Current state of the Dirac monopole problem

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The current state of the magnetic monopole problem is reviewed, and theoretical difficulties are discussed. Possible explanations for the negative results of the experimental search for monopoles are analyzed. There is a detailed discussion of the dual symmetry of electrodynamics and of various derivations of the charge-quantization condition. Applications of monopole theory, particularly in dyonium models of hadrons, are discussed.

INTRODUCTION

Dirac's hypothesis of the existence of isolated magnetic charge – the Dirac monopole – has stimulated both many theoretical studies and many attempts to detect this charge experimentally. General interest in the problem has intensified considerably in recent years.

The basic motivation for the introduction of magnetic charge was the attempt to extend to sources that symmetry of electrodynamics with respect to electric and magnetic quantities which is reflected in the invariance of the Maxwell field equations,[1])

$$\left.\begin{array}{l}\partial^{\nu}F_{\mu\nu}=0;\quad \partial^{\nu}\widetilde{F}_{\mu\nu}=0;\\ F_{\mu\nu}=1/2\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma};\quad \widetilde{F}_{\mu\nu}=-F_{\mu\nu};\quad \varepsilon_{0123}=-1,\end{array}\right\} \tag{1}$$

with respect to discrete duality transformations or Larmor transformations,

$$F_{\mu\nu}\rightarrow\pm\widetilde{F}_{\mu\nu};\quad \widetilde{F}_{\mu\nu}\rightarrow\mp F_{\mu\nu}. \tag{2}$$

From a different standpoint, on the quantum-mechanical level, the existence of the Dirac monopole leads to a quantization of electric charge,

$$eg=n\hbar c;\ n=\left\{\begin{array}{l}0,\ \pm 1/2,\ \pm 1\ldots\ (\text{acc. Dirac})\\ 0,\ \pm 1,\ \pm 2\ldots\ (\text{acc. Schwinger}),\end{array}\right. \tag{3}$$

where e is the electric charge and g is the magnetic charge. This latter circumstance is usually cited as the strongest argument in favor of the monopole concept, since it leads to a theoretical basis for the observed discreteness of electric charge.

The introduction of magnetic charge in electrodynamics presents serious difficulties. A Lagrangian formulation of the theory requires potentials, which can be preserved as dynamic field variables when magnetic sources are introduced only by introducing a singularity line (the "Dirac string"). Several crucial problems which arise can be dealt with satisfactorily only by imposing a charge-quantization condition. In turn, this condition leads to a large numerical value of the "magnetic" coupling constant ($g^2/\hbar_c \sim 34.25$ according to Dirac), which complicates calculations for events involving magnetic charge.

The Dirac monopole has not yet been detected experimentally. Although certain quite plausible arguments can be advanced to explain the negative results, it is still necessary to carefully analyze the premises for the introduction of magnetic charge in electrodynamics.

It turns out that the introduction of magnetic sources in the equations of electrodynamics (in an effort to retain dual symmetry) does not necessarily require the existence of new types of particles (monopoles). The assertion that all known charged particles can be thought of as being dually charged, i.e., simultaneously having electric (e) and magnetic (g) charges, turns out to cause no contradictions. If we assume that the ratio g/e is the same for all known particles, we find that experimentally we would observe an effective charge $q = (e^2 + g^2)^{1/2}$.

Analysis of dually charged particles for the general case leads to a generalization of the Dirac–Schwinger condition:

$$e_i g_j - e_j g_i = n_{ij}\hbar c;\quad n_{ij}=\left\{\begin{array}{l}0,\ \pm 1/2,\ \pm 1,\ \ldots,\\ 0,\ \pm 1,\ \pm 2,\ \ldots,\end{array}\right. \tag{4}$$

where i, j refer to different types of particles. The situation with regard to charge quantization is modified considerably; in particular, it becomes possible to introduce fractional as well as integral electric charges.

The charge-quantization condition can be derived by at least three logically independent methods: from the requirement for the single-valuedness of phase transformations, as a consequence of the quantization of magnetic flux, and from the invariance of the theory with respect to the spatial-rotation group. Although none of these methods is completely free of difficulties, on the whole this condition seems to be quite firmly established and necessary for internal consistency of the theory of magnetic charge. Actually, if the Dirac–Schwinger condition is not imposed, we are left with absolutely no "purely electrodynamic" grounds for introducing the model.

Analysis of dually charged particles using condition (4), the latest development in the Dirac-monopole problem, opens up nontrivial possibilities for the use of the magnetic-charge model in hadron physics. However, regardless of future progress in this direction, the monopole question cannot be reduced simply to a question of the presence or absence in nature of some new particle. The question of the possible existence of magnetic charge is primarily part of the problem of justifying the current form of electrodynamics.

1. QUANTUM THEORY OF THE DIRAC MONOPOLE

Singularity line and charge-quantization condition. As Dirac pointed out,[2] derivation of a theory which can be generalized to the quantum-mechanical case requires a Lagrangian formulation, which in turn is known to require the introduction of electromagnetic potentials.

The usual definition of the vector potential,

$$\mathbf{H}=\nabla\times\mathbf{A}, \qquad (5)$$

contradicts the equation $\nabla\mathbf{H}=g\delta(\mathbf{r})$ if the integral Gauss theorem must hold. Accordingly, definition (5) must be modified[2] to[2,12]

$$\nabla\times\mathbf{A}=\mathbf{H}+\mathbf{H}^{(f)}, \qquad (6)$$

where $\mathbf{H}=g\mathbf{r}/r^3$; $\mathbf{H}^{(f)}=f(\mathbf{r})\,\tau(\mathbf{r})$; $\nabla\mathbf{H}^{(f)}=-g\delta(\mathbf{r})$; $f(\mathbf{r})$ is a function which vanishes everywhere except on curve L, where it has a δ-function singularity, and $\tau(\mathbf{r})$ is the vector tangent to curve L. Accordingly, this contradiction can be removed if (5) is satisfied on at least some line L leading from the magnetic charge to infinity.

