$$

where l refers to a definite decay mode.

A detailed study of the 3π system has been made by Ascoli et al.[47] These authors made a partial-wave analysis of this system in the mass region from 0.8 to 2.0 GeV. They utilized data on the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ at nine energy values in the interval 5-25 GeV and on the reaction $\pi^+p \to p\pi^+\pi^+\pi^-$ for seven energy values in the interval from 5 to 8.5 GeV. All possible decay modes were considered in the analysis. The behavior of the various J^P states as a function of the mass of the (3π) system is shown in Fig. 23. It should be noted that neither the 0^- nor the 1^+ state has a clearly expressed resonance nature, in contrast to the 2^+ state in the A_2-meson region. A similar analysis was made by Antipov et al.[48] of work carried out in the CERN-IHEP boson spectrometer for the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ at a π^--momentum of 40 GeV/c. The results obtained in this work agree with the data of Ascoli et al.[47] on analysis of various J^P states. Comparison of the differential cross sections, however, is difficult, since in the two studies events with different t' intervals were investigated. Similar results have been obtained in several other studies which utilized the Dalitz-plot analysis or the method of angular momenta.[49]

On the basis of analysis of the data on the various decay modes (see Fig. 23) we can conclude the following:

a) The main contribution determining the shape of the entire mass spectrum of the (3π) system is the $(\rho\pi)$ system with a relative angular momentum $l = 0$ and the corresponding state with $J^P = 1^+$. The same spectrum shape is given by the $(\rho\pi)$ system with $l = 1(J^P = 0^-)$, although its contribution is significantly smaller. On the other hand, the $(\varepsilon\pi)$ system does not lead to a maximum, but gives a spectral shape which depends weakly on the mass of the (3π) system (Fig. 24).

b) The A_3 peak is due to the $(f\pi)$ system in the 2^- state, and here no contribution from the $(\rho\pi)$ system is observed. The mass and width values obtained for this peak in approximation by a Breit–Wigner formula are $M = 1656 \pm 11$ MeV and $\Gamma = 270 \pm 30$ MeV.

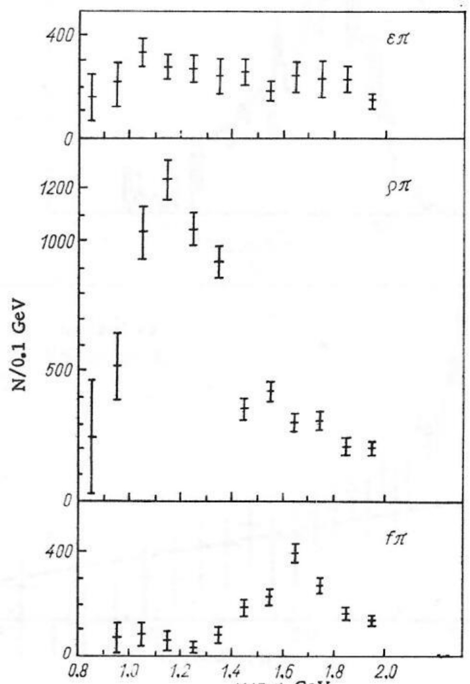

Fig. 24. Contributions of various decay modes of the (3π) system, summed over all J^P states, as a function of $M(3\pi)$ for the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ in the interval 11-25 GeV/c.

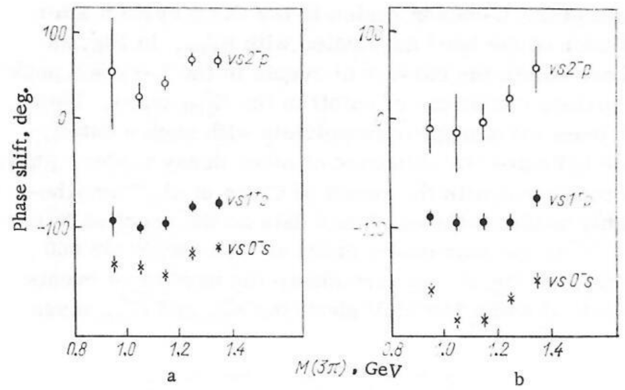

Fig. 25. Phase shift of the 1^+ s wave with respect to the 0^-s, 1^+ p, and 2^-s waves as a function of $M(3\pi)$ in the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$: a) at 40 GeV/c, $0.04 \leq |t| \leq 0.33$ (GeV/c)^2; b) at 5-7.5 GeV/c, $0 \leq |t| \leq 0.7$ (GeV/c)^2.

c) Calculations of the phase shifts of the amplitudes $1^+(\rho\pi)$, $0^-(\rho\pi)$, and $2^-(f\pi)$ associated with the observed peaks relative to other nonresonant amplitudes show that their phases do not change with change in mass of the (3π) system (Fig. 25).

d) From the analysis carried out by Ascoli et al.[47] it follows that the contribution of the 1^+ state with $l = 2$ (the d wave) is negligible in comparison with the s wave.

e) The differential cross sections for the 1^+ and 0^- states have a slope of the same order as in elastic scattering (Fig. 26).

f) The energy dependence of the cross section for production of the A_1 [$1.0 \approx M(3\pi) \leq 1.2$ GeV, $J^P = 1^+$] and A_3 [$1.5 \leq M(3\pi) \leq 1.8$ GeV, $J^P = 2^-$] in the reactions $\pi^-p \to p\pi^+\pi^-\pi^-$ and $\pi^+p \to p\pi^+\pi^+\pi^-$ can be represented in the form $\sigma(A_1) \sim p^{-0.4\pm0.11}$ and $\sigma(A_3) \sim p^{-0.82\pm0.46}$.

In investigation of the angular distributions in the $(K\pi\pi)$ system, simpler methods of analysis have been used to determine J^P. In some studies[50] the moments of the distribution of the normal to the plane of the decay have been calculated. It has been established that in the Q region the moments $\langle Y_{20}\rangle$, $\langle \mathrm{Re}Y_{21}\rangle$, and $\langle \mathrm{Re}Y_{22}\rangle$ are nonzero. This means that in the Q region the state with

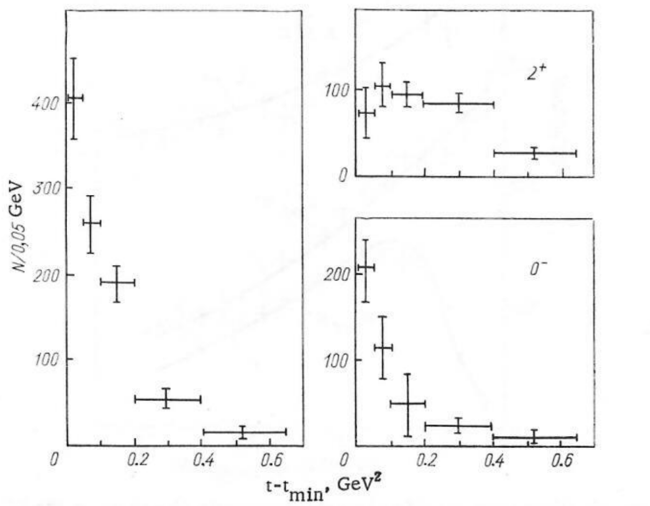

Fig. 26. Contributions of various J^P states as a function of momentum transfer for the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ in the interval 5-7.5 GeV.

$J^P = 1^+$ is realized preferentially. The choice of parity is due to the fact that in the $(K\pi)$ system there are no other resonances besides K^*_{1420}. On the basis of the analysis carried out, Davis et al.[50] draw the following conclusions:

a) In addition to the contribution of the 1^+ state there is a small nonvanishing contribution of the states 2^- and 0^-.

b) The value of ρ_{00} (on the assumption $J^P = 1^+$) is rather large but different from unity.

c) No appreciable variation of $\rho_{mm'}$ with $M(K\pi\pi)$ is observed in the Q region.

In the analysis carried out by means of Dalitz plots it is usually assumed that there is one definite J^P state and an incoherent background. In study of the Q region, states $J^P = 1^+$ with decay modes $K^*\pi$ and $K\rho$ were considered.[51] The ratio of the amplitudes of these decays can be expressed in the form $A(K\rho)/A(K^*\pi) = \alpha\exp(i\beta)$. The dependence of α on $M(K\pi\pi)$ is shown in Fig. 27. A substantial decrease in α is observed with increasing mass of the $(K\pi\pi)$ system. The dependence of β on $M(K\pi\pi)$ shows the existence of an appreciable change in the value of β with $M(K\pi\pi)$. This conclusion remains valid both for the $K^+\pi^+\pi^-$ system and for the $K^0\pi^+\pi^0$ system, and also for the combined consideration of these systems. Inclusion of 2^- states does not affect the observed dependence substantially.

