Rotational motion in odd-mass atomic nuclei

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The nonadiabatic description of rotational motion in odd-mass nuclei is reviewed. A theory of polarization effects leading to the renormalization of a number of one-particle matrix elements is developed. Rotational spectra and electrodynamic moments are considered in the intermediate coupling scheme. The dynamics of nuclear rotation is studied approximately.

INTRODUCTION

In nuclei, sets of nuclear levels whose energies follow exactly or approximately the law I(I+1), where I is the angular momentum, are called rotational bands. As a rule, the states in a band are connected by strong γ transitions, whose probabilities are much greater than for transitions between states of different bands. This definition is neither complete nor unique, especially from the point of view of the classification of the rotational bands with respect to the quantum numbers of the internal states, and it applies only in the case of weak interaction of states from different bands (adiabatic approximation). Indeed, strong departures from the I(I+1) law are observed in rotational bands in which the contribution of only one internal state predominates (for example, in bands with K = 1/2). On the other hand, cascade γ transitions can link the so-called yrast levels, λ i.e., states with minimal energy for a given value of the angular momentum. The levels of an yrast line may belong to different rotational bands.

The discovery of rotational bands in the spectra of atomic nuclei led to the idea of their deformation, i.e., to nonsphericity of the self-consistent field. This enables one to distinguish certain principal axes and think of the orientation of the nucleus as a whole. The motion of nucleons considered relative to the distinguished proper coordinate system, which is oriented in a certain manner in space, is called the internal motion (here and in what follows the vibrational excitations are regarded as forms of internal motion).

However, the presence of deformation is only a necessary but not a sufficient condition for the existence of a rotational spectrum. A second condition is one of adiabaticity, i.e., independence (exact or approximate) of the collective rotational motion and the internal motion.[2] In view of the noninertial nature of the rotational motion, an exact separation of the internal and rotational variables is impossible. In a rotating system, centrifugal and Coriolis forces arise, and these couple the internal and rotational degrees of freedom. Usually, it is assumed that the adiabaticity condition is well satisfied if the frequencies of the rotational motion are much lower than the frequencies of the internal excitations. Thus, a "fast" internal motion is realized in a "slowly" rotating deformed self-consistent field. We can make a classical estimate of the frequency of "slow" rotational motion:[3]

$$\omega_{\text{rot}} = \Delta\mathcal{E}_{I\to I-2}/2\hbar \approx 10^{20}\ \text{sec}^{-1}. \tag{1}$$

The vibrational cycle is shorter by a factor of approximately 2. Because of pairing correlations, adiabaticity for one particle motion is well satisfied in even–even nuclei. In odd and odd–odd nuclei, the adiabatic conditions are not well satisfied, and this leads in many cases to strong distortions of the rotational bands.

Our review does not include a discussion of all aspects of rotational motion. We restrict ourselves to rotational bands in odd-mass atomic nuclei, invoking data on even–even nuclei when necessary. We analyze the semiphenomenological rotator model, in which the rotation is described in terms of collective variables, as in the Bohr–Mottelson model[4-6] and the internal motion is studied by modern microscopic methods. We shall not here consider microscopic approaches to the description of the rotation. A fairly complete review of the literature on this question can be found in refs. 7-10.

1. NONADIABATIC ROTATIONAL MODEL

We shall consider deformed atomic nuclei in which well localized principal axes (an internal coordinate system) are distinguished. The spatial orientation of the system is determined by the Eulerian angles θ_i. In the generalized model of the nucleus it is assumed that the complete Hamiltonian consists of two parts:

$$H = H_{\text{intr}} + H_{\text{rot}}, \tag{2}$$

where H_{intr} describes the internal motion of the nucleons, i.e., it depends only on the coordinates of the nucleons in the internal coordinate system, and H_{rot} can be represented[4] in the form ($\hbar = 1$)

$$H_{\text{rot}} = \sum_{i=1}^{3} \frac{1}{2J_i}(I_i - j_i)^2. \tag{3}$$

Here, J_i are the moments of inertia with respect to the principal axes; I_i and j_i are the components of the total and internal angular momenta. Such a form of the kinetic energy operator presupposes that the total angular momentum **I** of the nucleus (an integral of the motion) is the vector sum of the angular momenta of the collective motion (rotation of the nucleus as a whole) and the internal angular momentum **j** formed by one or several unpaired nucleons.[1)] The operators I_i act in the space of collective variables (Eulerian angles) and the operators j_i in the space of the internal variables of the nucleons, which implicitly depend on the total angular momentum. Because of the independence of these spaces, the operators I_i and j_i commute. As quantum numbers characterizing the rotational states we can use the total angular momentum I, its projection M onto the laboratory axis z, and the projection K onto one of the principal axes. The first two are integrals of the motion, while K is not conserved

in the general case. This corresponds to what is known as two-dimensional rotation, i.e., rotation about a fixed axis.

For the following treatment, we must give a further particularization of the self-consistent field and the scheme of coupling of the internal and rotational angular momenta. In the special case of an axially symmetric form of the self-consistent field, the nucleus has a distinguished symmetry axis, along which we can take axis 3 of the internal coordinate system. The scheme of coupling the angular momenta is determined by the relationship of the interaction energy of an external nucleon (nucleons) with the nonspherical part of the self-consistent field of the core nucleus (even–even nucleus) and the energy of rotation of the nucleus. In the case of strong coupling, the first interaction energy is much greater than the difference of the energies of the rotational levels, i.e., the rotation can be regarded as a perturbation. With increasing angular momentum of the rotation, the strong coupling scheme may break down.

An active role in this process is played by the Coriolis forces, which act on an outer nucleon in the rotating frame (see below). These forces tend to align the rotational and internal angular momenta. Such a decoupling process is well known in rotational bands with K = 1/2 (refs. 5 and 6). In the limiting case when the angular momentum of rotation and the internal angular momentum of the outer particle are almost parallel, the coupling of the nucleon to the rotation of the core through the Coriolis interaction is greatly weakened. Then the energy differences in a rotational band of an odd-mass nucleus are approximately equal to the corresponding energy differences in the rotational band of the ground state of a neighboring even–even nucleus.[11] The realization of such a coupling scheme in a number of odd-mass lanthanum isotopes has been discussed in ref. 12. A similar spectrum arises in odd-mass nuclei in the weak coupling scheme, when the interaction of the outer nucleon with the core is small compared with the kinetic energy of rotation[13] (for example, at small deformations). In this case, j can be approximately assumed to be an integral of the motion.

In what follows we shall discuss a variant of the rotator model in an intermediate coupling scheme for an axially symmetric deformation. As the axis of rotation we can choose any axis perpendicular to the symmetry axis. Since the rotational angular momentum is directed along the axis of rotation,

$$I_3 - j_3 = 0;\ J_3 = 0;\ J_1 = J_2 = J \tag{4}$$

and the rotational Hamiltonian takes the simple form

$$H_{\text{rot}} = [(I_1 - j_1)^2 + (I_2 - j_2)^2]/2J. \tag{5}$$

For the subsequent discussion, it is convenient to represent this Hamiltonian in the form

$$\left.\begin{aligned} H_{\text{rot}} &= H^0_{\text{rot}} + H_C + H_j; \\ H^0_{\text{rot}} &= [I(I+1) - I_3^2]/2J; \\ H_C &= -2[I_1 j_1 + I_2 j_2]/2J; \\ H_j &= [j_1^2 + j_2^2]/2J. \end{aligned}\right\} \tag{6}$$

Here, H_C is the Coriolis interaction of the outer nucleon (nucleons) with the rotation of the core; H_j, which does not depend on the total angular momentum, can be included in the internal Hamiltonian (see below for discussion of the role of H_j).

The internal angular momentum j in a deformed nucleus is not an integral of the motion (unless it coincides with I). However, in a standing nucleus (fixed orientation), the projection $j_3 = K$ of this angular momentum onto the symmetry axis is a good quantum number (strong coupling scheme). This quantum number is usually used to classify the internal states. We denote the additional quantum numbers (parity π, principal quantum number N, etc.) by ρ. We shall assume known the eigenfunctions and eigenvalues of the internal motion:

$$(H_{\text{intr}} + H_j)\,\Phi_{K\rho} = \mathscr{E}_{K\rho}\Phi_{K\rho}. \tag{7}$$

Then the complete symmetrized wave function of the nucleus in a state with the quantum numbers I and M can be sought in the form

$$|IM\rangle = \sum_{K,\rho} C^I_{K\rho}\,|IMK\rho\rangle, \tag{8}$$

where the adiabatic wave functions are[4]

$$\begin{aligned} |IMK\rho\rangle = \sqrt{(2I+1)/16\pi^2}\,\{&\Phi_{K\rho}\mathscr{D}^I_{MK}(\theta_i) \\ &+ (-1)^{I+l+K}\Phi_{\tilde{K}\rho}\mathscr{D}^I_{M,-K}(\theta_i)\}, \end{aligned} \tag{8a}$$

and the mixing amplitudes $C^I_{K\rho}$ satisfy the normalization

$$\sum_{K,\rho} (C^I_{K\rho})^2 = 1. \tag{9}$$

In Eq. (8a), the states $\Phi_{K\rho}$ and $\Phi_{\tilde{K}\rho}$ are conjugate with respect to time, the time-inversion operation being defined as in ref. 14:

$$T\Phi_{K\rho} = \Phi_{\tilde{K}\rho} = (-1)^{K+1/2}\Phi_{-K,\rho}, \tag{10}$$

while the function $\Phi_{-K,\rho}$ is determined by the action of the operator of rotation through an angle π about an axis perpendicular to the symmetry axis:[15]

$$R_1\Phi_{K\rho} = (-1)^{K+l+1/2}\Phi_{-K,\rho}. \tag{11}$$

Here, the phase $(-1)^l$ characterizes the parity of the state.

The eigenvalues of the Hamiltonian (2) and the amplitude $C^I_{K\rho}$ can be found by directly diagonalizing the Hamiltonian in the space of states (8a). Using the well-known properties of the $\mathscr{D}$ functions (see, for example, refs. 13 and 15), we write a diagonal matrix element of the Hamiltonian (2) in the form[14]

$$\left.\begin{aligned} \mathscr{E}(I) &= \sum_{K,\rho} (C^I_{K\rho})^2\,\mathscr{E}_{K\rho} + g(I)/2J; \\ g(I) &\equiv I(I+1) - \sum_{K,\rho} (C^I_{K\rho})^2 K^2 \\ &\quad + (-1)^{I+1/2}(I+1/2)\,a(I). \end{aligned}\right\} \tag{12}$$

Here we have introduced the generalized decoupling parameter $a(I)$, defined by

$$a(I) = (-1)^{I+1/2}(I+1/2)^{-1}\,2J\,\langle IM|H_C|IM\rangle. \tag{13}$$

In the general case, the decoupling parameter is a function of the spin and is nonvanishing in any rotational band. Note that exact diagonalization of the Coriolis interaction essentially amounts to our abandoning the strong coupling scheme and going over to an intermediate coupling. The transition to the limit discussed in refs. 11 and 12 will be considered below. In all the equations that we derive, the parameter J is the effective moment of inertia corresponding to the collective rotational motion, i.e., it is the moment of inertia of the even–even core.

2. ADIABATIC APPROXIMATION

Let us consider the simplest variant of the model formulated in the foregoing section, namely, when one considers the motion of one particle in the field of an axially symmetric rotator that does not have internal degrees of freedom.[5] The Hamiltonian of the internal motion reduces in this case to a one-particle Hamiltonian that describes the motion of a particle in a potential well, whose role is played by the potential energy of the interaction between the particle and the rotator:

$$H_{\text{intr}} = \mathbf{P}/2m + V(\mathbf{r}) + \mathbf{j}^2/2J. \tag{14}$$

It is generally assumed that the rotator and the potential $V(\mathbf{r})$ have the same symmetry. In particular, $V(\mathbf{r})$ can be chosen in the form of the Nilsson potential or of the Woods–Saxon type.[17,18] The influence of the last term in (14) on the one-particle spectra has been discussed in refs. 19.

If the Coriolis interaction in the Hamiltonian (6) is regarded as a weak perturbation, the spectrum of a particle in the field of the rotator is described as follows:[2)]

$$\mathcal{E}(I, K\rho) = \varepsilon_{K\rho} + [I(I+1) - 2K^2 + \delta_{K,1/2}(-1)^{I+1/2}(I+1/2)\,a_{sp}]/2J\ , \tag{15}$$

where $\varepsilon_{K\rho}$ are the eigenvalues of the Hamiltonian (14). The last term in the square brackets in (15) is the contribution of the Coriolis interaction in the first order of perturbation theory, and the one-particle decoupling parameter a_{sp} is obtained from the definition (13) in the case of an isolated band with $K = 1/2$:

$$a_{sp} \equiv (-1)^l \langle \chi_{-1/2,\rho}|j_-|\chi_{1/2,\rho}\rangle, \tag{16}$$

where $\chi_{K\rho}$ are the eigenfunctions of the Hamiltonian (14) (the phases are chosen in accordance with ref. 15). The states of the system are then described by the adiabatic wave functions (8a), in which $\Phi_{K\rho}$ is replaced by $\chi_{K\rho}$.

Allowance for the pairing correlations between nucleons of the core (see, for example, the extensive literature on this question in Solov'ev's monograph, ref. 20) reduces in this variant of the model to replacing the one-particle spectrum by the one-quasiparticle spectrum if one does not take into account the value of the moment of inertia J, which figures here as a parameter.

In certain cases it is necessary to take into account the Coriolis interaction in higher perturbation orders and sometimes to diagonalize exactly H_C in the space of states (8a) (for example, in the case of levels of one parity and nearly equal energy with $|\Delta K| = 1$ (see ref. 21) or when the one-particle matrix elements of the operator j_- are large (ref. 22). A systematic analysis of these corrections in the framework of perturbation theory has been developed in ref. 23, which gives the following equation, widely used by experimentalists:

$$\mathcal{E}(I, K\rho) = \varepsilon_{K\rho} + A[I(I+1) - K^2] + B[I(I+1) - K^2]^2 + \ldots + (-1)^{I+K}[(I+K)!/(I-K)!] \times \{A_{2K} + B_{2K}[I(I+1) - K^2] + \ldots\}, \tag{17}$$

where A, B, A_{2K}, ... are regarded as parameters.

A more detailed discussion of the adiabatic model can be found, for example, in refs. 2, 5, 6, 15, 24. Information on the electromagnetic moments in this model will be given in the subsequent sections whenever necessary.

3. INTERNAL MOTION

Any generalization of the particle + rotator model considered in the foregoing section entails the splitting of an odd-mass nucleus into a core and outer nucleons; more precisely, one must define a means of splitting the total angular momentum into the rotational and the internal momentum. We write down the operator of the internal angular momentum in the second-quantization representation as the one-particle operator

$$j_\mu = \sum_{\nu,\nu'} \langle \nu|j_\mu|\nu'\rangle\, a_\nu^+ a_{\nu'}, \tag{18}$$

where $\langle\nu|j_\mu|\nu'\rangle$ are the one-particle matrix elements for the μ-th component of the angular momentum; a_ν^+ and a_ν are the operators of creation and annihilation of a particle in the state $|\nu\rangle$, respectively.