Let us derive the charge-quantization condition for the motion of an electron in the field of a fixed monopole. All the specific features of the Dirac approach[1,2] are completely retained in this model. In the Coulomb gauge, we can write the potential **A** as

$$\mathbf{A}=-g\int_L d\mathbf{a}\times\nabla\,(1/(r-a))=-\int_L d\mathbf{a}\times\mathbf{H}\,(\mathbf{r}-\mathbf{a}). \qquad (7)$$

The transformation from the given line L to some other line L' generates a change in the potential **A** which is proportional to the gradient of the solid angle Ω on the LL' contour (see the discussion below for more details). The electron wave function thus must satisfy the gauge transformation

$$\psi\to\psi\exp\,(-ieg\Omega/4\pi). \qquad (8)$$

If line L describes a closed surface in the space and returns to its initial position, there is a change in the phase of function ψ corresponding to the total solid angle of 4π, i.e., we have

$$\psi\to\psi\exp\,(-ieg). \qquad (9)$$

The requirement that ψ be single-valued leads to the Dirac charge-quantization condition: $eg = 2\pi n$, $n = 0, \pm 2, \ldots$. Line L is the singularity line for the electromagnetic potential: the Dirac string.

Singular potentials can be used to construct an action integral which leads to the Maxwell equations and the equations of motion. However, this approach requires the use of an additional condition (the "Dirac veto"), which states that electrically charged particles can never intersect a singularity line connected to the magnetic charge.[2,12,21,34]

The singularity line (or lines) can be an arbitrary curve. If we choose it for simplicity to be a straight line, we can write the potential for a point monopole in Dirac theory as

$$\mathbf{A}=(g/r)\,[\hat{\mathbf{n}}\times\mathbf{r}/(r-\hat{\mathbf{n}}\cdot\mathbf{r})], \qquad (10)$$

which corresponds to a semiinfinite singularity line.

Schwinger[8a] uses the potential

$$\mathbf{A}=\frac{g}{2r}\left(\frac{\hat{\mathbf{n}}\times\mathbf{r}}{r-\hat{\mathbf{n}}\cdot\mathbf{r}}-\frac{\hat{\mathbf{n}}\times\mathbf{r}}{r+\hat{\mathbf{n}}\cdot\mathbf{r}}\right). \qquad (11)$$

The singularity line of this potential is infinite.

The Schwinger quantum-field approach. We can avoid the use of the Dirac veto in constructing a quantum theory of magnetic charge by taking into account the following considerations. The specification of an explicit scalar Lagrangian $\mathscr{L}$ is known to lead to relativistic invariance of quantum theory. However, it is not necessary that $\mathscr{L}$ be a scalar, and this condition can be replaced by more general conditions consistent with an action principle. A system invariant with respect to three-dimensional rotations is Lorentz invariant if the density of the energy–momentum tensor satisfies the equal-time commutation relation[D1]

$$-\mathrm{i}\,[T^{00}(x),\ T^{00}(x')]=-(T^{0k}(x)+T^{0k}(x'))\,\partial_k\delta(\mathbf{x}-\mathbf{x}'). \qquad (12)$$

This is a sufficient condition.

In magnetic-charge theory the corresponding energy-density operator is[8,9]

$$\left.\begin{aligned}T^{00}&=(\mathbf{E}^2+\mathbf{H}^2)/2+\bar{\psi}_e\gamma\,(-\mathrm{i}\nabla-eA^T-eA_g)\,\psi_e+m_e\bar{\psi}_e\psi_e+\\&\quad+\bar{\psi}_g\gamma\,(-\mathrm{i}\nabla-g\mathbf{B}^T-g\mathbf{B}_e)\,\psi_g+m_g\bar{\psi}_g\psi_g;\\ \mathbf{A}_g(x)&=\int d\mathbf{x}'\,\mathbf{a}\,(\mathbf{x}-\mathbf{x}')\,j_e^0(x');\\ \mathbf{B}_e(x)&=-\int d\mathbf{x}'\,\mathbf{a}\,(\mathbf{x}-\mathbf{x}')\,j_{(g)}^0(x'),\end{aligned}\right\} \qquad (13)$$

where $\mathbf{a}(\mathbf{x})$ is a vector function satisfying

$$\nabla\times\mathbf{a}\,(\mathbf{x})=\mathbf{h}\,(\mathbf{x});\ \ \nabla\cdot\mathbf{a}\,(\mathbf{x})=0;\ \ \nabla\cdot\mathbf{h}\,(\mathbf{x})=-\delta\,(\mathbf{x});$$

to describe the free field we use the two transverse potentials $\mathbf{A}^T$ and $\mathbf{B}^T$:

$$\mathbf{H}^T=\nabla\times\mathbf{A}^T,\quad \mathbf{E}^T=-\nabla\times\mathbf{B}^T, \qquad (14)$$

$$\left.\begin{aligned}\mathbf{E}&=\mathbf{E}^T-\nabla\phi_e,\quad \phi_e=\int(d\mathbf{x}')\,D\,(\mathbf{x}-\mathbf{x}')\,j_e^0(x');\\ \mathbf{H}&=\mathbf{H}^T-\nabla\phi_g,\quad \phi_g=\int(d\mathbf{x}')\,D\,(\mathbf{x}-\mathbf{x}')\,j_g^0(x);\end{aligned}\right\} \qquad (15)$$

where $D(\mathbf{x}) = 1/4\pi|\mathbf{x}|$. Here it is assumed that the following canonical commutation relations hold:

$$\left.\begin{aligned}&\mathrm{i}\,[A_i^T(x),\ B_j^T(x')]=\varepsilon_{ijk}\partial_k D\,(\mathbf{x}-\mathbf{x}');\\ &\mathrm{i}\,[E_i^T(x),\ H_j^T(x')]=\varepsilon_{ijk}\partial_k\delta\,(\mathbf{x}-\mathbf{x}');\\ &\mathrm{i}\,[A_i^T(x),\ E_j^T(x')]=\mathrm{i}\,[B_i^T(x),\ H_j^T(x')]\\ &\quad=\delta_{ij}\delta\,(\mathbf{x}-\mathbf{x}')-\partial_i\partial_j D\,(\mathbf{x}-\mathbf{x}').\end{aligned}\right\} \qquad (16)$$

The charge-quantization condition follows from the requirement that the gauge transformations corresponding to vector potentials be single-valued in transformations from one singularity line to another.