The decrease of α with increasing $M(K\pi\pi)$ is inconsistent with the assumption that the observed peak in the Q region is due to one resonance. In terms of the quark model it can be assumed that in the Q region there are two

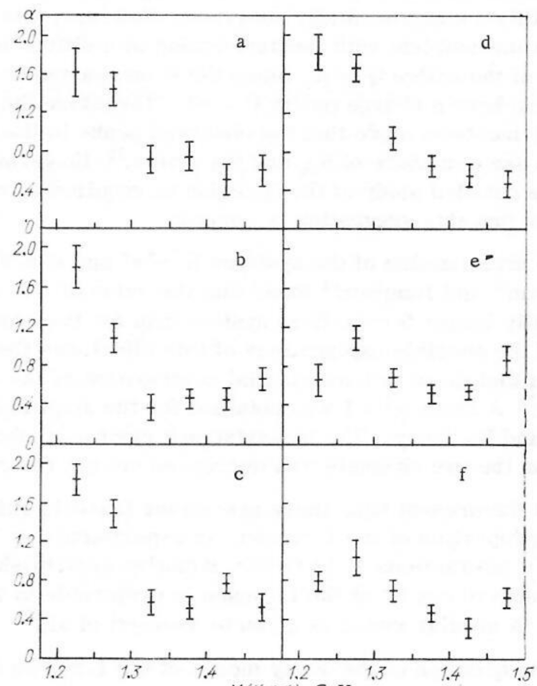

Fig. 27. Modulus of the ratio of amplitudes α for $K\rho$ and $K^*\pi$ decays in various momentum intervals p_{in} for the reactions $K^+p \to K^+p\pi^+\pi^-$ (a-d) and $K^+p \to pK^0\pi^+\pi^0$ (e-f) as a function of $M(K\pi\pi)$: a) $2.5 \leq p_{in} \leq 4.4$ GeV/c; b) $4.4 \leq p_{in} \leq 7.0$ GeV/c; c) $7.0 \leq p_{in} \leq 9.5$ GeV/c; d) $9.5 \leq p_{in} \leq 12.7$ GeV/c; e) $7.0 \leq p_{in} \leq 12.7$ GeV/c.

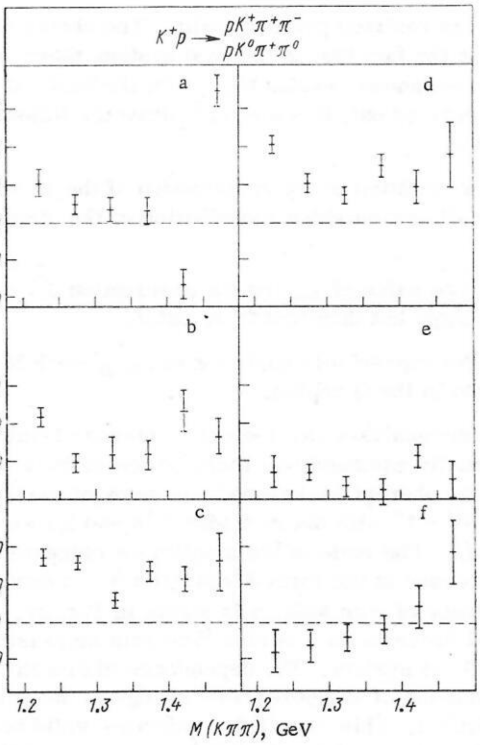

Fig. 28. Dependence of the phase-shift β of the ratio of the amplitudes of Kρ and K*π decays in the reactions $K^+p \to K^+p\pi^+\pi^-$ (a-d) and $K^+p \to pK^0\pi^+\pi^0$ (e and f). The designations are the same as in Fig. 27.

resonances, one of which (K_A) belongs to the SU(3) multiplet A_1 and the second (K_B) to the multiplet B (ref. 52). Then, for K_A, constructive interference should be observed with the decay modes $K^*\pi$ and $K\rho$, and for K_B, destructive interference, which leads to a strong dependence of β on $M(K\pi\pi)$ experimentally observed. However, this picture is inconsistent with the assumption of a diffraction origin of the entire Q peak, since the K meson and the pomeron have a charge parity C = +1. Therefore the suggestion has been made that the observed peaks in the Q region are a mixture of K_A and K_B states.[52] However, a more detailed study of the Q region is required to make it clear that this suggestion is correct.

In investigation of the systems $K^+\pi^+\pi^-$ and $K^0\pi^+\pi^0$ Barnham[51] and Bingham[51] found that the value of α is substantially larger for the first system than for the second. One of the possible explanations of this effect was the suggestion that there is a substantial contribution of the Kε decay.[4)] A ratio $|\gamma| = 1$ was obtained for the amplitudes of Kε and Kρ decay. The inconsistency mentioned above between the two channels was decreased but not removed.

At the present time there are rather few data relating to the properties of the L meson. In experiments on K^+p and K^-p interactions at 10 GeV/c, it has been established[54,55] that a spin parity 2^- of the L meson is preferable to 1^+ or 3^+. A similar result is given by Denegri et al.[56]

The question of the decay modes of the L meson is extremely important. In analysis of Dalitz plots,[54] estimates have been obtained of the contribution of the decays $K^*_{1420}\pi$, $K^*_{890}\pi$, and $K\rho$ of 20 ± 15%, 35 ± 12%, and 11 ± 9%, respectively, which agrees with the data[55] of work at 10 GeV/c.

Bartsch et al.[57] carried out a study of the excess of events in the L-meson region in the (Kππ) system after exclusion of the band associated with K^*_{1420}. In Fig. 29 we have shown the number of events in the L-meson peak for various conditions of cutoff in the K^*_{1420} mass. This peak does not disappear completely with such a cutoff, which indicates the existence of other decay modes. This is inconsistent with the result of Chien et al.,[58] who thoroughly analyzed the combined data on the reaction $K^+p \to pK^+\pi^+\pi^-$ in the momentum interval 7-13 GeV/c (65 000 events). In Fig. 30 we have shown the number of events in bands of width 100 MeV about the K^*_{890} and K^*_{1420} mass

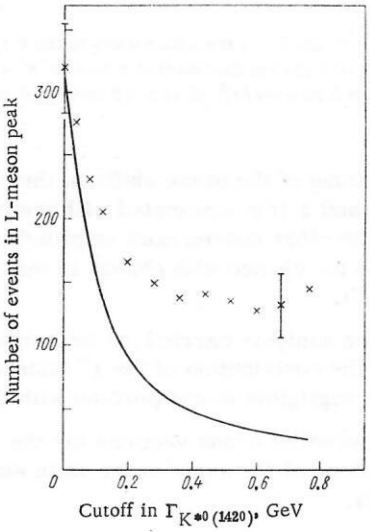

Fig. 29. Dependence of the number of events in the L-meson peak after exclusion of K^{*0}_{1420} for the reaction $K^-p \to pK^-\pi^+\pi^-$ at 10 GeV/c on the width of the band corresponding to K^*_{1420}. The smooth curve shows the expected number of events in the L peak on the assumption that only the decay $L \to K^{*0}_{1420}\pi$ occurs.

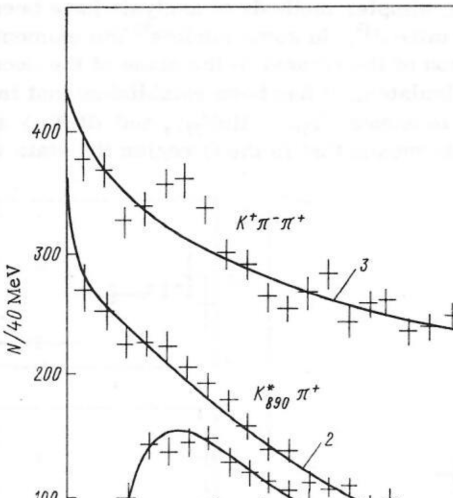

Fig. 30. Distribution in $M(K\pi\pi)$ for events in which $M(K\pi)$ lies in the K^{*0}_{1420} band (1), in the K^{*0}_{890} band (2), and outside these bands (3) for the reaction $K^+p \to K^+\pi^-\pi^+p$ for the interval 7-13 GeV/c. The bandwidth is 100 MeV.

values and outside of these bands as a function of $M(K\pi\pi)$. The absence of any peak at the K^*_{890} mass in the L-meson region in contrast to the results presented above strongly suggests that the L-meson decay occurs preferentially by the $K^*_{1420}\pi$ mode. This hypothesis is confirmed also in several other studies.[59] For a final solution of this problem it would be necessary to make an analysis with substantially better statistics.