Obviously, the posed problem reduces to determining the configuration space of the states on which the operators j_μ act. If the summation in (18) is extended over all neutron and proton states, the internal angular momentum must coincide with the total angular momentum of the nucleus. In odd-mass nuclei, it is reasonable to sum over only the space of odd-nucleon states coupled by the Coriolis interaction, i.e., over the space of states of one parity equal to that of the considered rotational band. In special cases, when the odd nucleon is in one-particle orbits that derive from spherical subshells with large j, one can restrict oneself to the set of states of this subshell.

Thus, it is impossible (and not necessary) to distinguish a priori the outer nucleons. This process is dynamical, depending on the characteristics of the internal state and the total angular momentum, since even more nucleons are involved in the Coriolis interaction when the

latter increases. By way of an analogy, we may cite the difficulty in separating the closed shells in deformed nuclei, which is important for calculating effective charges. By increasing the configuration space for the quadrupole–quadrupole interaction, we can, for example, obtain B(E2) values that greatly exceed the experimental values even without allowance for effective charges.

We now turn to the description of internal motion in odd-mass nuclei, using modern microscopic models. One of the important questions is the choice of the residual interactions. We approach this question from the point of view of the influence that the various residual interactions have on the results of diagonalizing the matrix of the Coriolis interaction; these depend on the density and quantum numbers of the low-lying internal excitations and also on the one-particle matrix elements of the operators $j_{\pm}$.

Thus, it is necessary to take into account pairing correlations, which essentially determine the density of quasiparticle excitations (see, for example, the discussion in ref. 20). The residual quadrupole (octupole) forces have rather little effect on the structure of the low-lying excitations and their density. The wave functions of these states acquire small three-quasiparticle (or quasiparticle plus phonon) admixtures, which are only manifested if they participate coherently in a process. For example, quasiparticle + quadrupole phonon admixtures can appreciably perturb the probabilities of E2 transitions between internal states.[20,25] These effects can be regarded as due to the renormalization of the one-particle quadrupole moments, the renormalization depending on the frequency of the collective excitations of the even–even core.

It is well known that the one-particle matrix elements of the spin operators σ_μ are renormalized by the residual spin isovector forces (spin polarization effects).[26,27] The spin polarization effects can be regarded as generated by the interaction of quasiparticles with 1^+ (K = 0 and 1) excitations of the even–even core.[28] And although the total three-quasiparticle admixtures are small in this case (1-3%), they lead to coherent renormalization of the gyromagnetic g_s factors in the magnetic moments and the probabilities of M1 transitions.

The list of polarization effects could be considerably extended, but this is not the aim of our review. Here, we wish only to emphasize that when one studies the Coriolis interaction it is necessary to separate out accurately all the residual forces that lead to coherent renormalization of the one-particle matrix elements of the operators $j_{\pm}$. We therefore turn to a consideration of the term H_j in the Hamiltonian (6). Having in mind the representation (18), we shall regard it as the residual centrifugal interaction (recoil interaction in the Bohr–Mottelson terminology[2]) between outer nucleons. The source of these forces is the rotation of the core. Together with the spin interactions, the centrifugal forces foster the appearance of a collective 1^+ branch of internal excitations in the core and affect the renormalization of the matrix elements of the operators $j_{\pm}$ and $\sigma_{\pm}$ (see ref. 14).

Thus, to describe the internal motion we use the Hamiltonian

$$\mathscr{H}_{\text{intr}} = H_{sp} + H_{\text{pair}} + H_\sigma + H_j. \tag{19}$$

The individual terms in its second-quantization representation have the form

$$H_{sp} = \sum_{\nu>0} (\varepsilon_\nu - \lambda)(a_\nu^+ a_\nu + a_{\tilde\nu}^+ a_{\tilde\nu}); \tag{19a}$$

$$H_{\text{pair}} = -G\Gamma^+\Gamma;\ \Gamma \equiv \sum_{\nu>0} a_{\tilde\nu} a_\nu; \tag{19b}$$

$$\left.\begin{aligned} H_\sigma &= (1/8)\varkappa \sum_{p=\pm} (F_\sigma^{(p)})^+ F_\sigma^{(p)}; \\ F_\sigma^{(p)} &\equiv \sum_{\nu,\nu'} \langle \nu | \sigma_+ - p\sigma_- | \nu' \rangle a_\nu^+ a_{\nu'}; \end{aligned}\right\} \tag{19c}$$

$$\left.\begin{aligned} H_j &= (1/2J)(1/4) \sum_{p=\pm 1} (F_j^{(p)})^+ F_j^{(p)}; \\ F_j^{(p)} &\equiv \sum_{\nu,\nu'} \langle \nu | j_+ - pj_- | \nu' \rangle a_\nu^+ a_{\nu'}. \end{aligned}\right\} \tag{19d}$$

Here, ε_ν are the one-particle energies; λ is the chemical potential of the system; G and $\varkappa$ are the force parameters of the pairing and spin interactions.[3)] The remaining notation is standard. Note that the configuration space in the operator $F^{(p)}$ is bounded in accordance with the definition (18), but in the operator $F_\sigma^{(p)}$ there are no such restrictions.

In what follows, we use the following symmetry properties of the matrix elements of the operator $j^{(p)} \equiv j_+ - pj_-$:

$$j_{\nu\nu'}^{(p)} \equiv \langle \nu | j^{(p)} | \nu' \rangle = p \langle \tilde\nu | j^{(p)} | \tilde\nu' \rangle = -p \langle \nu' | j^{(p)} | \nu \rangle. \tag{20}$$

The matrix elements of the spin operators satisfy the same symmetry relations. Note also that the matrix elements $j_{\nu\nu'}^{(+)}$ and $j_{\nu\nu'}^{(-)}$ differ only in phase, i.e.,

$$|j_{\nu\nu'}^{(+)}|^2 = |j_{\nu\nu'}^{(-)}|^2 \equiv j_{\nu\nu'}^2. \tag{20a}$$

The scheme for solving the problem of the internal motion is as follows.

1. A canonical (u, v) Bogolyubov transformation is applied to the quasiparticles.[29] From the Hamiltonian $H_{sp} + H_{\text{pair}}$ we separate the one-quasiparticle part

$$H_{sqp} \equiv \sum_{\nu>0} E_\nu (\alpha_\nu^+ \alpha_\nu + \alpha_{\tilde\nu}^+ \alpha_{\tilde\nu}), \tag{21}$$

where α_ν are quasiparticle operators; $E_\nu = [(\varepsilon_\nu - \lambda)^2 + \Delta^2]^{1/2}$ are one-quasiparticle energies (Δ is the energy gap).

In the quasiparticle representation, the operators $F_j^{(p)}$ have the form[14]

$$\left.\begin{aligned} F_j^{(p)} &= (F_j^{(p)})_B + (F_j^{(p)})_A; \\ (F_j^{(p)})_B &\equiv \sum_{\nu,\nu'>0} p j_{\nu\nu'}^{(p)} M_{\nu\nu'} B_{\nu\nu'}^{(p)}; \\ (F_j^{(p)})_A &= (1/2) \sum_{\nu,\nu'>0} p j_{\nu\nu'}^{(p)} L_{\nu\nu'} [A_{\nu\nu'}^{(p)+} - pA_{\nu\nu'}^{(p)}]. \end{aligned}\right\} \tag{22}$$

Here we have introduced the notation

$$B^{(p)}_{\nu\nu'} = \alpha^+_\nu \alpha_{\nu'} + p\alpha^+_{\tilde\nu}\alpha_{\tilde\nu'};\ A^{(p)}_{\nu\nu'} = \alpha_\nu \alpha_{\tilde\nu'} - p\alpha_{\tilde\nu}\alpha_{\nu'}; \tag{23}$$

$$M_{\nu\nu'} = u_\nu u_{\nu'} + v_\nu v_{\nu'};\ L_{\nu\nu'} = u_\nu v_{\nu'} - u_{\nu'} v_\nu. \tag{24}$$

Obviously, $[F^{(p)}_j]_B$ and $[F^{(p)}_j]_A$ are, respectively, the one-quasiparticle and the boson part of the internal angular momentum operator (in the operators $A^{(p)}_{\nu\nu'}$ the quasiparticles are coupled in such a way that their total value is $K^\pi = 1^+$). The operator $F^{(p)}_\sigma$ has the same form as (22) if $j^{(p)}_{\nu\nu'}$ is replaced by $\sigma^{(p)}_{\nu\nu'} \equiv \langle\nu|\sigma_+ - p\sigma_-|\nu'\rangle$, where $\sigma_\pm = \sigma_1 \pm i\sigma_2$.

2. The final spectrum $\mathscr{E}_{K\rho}$ of internal excitations is obtained as the set of eigenvalues of the approximate Hamiltonian

$$\begin{aligned}\mathscr{H}'_{\text{intr}} = H_{sqp} &+ (1/8)\varkappa \sum_{p=\pm1} \{(F^{(p)}_\sigma)^+_A (F^{(p)}_\sigma)_A \\ &+ (F^{(p)}_\sigma)^+_A (F^{(p)}_\sigma)_B + (F^{(p)}_\sigma)^+_B (F^{(p)}_\sigma)_A\} \\ &+ (1/2J)(1/4) \sum_{p=\pm1} \{(F^{(p)}_j)^+_A (F^{(p)}_j)_A \\ &+ (F^{(p)}_j)^+_A (F^{(p)}_j)_B + (F^{(p)}_j)^+_B (F^{(p)}_j)_A\}.\end{aligned} \tag{25}$$

To find the eigenvalues one can use the Tamm–Dancoff method with trial wave functions of the type used in ref. 28 (in what follows, we omit the subscript ρ as unimportant and assume throughout that $K > 0$):

$$\begin{aligned}\Phi_K = \{N_K \alpha^+_K &+ (1/2) \sum_{p=\pm1} \sum_{K'} \sum_{\lambda\lambda' \neq K'} D^{KK'}_{\lambda\lambda'}(p) \\ &\times [\alpha^+_{K'} A^{(p)+}_{\lambda\lambda'} + \delta_{K,1/2}\delta_{K',1/2}\alpha^+_{\tilde K'} A^{(p)+}_{\lambda\lambda'}]\}|0\rangle,\end{aligned} \tag{26}$$

where $|0\rangle$ is the quasiparticle vacuum; N_K and $D^{KK'}_{\lambda\lambda'}$ are variational amplitudes, which satisfy the normalization

$$N^2_K + \sum_{p=\pm1} \sum_{K'} \sum_{\lambda\lambda' \neq K'} [D^{KK'}_{\lambda\lambda'}(p)]^2 = 1. \tag{27}$$

The wave function (26) takes into account the interaction of the quasiparticles with 1^+ excitations of the even–even core (Tamm–Dancoff phonons), and the blocking effect in the phonons is taken into account explicitly ($\lambda\lambda' \neq K'$). Note that $K' = K \pm 1$, so that for $K = 1/2$ it is possible for K' to have the same value as K. Omitting the details of the derivation, we write down a dispersion equation for the eigenvalues:[14]

$$E_K - \mathscr{E}_K = \sum_{K'} M^2_{KK'} \left\{ \frac{1}{2J} j^2_{KK'} \frac{\chi^{(\sigma)}_{KK'}}{2J + \chi^{(\sigma)}_{KK'}} + \frac{1}{2}\varkappa\sigma^2_{KK'} \frac{\varkappa S^{(j)}_{KK'}}{1 + \varkappa S^{(\sigma)}_{KK'}} \right\}; \tag{28}$$

$$\chi^{(\sigma)}_{KK'} \equiv (1/2) \sum_{\lambda\lambda' \neq K'} [j^{(p)}_{\lambda\lambda'} \overline{j^{(p)}_{\lambda\lambda'}} L^2_{\lambda\lambda'} / (E_\lambda + E_{\lambda'} + E_{K'} - \mathscr{E}_K)]; \tag{28a}$$

$$S^{(j)}_{KK'} \equiv (1/4) \sum_{\lambda\lambda' \neq K'} [\sigma^{(p)}_{\lambda\lambda'} \overline{\sigma^{(p)}_{\lambda\lambda'}} L^2_{\lambda\lambda'} / (E_\lambda + E_{\lambda'} + E_{K'} - \mathscr{E}_K)]. \tag{28b}$$

Here, we have introduced the effective matrix elements

$$\left.\begin{aligned}\overline{j^{(p)}_{\lambda\lambda'}} &= j^{(p)}_{\lambda\lambda'} - \sigma^{(\sigma)}_{\lambda\lambda'} \varkappa X^{(p)}_{KK'} / (1 + \varkappa S_{KK'}); \\ \overline{\sigma^{(p)}_{\lambda\lambda'}} &= \sigma^{(p)}_{\lambda\lambda'} - j^{(p)}_{\lambda\lambda'} X^{(p)}_{KK'} / (2J + \chi^{(\sigma)}_{KK'});\end{aligned}\right\} \tag{29}$$

$$X^{(p)}_{KK'} = (1/4) \sum_{\lambda\lambda' \neq K'} [\sigma^{(p)}_{\lambda\lambda'} j^{(p)}_{\lambda\lambda'} L^2_{\lambda\lambda'} / (E_\lambda + E_{\lambda'} + E_{K'} - \mathscr{E}_K)]. \tag{29a}$$

The expressions for $\chi_{KK'}$ and $S_{KK'}$ are identical with (28a) and (28b) if the effective matrix elements in the latter are replaced by the one-particle elements.

We also write down an expression for the amplitude of the three-quasiparticle admixtures:

$$\begin{aligned}D^{KK'}_{\lambda\lambda'}(p) = [&-(1/2) N_K M_{KK'} L_{\lambda\lambda'} / (E_\lambda + E_{\lambda'} + E_{K'} - \mathscr{E}_K)] \\ &\times [\overline{j^{(p)}_{KK'}} j^{(p)}_{\lambda\lambda'} / (2J + \chi^{(\sigma)}_{KK'}) \\ &+ (1/2) \overline{\sigma^{(p)}_{KK'}} \sigma^{(p)}_{\lambda\lambda'} \varkappa / (1 + \varkappa S^{(j)}_{KK'})].\end{aligned} \tag{30}$$

The shift of the levels of the one-quasiparticle spectrum $E_K - \mathscr{E}_K$ in (28) is entirely due to the three-quasiparticle admixtures. The solution of the problem in the random-phase method does not lead to any qualitative modifications in the spectrum of the internal excitations.

Thus, in contrast to the ordinary approaches we shall take into account the influence of the collective branch of 1^+ excitations in the even–even core. This leads to a renormalization of the one-particle matrix elements of the operators $j^{(p)}$ and $\sigma^{(p)}$ (polarization effects), which we shall discuss below. Here we should like to make some comments on the properties of 1^+ excitations. There have been many investigations of these states (see, for example, refs. 30-34). It has been found that the physical 1^+ excitations in the spectra of even–even nuclei lie above the energy gap. The coherent branch of these excitations (magnetic dipole resonance) is expected at an energy of order 8-10 MeV. States of this type were recently discovered in ^{208}Pb (see ref. 35) and in nuclei with A = 140 (see ref. 36) in this region of energies. They are characterized by strong M1 transitions to the ground state [radiative width $\Gamma_\gamma(\text{M1}) \sim 10^2$ eV].