Zwanziger[21] gave a Lagrangian formulation of the Schwinger quantum-field theory. Rabl[18] used the Schwinger approach to construct a Lorentz-invariant S matrix; in this case the propagator function for proton exchange between electric and magnetic charges is[203]

$$D_{\mu\nu}(k)=(k^2+\mathrm{i}\varepsilon)^{-1}\,\varepsilon_{\mu\nu\rho\sigma}k^{\rho}n^{\sigma}/nk. \qquad (17)$$

This propagator function is covariant, but it depends on the singularity line [in the quantization system $\hat{n} = (0, \hat{\mathbf{n}})$, where $\hat{\mathbf{n}}$ is the unit vector in the direction of the singularity line]. An appropriate averaging must be carried out to remove the dependence on the $\hat{n}$ direction in the complete expansion.

Difficulties in the theory of the Dirac monopole. The introduction of a Dirac string thus allows us to construct both quantum[2,8,9,11,16,18,21] and classical[2,21,34] theories of magnetic charge. On the quantum level the charge-quantization condition leads to no contradictions, but the use of the singularity line meets with several objections, among which the following are the most important:

a) The singular nature of the potential A_μ does not correspond to any physical features of the electromagnetic field $F_{\mu\nu}$.[3]

b) The Dirac veto does not follow from a variational principle, but is an additional requirement.[33]

c) The Dirac theory does not describe a point magnetic charge but a semiinfinite thin solenoid.[28,36]

d) The operator generating gauge transformations for charged fields related to a change in the singularity line is not unitary;[D2] the nongradient nature of the transformations corresponding to a change in the position of the singularity line disrupts the self-consistency of the theory.[208]

e) Introduction of magnetic charge violates the Lorentz invariance of the theory.[5-7]

f) The charge-quantization condition is a specific result of the use of the singularity line, so it is not a necessary part of magnetic-charge theory.[36,40]

While objections a and b deal essentially with technical aspects of the theory, the validity of any of the other objections (which are in fact extremely closely related) would mean that at present we have no consistent magnetic-charge theory. Let us briefly discuss these objections; we can discuss the first two on the basis of classical theory.

a) We can seek a solution of the Maxwell equations by introducing the two independent potentials A_μ and B_μ[3] (see also refs. 139 and 204):

$$\left.\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - \varepsilon_{\mu\nu\rho\sigma}\partial^\rho B^\sigma; \\ F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + \varepsilon_{\mu\nu\rho\sigma}\partial^\rho A^\sigma. \end{aligned}\right\} \tag{18}$$

However, it is not possible to construct an action integral which would lead simultaneously to both the field equations and the equations of motion in this case.[27,31,33,43] To construct such an integral, we would have to either restrict the treatment to "purely kinematic" theory, in which there is no interaction between electric and magnetic charges,[7,27,135] or alter the equations of motion for the monopole.[38,42] In the latter case, however, the motivation for introducing magnetic charge on the basis of dual-symmetry considerations would vanish completely, since this symmetry includes, in addition to the symmetry of the Maxwell equations, symmetric behavior of electric and magnetic charges in electric and magnetic fields.

As was pointed out in ref. 34, a source of difficulties in constructing the action integral in the classical theory is that the use of two potentials increases the number of degrees of freedom of the electromagnetic field and causes all six $F_{\mu\nu}$ components to become independent dynamic variables (see also refs. 43 and D3).

In the quantum theory for such a system the energy need not be positive definite,[D1] and it is completely possible that a similar situation may occur on the classical level. Accordingly, field schemes in magnetic-charge theory which are not based on singular potentials can scarcely compete with the Dirac–Schwinger approach at present because of several specific difficulties (the appearance of additional degrees of freedom of the field, the impossibility of constructing an action integral, etc.).

b) Although the Dirac veto arises as an additional condition which may turn out to be extremely stringent in several cases (see, e.g., ref. 33), it is physically plausible. Classically, when the trajectories of charged particles can in principle be determined exactly, the position of the singularity line can be chosen in each specific problem in such a manner that the Dirac veto is a consequence of the equations of motion. This choice can be made unless the electric field is moving along a magnetic line of force (head-on collisions), but such initial conditions are ruled out.

Jan[34] showed that by defining a nonlocal action integral by means of limiting procedures one can construct a definite position for the singularity line and establish the equivalence of all space–time points without exception. However, a necessary condition for the self-consistency of this approach is the classical quantization condition $eg = 2\pi nkc$, where k is a constant having the dimensionality of action and where $n = 0, \pm 1, \pm 2, \ldots$.

c) To get a clearer picture of the macroscopic analog of the system described by (10), we note that this potential satisfies the relations $\nabla \times \mathbf{A} = g\mathbf{r}/r^3$ for $\mathbf{r} \nparallel \hat{\mathbf{n}}$ and $\oint_C \mathbf{A}\cdot d\mathbf{l} = -g$, where C is a small contour around the line $\mathbf{r} \parallel \hat{\mathbf{n}}$. The potential $\mathbf{A}$ thus describes a field which is produced, not by an isolated charge, but by a line of magnetic flux extending from the origin to infinity along the direction $\mathbf{r} \parallel \hat{\mathbf{n}}$. In electrodynamics with electric charges, the most natural physical model for a magnetic charge is thus a semiinfinite thin solenoid.[29,30,32,41]

The Dirac monopole can be treated as a point particle if we stipulate that the magnetic flux along $\hat{n}$ cannot lead to observable effects; this situation can be arranged by using the Dirac veto. At the quantum level the charge-quantization condition plays an analogous role, leading to a cyclic phase of the function ψ and contracting the region in which the Hamiltonian and angular-momentum operators are defined (see, e.g., refs. 21, 146, and 149).