Analysis of angular distributions of secondary particles has been carried out also for baryon systems. The most detailed study has been made of the $(N\pi)$ system by Lissauer[60] in the reaction $K^+n \to K^+(p\pi^-)$ at 12 GeV/c. The mass spectrum obtained agrees with the data of other studies.[61,62] In analysis of the angular distribution the values of $\langle Y_{lm}(\vartheta,\varphi)\rangle$ were determined, where ϑ and φ are the angles in the Gottfried–Jackson system, and it was shown that for $m \neq 0$ all $\langle Y_{lm}\rangle$ are practically equal to zero.[5)] In analysis of the data it was assumed that only P, D, and F waves are important, the D and F waves being described by a Breit–Wigner formula with fixed values $M = 1500$ MeV, $\Gamma = 120$ MeV, and $M = 1680$ MeV, $\Gamma = 130$ MeV, respectively. The P-wave contribution gives a maximum at $M \sim 1250$ MeV with the width $\Gamma \sim 300$ MeV (Fig. 31). Here the phase varies from 105 to 135° in the mass region $1.3 \leq M(p\pi^-) \leq 1.5$ GeV. Lissauer et al.[60] state on the basis of analysis of $K^+n \to \Delta^{++}K^0$ that the contribution to this region of Δ^{++}_{1238} is negligible. The amplitude P_{11} determined from phase-shift analysis has the same shape but with a maximum shifted by about 220 MeV toward larger mass of the (πN) system. This may mean that there are two P_{11} peaks corresponding to different resonances, or that a shift is observed similar to the shift in the maximum in ρ-meson photoproduction. The number of events corresponding to the contributions of various waves in various intervals of $|t'|$ are listed in Table 5.

These data indicate that the P-wave contribution behaves peripherally, while the D-wave contribution is approximately the same up to $|t'| = 0.3\ (\mathrm{GeV/c})^2$. The F-wave contribution increases with increasing $|t'|$.

5. SELECTION RULES IN DIFFRACTION PROCESSES

Determination of the spin and parity of a system formed in diffraction processes is extremely important since it

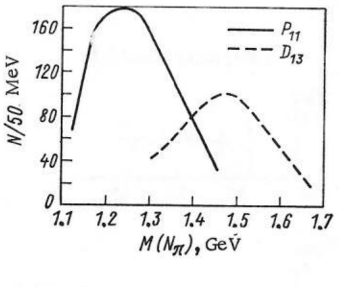

Fig. 31. Contribution of P_{11} and D_{13} states to the mass spectrum $M(p\pi^-)$ for the reaction $K^+n \to K^+(p\pi^-)$ at 12 GeV/c, $t \leq 0.1\ (\mathrm{GeV/c})^2$.

TABLE 5

Number of events	$\lvert t'\rvert \leq 0.1$, (GeV/c)²	$0.1 \leq \lvert t'\rvert \leq 0.3$, (GeV/c)²	$\lvert t'\rvert \geq 0.3$, (GeV/c)²
N (P)	925	0	0
N (D)	685	1447	740
N (F)	0	0	370
N (total)	1610	1447	1110

permits a comparison to be made with the predictions of various models which provide definite selection rules. One of these rules, the so-called Gribov–Morrison rule, relates the primary-particle parity P_i with the parity P_f of the final system and the spin change ΔJ of the initial and final states,

$$P_f = P_i(-1)^{\Delta J}. \qquad (24)$$

The selection rule (24) has been rigidly established only for collision of spinless particles and has not been proved in the more general case. Therefore it must be considered as empirical for interaction of particles with spins.

According to (24), diffraction processes are possible only for the transitions $0^- \to 0^-, 1^+, 2^-, 3^+, \ldots, 1/2^+ \to 1/2^+, 3/2^-, 5/2^+, 7/2^- \ldots$ and so forth. Therefore the reaction $\pi p \to A_2 p$ and similar reactions cannot occur as the result of diffraction processes and, in particular, by exchange of a pomeron. The boson systems A_1, A_3, Q, and L, whose cross-section dependence is characterized by a power n (see Table 1), satisfy this rule. However, its universality is subject to some doubt as a result of the further study of the A_2 meson, a resonance with established values $J^P = 2^+$ and satisfactorily described by a Breit–Wigner formula.

Caroll et al.[63] showed for the first time that at high energies in the reaction $\pi^-p \to pA_2^-$ the dependence of the cross section on energy is substantially less than for the reaction $\pi^+n \to pA_2^0$. In study of the $\pi^-p \to pA_2^-$ reaction with the CERN-IHEP boson spectrometer[48] a value $n = 0.7 \pm 0.3$ was obtained in the energy interval 25-40 GeV.

The Illinois group[64] carried out a partial-wave analysis for the reaction $\pi^\pm p \to p\pi^\pm\pi^+\pi^-$. The partial wave $J^P = 2^+$ has a resonance nature with a mass $M = 1315 \pm 5$ MeV and a width $\Gamma = 110 \pm 20$ MeV. In the mass region $1.2 \leq M(3\pi) \leq 1.4$ GeV for the 2^+ state (the A_2 meson) they obtained the value $n = 0.79 \pm 0.08$. Here the natural part of the cross section is described by the function $\sigma_N \sim p^{-0.57\pm0.03}$, and the unnatural part[6)] by $\sigma_U \sim p^{-1.85\pm0.45}$. The distribution in momentum transfer t' for the reaction $\pi^-p \to pA_2^-$ is shown in Fig. 26. It can be represented in the form $d\sigma/dt' \sim t' \exp(-Bt')$, where $B = 6.41 \pm 0.33\ (\mathrm{GeV/c})^{-2}$ for primary momenta $p_{in} < 10$ GeV/c and $B = 9.04 \pm 0.92\ (\mathrm{GeV/c})^{-2}$ for $p_{in} > 10$ GeV/c.

Representing the differential cross section in the form

$$d^2\sigma/dt\,ds = f(t)\, s^{\alpha(t)-2}, \qquad (25)$$

where $\alpha(t)$ is understood as some effective trajectory, and using this expression to approximate the data in various intervals in t, the Illinois group[64] obtained $\alpha(t) = (0.91 \pm 0.12) + (1.31 \pm 0.49)t$, and with further restriction $\alpha'(t) = 1 - \alpha(t) = 0.84 \pm 0.05 + t$.

The isospin characteristics of the exchange system have been investigated in detail in the energy region below 10 GeV. The equality $\sigma(A_2^+) = \sigma(A_2^-)$ has been established experimentally, which means that the interference of the amplitude with isoscalar and isovector exchange is small. This permits us to neglect interference, to separate the cross sections into parts corresponding to isoscalar and isospin exchange, using here additional data on the $\pi^+d \to ppA_2^0$ reaction, and to show that the fraction of the cross section due to isoscalar exchange, which is characterized, for example, by the ratio $R = \sqrt{\sigma_{isosc}/\sigma_{tot}}$, is large ($R = 0.8$ for 5 GeV).

The behavior of the trajectories responsible for production of A_1 and A_2 mesons can be investigated in analysis of the interference effects of partial waves with various J^P values. In refs. 48 and 64 the authors studied the dependence of the relative phase of the spin density matrix element ρ_{01}^{1+2+} on t, which characterizes the interference of the 1^+ and 2^+ partial waves. The observed independence of ρ_{01}^{1+2+} of t indicates that the trajectories taking part in production of A_1 and A_2 mesons are parallel.

The data considered strongly suggest that production of the A_2 meson (at least for the decay mode $A_2 \to \pi^+\pi^-\pi^-$) can be described as a diffraction process. If this statement is true, then we have in this case a violation of the rule (24).

Some authors have also discussed other possible selection rules for diffraction processes. Carlitz et al.[65] proposed the conservation of quark spin and of the SU_3 quantum numbers in the case of pomeron exchange. In this case production of the A_1 with pomeron exchange is a forbidden process, which is hardly valid according to the experimental results considered above. Another selection rule suggested by Freund et al.[66] is based on the assumption of $f-f'$ coupling of the pomeron. In exchange of a pomeron all J^P states are allowed in which $f-f'$ exchange (A_1, A_2) is permitted. On the basis of the quark model Le Yaouanc et al.[67] established the rule $P_iP_f = (-1)^{\Delta s}$, where Δs is the total spin change of the quark, which permits A_1 and A_2 production to be considered as diffraction processes. Calculations on the basis of this model predict a decrease in the cross section for production of A_1 for $t' = 0$, associated with the spin-flip amplitude. The experimental distribution in $d\sigma/dt$ is inconsistent with this statement.

6. VERIFICATION OF HELICITY CONSERVATION

We will discuss diffraction dissociation as a quasiparticle process $a + b \to c + d$. The amplitude of this process can be represented in the form of a coherent sum of states with different J^P values, each of which is represented in the form of a series in helicity amplitudes in the s or t channel: $F^s_{\lambda_c\lambda_d\lambda_a\lambda_b}$, $F^t_{\lambda_c\lambda_d\lambda_a\lambda_b}$. We will represent these amplitudes in a form free from kinematic singularities, for example in the Reggeized form:

$$F^s_{\lambda_c\lambda_d\lambda_a\lambda_b} = (\sqrt{-t/s_0})^{|\lambda_c-\lambda_a|+|\lambda_d-\lambda_b|} g_{\lambda_c-\lambda_a,\,\lambda_d-\lambda_b} (s/s_0)^{\alpha} \varphi_\alpha. \quad (26)$$

Conservation of helicity in the s channel means that for any λ, $u \neq 0$ the quantity $g_{\lambda u} = 0$. The same condition for F^t must be satisfied for conservation of helicity in the t channel.