4. POLARIZATION EFFECTS

Three-quasiparticle admixtures in the wave functions (26) significantly perturb the one-particle matrix elements of the angular momentum and spin operators. For example, for the matrix elements of the operator $F^{(p)}_j$ we obtain in the quasiboson approximation[37,38]

$$\langle\Phi_K | F^{(p)}_j | \Phi_{K'}\rangle = -p j^{(p)}_{KK'} M_{KK'} R^{(\sigma)}_j(K, K'), \tag{31}$$

where the polarization factor is

$$\begin{aligned}R^{(\sigma)}_j(K, K') = N_K N_{K'} \{&(2J/j^{(p)}_{KK'}) [\overline{j^{(p)}_{KK'}} / (2J + \chi^{(\sigma)}_{KK'}) \\ &+ \overline{j^{(p)}_{K'K}} / (2J + \chi^{(\sigma)}_{K'K})] - 1\}.\end{aligned} \tag{31a}$$

The quantity in the braces differs from unity because of the contribution of the boson part of the operator $F^{(p)}_j$.

One can obtain a simple quasiclassical estimate (31a) in the case $\varkappa = 0$ (effects of the centrifugal interaction). For this, we consider interaction of closely spaced levels with $|\Delta K| = 1$ ($|E_K - E_{K'}| \ll E_\lambda + E_{\lambda'}$) and we ignore the blocking effect in Eqs. (28a) and (28b). Then we obtain

$$\mathcal{I}^{(\sigma)}_{KK'}|_{\varkappa=0} \approx (1/2) \sum_{\lambda\lambda'>0} [j^2_{\lambda\lambda'} L^2_{\lambda\lambda'}/(E_\lambda + E_{\lambda'})] \equiv \mathcal{I}_c, \qquad (32)$$

the summation being restricted to the space of states for the internal angular momentum. Obviously, $\mathcal{I}_c$ is the contribution of the outer nucleons to the moment of inertia (cranking model with pairing[26,39]). For the polarization factor, we obtain the simple estimate

$$R^{(\sigma)}_j(K, K')|_{\varkappa=0} \approx N_K N_{K'} (2J - \mathcal{I}_c)/(2J + \mathcal{I}_c) \leqslant 1. \qquad (33)$$

The amplitudes N_K are near unity for low-lying states but decrease strongly near the threshold of three-quasiparticle excitations.

To get a clear picture of the influence of the renormalization (31) due to the Coriolis interaction, we write H_C in the form

$$H_C = (-1\ 2J)(1/2) \sum_{p=\pm1} (I_+ - pI_-)(F^{(p)}_j)^+. \qquad (34)$$

The replacement of the one-particle matrix elements of the internal angular momentum operator by the elements renormalized in the quasiclassical approximation reduces to renormalization of the rotational parameter:[4)]

$$1/2J \to R^{(\sigma)}_j/2J. \qquad (35)$$

Thus, the polarization effects lead to a weakening of the Coriolis interaction, at least between low-lying configurations.

The matrix elements of the spin operators are also renormalized. For example, the matrix element of the operator $F^{(p)}_\sigma$ has the form[37,38]

$$\langle \Phi_K | F^{(p)}_\sigma | \Phi_{K'} \rangle = -p\sigma^{(p)}_{KK'} M_{KK'} R^{(j)}_\sigma(K, K'); \qquad (36)$$

$$R^{(j)}_\sigma(K, K') = N_K N_{K'} \{(1/\sigma^{(p)}_{KK'})[\overline{\sigma^{(p)}_{KK'}}/(1 + \varkappa S^{(j)}_{KK'}) + \overline{\sigma^{(p)}_{K'K}}/(1 + \varkappa S^{(j)}_{K'K})] - 1\}, \quad |K - K'| = 1. \qquad (36a)$$

Here, the renormalization is essentially due to spin forces. The matrix elements of the operator σ_3 are renormalized by only the spin forces.[28] The corresponding polarization factor is

$$R_\sigma(K, K') = N_K N_{K'} \{1 - \varkappa S^{(0)}_{KK'}/(1 + \varkappa S^{(0)}_{KK'}) - \varkappa S^{(0)}_{K'K}/(1 + \varkappa S^{(0)}_{K'K})\}, \qquad (37)$$

$$K - K' = 0;$$

$$S^{(0)}_{KK'} \equiv \sum_{\lambda\lambda' \neq K'} [(\sigma_3)^2_{\lambda\lambda'} L^2_{\lambda\lambda'}/(E_\lambda + E_{\lambda'} + E_{K'} - \mathcal{E}_K)]. \qquad (37a)$$

Note that the polarization factors $R^{(j)}_\sigma$ and R_σ may be very different since they arise from the interaction of quasiparticles with 1^+ K = 1 and 1^+ K = 0 phonons, respectively, i.e., the matrix elements of the operators $\sigma^{(p)}$ and σ_3 are renormalized independently. This is why one distinguishes transverse and longitudinal spin polarization effects.[27,28] It is important to allow for the spin polarization for the magnetic moments and the probabilities of M1 transitions.

5. ROTATIONAL INVARIANCE

The polarization effects considered in the foregoing section arise from the interaction of quasiparticles with the internal motion in the core, but not with the rotation. However, because of the rotational noninvariance of a deformed self-consistent field, the internal 1^+ excitations of the core include one "ghost" state belonging to the rotational branch of excitations.[40,41] To restore rotational invariance of the internal Hamiltonian, we can augment it with effective residual interactions,[5)] whose characteristics (force parameter and one-particle matrix elements) are determined from the condition that the total Hamiltonian commutes with the angular momentum operator in the internal coordinate system.[41,42] This leads to the separation of a "ghost" branch of 1^+ excitations with internal energy $\omega = 0$. The corresponding term of the Hamiltonian has the structure of rotational energy. At the same time, all the physical 1^+ states are automatically collectivized, even in the absence of other residual interactions. In odd-mass nuclei, the very procedure for restoring rotational invariance renormalizes the matrix elements of the internal angular momentum operator (here it is important that **j** is not an integral of the motion). Using the results of ref. 42, we obtain in the random-phase method the following expression for the polarization factor[38] ($\varkappa = 0$):

$$R_j(K, K') = N_K N_{K'} \{1 + \sum_{i \neq 0} (\partial \mathcal{I}(\omega_i)/\partial\omega_i)^{-1} \times \mathcal{I}_c(\omega_i)(E_K - E_{K'})/\omega_i [1/(E_{K'} + \omega_i - \mathcal{E}_K) - 1/(E_K + \omega_i - \mathcal{E}_{K'})]\}, \qquad (38)$$

where the energies ω_i of the 1^+ excitations of the even–even core are solutions of the dispersion relation

$$\mathcal{I}(\omega_i) \equiv (1/2) \sum_{\lambda\lambda'>0} \{(E_\lambda + E_{\lambda'}) L^2_{\lambda\lambda'} j^2_{\lambda\lambda'}/[(E_\lambda + E_{\lambda'})^2 - \omega_i^2]\} = 0. \qquad (39)$$

The function $\mathcal{I}_c(\omega_i)$ has the same form as (39), but the summation in it is restricted to the space of states on which the internal angular momentum operator acts. Thus, the sign of this function is not known a priori, and it is therefore impossible in the general case to estimate the magnitude of the factor $R_j(K, K')$. It follows from (38) however that if the angular momentum **j** goes over into the total angular momentum of the nucleus, then $\mathcal{I}_c(\omega_i) \to \mathcal{I}(\omega_i) = 0$, and the polarization effect disappears, which one would expect. The numerical estimates (38) show that for low-lying states with $|E_K - E_{K'}| \ll \omega_i$ the polarization effects due to the rotational noninvariance of the self-consistent field are negligibly small.

6. EFFECTIVE MOMENTS OF INERTIA OF ODD-MASS NUCLEI

We have already mentioned that the parameter J used in the nonadiabatic description characterizes the effective moment of inertia of the core. The effective moment of inertia of an odd-mass nucleus can be introduced by relating the observed rotational spectrum to an adiabatic equation of the type (15). We then obtain the following relation, which is frequently used in the evaluation of experimental spectra:

$$[\mathcal{E}(I+1)-\mathcal{E}(I)]/[2(I+1)]=(1/2J^{\text{eff}})[1-\delta_{K,1/2}(-1)^{I+1/2}a_{sp}]. \tag{40}$$

It is the quantity J^{eff} that the experimentalists frequently call the effective moment of inertia of an odd-mass nucleus and that is compared with the moments of inertia that are usually calculated in the cranking model (see, for example, ref. 43). Let us consider under which conditions J and J^{eff} can have similar values. Using Eq. (12), we obtain

$$[\mathcal{E}(I+1)-\mathcal{E}(I)]/[2(I+1)]=1/2J_{\text{in}}^{\text{eff}}+1/2J_{\text{rot}}^{\text{eff}}, \tag{41}$$

where, by definition,

$$1/2J_{\text{in}}^{\text{eff}}\equiv[1/2(I+1)]\sum_{K,\rho}\mathcal{E}_{K\rho}[(C_{K\rho}^{I+1})^2-(C_{K\rho}^{I})^2]; \tag{41a}$$

$$\begin{aligned}1/2J_{\text{rot}}^{\text{eff}}&\equiv(1/2J)(1/2(I+1))[g(I+1)-g(I)]\\&=(1/2J)\{1-(-1)^{I+1/2}[1/2(I+1)][(I+3/2)a(I+1)\\&+(I+1/2)a(I)]-[1/2(I+1)]\sum_{K\rho}K^2[(C_{K\rho}^{I+1})^2-(C_{K\rho}^{I})^2]\}.\end{aligned} \tag{41b}$$

Here, $J_{\text{in}}^{\text{eff}}$, which is due to the fluctuation of the expansion amplitudes $C_{K\rho}^{I}$ in the rotational band, is essentially the contribution due to the rearrangement of the internal spectrum as a result of rotation.[6)] The quantity $J_{\text{rot}}^{\text{eff}}$ arises from the variation of the kinetic energy of rotation. Under definite restrictions on the generalized decoupling parameter,

$$a(I)=\delta_{K,1/2}a_{sp}, \tag{42}$$

this quantity can be equal to J^{eff} defined by (40). The condition (42) can only be realized in the case of a completely isolated rotational band with K = 1/2. Thus, we can expect in practically all cases that the values of J used in the nonadiabatic approach and of J^{eff} extracted from the experimental energies by means of the relation (40) will be different. From general considerations we can also expect that the J values will be close to the values of the effective moment of inertia in the neighboring even–even nuclei.

In order to give an impression of the spread of J^{eff} values in odd-mass nuclei, we reproduce in Figs. 1 and 2 data obtained from an evaluation of observed spectra in accordance with (40). We give data for all the known rotational bands, these data being taken basically from ref. 44, and also from the current literature. For comparison, we give observed values of the effective moments of inertia in even–even nuclei (from the energies of the first 2^+ levels) taken from refs. 45 and 46 and individual cases from the current literature. A rigid-body estimate of the moment of inertia was made in accordance with the usual equation:[47]

$$J_{\text{rig}}=(2/5)AmR_0^2(1+0.31\beta), \tag{43}$$

assuming $R_0 = 1.2A^{-1/3}$ F and $\beta \approx 0.3$.

It can be seen that in the odd-mass nuclei there is an entire spectrum of J^{eff} values between the rigid-body value and the values in the even–even nuclei. Values of J^{eff} near the estimate (43) correspond to the case of strong Coriolis interaction in the bands. Such are the rotational bands associated with positive-parity levels in nuclei with odd N (see Fig. 1) and negative-parity levels in nuclei with odd Z (see Fig. 2). When $J^{\text{eff}} > J_{\text{rig}}$, expressions of the type (15) or (17) become quite unsuitable for evaluating a spectrum because of the strong distortions and the J^{eff} values extracted from them have no more than illustrative value.

To conclude this section, we point out that the definition of J^{eff} by Eq. (40) is not the only possible one. In

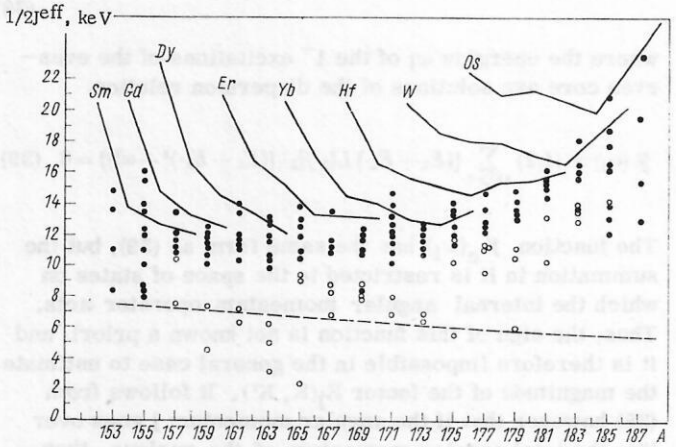

Fig. 1. Empirical values of the rotational parameters $1/2J^{\text{eff}}$ in even–even nuclei and in nuclei with odd N (●) calculated in accordance with the definition (40). For each odd-mass nucleus, data on all known rotational bands are given: ○ corresponds to rotational bands associated with positive-parity levels; ●, to bands associated with negative-parity levels; a continuous line corresponds to even–even nuclei; a dashed line to the rigid-body estimate ($1/2\,J_{\text{rig}}$).

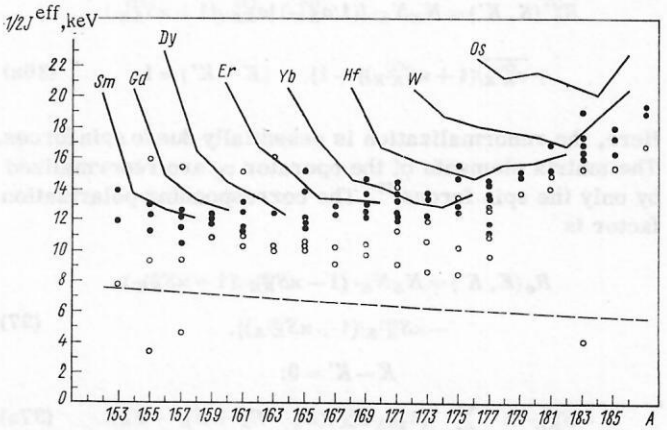

Fig. 2. The same as Fig. 1 for nuclei with odd Z: ○ corresponds to rotational bands associated with negative parity levels.

ref. 48 we have pointed out that one can introduce a "smoother" definition, which does not include Coriolis interaction effects in the first order, namely:[7)]

$$1/J_1^{\text{eff}} \equiv (1/8)\{\mathscr{E}(I+2)+\mathscr{E}(I-2)-2\mathscr{E}(I)\}. \quad (44)$$

In the adiabatic approximation (without allowance for the dynamics of the internal motion) the two expressions (40) and (44) must give the same value $J^{\text{eff}} = J_1^{\text{eff}}$, which is false even for levels that are the lowest with regard to the spin. Naturally, this leads to ambiguity when one compares the moments of inertia of odd-mass nuclei calculated in the cranking model with the experimental data. Evidently, such calculations are only qualitative. In the following sections, we shall discuss the electromagnetic moments of rotational states.