d) Let us consider a potential transformation which leads to a transformation between two different singularity lines. Using the familar equations of vector analysis, we find

$$\mathbf{A}(L_1) - \mathbf{A}(L_2) = \Big(\int_{L_1} - \int_{L_2}\Big)\, d\mathbf{a} \times \mathbf{H}(\mathbf{r}-\mathbf{a}) = \oint d\mathbf{a} \times \mathbf{H}(\mathbf{r}-\mathbf{a})$$
$$= \int_\sigma (d\boldsymbol{\sigma} \times \nabla) \times \mathbf{H}(\mathbf{r}-\mathbf{a}) = \int_\sigma \nabla (\mathbf{H}(\mathbf{r}-\mathbf{a})\, d\boldsymbol{\sigma}) -$$

$$-\int_{\sigma}(\nabla\cdot\mathbf{H}(\mathbf{r}-\mathbf{a}))\,d\sigma=\nabla\Lambda-g\int_{\sigma}\delta(\mathbf{r}-\mathbf{a})\,d\sigma, \qquad (19)$$

where σ is a surface bounded by the line $L_1 - L_2$ (supplemented by the line to infinity if L_1 and L_2 go to infinity in different directions), and

$$\Lambda(r)=\int_{\sigma}\mathbf{H}(\mathbf{r}-\mathbf{a})\,d\sigma=-g\nabla\int_{\sigma}d\sigma/(r-a). \qquad (20)$$

The presence in (19) of a singular term in addition to the gradient term shows that the definition of quantities such as T^{00} and T^{0k} must be refined, since the potentials are singular along the Dirac string, and the field operators ψ and $\bar{\psi}$ have an indefinite phase on this string. It must be taken into account for this purpose that products of local field operators should be understood[D4] as the limits of products defined for noncoincident points.[3)] For example, we could require that the following condition hold:

$$-\bar{\psi}(x)\,\gamma(\nabla-ie\mathbf{A}(x))\,\psi(x)=\lim/|\varepsilon|\to 0$$
$$\times\left[\psi(x+\varepsilon/2)\,(3\gamma\varepsilon/\varepsilon^2)\,\psi(x-\varepsilon/2)\exp\left\{ie\int_{x-\varepsilon/2}^{x+\varepsilon/2}dx'\,\mathbf{A}(x')\right\}\right], \qquad (21)$$

where the integration path is the straight line connecting the two simultaneous points, and the averaging over all ε directions should be carried out before going to the limit $|\varepsilon|\to 0$.

Because of the charge-quantization condition, the exponential function in (21) is defined unambiguously, since if the integration path intersects the singularity line the exponent changes by $2\pi n$, and the exponential remains the same. Therefore the operator U which generates the gauge transformations for the charge fields:

$$U\psi_e U^{-1}=\exp\left\{ie\int_{\infty}^{x}dx'\,(\mathbf{A}_g'(x')-\mathbf{A}_g(x'))\right\}\psi_e(x);$$

$$U\psi_g U^{-1}=\exp\left\{ig\int_{\infty}^{x}dx'\,(\mathbf{B}_e'(x')-\mathbf{B}_e(x'))\right\}\psi_g(x),$$

leaves invariant those expressions for T^{00} and T^{0k} which supplement the definitions of these quantities, does not alter the commutation relations for the fields, and is thus a unitary operator.

This analysis shows that, as expected, the difference $\mathbf{A}(L_1)-\mathbf{A}(L_2)$ cannot be written as a gradient at all points. By using the appropriate supplementary relation for the products of field operators and using the charge-quantization condition, however, we can avoid this difficulty.

e) The studies in refs. 5-7 of the Lorentz invariance of the theory including magnetic charges do not refer to the theory of the Dirac monopole, however, since they do not incorporate the charge-quantization condition (see also refs. 8 and 16). The results of these studies apparently show that a relativistically covariant quantum theory cannot be constructed with arbitrary values of e and g within the framework of current concepts (see also ref. 8b).

f) Whether the Dirac approach is correct depends on the validity of the charge-quantization condition, and nearly all attempts which have been made to experimentally detect magnetic charge have been based on this condition. In Section 4 we will discuss in detail the extent to which this condition is necessary to a systematic theory of the monopole.

2. EXPERIMENTAL SEARCH FOR THE DIRAC MONOPOLE

Properties of the Dirac monopole. Two basic properties of the monopole follow from the theory of magnetic charge:

1. The magnitude of the magnetic charge can have the values

$g\approx 68.5en$; $g^2/\hbar c\approx 34.25$ $(n=1)$ (acc. Dirac) [1, 2];

$g\approx 137en$, $g^2/\hbar c\approx 137$ $(n=1)$ (acc. Schwinger) [8a].

2. The behavior of the monopoles is analogous to that of electrically charged particles (when the appropriate substitutions are made: $e\to g$, $\mathbf{E}\to\mathbf{H}$, $\mathbf{H}\to-\mathbf{E}$).

The Dirac–Schwinger theory yields no rigorous predictions regarding the mass of the monopole. Frequent use is made of the "canonical" mass $m_g = g^2/r_e c^2$, equal to $2.4m_p$ according to Dirac or $9.6m_p$ according to Schwinger ($r_e = e^2/m_e c^2$); this mass value is derived by setting the classical radius of the magnetic monopole equal to that of the electron.

The ionizing effect of the monopole must be approximately equivalent to that of heavy nuclei (in the relativistic treatment) having an electric charge of[65–68,71,81] (137/4)e [the ionization loss in matter is ~ 8 GeV/(g/cm²)]. A specific feature of the ionization loss of a monopole is its independence of the monopole velocity.[4)]

Monopoles can be detected experimentally[70,73,78] on the basis of either Cerenkov radiation[5)] or the quite intense radiation emitted as monopoles move across the interface between two media.[72,80]

Theoretical estimation of the cross section for the production of monopole–antimonopole pairs is extremely difficult for two reasons: First, the large coupling constant of the monopole casts doubt on the validity of using perturbation-theory methods; second, there is uncertainty regarding the mechanism for the electromagnetic interaction between electrically and magnetically charged particles (see, e.g., ref. 83). Nevertheless, Cabibbo and Ferrari[204] made some rough estimates. For the reaction $p+p\to p'+p'+g^{+}+g^{-}$ they carried out a calculation for the diagram describing the production of a monopole–antimonopole pair resulting from a purely Coulomb interaction between the colliding protons. With a monopole mass of $m_g\approx 2.5m_p$ they found the cross section to be on the order of $\sigma\sim 10^{-34}$ cm²/nucleon.