It is well known that the conservation of angular momentum and parity results in the following relation for the helicity amplitudes:

$$F^s_{\lambda_c\lambda_d\lambda_a\lambda_b} = \sigma_b\sigma_a\eta_b\eta_d(-1)^{s_a-s_b}(-1)^{\lambda_d-\lambda_b}F^s_{\lambda_c-\lambda_d,\,\lambda_a-\lambda_b}, \quad (27)$$

where σ_i is the naturalness of particle i.

Since in forward scattering $\lambda_a = \lambda_c$ and $\lambda_b = \lambda_d$, helicity conservation in the s channel for $t = 0$ is a trivial consequence of conservation of J^P. Furthermore, for $s_a = 0$ the Gribov–Morrison rule for $t = 0$ automatically follows from Eq. (27).

In the work of Erbe et al.[68] and Ballam et al.[69] on study of the reaction $\gamma p \to \rho p$ with polarized photons it was shown that the ρ meson is formed with the same polarization as the primary photon, independently of the emission angle. On the basis of these results and analysis of elastic πp and pp scattering, Gilman et al.[70] made the statement that helicity conservation in the s channel is a general property of diffraction processes with participation of hadrons. However, the results of subsequent studies cast doubt on this statement. Actually, it follows from data on the $\gamma p \to \rho p$ reaction with polarized photons[24] at energies of 2.8, 4.7, and 9.3 GeV that $\mathrm{Re}\,\rho_{10}^0$ and $\mathrm{Re}\rho_{11}^0$ in the region $0.18 \leq |t'| \leq 0.8$ (GeV/c)^2 are different from zero. This indicates a change in helicity in the s channel.

The main contribution to $\mathrm{Re}\rho_{10}^0$ is due to natural exchange. The dependence of this quantity on the primary energy is extremely weak. The values of $2\mathrm{Re}\rho_{10}^N = \mathrm{Im}\, T_{01}^N |T_{11}^N|$, where T_{ij}^N is the helicity amplitude corresponding to natural exchange, at energies $E_\gamma = 2.8$, 4.7, and 9.3 GeV are 0.16 ± 0.03, 0.12 ± 0.03, and 0.14 ± 0.02, respectively.

A similar analysis has been carried out for systems arising in diffraction dissociation of hadrons. The Illinois group[47] determined the values of $\rho^{1+1+}_{M_1M_1}$ as a function of t for the reaction $\pi^-p \to p(3\pi)^-$ in the A_1 region. For small t the values obtained for ρ_{00}^{1+1+} agree with the assumption of helicity conservation in the s and t channels.[7] For large t the t dependence ρ_{00}^{1+1+} (t) indicates such conservation only in the t channel. However, a more detailed study of the other matrix elements, for example the difference from zero of $\mathrm{Re}\rho_{01}^{11}$, shows that the statement that helicity is conserved in the t channel for this reaction is not valid. This conclusion is confirmed by the CERN-IHEP data[48] (Fig. 32). The same result was also obtained in study of the A_3 region.

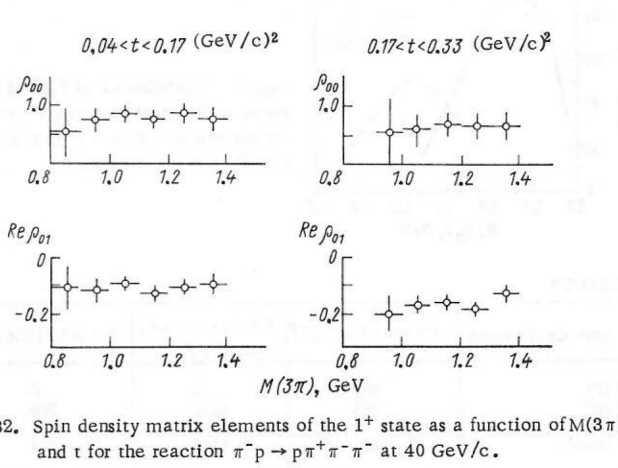

Fig. 32. Spin density matrix elements of the 1^+ state as a function of $M(3\pi)$ and t for the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ at 40 GeV/c.

Beaupre et al.[71] analyzed the angular distribution in the respective systems by the method of moments. Determination of the contribution of states with different L and M was carried out by minimization of the functional

$$\chi^2(\beta)=\sum_{L=1}^{5}\sum_{M=1}^{L}\frac{(Y_{LM}(\vartheta,\varphi))^2}{\sigma^2}, \tag{28}$$

where ϑ and φ are the azimuthal and polar angles in the center of mass of the system under study between the normal to the decay plane (or one of the decay particles) and the Z axis; $Y_{LM}(\vartheta, \varphi)$ are spherical harmonics; β is the angle of rotation of the Z axis relative to the direction of the incident particle in the same system.

The reaction $K^-p \to pK^-\pi^+\pi^-$ was studied at 10 GeV/c and the reaction $\pi^+p \to p\pi^+\pi^+\pi^-$ at 16 GeV/c in the Q and A_1 regions, respectively. As a result of the analysis it was shown that the angle β at which the functional reaches its minimum value does not correspond either to the Gottfried–Jackson system or the helical system. This means that for A_1 and Q mesons helicity is not conserved either in the t or s channel.

Study of the baryon system has been carried out both in reactions of the type[71] $\frac{\pi}{K}p(p\pi\pi)$ and in the coherent processes[46] $dp \to dp\pi^+\pi^-$ at 25 GeV. In most of the studies helicity conservation has not been established either in the s or t channel.

7. ANALYSIS OF DIFFRACTION DISSOCIATION PROCESSES BY THE LONGITUDINAL PHASE-SPACE METHOD

One of the possible means of identifying diffraction processes involves analysis in longitudinal phase space as suggested by Van Hove.[72] The assumption is quite often made that different regions of longitudinal phase space correspond to different exchange diagrams. For example, in a boson–baryon interaction particles with positive values of longitudinal momentum are associated with the meson vertex, and particles with negative values with the baryon vertex. This analysis has been used to study the reaction $\pi^+p \to \pi^+(N\pi)^+$ in the momentum interval from 4 to 16 GeV/c after separation of the $(N\pi)$ system on the basis of isospin.[73] The correspondence between the regions of longitudinal phase space and the various exchange diagrams is shown in Fig. 33. For various regions of longitudinal phase space, characterized by the angle ω, the dependence of the cross section on the primary momentum was determined in the form $\sigma \sim p^{-n}$ (Fig. 34). It can be seen that the cross section is practically constant (i.e., $n \approx 0$) for the region $90° \leqslant \omega \leqslant 120°$, which corresponds to the diagram $\pi^+p \to \pi^+(N\pi)^+$. We note that at 16 GeV/c in this region the contribution of states I = 1/2 for the $(N\pi)$ system amounts to about 70%.

A similar analysis was made for the quasi three-particle reactions $K^+p \to \pi^-K^+\Delta^{++}$ and $K^+p \to \pi^0K^0\Delta^{++}$ at momenta from 5 to 12.7 GeV/c and for the reaction $pp \to p\pi^-\Delta^{++}$ at momenta from 4 to 25 GeV/c (refs. 74, 75). Results of the analysis agree with the data for the π^-p interaction (see Fig. 34). It should be noted, however, that for $\omega = 120°$ the exponent n for the reaction $K^+p \to$

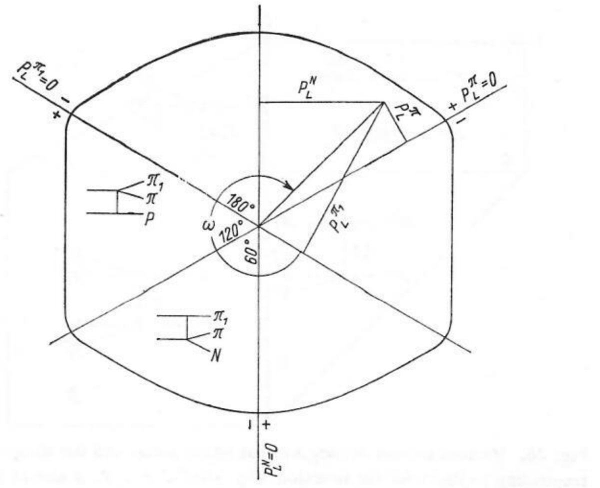

Fig. 33. Van Hove diagram for a three-particle final state and the suggested correspondence of its sectors to Feynman diagrams.

$K^0\pi^0\Delta^{++}$ is somewhat larger than in the case $K^+p \to K^+\pi^-\Delta^{++}$, which is due to the fact that pomeron exchange is forbidden. Analysis of four-particle final states on the basis of longitudinal phase space was first carried out by Kittel, Ratti, and Van Hove[76] and subsequently has been used in many studies.