7. MAGNETIC MOMENTS

In the generalized model, the operator of the magnetic dipole moment is[5]

$$\boldsymbol{\mu} = g_R^0(\mathbf{I}-\mathbf{j}) + g_s\mathbf{s} + g_l\mathbf{l}, \quad (45)$$

where **s** and **l** are the spin and orbital angular momenta of the outer nucleons; g_R^0, g_S, and g_l are the gyromagnetic ratios.

The magnetic moment of a nucleus in the state $|IM\rangle$ is

$$\mu_I = \frac{\langle IM|\boldsymbol{\mu}\mathbf{I}|IM\rangle}{I+1}. \quad (46)$$

Using the wave functions (8), we obtain[37,38,49]

$$\mu_I = (I+1)^{-1}\{(g_R^0-g_l)G(I)+g_lI(I+1) + (g_s-g_l)\langle IM|\mathbf{sI}|IM\rangle\}; \quad (47)$$

$$G(I) \equiv g(I)-(1/2)(-1)^{I+1/2}(I+1/2)a(I); \quad (47a)$$

$$\langle IM|\mathbf{sI}|IM\rangle = \sum_{K,K'} C_K^I C_{K'}^I M_{K,K'}$$

$$\times\{\delta_{K,K'}(1/2)(\sigma_3)_{K,K}R_\sigma(K,K)+(1/8)\sum_{p=\pm1} p[\sigma_{K,K'}^{(p)}R_\sigma^{(j)}(K,K')$$

$$\times(\delta_{K',K+1}\sqrt{(I-K)(I+K+1)}$$

$$+p\delta_{K',K-1}\sqrt{(I+K)(I-K+1)})$$

$$+(1/2)\delta_{K,1/2}\delta_{K',1/2}(-1)^{I+l+1/2}(I+1/2)\sigma_{K,K'}^{(p)}R_\sigma^{(j)}(K,K')]\}. \quad (47b)$$

This somewhat complicated nonadiabatic expression for the magnetic moment, which contains one free parameter, g_R^0, of the core, can be reduced to the well-known adiabatic approximation[8)] (see ref. 16)

$$\mu_I^{ad} = (I+1)^{-1}\{(g_R^{\text{eff}}-g_l)G_K(I)+g_lI(I+1) + (g_s^{\text{eff}}-g_l)\langle IMK|\mathbf{sI}|IMK\rangle\}; \quad (48)$$

$$G_K(I) \equiv I(I+1)-K^2+(1/2)\delta_{K,1/2}(-1)^{I+1/2}(I+1/2)a_{sp}; \quad (48a)$$

$$\langle IMK|\mathbf{sI}|IMK\rangle = K(\sigma_3)_{K,K}/2 + \delta_{K,1/2}(-1)^{I+1/2}(I+1/2)(\sigma_+)_{K,-K}/4 \quad (48b)$$

by renormalizing the gyromagnetic factors g_R^0 and g_S.

Comparing (47) and (48), we obtain the simple relations

$$g_R^{\text{eff}} - g_l = (g_R^0 - g_l)(G(I)/G_K(I)); \quad (49)$$

$$g_s^{\text{eff}} - g_l = (g_s - g_l)\frac{\langle IM|\mathbf{sI}|IM\rangle}{\langle IMK|\mathbf{sI}|IMK\rangle}. \quad (50)$$

Naturally, in the nonadiabatic approach g_s^{eff} and g_R^{eff} will depend on the spin and are not constants of a rotational band. The experimentalists generally operate with these factors, which are extracted in the adiabatic approximation from the values of the magnetic moments and the B(M1) values in the rotational band. Let us point out some consequences of Eqs. (49) and (50). It is immediately evident that g_R^{eff} will differ in nuclei with odd N and Z (since g_l = 0 and 1, respectively, for n and p). For the states of a rotational band of given parity with minimal energy (yrast levels) we obtain the inequalities

$$g_R^{\text{eff}}(N) \leqslant g_R^0 \leqslant g_R^{\text{eff}}(Z), \quad (51)$$

which agree well with the theoretical estimates in the cranking model[43,50,51] and empirical data.[24,25] For the same levels, one can obtain the asymptotic estimate

$$(g_R^{\text{eff}} - g_l)_{I\gg1} \approx (g_R^0 - g_l)[1-|a(I)|/2I], \quad (52)$$

i.e., with increasing spin, g_R^{eff} tends asymptotically to g_R^0, since $|a(I)|$ changes slowly in this process.[48,53]

The definition of g_s^{eff} by Eq. (50) differs from the renormalization used in the adiabatic approach[27,28] in that it includes the Coriolis interaction effects. The latter do not enable one to distinguish the longitudinal and transverse spin polarization effects, to which there correspond different factors R_σ and $R_\sigma^{(j)}$ [Eqs. (36a) and (37)]. Therefore, in the nonadiabatic approach g_s^{eff} may vary strongly in a rotational band, which we shall show below.

To conclude this section, we note that in the nonadiabatic model the magnetic moment is intimately related to the spectrum since the spectral function g(I) also occurs in the expression for μ_I. It is this circumstance that enables us in calculations of the magnetic moments to restrict ourselves to the single free parameter g_R^0 (it is expected that this parameter has a value near its values in the neighboring even–even nuclei). In the adiabatic approach, one must use two parameters (in the case K = 1/2, three parameters) and the relationship to the spectrum is essentially lost.

8. ELECTROMAGNETIC TRANSITIONS

Let us consider M1 and E2 transitions between the levels of one rotational band. The operator of an M1 transition has the following form in the laboratory frame:

$$\mathfrak{M}(M1,\nu) = \sqrt{3/4\pi}\,D_{\nu}\mu_N \quad (\mu_N \equiv e\hbar/2mc); \quad (53)$$

$$D_0 = (g_s - g_l)\,\sigma_z/2 + (g_l - g_R^0)\,j_z; \tag{53a}$$

$$D_{\pm 1} = \mp (1/\sqrt{2})\,[(g_s - g_l)\,\sigma_\pm/2 + (g_l - g_R^0)\,j_\pm]. \tag{53b}$$

For the reduced transition probability one readily obtains in the boson approximation the expression[37,38]

$$\begin{aligned} B(M1, I' \to I) = (3/4\pi)\,\mu_N^2 \Big| \sum_{K,K'} & C_K^I C_{K'}^{I'} M_{KK'} \\ \times \{ & \delta_{K,K'} \langle I'1K0 \,|\, IK\rangle [(g_s - g_l)(\sigma_3)_{KK} R_\sigma(K,K)/2 + K(g_l - g_R^0)] \\ & + (1/2\sqrt{2}) \sum_{p=\pm 1} [(g_s - g_l)\,\sigma_{K,K'}^{(p)} R_\sigma^{(j)}(K,K')/2 \\ & + (g_l - g_R^0)\, j_{K,K'}^{(p)} R_j^{(\sigma)}(K,K')]\,[p\delta_{K',K+1}\langle I'1K+1-1 \,|\, IK\rangle \\ & - \delta_{K',K-1}\langle I'1K-1\,1 \,|\, IK\rangle] \\ & + \delta_{K,1/2}\delta_{K',1/2}(-1)^{I+l+1/2}\langle I'1\,1/2-1 \,|\, I-1/2\rangle \\ & \times (1/2\sqrt{2}) \sum_{p=\pm 1} p\,[(g_s - g_l)\,\sigma_{K,K'}^{(p)} R_\sigma^{(j)}(K,K')/2 \\ & + (g_l - g_R^0)\, j_{K,K'}^{(p)} R_j^{(\sigma)}(K,K')]\}\Big|^2. \end{aligned} \tag{54}$$

In the adiabatic approximation, this quantity can be expressed in terms of the same effective parameters as μ_I^{ad}, namely[6,15,54]

$$\begin{aligned} B(M1, I'K \to IK) = (3/4\pi)\,\mu_N^2 & |\langle I'1K0 \,|\, IK\rangle \\ \times K(g_K - g_R^{eff})\,[1 + \delta_{K,1/2} b_0 (-1)^{I>+1/2}]\,|^2, \end{aligned} \tag{55}$$

where

$$(g_K - g_R^{eff})\, b_0 = -(g_l - g_R^{eff})\, a_{sp} + \delta_{K,1/2}(g_s^{eff} - g_l)(\sigma_+)_{K,-K}/2. \tag{55a}$$

The definition of B(M1) here includes the three parameters g_R^{eff}, $g_K(g_s^{eff})$, and b_0. Obviously, the nonadiabatic equation (54) cannot be reduced by the renormalizations (49) and (50) to Eq. (55). In other words, the effective parameters $g_s^{eff}(g_K)$ and g_R^{eff} extracted from the experimental B(M1) values (or their ratios) will differ in the general case from the corresponding values obtained from the magnetic moments. Thus, the wide use of the parameter $g_s^{eff}/g_S \approx 0.6$ in calculations of μ_I and B(M1), and also the neglect of the renormalization of the matrix elements $j_{KK'}^{(p)}$ are completely unjustified and actually lead in some occasions to contradictory results (see the discussion in Sec. 10).

We also give simple nonadiabatic expressions for the spectroscopic quadrupole moment Q_I and the probabilities of E2 transitions in the band:

$$Q_I = Q_0 [3 \sum_K (C_K^I)^2 K^2 - I(I+1)]/(I+1)(2I+3); \tag{56}$$

$$B(E2, I' \to I) = (5/16\pi)\, e^2 Q_0^2 \Big| \sum_K C_K^{I'} C_K^I \langle I'2K0 \,|\, IK\rangle \Big|^2. \tag{57}$$

Here, Q_0 is the internal quadrupole moment of the core, regarded as a parameter. Its value must be close to its value in the neighboring even–even nuclei (a certain difference may arise from the contribution of the odd nucleon and its interaction with the quadrupole vibrations of the core).

Usually, nonadiabatic effects are clearly manifested in the value of the mixing amplitude of M1 and E2 transitions:

$$\delta_I^2 \equiv T_\gamma(E2, I \to I-1)/T_\gamma(M1, I \to I-1), \tag{58}$$

where T_γ is the probability of a γ transition.

In the adiabatic approach, this quantity can be expressed[15,24,54] in terms of the parameters g_K, g_R^{eff}, b_0, and Q_0:

$$|\delta_I| \sim \left| \frac{g_K - g_R^{eff}}{Q_0} (1 + \delta_{K,1/2} b_0 (-1)^{I>+1/2}) \right|. \tag{58a}$$

This enables one to extract these parameters from correlation experiments. In the nonadiabatic approach, one can also separate an effective quantity of the type[49]

$$\left| \frac{g_K - g_R}{Q_0} \right|_{eff}^2 \equiv (1/\delta_I^2)\,(E_\gamma^+[\text{MeV}]/0.287\,(2I+2)\,(2I-1)), \tag{58b}$$

which, however, is not a constant in a rotational band because of the Coriolis interaction.

Besides δ_I^2, it is convenient to use the intensity ratio[15]

$$\lambda_I \equiv \frac{T_\gamma(E2, I \to I-2)}{T_\gamma(E2, I \to I-1)} \cdot \frac{\delta_I^2}{1+\delta_I^2}, \tag{59}$$

which can be measured much more readily than δ_I^2.

The E2 transitions between levels of different bands may be strongly perturbed by the interaction of quasiparticles with quadrupole vibrations of the core, which are not taken into account in the approach described here. Therefore, we shall not consider them in what follows.

It is of interest to investigate the effect of the Coriolis forces on the intensities of the E1 transitions between bands. It is well known that in many cases Alaga's rules[55] for these transitions are strongly broken. In the adiabatic approach it follows from Alaga's rules that (except for the case K = K' = 1/2)

$$\frac{B(E1, IK \to I'K')}{B(E1, IK \to I''K')} = \left| \frac{\langle I1KK'-K \,|\, I'K'\rangle}{\langle I1KK'-K \,|\, I''K'\rangle} \right|^2. \tag{60}$$

Many attempts have been made in the literature to correct these relations by taking into account effects due to coupling between the rotational and the internal motions. In particular, a very felicitous parametrization of these effects was proposed in refs. 56 and 57. A theoretical calculation of the parameters entails allowing for the dynamics of the core rotation, the collective excitations in the core (for example, octupole), and the Coriolis interaction. Basically, the coupling of the quasiparticles to the octupole vibrations affects the absolute magnitude of the matrix element of an E1 transition, and the Coriolis interaction is very important for the ratio of the transition intensities. Detailed calculations of these effects have been made in ref. 58. In all investigations, the Coriolis interaction has generally been regarded as a perturba-

tion and taken into account in the lowest orders, the number of model parameters increasing with increasing order (for example, when H_C is allowed for in the second order the minimum number of parameters is three.[59])

In the nonadiabatic approach, the reduced probability of an E1 transition is[60]

$$B(E1, I \to I') \sim \Big| \sum_{K, K'} C_K^I C_{K'}^{I'} [\langle I1K, K'-K \,|\, I'K'\rangle \times \langle \Phi_{K'} \,|\, \mathfrak{M}(E1, K'-K) \,|\, \Phi_K\rangle + \delta_{K, 1/2}\delta_{K', 1/2} \times (-1)^{I'+1'+K'} \langle \Phi_{\bar{K}'} \,|\, \mathfrak{M}(E1, -K'-K) \,|\, \Phi_K\rangle \times \langle I1K - K' - K \,|\, I' - K'\rangle]\Big|^2, \quad (61)$$

where the matrix element is given by

$$\langle \Phi_{K'} \,|\, \mathfrak{M}(E1, K'-K) \,|\, \Phi_K\rangle \approx N_N N_{K'} (u_K u_{K'} - v_K v_{K'}) (G_{E1})_{KK'}. \quad (61a)$$

An expression for $(G_{E1})_{KK'}$ can be found, for example, in ref. 16.

As can be seen from Eqs. (61), the B(E1) value here does not contain any parameters, since the amplitudes C_K^I are known from calculations of the spectrum. Of course, if octapole vibrations are ignored, one can hardly hope to obtain the correct absolute value of B(E1), though departures from Alaga's rules for transitions between low-lying levels are reproduced entirely satisfactorily (see Sec. 12).