Experimental attempts to find Dirac monopoles have been designed exclusively on the basis of the electromagnetic interactions of these monopoles.

Experimental procedure.[6)] The experimental attempts to detect magnetic monopoles can be classified into two groups:

1. Attempts in accelerators, through use of the reactions

$p+N \to p+N+g^{+}+g^{-}+\text{anything}$ [87–90, 104].

2. Attempts to find monopoles under natural conditions, either by: a) directly detecting monopoles in cosmic rays,[84,86,94,101,103,105,108] or b) searching for magnetic charges in terrestrial matter,[85,92,96–99,109] in meteorites,[91,93,110] or in lunar rock,[106,107,110] where they could be produced as a result of the interaction of cosmic rays[7)] with matter or as the result of direct capture by matter.

All attempts which have been made to experimentally detect magnetic charge have yielded negative results; the overall tendency is for a progressive decrease in the upper limits on the production cross section for monopoles of progressively larger mass.[8)] For example, the first bevatron experiment at Berkeley (1959) resulted in an estimate of $\sigma \le 10^{-35}$ cm²/nucleon as the cross section for the production of a singly charged monopole having a mass on the order of the proton mass, while an estimate of $\sigma \le 10^{-43}$ cm²/nucleon for $m_g \le 5_{mp}$ was found at Serpukhov (1970) for the production of monopole–antimonopole pairs. The situation with regard to experiments of the second type is approximately the same. In the pioneering study of Malkus[84] a cross section of $\sigma \le 3 \cdot 10^{-35}$ cm²/nucleon was found, while more recent experiments by Alvarez et al.,[106] who searched for monopoles in lunar material, yielded an upper limit of $\sigma \le 10^{-41}$ cm²/nucleon for $m_g \le 5m_p$ and $\sigma \lesssim 10^{-35}$ cm²/nucleon for $m_g \le 1000$ m_p.

Recent experiments[97–99,105,108] have established an upper limit on the flux of cosmic monopoles penetrating the earth's surface: $N \le 8.4 \cdot 10^{-18}$ cm$^{-2}\cdot$sec^{-1}. This result means that over the entire history of the earth ($4.6 \cdot 10^9$ yr) at best a single monopole has been incident on each area of 2 cm².

Discussion. In the absence of a sufficiently reliable estimate of the mass and production cross section of the monopole, it is difficult to judge how strong an argument these results constitute against the existence of the monopole. In principle, several considerations could be advanced to explain the negative experimental results. We will discuss some of these considerations.

1. The simplest (and least substantial) argument is that the mass of magnetically charged particles is very high, and the threshold energy required for the production of these particles has not yet been achieved.

Since the theory of magnetic charge, as mentioned above, contains no ironclad predictions of the possible monopole masses, the lower limit on this mass could in principle be made arbitrarily large.[9)]

2. It is tempting to seek a general principle which would rule out the production and experimental detection of monopoles.

The simplest way to formulate this prohibition might be to assign the monopole some property which would lead to the violation of one of the well-established conservation laws. For example, if we assume that the only contribution of the magnetic current is a pseudovector increment to the Lagrangian of the interaction, we would find that the observed degree of parity conservation in electromagnetic interactions could strongly suppress all processes involving monopoles, thereby leading to a simple explanation for the negative experimental results.[155] Unfortunately, for this purpose we would have to treat the magnetic current as a vector, although this would not be necessary otherwise. This ambiguity seems to be a necessary corollary of prohibitions of this type. The hypothetical object could be assigned some special property, which would then itself have to be justified.

3. It is quite probable that the estimates of Cabibbo and Ferrari for the cross section for the production of monopole–antimonopole pairs ($\sigma \ge 10^{-35}$ cm²/nucleon for a mass of ~2.5 GeV) are much too high because of improper account of the large constant for the g–g interaction.

For example, it was shown in ref. 177 that because of the extremely strong magnetic Coulomb attraction the energy required for the production of an unbound g^+g^- pair should be far above the production threshold. This interaction could lead to a rapid recombination of monopole pairs (and their annihilation) after the production of the pairs, but before the particles leave the interaction region. This mechanism greatly reduces the probability for the production of unbound monopoles. The introduction of such a superstrong interaction in the final state of the particle production was analyzed in ref. 82; it was found possible to reinterpret the experimental data obtained in the Serpukhov accelerator. The upper limit on the production cross section, $\sigma \lesssim 10^{-43}$ cm²/nucleon, could then refer to monopole masses roughly half as large.

The interpretation of the experimental results found in the search for free monopoles produced in the interaction of primary cosmic rays with the atmosphere is thus radically altered if this mechanism for the production of g^+g^- pairs is actually valid. Furthermore, long exposure times would be required to detect isolated high-energy monopoles in the primary cosmic rays.

4. In an evaluation of the results of experiments designed to detect monopoles bound with matter, account should be taken of the fact that there may be certain features of the monopole-capture processes and of the behavior of a monopole in matter which are insufficiently well understood.[10)]

It is usually assumed that the monopoles can be thermalized in matter and that they can migrate and become magnetostatically bound in ferromagnets and paramagnets. On the other hand, monopoles must not be bound to atoms or nuclei in nonferromagnets. It is also assumed that the monopoles can be extracted from matter by strong magnetic fields.