If we divide the longitudinal phase space into sectors corresponding to definite combinations of particles emitted in the forward or backward directions in the center of

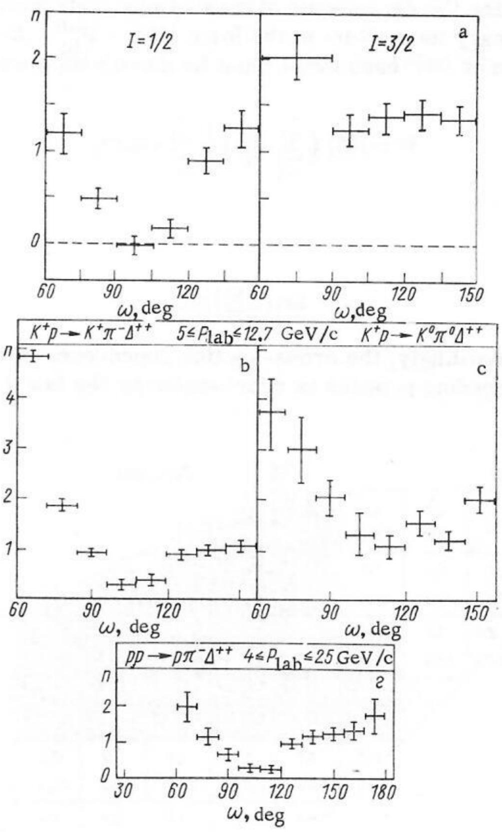

Fig. 34. Value of the exponent n in the equation $\sigma \sim p^{-n}$ as a function of the angle ω for various reactions: a) for the reaction $\pi^+p \to \pi(N\pi)_I$ at 4-16 GeV/c; b) for the reaction $K^+p \to K^+\pi^-\Delta^{++}$ at $3 \leqslant P_{lab} \leqslant 12.7$ GeV/c; c) for the reaction $K^+p \to K^0\pi^0\Delta^{++}$ at $5 \leqslant P_{lab} \leqslant 12.7$ GeV/c; d) for the reaction $pp \to p\pi^-\Delta^{++}$ at $4 \leq P_{lab} \leq 25$ GeV/c.

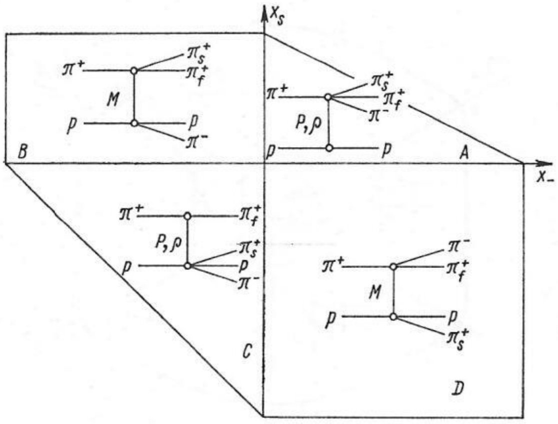

Fig. 35. Various sectors of longitudinal phase space and the diagrams corresponding to them for the reaction $\pi^+p \to p\pi^+\pi^+\pi^-$: P, ρ and M indicate the possibility of exchange of a pomeron, a meson with G = + 1, and a meson with G = −1.

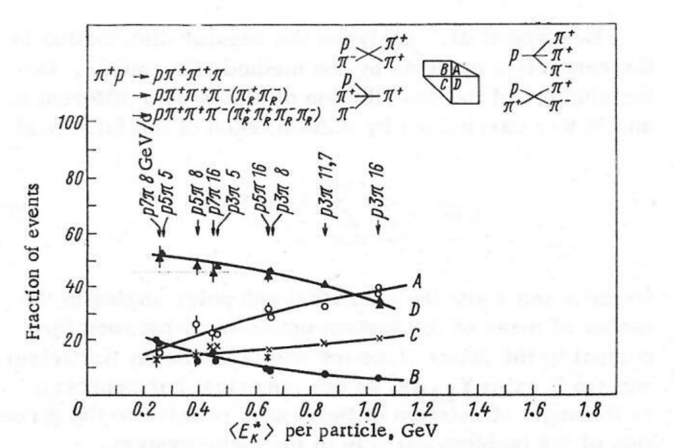

Fig. 37. Fraction of events in sectors A, B, C, and D for the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ at various energies. In reactions with a large number of π mesons the very slow $\pi^+\pi^-$ meson pairs in the center of mass, shown in parentheses, have been excluded.

mass, there will be 14 kinematically possible different cases for four-particle final states. However, in practice the number of these possibilities is substantially smaller. For example, Beaupre et al.[77] observed emission of a proton into the forward hemisphere only in 7.8% of the events in the reaction $\pi^+p \to p\pi^+\pi^+\pi^-$ at 8 GeV/c. For a primary momentum of 16 GeV/c the fraction is only 2.3%. On the basis of these data, it is easy to reduce the number of significant sectors of phase space. For these sectors and the diagrams corresponding to them (Fig. 35) we can determine the dependence of the averaged matrix elements on primary momentum in the form $|M|^2 \sim P_{in}^{-n}$. In calculation of $|M|^2$ each event must be taken with a weight

$$W = \Pi E_i^* \left(\sum_{i=1}^{n} \frac{x_i}{E_i^*} \right) \frac{1}{2} \sum_{i=1}^{n} |p_i^*|^{-1}, \qquad (29)$$

where

$$x_i = 2p_{i\parallel} / \sum_j |p_{j\parallel}| .$$

Accordingly, the cross-section dependence of the corresponding process is represented by the law $\sigma \sim p^{-n_W}$. It can be seen (Fig. 36) that for the sectors corresponding to diffraction processes, i.e., sector A($\pi^+p \to p\pi^+\pi^+\pi^-$) and sector C($\pi^+p \to p\pi^+\pi^-\pi^-$), the value of n_W is substantially less than n_W for the sectors corresponding to meson exchange, namely: sector D ($\pi^+p \to p\pi^+\pi^+\pi^-$ and sector B ($\pi^+p \to p\pi^-\pi^+\pi^+$). Here the values of n_W for sectors A and C are about the same. On the basis of the results obtained it can be concluded that the cross section for diffraction dissociation decreases with energy in the same way as the elastic-scattering cross section (n_{el} = 0.27 ± 0.06) in the energy region considered, excluding here the sector boundaries. The fraction of events corresponding to the various sectors of longitudinal phase space[78] is shown in Fig. 37. We note that for the reaction $\pi p \to p(3\pi)$ the fraction of events which belong to the sectors corresponding to diffraction dissociation, i.e., A and C, increases, while the population of sectors B and D drops with energy. At an energy of 16 GeV sector A is the most populated. In addition, the cross section for dissociation of π mesons turns out to be approximately 1.8 times larger than the cross section for dissociation of protons.

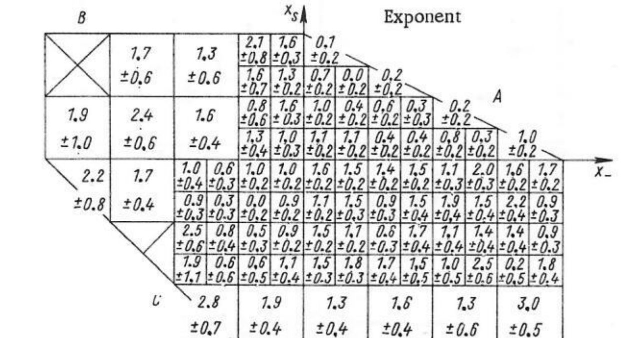

Fig. 36. The quantity n_W characterizing the energy dependence of the weighted longitudinal phase-space distribution for the reaction $\pi^+p \to p\pi^+\pi^+\pi^-$ in the energy interval from 8 to 16 GeV.

The effective-mass distributions of systems corresponding to the vertices in each of the sectors[37] are shown in Fig. 38. The observed effective-mass distributions in the diffraction sectors A and C are similar to the distributions discussed in section 1. The same result has also been obtained for other reactions.[79]

In study of diffraction processes with charge-conjugate primary particles we should expect equality of the cross sections. Tomasini[80] made a comparison of the cross sections for the reactions $\pi^+p \to p\pi^+\pi^+\pi^-$ and $\pi^-p \to p\pi^-\pi^+\pi^-$ in sectors A and C. The results of the comparison, which are listed in Table 6, indicate the equality of the cross sections for these reactions. We note that for elastic scattering the cross-section ratio is R = $\sigma_{el}(\pi^-p)/\sigma_{el}(\pi^+p)$ = 1.03 ± 0.02 for a momentum of 16 GeV/c.

The method of analysis developed above can also be applied to processes with high multiplicity for study of their general characteristics such as the cross section, its dependence on energy, the distribution of effective mass in the respective sectors, and so forth.