To conclude this section, we mention that Alaga's rules for E2 transitions within a rotational band are broken only slightly by nonadiabatic effects. Appreciable deviations from Alaga's rules are only expected in bands with $K \sim 1/2, 3/2$ in the case of a strong Coriolis interaction.[61]

9. DYNAMICAL EFFECTS

As the angular momentum of a rotational state changes, so do the characteristics of the self-consistent field and, therefore, the collective parameters J, g_R^0, and Q_0. We have already taken into account the rearrangement of the internal spectrum, but only in the static limit with respect to pairing. However, it is known that antipairing effects, which weaken the pairing field, arise as a result of rotation.[62,63] In recent years, intensive investigations have been made of dynamical effects in the framework of the cranking model (see, for example, the discussions in refs. 8 and 64 and also in refs. 65 and 66). At large angular momenta, one expects a phase transition of the nucleus into the normal state, which has a strong effect on the structure of the rotational spectrum. The moment of inertia of the nucleus then takes the rigid-body value. A very simple estimate of the critical moment I_c can be obtained from an equation that follows from the idea that the nucleus can exist in a normal and a superfluid phase:[63,67]

$$(1/2J - 1/2J_{\text{rig}})\, I_c (I_c + 1) = \Delta^2/d, \quad (62)$$

where $d \approx 63/A$ (MeV) is the mean distance between the one-particle levels. Substituting medium values, $A \sim 160$, into this equation, we obtain the estimate $I_c \sim 16$. The accurate theoretical treatment of the phase transition,[68] which differs qualitatively from our above treatment, does not essentially alter this estimate. However, the reliability of the estimate is still low ($\sim 20\%$).

Analysis of rotational bands to $I \sim 20$ made for some rare-earth nuclei did indeed reveal an anomalous behavior of the moment of inertia at angular momenta $I \geqslant 12$ (see ref. 69). However, it is as yet impossible to identify this anomaly uniquely with a phase transition. The possibility cannot be excluded that the discontinuities in the moment of inertia are also due to reorientation of the one-particle orbits at large angular momenta (see the discussions in refs. 64, 70, and 71).

Here, we are essentially interested in the dynamical corrections to the rotational spectrum at angular momenta below the critical value. They can be taken into account with good accuracy by means of the Chan–Valatin[72] equation, which in the case of an odd-mass nucleus can be written[48]

$$2/G = \sum_{\nu} \frac{1}{E_\nu(I)} - \sum_{K\rho} (C_{K\rho}^I)^2 \frac{1}{E_{K\rho}(I)} - \frac{\partial}{\partial \Delta^2} [g(I)/J(I)]. \quad (63)$$

In the derivation of this equation, we have ignored the shift of the quasiparticle spectrum due to the residual interactions H_σ and H_j, although polarization effects are contained in g(I). The second term on the right-hand side of (63) is due to the blocking effect. The main assumption that is made in the solution of Eq. (63) is the definition of the form of the dependence $J(\Delta)$, since in the foregoing treatment this quantity appeared as a parameter. In order to solve such a problem exactly, self-consistency is required. Here, we assume that $J(\Delta)$ has the usual form in the cranking model with pairing. Equation (63) is solved by perturbation theory in ref. 48 and by iteration in ref. 73. Finally, the case when the Coriolis interaction is taken into account by perturbation theory has been considered in ref. 74. The qualitative results of these investigations can be formulated as follows:

a) Dynamical effects in odd-mass nuclei are weakened by the Coriolis interaction between outer nucleons and the core. In particular, the parameter B in the correction term in the energy that is proportional to $I^2(I+1)^2$ can have the opposite sign to that in even–even nuclei;

b) in the case of a strong Coriolis interaction, the energy gap may even increase for a while with increasing spin (the initial value is taken to be the width of the gap with allowance for blocking), i.e., dynamical effects in the core are weaker than the effects due to the coupling of the outer nucleons and the rotation. It is this competition that stabilizes the gap right up to comparatively high spins;

c) as a whole, dynamical corrections in the spectra of odd-mass nuclei are appreciably smaller than in even–even nuclei.

In Sec. 12 we give an example of the solution of Eq. (63), and we discuss dynamical effects in more detail.

10. PHENOMENOLOGICAL APPROXIMATIONS

Nonadiabatic effects in the rotational motion have been allowed for by many authors, beginning with Kerman,[21]

who diagonalized the Coriolis interaction exactly in the bounded space of one-particle states. In later papers, this scheme was transferred to the space of quasiparticle states, i.e., pairing correlations were taken into account. Depending on the extent of known experimental data, the parameters used in the diagonalization are taken to be the energies of the internal states, the matrix elements $j^{(p)}_{K,K'}$, the decoupling parameter a_{sp}, etc. As an example of detailed calculations, we may mention refs. 75-78. In these papers, the experimental data on the energies and the electromagnetic moments are used to find the experimental values of the matrix elements and the effective parameters, i.e., the polarization and dynamical effects and occasionally also the Coriolis interaction effects, are essentially parametrized. An extensive set of such parameters is collected in ref. 24.

Particularly intensive calculations of this kind have been made in recent years in connection with an investigation of "long" rotational bands in reactions with α particles and heavy ions (see, for example, refs. 79-86). In all these investigations it has been noted that a satisfactory description of the rotational energies can be obtained in the Kerman scheme only if the Coriolis interaction is renormalized. In particular, for rotational bands associated with one-particle levels it is found on the basis of spherical subshells with large j ($h_{11/2}$, $i_{13/2}$, etc.) that the experimental matrix elements are systematically smaller than the one-particle values of $j^{(p)}_{K,K'}$ (i.e., in terms of polarization effects we have $R^{(\sigma)}_{j} < 1$). Therefore, it is customary to introduce appropriate cutoff factors as fitting parameters. However, apart from rare exceptions, a unique choice of these parameters from the fitting procedure applied to the experimental data appears impossible. Usually, the one-particle decoupling parameters are not completely renormalized. Further, from calculations of the magnetic moments, it is known that the g_S factor must be renormalized, and one usually chooses $g_S^{eff} \approx 0.6 g_S$. This same procedure is automatically transferred to B(M1) calculations, in which the matrix elements $j^{(p)}_{KK'}$ are generally not renormalized. As a result, it has been found that in certain nuclei a renormalization of g_S is very necessary,[77,84,86] while in others it only worsens the agreement with experiment.[81,82,85] The reason for this discrepancy is that the parametrization of the polarization effects made for the magnetic moments does not apply in the general case in calculations of B(M1). In addition, in the general case, g_S^{eff} does not remain constant at all in a rotational band (see Sec. 12).

In some investigations, the dynamical corrections in the rotational spectrum have been parametrized by the introduction of a spin dependence of the rotational parameter:[81,84-86]

$$1/2J(I) = 1/2J_0 + B[I(I+1) - K^2]. \tag{64}$$

As yet, no theoretical justification for such a spin dependence has been found.

From the point of view of a description of the energetics of rotational bands, the phenomenological approaches have been successful. However, this has been achieved, as a rule, at the price of using a large number of parameters (sometimes as many as 10-12). Of course, this greatly reduces the amount of useful information that can be obtained and the heuristic value of the phenomenological analysis.

Attempts have been made to improve the Bohr–Mottelson parametrization of rotational energies in odd-mass nuclei [see Eq. (17)]. One such attempt uses the projection method.[87] The internal wave function $\Phi_{K\rho}$, which does not have definite angular momentum, is projected onto the space of states with definite quantum numbers I and M. Then the variational problem for the original microscopic Hamiltonian is solved with such a function. The overlap integrals are not calculated exactly but rather parametrized. This method enables one to give a better description of rotational bands weakly perturbed by the Coriolis interaction than can be obtained with Eq. (17) for the same number of parameters.

Another attempt is associated with a generalization of a model with variable moment of inertia to the case of odd-mass nuclei.[88] However, as in the foregoing case, it can only be applied for a weak Coriolis interaction.

11. LIMITING CASE IN THE NONADIABATIC APPROACH

Let us consider the limiting case, when the rotational spectrum in the case of strong mixing of internal states with respect to K can be described by the simple rotator equation. It was shown in refs. 48, 53, and 89 that there is then a splitting of the rotational band into two groups of levels, which are displaced with respect to one another, with even and odd values of $1+1/2$. Obviously, the lowest group is the one in which [see Eq. (12)]

$$(-1)^{I+1/2} a(I) < 0. \tag{65}$$

This group of levels with $\Delta I = 2$ forms a yrast line of the spectrum of given parity. Analysis of $|a(I)|$ shows that at sufficiently high spins it becomes almost constant in such a band, i.e.,

$$(-1)^{I+1/2} a(I) \approx -a = \text{const}. \tag{66}$$

On the other hand, the amplitudes C^I_K also change smoothly and therefore

$$\sum_K (C^I_K)^2 \mathscr{E}_K \approx \text{const} \qquad (\Delta I = 2). \tag{67}$$

As a result, the energy spectrum takes the very simple form

$$\begin{aligned} \mathscr{E}(I) &\approx \text{const} + (1/2J)\{I(I+1) - a(I+1/2)\} \\ &= \text{const} + (1/2J) R(R+1), \end{aligned} \tag{68}$$

where

$$R \equiv I - a/2. \tag{68a}$$

This corresponds to the limiting case of decoupling, when the energy spectrum is determined by the rotation of the

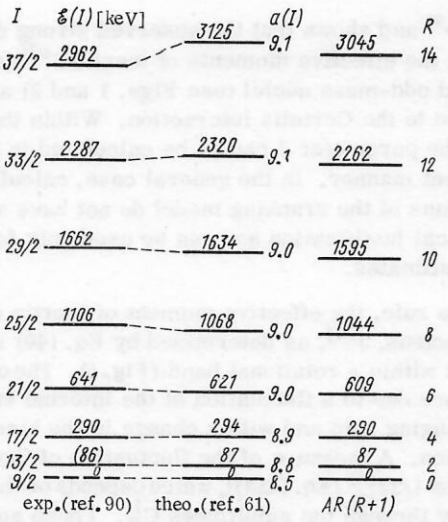

Fig. 3. Yrast band of positive parity in the ^{159}Er nucleus.

even–even core, and the odd "particle" is in a state with effective angular momentum

$$j_{\text{eff}} = a/2, \tag{69}$$

and the internal angular momentum is approximately conserved. It is a spectrum such as this that was discussed in refs. 11 and 12.

The realization of a spectrum of the type (68) in the ^{159}Er nucleus is shown in Fig. 3, in which we reproduce exact calculations from ref. 61 and their approximation at the values

$$A = 14.5 \text{ keV}, \quad a = 9. \tag{70}$$

In ref. 61 the calculations were made of 1/2J = 15 keV and the calculated values of a(I) are shown in Fig. 3. Naturally, the approximation (68) is suitable for describing only the separated part of the spectrum of excitations of given parity.

Thus, the decoupling process (reorientation of the one-particle orbits) is a special case of the general nonadiabatic model. Note that the band considered does not correspond to any definite internal state Φ_K. In reality, we are here concerned with a complicated internal state, which includes a mixture of states of the subshell $i_{13/2}$ with[61] K = 1/2, 3/2, 5/2, ... The effective energy of the corresponding quasiparticle can be defined as

$$E_{\text{eff}} \approx \sum_K (C_K^I)^2 \mathscr{E}_K - \frac{1}{2J} \cdot \frac{a^2}{4}. \tag{71}$$

The rotator spectrum shown in Fig. 3 differs strongly from the spectrum of the neighboring nuclei ^{158}Er and ^{160}Er (see, for example, ref. 69). Note finally that the case we have considered is not an exception. Similar yrast bands can be separated out approximately in the neighboring nuclei ^{157}Dy, ^{157}Er, and others.

12. CALCULATIONS IN THE NONADIABATIC MODEL AND COMPARISON WITH EXPERIMENTS

The nonadiabatic rotational model has been successfully used to describe rotational spectra and electromagnetic moments in a large number of nuclei in the rare-earth region. These results are described in detail in refs. 37, 38, 49, 61, 91-93. In this section, we give only some general conclusions and also mention some essentially new results in order to illustrate the theory set forth in the foregoing sections.

Choice of parameters and general remarks. In all calculations the scheme of one-particle levels in the Woods–Saxon potential obtained in ref. 18 was used. The actual parameters of the potential are given in ref. 25.

The problem of pairing correlations is solved without allowance for the blocking effect (to simplify the calculations), which gives the spectrum of quasiparticle excitations with energy E_K. The spectral shifts $E_K - \mathscr{E}_K$ [Eq. (28)] due to the residual centrifugal and spin forces are small ($\lesssim 100$ keV) and almost homogeneous for all levels of one parity with energy $\lesssim 2$ MeV, i.e., they have little effect on the results of diagonalizing the Coriolis interaction matrix. The total amount of the three-

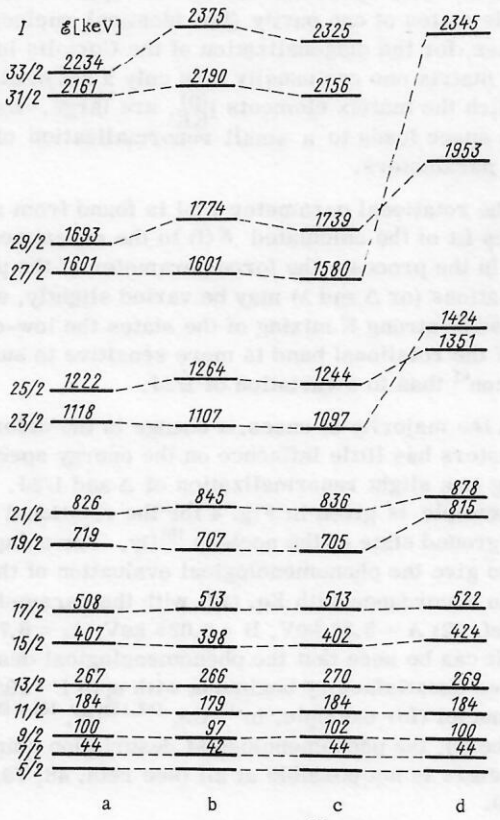

Fig. 4. Ground-state rotational band in the ^{161}Dy nucleus: a) experimental data,[84] b) nonadiabatic theory at the parameter values $\beta_{20} = 0.32$, $\beta_{40} = 0$, $\Delta_n = 0.58$ MeV, and 1/2J = 1.21 keV; c) nonadiabatic theory at the parameter values $\beta_{20} = 0.324$, $\beta_{40} = 0.04$, $\Delta_n = 0.67$ MeV, and 1/2J = 12 keV; d) calculation in accordance with the phenomenological equation (17). The parameters are given in the text.

quasiparticle admixtures in the wave functions Φ_K [Eq. (26)] does not usually exceed 1-3%, although because of the coherent character of their contribution the polarization factors are appreciably different from unity. The polarization factors were calculated in accordance with Eqs. (31a), (36a), and (37), in which sums of the type $\mathscr{E}^{(\sigma)}_{KK'}$, $S^{(j)}_{KK'}$, and $X^{(p)}_{KK'}$ were calculated in the quasiclassical approximation (without allowance for blocking).