Most of the experiments which have been carried out have been based on this picture of the interaction of monopoles with matter. For example, wide use has been made of the results reported by Malkus,[84] which show that the energy with which a monopole is bound in matter does not exceed a few electron volts. However, as was shown in ref 79, monopoles can in principle be strongly bound with free nuclei having a magnetic dipole moment (the binding energy may reach 1 MeV). If so, experimental attempts to detect magnetic charges in, e.g., geological materials which are potential collectors of monopoles, may be unsuccessful. It should also be pointed out that the interpretation of experiments based on the assumption of simply the existence of monopoles of two signs[93,106,110] also contains a crucial assumption: that monopoles of a given sign predominate in the material being studied (the meteorite or the lunar rock).

5. Finally, and more radically, it could be argued that the existing theory for the Dirac monopole does not given an adequate description of the actual properties of magnetic charge. If so, either the quantum theory of the magnetic charge in its present form is incorrect, and thus the Dirac–Schwinger condition is incorrect,[36,40] or the expected classical properties of the monopole cannot be reconciled with the picture of a point magnetostatic source[38,42,59,60] (or both explanations may be valid simultaneously).

The experimental situation thus demands a critical analysis of two fundamental points in the current theory of magnetic charge: the dual symmetry of electrodynamics and the charge-quantization condition.

3. DUAL SYMMETRY IN THE THEORY OF MAGNETIC CHARGE

The free field. The Larmor transformation $E \to \pm H$, $H \to \mp E$ is a particular case of the more general transformations (dual rotations)[111–114]

$$\left.\begin{aligned} F_{\mu\nu} &\to F_{\mu\nu}\cos\theta + \widetilde{F}_{\mu\nu}\sin\theta;\\ \widetilde{F}_{\mu\nu} &\to -F_{\mu\nu}\sin\theta + \widetilde{F}_{\mu\nu}\cos\theta, \end{aligned}\right\} \tag{22}$$

which leave the free-field Maxwell equations invariant.

The dual symmetry of the free-field Maxwell equations can be associated with the following conserved pseudovector (the "dual current"):[117,119,126]

$$\Pi_\mu = F_{\mu\nu}B^\nu - \widetilde{F}_{\mu\nu}A^\nu, \tag{23}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\widetilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

It is not difficult to show that in the momentum representation the components of dual current Π_μ are proportional to the difference between the numbers of right-hand and left-hand polarized photons.[11)] The fourth component of Π_μ

$$\Pi_4 = i\Pi_0 = \int d^3x\,(\mathbf{A}^T\cdot\mathbf{H}^T - \mathbf{E}^T\cdot\mathbf{B}^T) \tag{24}$$

is the generator of dual transformations. Dual transformations (22) are generated by the following unitary operator:

$$U(\theta) = \exp\left\{ i\theta/2 \int (\mathbf{A}^T\cdot\mathbf{H}^T - \mathbf{E}^T\cdot\mathbf{B}^T)\,d^3x \right\}. \tag{25}$$

It was shown in refs. 116 and 117 that a joint formulation of relativistic and dual invariance requires classification of the field quantities in terms of the irreducible representations of an expanded 11-parameter group which includes, in addition to the Lorentz transformations and displacements, dual transformations.

In quantum theory the existence of a dual group leads to the impossibility of experimentally determining the absolute polarization plane of linearly polarized light and thus the impossibility of determining the relative phase for left-hand and right-hand polarized light.

Classical field with sources. The Maxwell equations in the presence of two types of sources,

$$\partial^\nu F_{\mu\nu} = J_\mu^{(e)};\quad \partial^\nu \widetilde{F}_{\mu\nu} = J_\mu^{(g)} \tag{26}$$

are invariant not only with respect to expanded Larmor transformations ($F_{\mu\nu} \to \pm\widetilde{F}_{\mu\nu}$, $\widetilde{F}_{\mu\nu} \to \mp F_{\mu\nu}$, $J_\mu^{(e)} \to \pm J_\mu^{(g)}$, $J_\mu^{(g)} \to \mp J_\mu^{(e)}$), but also with respect to transformations (22) and the corresponding transformations for the currents:

$$\left.\begin{aligned} J_\mu^{(e)} &\to J_\mu^{(e)}\cos\theta + J_\mu^{(g)}\sin\theta;\\ J_\mu^{(g)} &\to -J_\mu^{(e)}\sin\theta + J_\mu^{(g)}\cos\theta; \end{aligned}\right\} \tag{27}$$

where $J_\mu^{(e)} = eJ_\mu$, $J_\mu^{(g)} = gJ_\mu$, and J_μ is the particle density. If we introduce the idea of dually charged particles, i.e., particles which may simultaneously carry electric (e) and magnetic (g) charges, we could define the Lorentz-force density as

$$f^\nu = J_\mu^{(e)}F^{\mu\nu} + J_\mu^{(g)}\widetilde{F}^{\mu\nu}. \tag{28}$$

This measure would ensure that the symmetry of the field equations (26) was consistent with that of the equations of motion.

Transformations (22) and (27) leave Eqs. (26), Lorentz force (28), and the energy–momentum tensor of the electromagnetic field invariant. Accordingly, the question of defining the parameter θ and fixing its value is one of reaching an agreement rather than finding an experimental answer. If we now analyze the set of dually charged particles, assuming that for each particle the ratio g/e has the same arbitrary value, we can relate the parameter θ to this ratio by defining it, e.g., in the following manner:

$$\theta = \mathrm{arctg}\,(g/e). \tag{29}$$

Then we can use the dual rotation

$$\left.\begin{aligned} \mathcal{F}_{\mu\nu} &= (eF_{\mu\nu} + g\widetilde{F}_{\mu\nu})/q = F_{\mu\nu}\cos\theta + \widetilde{F}_{\mu\nu}\sin\theta;\\ \widetilde{\mathcal{F}}_{\mu\nu} &= (e\widetilde{F}_{\mu\nu} - gF_{\mu\nu})/q = -F_{\mu\nu}\sin\theta + \widetilde{F}_{\mu\nu}\cos\theta, \end{aligned}\right\} \tag{30}$$

where $q = (e^2+g^2)^{1/2}$, to find the Maxwell equations with sources of a single type,

$$\partial^\nu \mathcal{F}_{\mu\nu} = qJ_\mu,\quad \partial^\nu \widetilde{\mathcal{F}}_{\mu\nu} = 0 \tag{31}$$

and the ordinary Lorentz force on a test charge q,

$$f^\nu = qJ_\mu\mathcal{F}^{\mu\nu}. \tag{32}$$

Accordingly, starting from the electrodynamics of dually charged particles with a universal ratio g/e we can always find some linear transformation of the field components to transform to the usual Maxwell form with a single effective charge q; i.e., the two forms of these equations are essentially equivalent.[12)] That we can formally carry out this transformation has a profound physical meaning.[13)]