TABLE 6

Sector	Particles emitted forward	Particles emitted backward	σ, μb	Ratio of σ in diffraction sectors
A^+	$\pi^+\pi^+\pi^-$	p	472 ± 52	$A^+/A^- = 0.90\pm0.05$
A^-	$\pi^+\pi^-\pi^-$	p	458 ± 50	
C^+	π^+	$p\pi^+\pi^-$	274 ± 30	$C^+/C^- = 0.89\pm0.13$
C^-	π^-	$p\pi^+\pi^-$	242 ± 27	
B^+	$\pi^+\pi^+$	$p\pi^-$	110 ± 12	
B^-	$\pi^-\pi^-$	$p\pi^+$	176 ± 19	
D^+	$\pi^+\pi^-$	$p\pi^+$	582 ± 64	
D^-	$\pi^+\pi^-$	$p\pi^-$	264 ± 29	

In particular, Kittel et al.[76] used the longitudinal phase space method to carry out an analysis of the double diffraction dissociation reactions $\pi^-p \to (p\pi^0)(\pi^+\pi^-\pi^-)$ and $\pi^-p \to (n\pi^+)(\pi^+\pi^-\pi^-)$ at 16 GeV/c. The effective-mass distributions in the respective sectors are shown in Fig. 39. In their general features they agree with the distributions obtained in reactions with single dissociation. However, differences are observed which are due to production of the Δ^{++}_{1238} and A_2 mesons and indicate the existence of other exchange mechanisms. The ratio R of the cross sections for production of $(p\pi^0)$ and $(n\pi^+)$ systems turned out to be $R = 0.86 \pm 0.13$. Therefore, in addition to the state with $I = 1/2$, which gives a value $R = 1/2$, the state with $I = 3/2$ ($R = 2$) can contribute to formation of the $(N\pi)$ system. After identification of states with $I = 1/2$, a value 0.66 ± 0.11 was obtained for R.

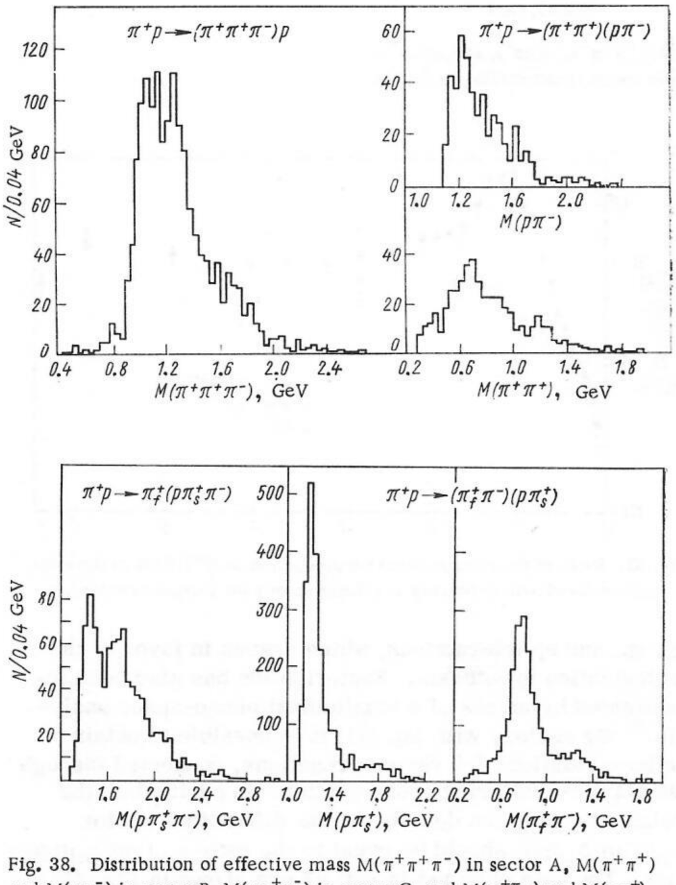

Fig. 38. Distribution of effective mass $M(\pi^+\pi^+\pi^-)$ in sector A, $M(\pi^+\pi^+)$ and $M(p\pi^-)$ in sector B, $M(p\pi^+_s\pi^-)$ in sector C, and $M(\pi^+_f\pi^-)$ and $M(p\pi^+_s)$ in sector D for the reaction $\pi^+p \to p\pi^+\pi^+\pi^-$ at 8 GeV/c. The subscripts f and s indicate fast and slow π mesons in the laboratory system.

The German–English collaboration[81] has carried out an analysis of the angular momenta of the (3π) system in the reactions $\pi^+p \to (N\pi^+)(\pi^\pm\pi^+\pi^-)$. Identification of the double diffraction dissociation process was accomplished on the basis of a longitudinal phase space analysis. The results obtained are shown in Fig. 40a. Data for the single diffraction dissociation process $\pi^\pm p \to p(\pi^\pm\pi^+\pi^-)$ are shown in Fig. 40b. It can be seen that the dependence of the angular momenta on the mass of the (3π) system is practically the same in the two cases.

Indications of the existence of double diffraction dissociation processes also exist in the reaction $\gamma p \to p\pi^+\pi^+\pi^-\pi^-$, for which the mass spectra of $M(\pi^+\pi^-)$ and $M(p\pi^+\pi^-)$ in certain spectra are similar in shape to the spectra for single dissociation.[82]

It should be noted, however, that a detailed study of double diffraction dissociation processes will require a substantial increase in the statistics over a wide range of energies.

8. FACTORIZATION

As was noted by Van Hove,[83] factorization in the direct and crossing channels is possible for the transition amplitudes $T_{AB \to A'B'}$. Factorization in the t channel means that the invariant amplitude can be expressed in the form of a product of the vertex factor and the propagator

$$T_{AB \to A'B'} = f(s, t)\, \gamma(t, X_A, X'_{A'})\, \gamma(t, X_B, X'_{B'}). \qquad (30)$$

This expression is valid if the existence of simple poles is sufficient for description of the process. However, there are many indications that, in addition to poles, other singularities not permitting factorization take on substantial importance. Therefore the experimental verification of the predictions of the factorization hypothesis can give information on how important are these singularities.

Such a check was made by Freund[84] for the reactions $\pi p \to \pi N^*$ and $pp \to pN^*$. Assuming that these reactions are diffraction processes and consequently can be described by a mechanism with exchange of the same pole, Freund obtained a relation connecting the cross sections for these reactions with the cross sections of the corresponding elastic reactions:

$$\frac{d^2\sigma^{\pi p}/dt\, dM_{\pi N}}{d^2\sigma^{pp}/dt\, dM_{\pi N}} = \left(\frac{d\sigma^{\pi p}_{el}}{dt}\right) \Big/ \left(\frac{d\sigma^{pp}_{el}}{dt}\right). \qquad (31)$$

Subsequently the assumption was made that the dependence of $d\sigma/dt$ on t is identical for elastic πp and pp scattering. Then we should expect that the left-hand side of Eq. (31) should be independent of t, which is in good agreement with experiment. A similar result was obtained by Bari et al.[85] for different values of $M_{\pi N}$.

More careful investigations were made by Amaldi et al.[86] on the basis of the experimental data on pp interactions and the data of Anderson et al. on π^-p interactions.[22] The ratio of the cross sections for production of $N^*(1688)$ to the corresponding value for elastic scattering was determined for various values of t. This quantity is shown in Fig. 41. For $|t| \le 0.5$ (GeV/c)^2, i.e., for the re-

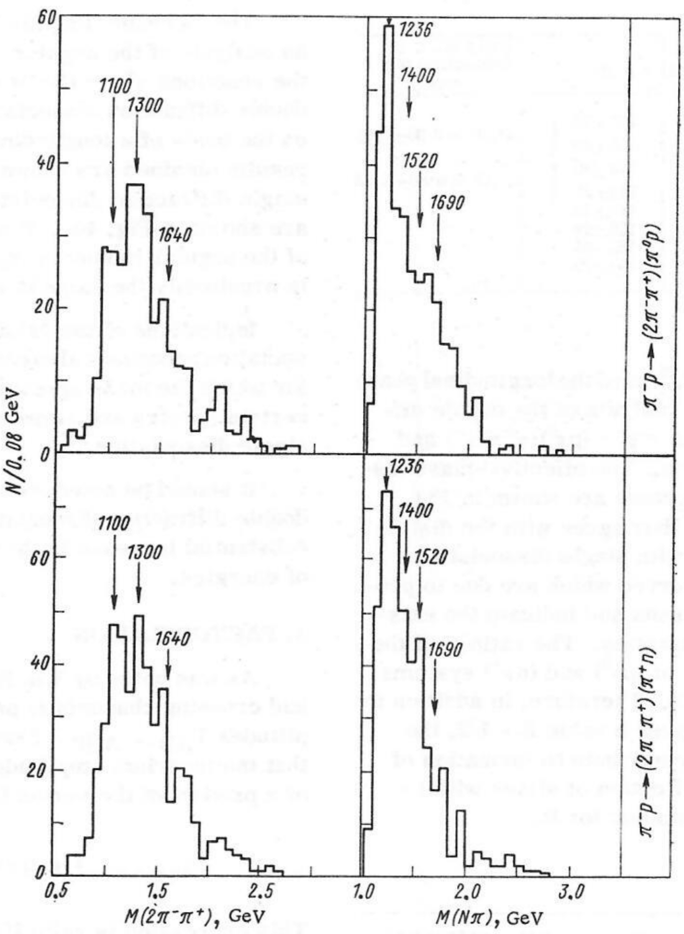

Fig. 39. Distribution of effective mass $M(\pi^+\pi^-\pi^-)$, $M(p\pi^0)$, and $M(n\pi^+)$ for the reaction $\pi^-p \to \pi^+\pi^-\pi^-(N\pi)$ in the sector corresponding to double dissociation, at 16 GeV/c.