The values of the factor $R_j^{(\sigma)}$ depend on the configuration space of the outer nucleons. In particular, for rotational bands associated with levels of positive parity (of the subshell $i_{13/2}$) in nuclei with odd N, typical values are $R_j^{(\sigma)} \sim 0.6$-0.8 (see ref. 61). In the nuclei in which the Coriolis interaction is weak, effects of centrifugal polarization are manifested weakly. Centrifugal polarization effects are also small in the case of strong mixing with respect to K of just two levels, expecially in nuclei with odd Z. For example, for the band $1/2^-[541]$ in the ^{171,173}Lu nuclei we have $R_j^{(\sigma)} \approx 0.96$ (see ref. 91). In other words, centrifugal polarization is essentially dynamic, and depends strongly on the state. In contrast to the centrifugal polarization, the spin polarization (the factors $R^{(j)}_{(\sigma)}$ and R_σ) depends weakly on the quantum numbers of the states. Typical values are $R_\sigma \sim 0.6$-0.7 (longitudinal spin polarization[28]). Effects of transverse spin polarization are usually more strongly manifested [$R_\sigma^{(j)} \sim 0.4$-0.6].

As an outer space one can use the space of all one-particle states of one parity (for identical nucleons). However, for the diagonalization of the Coriolis interaction matrix one can usually take only a set of states for which the matrix elements $j^{(p)}_{KK'}$ are large. Extension of this space leads to a small renormalization of the model parameters.

The rotational parameter 1/2J is found from a least-squares fit of the calculated $\mathscr{E}(I)$ to the experimental data. In the process, the force parameter of the pairing correlations (or Δ and λ) may be varied slightly, since in the case of strong K mixing of the states the low-energy part of the rotational band is more sensitive to such a variation[61] than to a variation of 1/2J.

In the majority of cases, a change in the deformation parameters has little influence on the energy spectrum, leading to a slight renormalization of Δ and 1/2J. A typical example is given in Fig. 4 for the rotational band[9] of the ground state of the nucleus ^{161}Dy. For comparison, we also give the phenomenological evaluation of the spectrum in accordance with Eq. (17) with the parameters (see ref. 22) A = 5.53 keV, B = 0.025 keV, $A_5 = 6.77 \cdot 10^{-4}$ keV. It can be seen that the phenomenological description becomes unsatisfactory beginning with spin I = 19/2. In some nuclei (for example, in ^{155}Gd, ^{157,159}Dy, $^{161-165}$Er, and others), the phenomenological description using a few parameters is not possible at all (see refs. 48, 79, 80, and 86).

Effective moments of inertia. We have already pointed out that the parameter J is the effective moment of inertia of the core, and we can therefore expect that it has a value near the ones in the neighboring even–even nuclei. Systematic calculations in the region of the rare-earth nuclei have confirmed this assumption[49,61,93] and shown that the observed strong difference between the effective moments of inertia J^{eff} in even–even and odd-mass nuclei (see Figs. 1 and 2) are basically due to the Coriolis interaction. Within the model itself, the parameter J cannot be calculated in a self-consistent manner. In the general case, calculations of J by means of the cranking model do not have a sound theoretical justification and can be used only for qualitative estimates.

As a rule, the effective moment of inertia of an odd-mass nucleus, J^{eff}, as determined by Eq. (40) is not a constant within a rotational band (Fig. 5). The oscillations of J^{eff} are due to a fluctuation of the internal energy (67) with changing spin and with a change in the kinetic energy of rotation. A measure of the fluctuation of the internal energy is $1/2J^{eff}_{in}$ [Eq. (41a)], which depends on the parameter 1/2J only through the amplitudes C^I_K. These amplitudes are essentially determined by the form of the internal spectrum and the values of the matrix elements $j^{(p)}_{KK'}$. The change in the kinetic energy can be characterized by $1/2J^{eff}_{rot}$ [Eq. (41b)], which is directly proportional to 1/2J. Therefore, in the cases when the contribution of $1/2J^{eff}_{in}$ to $1/2J^{eff}$ is dominant, the entire spectrum depends weakly on the choice of the parameter 1/2J, but it is more sensitive to a variation of the energy gap Δ. A typical example is shown in Fig. 5. It can be seen that here the contribution of $1/2J^{eff}_{in}$ is dominant for $1+1/2 = 2n$. If the fluctuation of the internal energy is ignored, the doublet states in the spectrum (for example, the states 15/2 and 17/2, 19/2 and 21/2, etc., see Fig. 4) are almost degenerate. Such degeneracy of doublet states is observed in rotational bands associated with the state $1/2^+[411]$ in nuclei with odd Z and with the state $1/2^-$ [521] in nuclei with odd N (see ref. 24). In these bands, the mixing with respect to the quantum number K is weak, and therefore the fluctuations of the internal energy are small. In the case of a strong Coriolis interaction, they wash out the doublet structure of the spectrum in the lower part of the rotational band.

Systematization of magnetic moments. In calculations of the magnetic dipole moments, the nonadiabatic model acquires an additional parameter – the

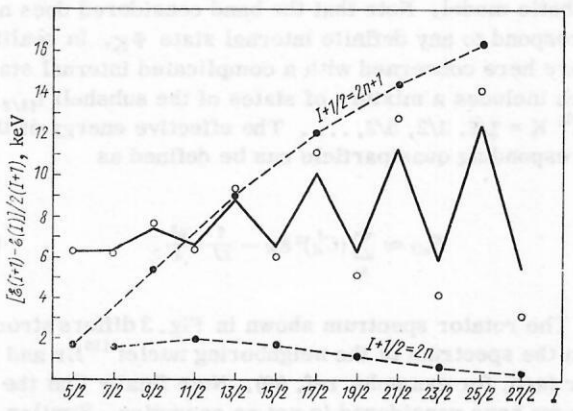

Fig. 5. Reciprocal of the effective moment of inertia in the ground-state rotational band of the ^{161}Dy nucleus: ○, experimental values (ref. 84); ——, calculation in accordance with nonadiabatic theory (corresponding to Fig. 4c); ●– – –● shows the behavior of $1/2J^{eff}_{rot}$ [Eq. (41b)].

TABLE 1. Magnetic Dipole Moments in Odd-Mass Deformed Nuclei (in units of μ_N)

Kernel	$I^\pi[Nn_z\Lambda]$	$\mathscr{E}(I)$, keV	g_R^0	g_R^{eff}	g_s^{eff}/g_s	μ_I theo.	μ_I exp.	Ref.
^{155}Gd	3/2⁻ [521]	0	0.33	0.29	0.74	−0.25	−0.26 (1)	[95]
	5/2⁻	60.0		0.30	0.59	0.27		
	3/2⁺	105.3		0.13	0.80	0.02	−0.38 (6)	[95]
							+0.13 (4)	
	5/2⁺	86.5		−0.08	1.51	−0.73	−0.53 (5)	[95]
^{157}Gd	3/2⁻ [521]	0	0.31	0.28	0.77	−0.33	−0.34 (1)	[95]
	5/2⁻	54.5		0.30	0.75	0.23		
^{159}Gd	3/2⁻ [521]	0	0.30	0.27	0.78	−0.34	−0.22 (5)	[96]
							−0.44 (3)	[97]
	5/2⁻	50.7		0.19	0.21	0.30		
^{155}Dy	3/2⁻ [521]	0	0.35	0.25	0.77	−0.34	−0.34 (3)	[98]
	5/2⁻	39.4		0.28	0.47	0.27		
^{157}Dy	3/2⁻ [521]	0	0.35	0.31	0.77	−0.31	−0.30 (3)	[98]
	5/2⁻	61.1		0.34	0.77	0.23		
^{161}Dy	5/2⁺ [642]	0	0.30	0.12	0.78	−0.48	−0.48 (4)	[98]
	7/2⁺	43.8		0.16	0.82	−0.13		
	5/2⁻ [523]	25.7		0.28	0.76	0.70	0.58 (2)	[95]
	7/2⁻	103.1		0.29	0.77	0.97		
	3/2⁻ [521]	74.6		0.27	0.77	−0.35	−0.39 (1)	[95]
	5/2⁻	131.8		0.24	0.26	0.29		
^{163}Dy	5/2⁻ [523]	0	0.30	0.29	0.64	0.67	0.66 (2)	[95]
	7/2⁻	73.4		0.28	0.60	0.93		
^{165}Dy	7/2⁺ [633]	0	0.35	0.23	0.68	−0.53	−0.52 (4)	[98]
	9/2⁺	84.5		0.25	0.69	−0.03		
^{161}Er	3/2⁻ [521]	0	0.31	0.27	0.82	−0.37	−0.37 (04)	[98]
	5/2⁻	59.5		0.29	0.79	0.21		
^{163}Er	5/2⁻ [523]	0	0.30	0.28	0.76	0.72	0.56 (2)	[98]
	7/2⁻	84.3		0.29	0.78	1.02		
^{165}Er	5/2⁻ [523]	0	0.30	0.26	0.77	0.69	0.65 (3)	[95]
	7/2⁻	77.6		0.27	0.75	0.96		
^{167}Er	7/2⁺ [633]	0	0.33	0.20	0.73	−0.59	−0.57 (02)	[95]
	9/2⁺	79.3		0.22	0.75	−0.14		
^{173}Yb	5/2⁻ [512]	0	0.27	0.25	0.75	−0.60	±0.68	[95]
	7/2⁻	78.7		0.20	0.31	0.18		
^{177}Hf	7/2⁻ [514]	0	0.30	0.26	0.71	0.92	0.78 (2)	[99]
	9/2⁻	113.0		0.27	0.75	1.24	1.10 (15)	[95]
	11/2⁻	249.7		0.27	0.79	1.54	1.43 (50)	[95]
	9/2⁺ [624]	321.3		0.17	0.66	−0.80	−0.73 (9)	[95]
	11/2⁺	426.6		0.19	0.71	−0.41		
^{179}Hf	9/2⁺ [624]	0	0.26	0.19	0.57	−0.66	−0.61 (2)	[100]
	11/2⁺	122.7		0.20	0.58	−0.22		
^{157}Tb	3/2⁺ [411]	0	0.35	0.40	0.73	1.98	2.0 (1)	[95]
	5/2⁺	60.8		0.39	0.75	1.99		
^{159}Tb	3/2⁺ [411]	0	0.35	0.40	0.72	1.96	1.99 (04)	[95]
	5/2⁺	58.0		0.39	0.74	1.98		
^{165}Ho	7/2⁻ [523]	0	0.35	0.61	0.67	4.07	4.11 (2)	[95]
	9/2⁻	94.7		0.57	0.66	4.11		

TABLE 1 (continued)

Kernel	$I^{\pi}[Nn_z\Lambda]$	$\mathcal{E}(I)$, keV	g_R^0	g_R^{eff}	g_s^{eff}/g_s	μ_I theo.	μ_I exp.	Ref.
^{169}Tm	1/2+ [411]	0	0.35	0.50	0.64	−0.22	−0.23	[24]
	3/2+	8.4		0.36	0.78	0.67	0.53 (1)	[24]
	5/2+	118.2		0.36	0.62	0.50	0.70 (5)	[24]
	7/2+	138.9		0.36	0.79	1.51	1.30 (5)	[24]
^{171}Tm	1/2+ [411]	0	0.35	0.50	0.63	−0.21	±0.23 (05)	[24]
	3/2+	5.1		0.36	0.77	0.67		
	5/2+	116.7		0.36	0.61	0.51	0.81 (37)	[24]
	7/2+	129.1		0.36	0.75	1.51	1.44 (14)	[95]
^{175}Lu	7/2+ [404]	0	0.35	0.46	0.71	2.00	2.23 (1)	[95]
	9/2+	113.8		0.42	0.69	2.31	1.81 (20)	[95]
	11/2+	251.4		0.41	0.68	2.65	2.0 (7)	[95]
^{177}Lu	7/2+ [404]	0	0.35	0.46	0.69	2.03	2.24 (1)	[95]
	9/2+	121.6		0.42	0.68	2.34		
^{181}Ta	7/2+ [404]	0	0.35	0.52	0.73	2.28	2.35 (1)	[95]
	9/2+	136.3		0.45	0.73	2.67		
	9/2− [514]	6.3		0.50	0.74	5.29	5.29 (11)	[95]
	11/2−	158.8		0.46	0.75	5.25		
	5/2+ [402]	482.1		0.37	0.74	3.14	3.28 (13)	[95]
	7/2+			0.36	0.74	3.01		
^{183}Re	9/2− [514]	496.0	0.35	0.51	0.75	5.31	5.31 (32)	[95]
	11/2−	663.6		0.47	0.76	5.28		
^{185}Re	5/2+ [402]	0	0.35	0.37	0.76	3.19	3.17	[95]
	7/2+	125.3		0.37	0.76	3.04		
^{187}Re	5/2+ [402]	0	0.35	0.37	0.75	3.17	3.20	[95]
	7/2+	134.2		0.37	0.75	3.03		

collective gyromagnetic ratio for the core: g_R^0. In refs. 37, 38, and 49 it was shown that the value of this parameter can be taken near the value in the neighboring even–even nuclei, i.e., of order 0.3–0.4. The force parameter of the spin interactions is chosen in a unified way for all nuclei in the form[32,37]

$$\varkappa = 1.5\,(N-Z)/A \text{ MeV}. \qquad (72)$$

The explicit isotopic dependence of $\varkappa$ indicates that the spin polarization effects are essentially due to isovector forces (for more details see ref. 94).

With this choice of the parameters, systematic calculations were made of the magnetic moments in odd-mass deformed nuclei, and the results of these calculations are shown in Table 1 together with the experimental data. Table 1 also includes the calculated effective factors g_R^{eff} and g_s^{eff} [see Eqs. (49) and (50)]. For each nucleus, the spectrum of rotational excitations was calculated, and then the values of the function G(I) [see Eq. (47a)]. Thus, the calculated values of μ_I were made consistent with the observed rotational spectrum, and this is something which is virtually impossible in the adiabatic approximation.

It can be seen from Table 1 that the nonadiabatic calculations agree well with the experimental data and that the spread of g_R^0 values is small. The adiabatic parameters[10] g_R^{eff} and g_s^{eff} (or g_K) can be calculated theoretically (see Table 1). Note, however, the relative importance of these quantities, since in many cases they are not constants in the rotational band.[37,38] To illustrate this assertion, we have plotted in Fig. 6 the results of a calculation of the parameters in the rotational bands of a number of nuclei (see also Tables 2–5). In all these cases, inequalities (51) are well satisfied. The difference between g_R^0 and g_R^{eff} is due primarily to the Coriolis interaction. It is this interaction that leads to the strong oscillation of g_R^{eff} in some rotational bands. The nonconstancy of g_R^{eff} in the band is the cause of the strong difference between the calculated values of μ_I and the adiabatic estimate μ_I^{ad}, as is shown in Table 2.