Since the presence of a field can be established only through its effect on some charged object, while the charge of any particle in turn can be determined (identified) only with the help of a field, only the effects of charge-field interactions – not the charges or fields individually – can be observed (measured) directly. It is therefore, in principle, impossible to establish an experimental difference between the results calculated for these effects on the basis of Eqs. (26), (28) and (31), (32) if we identify the effective charge $q = (e^2 + g^2)^{1/2}$ with the observable electric charge.

It is important to emphasize that a dual rotation like (30) leads to electrodynamics with a single charge for all sources simultaneously only if the ratio g/e is the same for all particles. Otherwise, we can carry out a transformation to a system with an effective charge only for particles having a single type of charge $[q_1 = (e_1^2 + g_1^2)^{1/2}]$. Particles having some other ratio g/e $(g_2/e_2 \neq g_1/e_1)$ in this system will have both an electric charge e' and a magnetic charge g', given by[121]

$$e' = q_2 \cos(\theta_2 - \theta_1);$$
$$g' = q_2 \sin(\theta_2 - \theta_1),$$

where $q_2 = (e_2^2 + g_2^2)^{1/2}$ and $\theta_i = \mathrm{arctg}(g_i/e_i)$ (i = 1, 2). We thus see that the universality of the ratio g/e is of critical importance in the electrodynamics of dually charged particles: If this ratio is the same for all particles, there is no observable magnetic charge.

The systematic exploitation of the dual symmetry of the equations of electrodynamics thus leads to the following statement of the problem.

Each particle is assigned both electric and magnetic charges a priori. As soon as agreement is reached that one of the particles (the "basis particle") has, e.g., only an electric (effective!) charge, the question of the existence of magnetically charged particles must be formulated in the following manner: What electric and magnetic charges does each new particle have with respect to the basis particle?

If we treat all known particles as dually charged, the question of whether these particles have a relative magnetic charge, i.e., the extent to which their ratios g/e differ, is amenable to a direct experimental check.[14)] Let us assume, e.g., that the electron, proton, and neutron are dually charged particles whose ratios g/e are different. Let us further assume that the observed charge of the electron is a purely electric effective charge. Then the proton and neutron have magnetic charge. From this point of view any electrically neutral system containing electrons, protons, and neutrons turns out to be magnetically charged; this result should lead to observable effects on the macroscopic scale, in particular, a radial component in the magnetic field intensities of the earth and sun. Since this component does not exceed 1 G, an upper limit on the magnitude of the magnetic charge for the proton and neutron can be established highly accurately. The corresponding estimates are[121,129,205] g_p, $g_n < 10^{-35}$ emu or, in units of electric charge, g_p, $g_n < 2 \cdot 10^{-26}$e.

A sensitive superconducting quantum interferometer was used in ref. 96 in a direct evaluation of the charges g_p and g_n: g_p, $g_n < 10^{-24}$e.

Preliminary estimates of the magnetic charge of the muon[96] yield $g_\mu < 10^{-5}$e.

Accordingly, the experimental information available can be thought of as evidence that the condition g/e = univ holds for all known particles. The fact that we can transform from the electrodynamics of dually charged particles to that with a single effective charge can thus be considered well established experimentally.

Quantum theory of dually charged particles. Schwinger[15] and Zwanziger[16,21] constructed a quantum theory for dually charged particles in which the following generalization of the charge-quantization condition arises:[15)]

$$e_i g_j - e_j g_i = n_{ij}\hbar c, \quad n_{ij} = 0, \pm 1, \pm 2, \ldots, \tag{33}$$

where i and j refer to different types of dually charged particles. It is not difficult to see that the charge-quantization condition changes significantly for dually charged particles. First, for a fixed ratio g_i/e_i condition (33) imposes no restrictions on the product $e_i g_i$. Here n_{ij} may vanish not only for $g_i = g_j = 0$, but also if the ratio g/e is universal, i.e., in the case $g_i \neq g_j \neq 0$. Finally, the generalized charge-quantization condition allows the atomic electric charge to take on values different from the electronic charge.[16)]

The independence of the physical consequences of the theory from the angular position in the charge (e,g) plane is important for a correct physical interpretation of dually symmetric electrodynamics. In classical theory this independence can be formulated as the principle that only dually invariant quantities are observables in electrodynamics.[126] On the quantum level, an analogous role is played by the assertion of the unitary equivalence of two Hamiltonians, one of which describes the system of dually charged particles with charges e_i, g_i, while the other describes a system of particles with charges $e_i' = e_i \cos\theta - g_i \sin\theta$, $g_i' = e_i \sin\theta + g_i \cos\theta$ (the Zwanziger theorem of "chiral" equivalence[16]).

A Lorentz-invariant quantum theory and an S matrix describing dually charged particles[17)] under the condition g/e = univ can be constructed even without the use of a Dirac string[136,138] (see also refs. 10 and 124). The Hamiltonian corresponding to particles having a single effective charge $q = (e^2 + g^2)^{1/2}$ are related[18)] by unitary dual transformation (25) with $\theta = \mathrm{arctg}\,(g/e)$.

The theory which treats both "purely electrically" and "purely magnetically" charged particles is a particular case of the theory of dually charged particles of two types, corresponding to the choice $g_i/e_i = -e_j/g_j$.