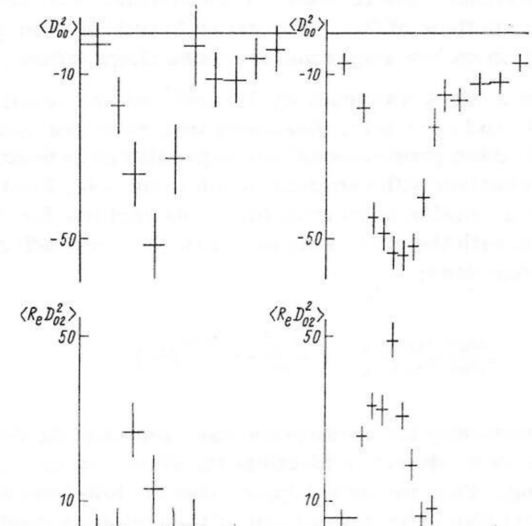

Fig. 40. Averaged angular functions $\langle D^J_{MM'} \rangle$ as a function of $M(3\pi)$ for single dissociation $\pi^{\pm}p \to (3\pi)^{\pm}p$ (a) and double dissociation $\pi^{\mp}p \to (3\pi)^{\pm}p$ (a) and double dissociation $\pi^{\pm}p \to (3\pi)^{\pm}(N\pi)^{+}$ (b) at 16 GeV/c.

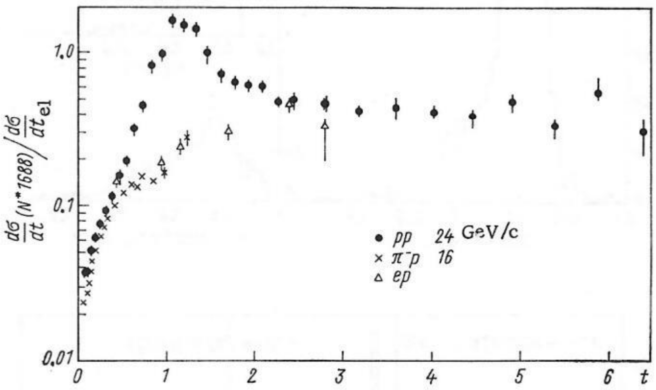

Fig. 41. Ratio of the cross sections for production of $N^*(1688)$ to the cross section for elastic scattering as a function of t for various reactions.

gion where we can expect the existence of diffraction dissociation, agreement of these quantities is observed for pp, πp, and ep interactions, which argues in favor of the factorization hypothesis. Factorization has also been investigated by means of a longitudinal phase-space analysis.[87] By analogy with Eq. (31) it is possible to obtain definite relations for various reactions, connected through the elastic-scattering cross section. In particular, the weighted population density in the diffraction sector $\Delta_\omega(\pi^-p)/\Delta_\omega(pp)$ should be equal to the ratio $\sigma_{el}(\pi p)/\sigma_{el}(pp) = 0.43$. The data obtained in ref. 87 are shown in Fig. 42. It can be seen that the value of the ratio in the central region is in good agreement with the predictions of fac-

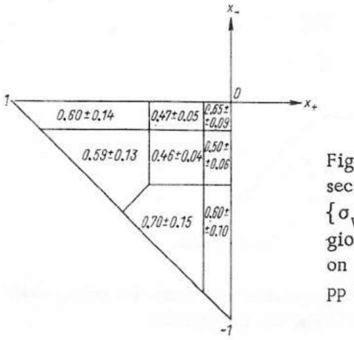

Fig. 42. Ratio of the weighted cross section $R = \sigma_W[\pi^-p \to \pi^-(p\pi^+\pi^-)] \cdot \{\sigma_W[pp \to p(p\pi^+\pi^-)]\}^{-1}$ in various regions of longitudinal phase space. Data on π^-p interactions at 16 GeV/c and pp interactions at 19 GeV/c were used.

torization. The deviations observed in the boundary regions are probably due to the effect of overlap of the sectors. Analysis of these same ratios $\Delta_\omega(\pi^\pm p)/\Delta_\omega(K^-p)$, $\Delta_\omega(K^+p)/\Delta_\omega(K^-p)$ and comparison of them with the corresponding elastic processes gave a similar result. The data presented indicate that for small momentum transfers no inconsistency with the consequences of the factorization hypothesis is observed in diffraction dissociation processes.

9. INTERPRETATION OF THE DIFFRACTION PEAKS

Since diffraction dissociation processes are similar in many respects to elastic scattering, their description in some models is based on this analogy. For example, in the vector-dominance model photoproduction of vector mesons is discussed as the conversion of a γ ray into a vector meson with subsequent elastic scattering of the latter by a nucleon or nucleus. Calculations made on the basis of this model are in rather good agreement with the experimental data. It should be emphasized that in diffraction dissociation processes produced by γ rays, resonances are produced which are produced also in reactions of other types.

The interpretation of systems arising as the result of hadron dissociation is much less clear. The relation between the singularities in the effective-mass spectrum of the boson systems A_1, A_3, Q, and L and also systems with baryon number B = 1 with resonance production is far from obvious. An indication of the resonance nature of these singularities would be their observation in reactions not involving diffraction dissociation. Some similar reactions with possible production of the A_1 meson have been discussed in the review by Garelik.[88] A peak in the A_1 region was observed in the reactions $K^+p \to K^+p\pi^+\pi^-\pi^0$ (ref. 89), $\bar{p}p \to \rho(5\pi)$ (ref. 90), $K^-p \to \Lambda(m\pi)$ (ref. 91), $\pi^-p \to p + (MM)^-$ in back scattering,[28] and $pp \to d + (MM)^+$ (ref. 92). The mass values obtained lie in the interval from 1030 to 1120 MeV, and the widths from 33 to 100 MeV. However, these data are inconsistent with the results of the phase-shift analysis in the A_1 region considered in section 4.

Peaks in the Q region have also been observed in reactions with formation of the $(K\pi\pi)$ system not involving diffraction dissociation. One of these cases is the C meson found in pp annihilation at rest.[93] In order to clarify the resonance origin of these peaks, it is necessary to make a phase-shift analysis of the $(K\pi\pi)$ systems in diffraction processes, which will permit establishment of whether, as in the case of the A_1 meson, this inconsistency exists.

Data have been obtained[41,94] in the study of coherent processes in nuclei with which it is possible to determine the cross section for interaction of $(\rho\pi)$ and $(K^*\pi)$ systems with protons. On the assumption that each of the particles of a system arising from one of the nucleons of the nucleus interacts with the nucleons of the same nucleus independently, this cross section should be approximately[8)] $2\,\sigma(\pi p)$. The experimental results are inconsistent with this relation: $\sigma[(3\pi)p] \approx \sigma(\pi p)$ and $\sigma[(K\pi\pi)p] \approx \sigma(Kp)$. This strongly suggests that in coherent processes bound systems are formed. However, interpretation of these systems as resonances is difficult, since the observed effect is practically independent of the mass of the system. For example, for the (3π) system these equalities are satisfied over the range 1.0-1.5 GeV. Furthermore, Van Hove,[96] Gottfried,[97] and E. L. Feinberg[98] put forward an explanation of this effect which does not require the formation of bound systems, particularly resonances.

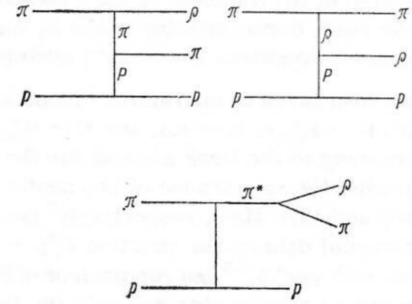

Fig. 43. Multi-Regge diagrams for the diffraction dissociation process $\pi p \to p\rho\pi$ according to the model of Ross and Yam.

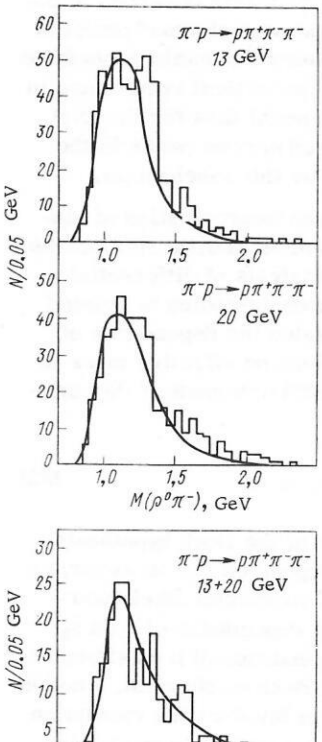

Fig. 44. Results of a calculation with the multi-Regge model for the reactions $\pi^-p \to p\rho\pi$ and $\pi^-p \to pf\pi$ at 13 and 20 GeV.