Transition probabilities. Allowance for nonadiabatic effects renormalizes the parameter of the internal quadrupole moment Q_0 extracted from the experimental data on the spectroscopic quadrupole moments Q_I or the values of B(E2) within the rotational band. In the general case, Q_0 is not a constant within a rotational band, although oscillations of its value become appre-

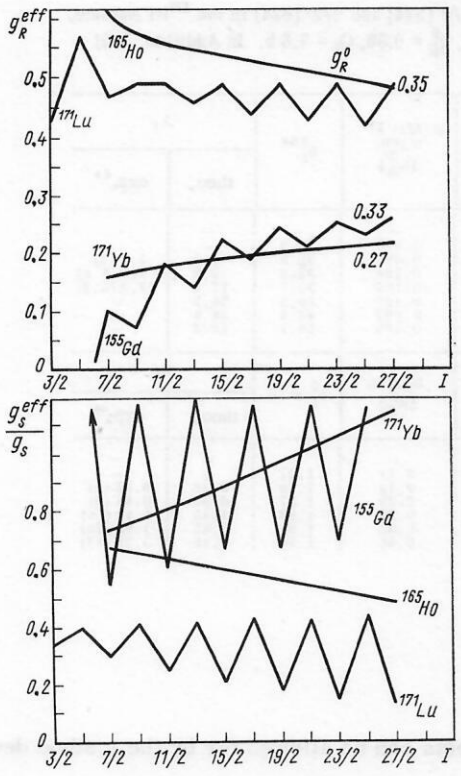

Fig. 6. Behavior of the effective parameters g_R^{eff} and g_s^{eff} in the rotational bands of the nuclei ^{171}Lu(1/2$^-$[541]), ^{165}Ho(7/2$^-$[523]), ^{171}Yb(7/2$^+$[633]), and ^{155}Gd (band of positive parity, see refs. 37 and 38).

ciable only in the case of strong K mixing of states[37,38] (for example, in ^{155}Gd they reach 20%). Therefore, Alaga's rules for E2 transitions within the band are usually very well satisfied.[61]

The quantity Q_0 is regarded as some effective parameter determined by matching the calculated values of B(E2), δ_I, and λ_I to the experimental data. It is expected that the values of Q_0 are close to the values in the neighboring even–even nuclei, though they need not necessarily be equal to them, as is confirmed by numerical calculations.[37,38]

In calculations of B(M1) values in rotational bands one requires no further parameters, so that a comparison of them with the experimental data serves to verify the validity of the nonadiabatic approach. In Tables 2-5 we have given the calculated values of B(M1, I → I − 1) for some rotational bands. Separating out the effective adiabatic parameter

$$(g_K - g_R)^2_{eff} \equiv \frac{4\pi}{3} \cdot \frac{I(2I+1)}{K^2(I-K)(I+K)} B(M1, I \to I-1) \qquad (73)$$

from these values, we can readily see that it is not constant within a rotational band (see Table 2). At high spins the fluctuations of this parameter reach 50%. The absolute B(M1) values that we calculated agree well with the few known experimental data (see the discussions in refs. 37 and 38 and also Tables 2 and 3).

Nonadiabatic effects are pronounced in the ratios of the transition probabilities λ_I and the mixing amplitudes δ_I. As an illustration, we have plotted in Fig. 7 the behavior of these quantities in the ground-state rotational band of the ^{161}Dy nucleus. It can be seen that the nonadiabatic calculations, using the single parameter Q_0 = 8.3 b, agree well with the experimental data. The adiabatic estimate was made for the parameter value $[(g_K - g_R)/Q_0]^2_{eff} = 2.75 \cdot 10^{-3}$ obtained from $\lambda_{9/2} = 0.29$ (ref. 84). In order to match the adiabatic calculations to the experimental δ_I data, it is necessary to assume strong variations of this parameter. For example, from $\delta_{17/2} = 0.13$ we obtain $[(g_K - g_R)/Q_0]^2_{eff} = 7.4 \cdot 10^{-3}$, i.e., the parameter is larger by a factor of three. The calculated values of λ_I and δ_I are compared with the experimental data in other nuclei in refs. 37, 38, and 91 (see also Table 3).

The Coriolis interaction strongly breaks Alaga's rules for El transitions between levels of rotational bands with $|\Delta K| = 1$. The nonadiabatic model enables one to

TABLE 2. Characteristics of the Ground-State Rotational Band in the ^{161}Dy nucleus. Calculations Made with the Parameter Values $\beta_{20} = 0.324$, $\beta_{40} = 0.04$, $1/2\mathcal{J} = 12$ keV, $\Delta_n = 0.67$ MeV, $g_R^0 = 0.30$

I^π	Mixing amplitudes C_K^I							$a(I)$	$\mathcal{E}(I)$, keV		μ_I, $[\mu_N]$	$\mu_I^{ad\,2*}$, $[\mu_N]$	g_R^{eff}	$B(M1)^{3*} \times 10$, $[\mu_N^2]$	$(g_K - g_R)^2_{eff}$
	[400]↑	[660]↑	[402]↓	[651]↑	[642]↑	[633]↑	[624]↑		theo.	exp.*					
5/2+	0.012	0.028	0.089	−0.156	0.983	—	—	1.10	0	0	−0.48	−0.46	0.12	—	—
7/2+	0.016	0.037	0.129	−0.222	0.958	0.118	—	−2.35	44	44	−0.13	−0.02	0.16	0.65	0.20
9/2+	0.038	0.089	0.165	−0.282	0.926	0.165	0.009	3.47	102	100	0.15	0.03	0.16	0.99	0.21
11/2+	0.030	0.068	0.183	−0.309	0.908	0.203	0.017	−3.89	184	184	0.47	0.21	0.18	1.11	0.20
13/2+	0.068	0.161	0.215	−0.359	0.863	0.222	0.023	4.91	270	267	0.70	0.37	0.18	1.34	0.23
15/2+	0.041	0.094	0.216	−0.360	0.865	0.253	0.031	−4.73	402	407	1.06	0.52	0.20	1.24	0.20
17/2+	0.098	0.231	0.249	−0.409	0.802	0.251	0.035	5.89	513	508	1.24	0.67	0.20	1.53	0.24
19/2+	0.049	0.113	0.239	−0.391	0.831	0.287	0.044	−5.23	705	719	1.65	0.80	0.22	1.23	0.19
21/2+	0.123	0.293	0.272	−0.441	0.748	0.264	0.044	6.56	836	826	1.80	0.93	0.21	1.66	0.25
23/2+	0.055	0.128	0.254	−0.410	0.804	0.312	0.057	−5.55	1097	1119	2.25	1.06	0.23	1.18	0.17
25/2+	0.144	0.344	0.287	−0.461	0.700	0.269	0.051	7.03	1244	1222	2.36	1.19	0.22	1.74	0.25
27/2+	0.060	0.139	0.264	−0.423	0.783	0.332	0.069	−5.77	1580	1602	2.85	1.31	0.24	1.11	0.16
29/2+	0.160	0.386	0.297	−0.474	0.661	0.269	0.057	7.36	1739	1693	2.94	1.43	0.23	1.79	0.26
31/2+	0.064	0.148	0.272	−0.432	0.766	0.348	0.081	−5.93	2156	2162	3.45	1.55	0.24	1.04	0.15
33/2+	0.173	0.421	0.304	−0.481	0.627	0.267	0.062	7.60	2325	2235	3.52	1.67	0.23	1.82	0.26

*Data of ref. 84.

2*Adiabatic estimate (Eq. 48) with parameters (ref. 101) $g_R^{eff} = 0.11$ and $g_K = -0.30$.

3*Corresponding experimental values (ref. 102) are equal to B(M1, 7/2 → 5/2) = 6.5 (9)·10^{-2} μ_N^2 and B(M1, 9/2 → 7/2) = 8.4 (2.5)·10^{-2} μ_N^2.

TABLE 3. Characteristics of Rotational Bands Associated with the States $7/2^-[514]$ and $9/2^+[624]$ in the ^{177}Hf Nucleus. Calculations Made with the Parameters $\beta_{20} = 0.24$, $\beta_{40} = 0$, $\Delta_n = 0.6$ MeV, $g_R^0 = 0.30$, $Q_0 = 7.8$ b. In Addition, $1/2\mathcal{J} = 13.4$ keV for the Band $7/2^-[514]$ and 14.9 keV for the Band $9/2^+[624]$

I^π	Mixing amplitudes C_K^I				$a(I)$	$\mathcal{E}(I)$, keV		μ_I, $[\mu_N]$	g_R^{eff}	$B(M1)$ 2* $\times 100$, $[\mu_N^2]$	δ_I^2 3*	λ_I	
	[523]↓	[512]↑	[514]↓	[505]↓		theo.	exp.*					theo.	exp.4*
7/2−	0.055	0.044	0.998	—	−0.23	0	0	0.92	0.26	—	—	—	—
9/2−	0.079	0.067	0.994	0.031	0.48	111	113.0	1.24	0.27	0.065	28.9	—	—
11/2−	0.102	0.087	0.990	0.046	−0.71	246	249.7	1.54	0.27	0.093	30.5	4.2	4.0 (4)
13/2−	0.123	0.106	0.985	0.059	0.91	406	409.4	1.85	0.27	0.104	32.8	9.9	7.0 (1.0)
15/2−	0.143	0.124	0.979	0.070	−1.10	590	591.3	2.15	0.28	0.106	35.7	17.2	17.3 (3.3)
17/2−	0.163	0.142	0.972	0.081	1.29	799	794.4	2.45	0.28	0.103	39.2	26.2	—
19/2−	0.181	0.159	0.965	0.091	−1.46	1033	1017.7	2.74	0.28	0.097	43.4	36.8	—
21/2−	0.199	0.176	0.957	0.100	1.63	1291	1260.3	3.04	0.28	0.090	48.1	49.2	—

I^π	Mixing amplitudes C_K^I				$a(I)$	$\mathcal{E}(I)$, keV		μ_I, $[\mu_N]$	g_R^{eff}	$B(M1)$, $[\mu_N^2]$	$\delta_I^2 \times 10$	λ_I	
	[642]↑	[633]↓	[624]↑	[615]↑		theo.	exp.*					theo.	exp.4*
9/2+	0.032	0.199	0.979	—	0.90	321	321.3	−0.80	0.17	—	—	—	—
11/2+	0.058	0.277	0.949	0.141	−2.02	424	426.6	−0.41	0.19	0.11	1.27	—	—
13/2+	0.083	0.332	0.920	0.191	2.84	551	555.1	−0.04	0.20	0.19	1.26	0.36	0.35 (4)
15/2+	0.107	0.374	0.893	0.224	−3.47	703	708.4	0.31	0.21	0.25	1.27	0.83	0.81 (8)
17/2+	0.131	0.409	0.868	0.246	3.97	881	882.8	0.65	0.22	0.29	1.22	1.45	1.42 (14)
19/2+	0.154	0.437	0.845	0.263	−4.36	1086	1086.9	0.98	0.22	0.32	1.29	2.05	1.95 (20)
21/2+	0.177	0.460	0.823	0.275	4.71	1319	1301.3	1.30	0.23	0.35	1.12	3.15	3.35 (42)

*Data of ref. 99

2* Experimental value (ref. 103) $B(M1, 9/2^- \to 7/2^-) = 0.073(3) \cdot 10^{-2}\ \mu_N^2$.

3* Experimental value (ref. 99) $\delta^2_{9/2} = 22.8(1.9)$.

4* Data of ref. 104.

make a parameter-free description (Fig. 8) of these effects, as we have pointed out in Sec. 8. It can be seen that the observed departure from Alaga's rules, which amounts to three orders of magnitude, can be explained satisfactorily.[11] However, allowance for the Coriolis interaction alone does not enable one in the general case to obtain the correct absolute magnitude of B(E1). Here, an important role is played by the interaction between quasiparticles and octupole vibrations of the core (see, for example, ref. 58). A certain role may also be played by effects due to the rotational noninvariance of the self-consistent field of deformed nuclei in the momentum space, i.e., the presence of a ghost state among the internal excitations with $K^\pi = 1^-$ in even–even nuclei, which corresponds to a motion of the center of mass of the nucleus.[40] These effects can be allowed for by the method developed in ref. 42.

Rotational spectra and dynamical effects. In this review we have not aimed to give a complete account of calculations of the rotational spectra in the nonadiabatic approach. The results of such calculations can be found in many papers. Here, we shall merely give some new results in order to demonstrate the efficacy of the nonadiabatic model. In Tables 2-5 we have given the amplitudes of K mixing of internal states,[12] the values of the generalized decoupling parameter, the calculated energies, and the electromagnetic moments. It is characteristic of all the bands that the parameter $a(I)$ has an appreciable value, although none of the bands is associated with a state with $K = 1/2$. It is this that is responsible

TABLE 4. Characteristics of Rotational Bands with Positive Parity in the ^{161}Er Nucleus. Calculations Made with the Parameters $\beta_{20} = 0.324$, $\beta_{40} = 0.04$, $\Delta_n = 1.1$ MeV, $1/2\mathcal{J} = 15$ keV, $g_R^0 = 0.30$, and $Q_0 = 8.3$ b

I^π	Mixing amplitudes C_K^I							$a(I)$	$\mathcal{E}(I)$, keV		μ_I, $[\mu_N]$	g_R^{eff}	$B(M1) \times 100$, $[\mu_N^2]$	δ_I^2	λ_I
	[660]↑	[400]↑	[402]↓	[651]↑	[642]↑	[633]↑	[624]↑		theo.	exp.*					
5/2+	0.212	0.093	0.251	−0.453	0.824	—	—	4.56	213	213	−0.96	−0.35	—	—	—
7/2+	0.131	0.057	0.271	−0.470	0.815	0.146	—	−5.12	219	217	−0.63	0.01	∼0.01	0.12	—
9/2+	0.361	0.155	0.312	−0.519	0.674	0.160	0.012	7.77	189	189	−0.62	0.02	—	—	—
11/2+	0.162	0.070	0.300	−0.497	0.761	0.227	0.026	−6.39	298	297	−0.09	0.12	0.70	0.71	1.46
13/2+	0.440	0.185	0.326	−0.524	0.597	0.187	0.025	8.77	273	268	−0.11	0.12	—	—	—
15/2+	0.178	0.076	0.311	−0.501	0.734	0.273	0.044	−6.91	484	509	0.50	0.16	1.19	1.04	0.77
17/2+	0.487	0.202	0.329	−0.520	0.551	0.198	0.034	9.20	469	466	0.46	0.15	—	—	—
19/2+	0.186	0.079	0.315	−0.500	0.716	0.304	0.061	−7.19	807	849	1.09	0.21	1.50	1.25	1.67
21/2+	0.517	0.213	0.330	−0.515	0.520	0.204	0.042	9.42	781	785	1.04	0.17	—	—	—
23/2+	0.191	0.080	0.317	−0.497	0.704	0.329	0.077	−7.37	1238	1308	1.69	0.21	1.72	1.36	2.64
25/2+	0.537	0.220	0.329	−0.510	0.499	0.208	0.048	9.56	1211	1208	1.63	0.19	—	—	—

*Data of refs. 79 and 105.