Another approach to the description of the singularities under discussion is based on consideration of kinematic effects. An explanation of the appearance of the peak in the A_1 region in terms of this approach was suggested by Deck.[99] For the $\pi^- p \to p\pi^+\pi^-\pi^-$ reaction he considered the first diagram shown in Fig. 43. It was suggested that the incident pion dissociates into a ρ meson and a pion. Then the pion is elastically scattered by the proton. For the latter process the characteristics of elastic πp scattering on the mass shell were used. The mass spectrum of $(\pi^+\pi^-\pi^-)$ systems obtained as a result of the calculation has a peak at $M(\pi^+\pi^-\pi^0) = 1.5$ GeV and its shape agrees with the experimental data. Subsequently Berger[100] carried out the reggeization of this diagram.

Ross and Yam[101] generalized the Deck mechanism, considering all three of the diagrams shown in Fig. 43. Comparison of the results of a calculation with the first two diagrams of Fig. 43 with the experimental data in the spirit of the double Regge model was carried out by Brandenburg et al.[102] (Fig. 44). Use of similar diagrams with replacement of the ρ meson by the f meson gives a peak at 1650 MeV, corresponding to the A_3 meson, in the effective-mass spectrum of $(\pi^+\pi^-\pi^-)$ systems.

The main processes of diffraction dissociation of K mesons are $K \to K^*_{890}\pi$, $K \to K\rho$, and $K \to K^*_{1420}\pi$. Calculations according to the Deck scheme for the first two processes predict the appearance of two peaks in the Q region at 1260 and 1370 MeV, respectively[7] (see Fig. 45). In the experimental data on the reaction $K^+p \to pK^+\pi^+\pi^-$ there are two such peaks,[103] but comparison of these peaks with the theoretical results obtained with the Deck model gives poorer agreement than in the case of the A_1 meson. In addition, this model cannot explain the decrease in the fraction of $K\rho$ decays in comparison with $K^*\pi$ decays with increasing mass of the $(K\pi\pi)$ system observed in the Q region (see Sec. 4). The appearance of broad peaks in the mass spectra of baryon systems also can be associated with the Deck mechanism. The theoretical results are in good agreement with the experimental data for the $(N\pi)$ system.[104] However, the observed narrow peaks in the $(N\pi\pi)$ system are not described by this mechanism.

Serious inconsistencies in the interpretation of singularities in the effective-mass spectra in the spirit of the Deck mechanism arise also in analysis of differential cross sections. One of these inconsistencies is pointed out by Miettinen,[105] who investigated the dependence of the differential cross-section slope on effective mass in the reaction $pp \to pn\pi^+$. The matrix element of this process (Fig. 46) is

$$M \sim \exp(bt)\, g(s_1, s_2, t_2), \qquad (32)$$

where $b = b(s_1, s_2, t_2)$. In terms of the Deck hypothesis, the matrix element should not depend on s_2. However, an analysis made by the method of maximum likelihood showed the existence of a strong dependence of b on s_2 (see Fig. 46). The observed dependence of b on effective mass cannot be explained by the Deck mechanism. Another argument against this mechanism involves the results on crossing of the differential cross sections in production of Q_0. In fact, let us consider the dissociation of the K^0 meson as the process shown in Fig. 43 with replacement

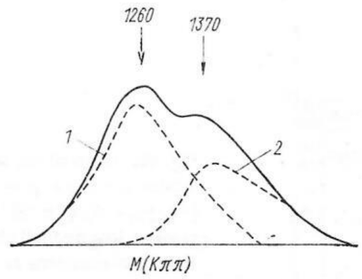

Fig. 45. Mass distribution in the Q region calculated from the Deck model for $K^*_{890}\pi$ systems (1) and $K\rho$ systems (2).

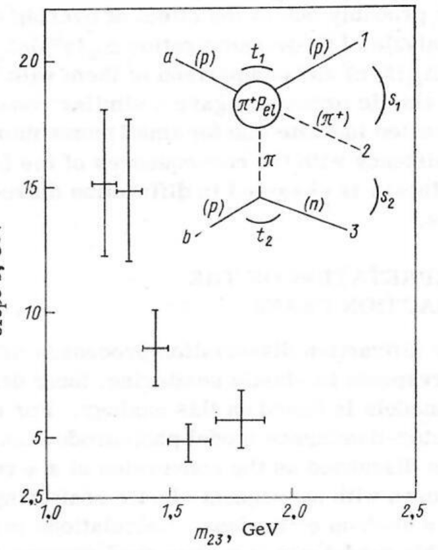

Fig. 46. Dependence of the slope b on $m_{23} = \sqrt{s_2}$ for the reaction $pp \to pn\pi^+$ at 19 GeV, obtained in ref. 105. The kinematic variables are indicated in the diagram.

of the ρ by K^*. Then for the first diagram (scattering of a virtual π meson by a proton) it follows that the direction of the crossing of the cross sections will correspond to the crossing of the cross sections in πp scattering, which is opposite to that observed experimentally (see Fig. 16). It is unlikely that the contribution of the second diagram (K^*p scattering) will be able to compensate and change this effect. With this explanation we would have to expect a large difference in the slopes for the reactions $K^+p \to pQ^+$ and $K^-p \to pQ^-$, which is not observed experimentally.

CONCLUSION

On the basis of the experimental data considered, it can be concluded that there is a special class of inelastic reactions, diffraction dissociation processes, which are observed both in the interaction of particles with protons and in coherent processes in nuclei. Identification of these reactions is based on study of the effective-mass spectra, longitudinal phase-space analysis, or behavior of definite isospin states. Diffraction dissociation processes are similar to elastic scattering in their main characteristics (cross-section dependence on energy, differential cross-section slopes, behavior of particle and antiparticle differential cross sections for fixed energy, and so forth).

In some cases, such as photoproduction reactions, the dissociated systems arising can be identified with known resonances. In dissociation of hadrons the situation turns out to be more complex. In particular, interpretation of the dissociated system in the A_1 region as a resonance is inconsistent with the phase-shift analysis results.

On the other hand, serious inconsistencies also arise in description of the singularities of dissociated systems as kinematic effects. Accordingly, for clarification of the mechanism of these processes, it is necessary to carry out a more detailed phase-shift analysis of such systems as $(K\pi\pi)$, $(N\pi)$, $(N\pi\pi)$, and so forth, and to study the energy dependence of their characteristics over the widest possible range of energies.

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[1]Reaction (6) can be considered as reaction (5) with replacement of the real photon by a virtual photon.

[2]This dependence follows from analysis of the electroproduction of resonances in reactions[19] ep = epx.

[3]It should be noted that production of the N^* (1400) also is not described by this scheme.

[4]The hypothesis of $K\varepsilon$ decay was advanced by Alexander et al.[53] to explain the observed ratio of decay modes $R = Q \rightarrow K^0\pi^+\pi^-/Q \rightarrow K^+\pi^+\pi^- = 1.3 \pm 0.06$ in the reaction $K^+p \rightarrow Qp$ and its correspondence to the isotopic ratios.

[5]It should be noted that this is inconsistent with the results of ref. 62.

[6]The naturalness N is defined as follows: $N = (-1)^J P$, where J and P are the spin and parity, or $N = P\tau$, where τ is the signature factor. The amplitude of a process $a + b \rightarrow c + d$ can be represented in the form of the sum of two terms corresponding to exchange of natural and unnatural systems:

$$M^{s\pm}_{\lambda_c\lambda_d\lambda_a\lambda_b} = M^s_{\lambda_c\lambda_d\lambda_a\lambda_b} \pm \varepsilon M^s_{-\lambda_c\lambda_d-\lambda_a\lambda_b},$$

where $\varepsilon = \eta_c\eta_a(-1)^{s_c - s_a}(-1)^{\lambda_c - \lambda_a}$; η are the internal parities of the particles; λ are the helicities. From this, for example, for the case of interaction of particles with $s_a = 0$ (π and K) it is easy to obtain

$$d\sigma^{\pm}_{\lambda_c}/dt = (d\sigma/dt)\,(\rho_{\lambda_c\lambda_c} \pm \varepsilon\rho_{\lambda_c-\lambda_c}).$$

[7]Determination of the values of $\rho^{nn'}_{mm'}$, in the Gottfried–Jackson system corresponds to the t channel, and in the helical system, to the s channel.

[8]More exact calculations with inclusion of screening give a ratio[95] $\sigma \cdot [(\rho\pi)p] \approx 1.7\sigma(\pi p)$.

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