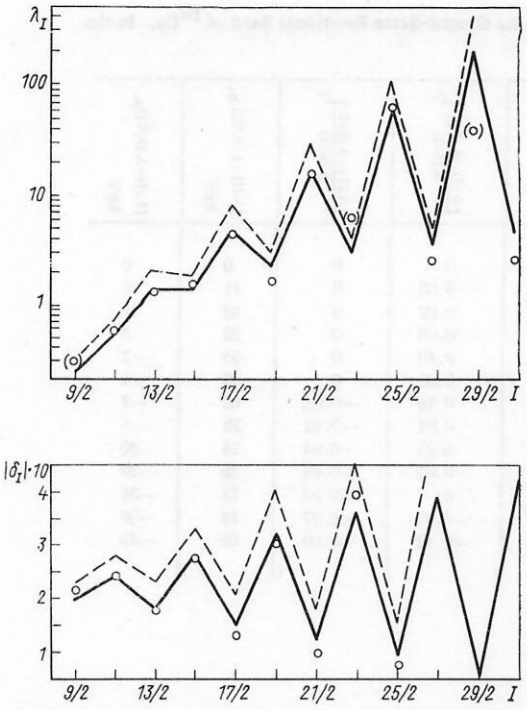

Fig. 7. Spin dependence of λ_I and $|\delta_I|$ in the ground-state rotational band of the ^{161}Dy nucleus: ○ gives the experimental data (ref. 84) (unreliable data given in parentheses); —— calculated data (corresponding to the results of Table 2); – – – – adiabatic estimate.

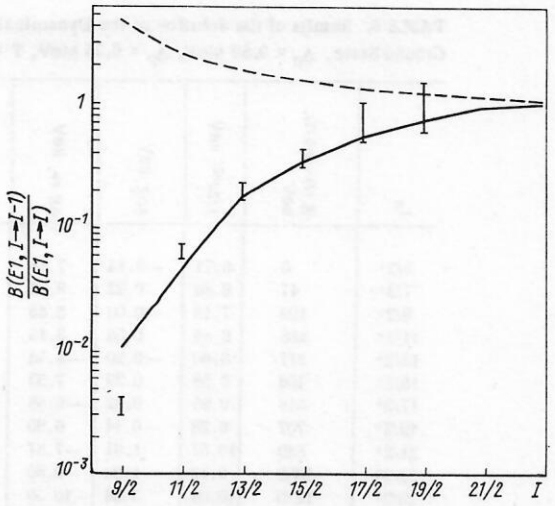

Fig. 8. Ratio of the reduced probabilities of E1 transitions between levels of rotational bands associated with the states $7/2^-[514]$ and $9/2^+[624]$ in the ^{177}Hf nucleus: – – – – adiabatic approximation (Alaga's rules); —— nonadiabatic calculations (from the results of Table 3); I) experimental data with error, from ref. 58.

for the pronounced difference between the J and J^{eff} values and also leads to the doublet structure of the rotational spectra, which is clearly manifested at large angular momenta.

All the spectra were calculated in the static approach, i.e., under the assumption of constancy of the parameters J, Δ, g_R^0, and Q_0 in the band. The calculations show that the stronger the Coriolis interaction, the better are the results of such a calculation at high spins. In weakly mixed rotational bands, the dynamical effects are manifested at lower spin values. Physically, this result can be understood, since the Coriolis interaction of an outer nucleon with the core hinders the development of dynamical effects in the core itself.[48]

If both processes are treated in the framework of perturbation theory, then in the lowest orders this competition leads to a change in the sign of the parameter B [Eq. (17)] in odd-mass nuclei as compared with even–even nuclei.[74] However, in the case of a strong Coriolis interaction, perturbation theory is no longer applicable and it is necessary to solve dynamical equations of the type (63). The internal energy of the nucleus in this case is given by the ordinary expression in a model with pairing and with allowance for the Coriolis mixing of states (the pairing interaction being replaced by a pairing field), i.e.,[13)]

TABLE 5. Characteristics of Rotational Bands Associated with the State $7/2^-[743]$ in the ^{235}U Nucleus. Calculations Made with the Parameters $\beta_{20} = 0.22$, $\beta_{40} = 0.08$, $\Delta n = 0.38$ MeV, $1/2J = 7$ keV, $g_R^0 = 0.20$, $Q_0 = 10$ b

I^π	Mixing amplitudes C_K^I					$a(I)$	$\mathcal{E}(I)$, keV		μ_I, $[\mu_N]$	g_R^{eff}	$B(M1)\times 10$ $[\mu_N^2]$	$B(E2)$, $[e^2b^2]$	δ_I^2	λ_I
	[761]↑	[752]↑	[743]↑	[734]↑	[725]↑		theo.	exp.*						
7/2⁻	0.007	0.095	0.995	—	—	−0.58	0	0	−0.26	0.14	—	—	—	—
9/2⁻	0.014	0.130	0.982	0.126	—	1.45	45	46.3	~10⁻³	0.14	0.36	3.33	0.13	—
11/2⁻	0.021	0.176	0.968	0.178	0.012	−2.15	101	103.1	0.23	0.14	0.57	3.37	0.13	0.48
13/2⁻	0.029	0.207	0.953	0.219	0.021	2.72	168	170.7	0.45	0.15	0.71	2.90	0.13	1.12
15/2⁻	0.037	0.235	0.938	0.252	0.030	−3.20	246	249.1	0.66	0.15	0.81	2.42	0.13	1.91
17/2⁻	0.046	0.260	0.923	0.278	0.039	3.61	337	338.7	0.86	0.15	0.88	2.01	0.13	2.74
19/2⁻	0.055	0.282	0.908	0.300	0.048	−3.96	438	438.5	1.06	0.15	0.94	1.68	0.13	3.85
21/2⁻	0.064	0.302	0.895	0.318	0.056	4.26	554	550.4	1.25	0.16	0.99	1.42	0.13	5.15
23/2⁻	0.073	0.321	0.881	0.333	0.065	−4.53	681	670.9	1.45	0.16	1.03	1.21	0.13	6.41
25/2⁻	0.082	0.338	0.869	0.345	0.073	4.76	821	804.9	1.65	0.16	1.06	1.04	0.13	8.09

*Data of ref. 77.

TABLE 6. Results of the Solution of the Dynamical Equations (76) for the Ground-State Rotational Band of ^{161}Dy. In the Ground State, Δ_n = 0.59 MeV, Δ_p = 0.95 MeV, T = 40.6 MeV^{-1}

I_π	$\mathscr{E}(I)-\mathscr{E}(5/2)$, keV	$1/2J^{eff}$, keV	δU_0^*, keV	δE^{2*}, keV	$1/2J^{eff}_{rot}$, keV	$\frac{J(I)-J(5/2)}{J(5/2)}$	$\left[\frac{\Delta(I)-\Delta(5/2)}{\Delta(5/2)}\right]_n$	$\left[\frac{\Delta(I)-\Delta(5/2)}{\Delta(5/2)}\right]_p$	$[\lambda(I)-\lambda(5/2)]_n$, keV	$[\lambda(I)-\lambda(5/2)]_p$, keV
5/2+	0	6.71	—0.14	7.28	—0.43	0	0	0	0	0
7/2+	47	6.89	0.22	9.56	—2.89	—0.02	0.03	0	11	0
9/2+	109	7.18	—0.09	5.45	1.82	—0.04	0.07	0	32	1
11/2+	188	6.84	0.54	3.15	3.15	—0.04	0.06	0	26	2
13/2+	277	8.60	—0.20	—4.73	13.53	—0.06	0.10	0	55	—2
15/2+	406	6.58	0.29	7.53	—1.24	—0.03	0.05	0	32	—1
17/2+	518	9.95	0.32	—6.68	16.31	—0.06	0.10	—0.02	68	—9
19/2+	707	6.29	—0.14	6.95	—0.52	—0.01	0.01	—0.01	32	—8
21/2+	839	10.57	1.61	—7.87	16.83	—0.04	0.07	—0.04	74	—20
23/2+	1082	6.12	—1.00	6.80	0.32	—0.05	—0.07	—0.04	29	—20
25/2+	1235	10.04	5.96	—10.59	14.67	—0.01	0	—0.07	72	—34
27/2+	1507	6.72	—3.69	8.03	2.38	0.21	—0.27	—0.07	19	—36
29/2+	1701	—	—	—	—	0.08	—0.12	—0.09	65	—45

* $\delta U_0 \equiv [U_0(I+1)-U_0(I)]/[2(I+1)]$.

2* $\delta E \equiv [1/[2(I+1)]] \sum_K [E_K(I+1)(C_K^{I+1})^2 - E_K(I)(C_K^I)^2]$.

$$\sum_K \mathscr{E}_K (C_K^I)^2 \equiv U_0 + \sum_K E_K (C_K^I)^2; \qquad (74)$$

$$U_0 = 2\sum_{\nu>0}(\varepsilon_\nu - \lambda)v_\nu^2 - 2\Delta\sum_{\nu>0} u_\nu v_\nu + \Delta^2/G + \lambda N, \qquad (74a)$$

where N is the number of particles in the system and E_K are the one-quasiparticle energies. The total energy of the nucleus in a state with spin I has the form (12). For the moment of inertia J of the core we use the ordinary expression in the cranking model, multiplied by some scale factor γ chosen from the condition of the best description of the observed spectrum:

$$J \equiv (\gamma/2) \sum_{\substack{\nu,\nu'>0 \\ (n,p)}} [j_{\nu\nu'}^2 L_{\nu\nu'}^2/(E_\nu + E_{\nu'})]. \qquad (75)$$

The scale factor γ is introduced because the moments of inertia usually calculated in the cranking model are smaller than the experimental values (see ref. 43) by 10-15%.

The total energy $\mathscr{E}(I)$ of the nucleus is minimized with respect to $\Delta_{(n,p)}$, $\lambda_{(n,p)}$, and C_K^I, i.e., one solves a system of variational equations

$$\partial\mathscr{E}(I)/\partial\Delta_{(n,p)} = 0; \quad \partial\mathscr{E}(I)/\partial\lambda_{(n,p)} = 0; \quad \partial\mathscr{E}(I)/\partial C_K^I = 0 \qquad (76)$$

with allowance for the normalization (9).

The results of solving Eq. (76) for the ground-state rotational band of the ^{161}Dy nucleus for γ = 1.15 are given in Table 6. In this table we give the calculated energies, the effective values of the rotational parameter $1/2J^{eff}$ [Eq. (40)], and the contributions to it that arise from the variation of the individual terms of the right-hand side of (74) (δU_0 and δE, respectively), and also the rotational energy $(1/2J^{eff}_{rot})$. In addition, we give results that characterize the behavior of the moment of inertia (75), the gap, and the chemical potential in the band.

It can be seen that the solution of Eqs. (76) gives approximately the same quantitative description of the rotational energies as the static approach. Dynamical effects in the values of the moment of inertia and the gap become appreciable only for $I \geqslant 27/2$. Up to a spin I = 21/2, the gap Δ_n even increases slightly from its ground-state value, i.e., the Coriolis interaction of the odd nucleon with the core is stronger than the antipairing effects in the core itself. Therefore, the potential energy U_0 of the core changes weakly up to large values of the spin, and the moment of inertia of the core also changes slowly. The mean values $\overline{\Delta_n(I)}$ and $\overline{J(I)}$ agree well with the parameter values used in the static calculations (see Table 2).

CONCLUSIONS

We have considered the possibility of describing rotational bands right up to high spin values in odd-mass atomic nuclei in the framework of a semiphenomenological nonadiabatic model that uses a small number of collective parameters. The main assumption of the model concerns the form of the Hamiltonian (3), which in the general case can also be used to describe rotational bands associated with the ground and excited states of even–even nuclei.[70,71] The relationship between this form of the Hamiltonian and the equations of the self-consistent-field method has been discussed in ref. 71. The freedom in the choice of the configuration space on which the operator of the internal angular momentum j acts reduces to the definition of the set of one-particle states coupled by the Coriolis interaction. From this point of view, the definition of j is dynamical, and it depends on the quantum numbers of the actually considered rotational band. The value of the rotational parameter 1/(2J) cannot be calculated self-consistently within the model. In the general case, J is

not the effective moment of inertia of the system in the usual sense of the cranking model. A relationship to the cranking model can only be obtained in the case of weak nonadiabaticity. As yet it remains obscure how the rotational part (3) is to be separated out from the many-particle microscopic Hamiltonian, although a number of constructive steps have already been taken in this direction (see, for example, refs. 7-9).

Numerous calculations have been made in the framework of the nonadiabatic model of the rotational spectra and the electromagnetic moments in odd-mass nuclei and satisfactory agreement has been achieved with the experimental data right up to large values of angular momentum.

Restrictions on the model arise from the neglect of the residual quadrupole and octupole forces and also from the assumption that the nuclei are axially symmetric. It is a purely technical problem to lift the first restriction, and one must merely use the basis of internal states developed, for example, in refs. 20 and 25. Nuclei may become dynamically nonaxial at large angular momenta. These effects were investigated approximately in refs. 13, 106, and 107, in which allowance was made for the effective departure from axial symmetry.

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1) In accordance with our definition of the internal motion, collective vibrations, for example, quadrupole and magnetic dipole vibrations, can also contribute to j.

2) The appearance of the coefficient 2 for K^2 in (15) is due to the inclusion of the $\hat{j}^2$ term in the average field.

3) Here, we use the same form of the spin interactions as in ref. 28. For simplicity, we assume that $\varkappa_{nn} = \varkappa_{pp} \equiv \varkappa$, $\varkappa_{np} = 0$, and that the isotopic dependence is included in this parameter. Interactions of the type $V(r_{12}) \cdot \sigma_3^{(1)} \sigma_3^{(2)}$ are not written down explicitly in the Hamiltonian but are allowed for in all the calculations. These forces generate $1^+(K = 0)$ excitations in the core and are responsible for longitudinal spin polarization effects. However, they do not affect the renormalization of the matrix elements of the operators $j^{(p)}$.

4) In this case, the one-particle matrix elements $j^{(p)}_{\nu'\nu}$, are used in the diagonalization of H_C. Note also that the rotational parameter in H^0_{rot} is not renormalized.

5) The interactions Hj and H_σ cannot serve as such interactions because of their repulsive nature. They affect the moment of inertia but cannot themselves generate a rotational branch of excitations.[42]

6) See also the discussion in Sec. 12.

7) This definition is convenient for bringing out discontinuities in the moment of inertia. For an equidistant spectrum $(1/2\, j_1^{eff}) = 0$ and at the point of phase transition this quantity becomes negative.

8) Here we use the effective g_s^{eff} factor instead of the g_K factor. They are related by $Kg_K = Kg_l\, (g_s^{eff} - g_l)(\sigma_3)_{K,K}/2$.

9) The configuration space for the outer nucleons in this case includes all states of the subshell $i_{13/2}$ and the states $1/2^+[400]$ and $3/2^+[402]$, which are coupled to it by the interaction with $\Delta N = 2$.

10) Note that in the nonadiabatic approach one can introduce a single effective factor g_s^{eff} instead of the two adiabatic magnetic parameters g_K and b_0 (see, for example, refs. 24 and 54).

11) Note that to describe E1 transitions it is important to allow accurately for blocking effects in the pairing interaction in odd-mass nuclei, which we have done in the present paper.

12) Only the maximum amplitudes have been given. The internal states are denoted by means of the asymptotic quantum numbers $[Nn_z\Lambda]\Sigma$.

13) The expressions are written down for an odd system. For an even system the internal energy is described simply by the expression U_0.

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