The introduction of magnetic charge in electrodynamics on the basis of dual-symmetry considerations is thus not only valid but necessary. This measure leads to a symmetry of the Maxwell equations with sources which is consistent with that of the free-field Maxwell equations. Significantly, however, symmetry of the equations of electrodynamics can be achieved without introducing a new type of particle. Treating known particles as dually charged particles turns out to be completely noncontradictory and can be reconciled with experimental data by treating these data as constituting evidence that all particles have the same ratio of electric and magnetic charges. From this standpoint, the traditional single-charge formulation of electrodynamics corresponds to a definite choice of "dual gauge."

Arguments in favor of the magnetic monopole must thus be based primarily not on the symmetry of the Maxwell equations, but on the possibility of achieving a theoretical explanation for the quantization of electric charge. If we discard the charge-quantization condition, we are left with no "purely electrodynamic" reasons for introducing particles having a ratio g/e different from that of known particles.

4. THE CHARGE-QUANTIZATION CONDITION

It is crucial to determine the extent to which the charge-quantization condition is independent of the specific formal apparatus of the theory of magnetic charge. This determination requires an analysis of various possible derivation methods.

Field theory. Particularly interesting are modifications of the Dirac–Schwinger approach in the theory of magnetic charge not involving the use of a singularity line.

Such a theory was first constructed by Cabibbo and Ferrari[3] (see also refs. 4, 10, 12, 18, 20, 165, and 204) on the basis of the Mandelstam formulation of quantum electrodynamics without potentials,[D5] in which the basic gauge-invariant dynamic variables are the field tensor $F_{\mu\nu}(x)$ and the field operators $\Phi(x, P)$, which depend, however, not only on the space–time point x but also on the space-like path P. In this case the Dirac–Schwinger relation arises as a condition for the self-consistency of the theory. The formalism without potentials can also be generalized[20,139] to the case of dually charged particles.[19)]

A specific difficulty arises in the Cabibbo–Ferrari approach: The derivatives of the field operators do not satisfy the Jacobi identity.[20)] In order to avoid this contradiction, we must subject the choice of space-like paths to certain restrictions which are very nearly equivalent to the Dirac veto.

Schwinger reconstructed the theory of magnetic charge on the basis of phenomenological and nonoperator approximations of the theory of sources.[15] He believes that this approach leads to a justification of the magnetic-charge concept which is independent of the nature of the limiting procedures and furnishes additional proof of the charge-quantization condition.

A version of magnetic-charge theory based on a hydrodynamic formulation of quantum mechanics without the use of a potential was analyzed in ref. 22. Internal consistency of the theory again requires the charge-quantization condition.

Group-theory approach. The first group-theory approach to the Dirac quantization condition was that of Fierz,[140] who showed that the eigenfunctions of the Hamiltonian for an electrically charged particle in the field of a fixed monopole (see Subsection 5.2) form a space of group representation 0(3) if and only if the parameter $\mu = eg/4\pi$ is an integer or a half-integer. That the charge-quantization condition is a necessary consequence of rotational invariance was analyzed earlier by Hurst[146] (see also refs. 21 and 58). An important consideration here is that the components of the angular-momentum operator J form O(3) algebra and commute with the Hamiltonian at all points of space except on the Dirac string. It is necessary to contract the regions in which H and J are defined, which is bounded by functions which fall off sufficiently rapidly on the Dirac string. Here H and J can be self-conjugate operators satisfying the commutation relations $[H, J] = 0$, $[J_k, J_l] = i\varepsilon_{k,ln}J_n$. This statement is true for any μ. However, if we also require that J_k not only generate the algebra of Lie group O(3) but also lead to finite representations of the rotation group, we conclude that μ can take on only quantized values: $\mu = 0, \pm 1/2, \pm 1, \ldots$.

Since the approach of Fierz and Hurst is based on the explicit use of a vector potential, there may be some doubt that the results are independent of the presence of a singularity line. It is therefore worthwhile to analyze the problem of the e–g interaction within the framework of a purely algebraic approach in the Heisenberg picture, in which it is sufficient to determine the temporal-translation operator and the commutation relations for the operators corresponding to the coordinates and kinetic momenta.[59,147,149]

Assuming that a point electric charge of mass m is moving in the field of a fixed monopole, we can write the energy operator as

$$H = \frac{1}{2m}\pi^2,$$

where kinetic momentum π is related by the usual relation π to the velocity. We choose the commutation relations for r and π in the form

$$[x_k, x_l] = 0;$$
$$[x_k, \pi_l] = i\delta_{kl};$$
$$[\pi_k, \pi_l] = -ie\varepsilon_{kln}H_n,$$

where $H = gr/r^3$ is a centrally symmetric magnetic field. This choice of energy operator and commutation relations leads to a correct expression for the Lorentz force in the equations[21)]

$$d\pi/dt = i[H, \pi] = (-e/2m)(\pi \times \mathbf{H} - \mathbf{H} \times \pi)$$

and allows us to construct a conserved angular momentum

$$\mathbf{J} = \mathbf{r} \times \pi - eg\hat{\mathbf{r}}/4\pi, \quad (34)$$

whose components satisfy the commutation relations for the O(3) generators,

$$[J_k, J_l] = i\varepsilon_{kln}J_n. \quad (35)$$

From (34) we find

$$\hat{J}\hat{r} = eg/4\pi. \quad (36)$$

Commutation relations (35) along with

$$[\hat{x}_k, \hat{x}_l] = 0;$$
$$[J_k, \hat{x}_l] = i\varepsilon_{kln}\hat{x}_n$$

(where $\hat{x}_k = x_k/r$ are the generators of unit translations in momentum space) form the algebra of three-dimensional Euclidean group E(3), while the operator $\hat{J}\hat{r}$ is one of the Casimir operators of this algebra. Since the numerical values j_0 of such an operator can be only integers or half-integers ($|j_0|$ = 0.1/2, 1, ...), we immediately find the Dirac quantization condition from (36).

Semiclassical analysis. Saha[143] and Wilson[141] proposed a simple and effective derivation of the Dirac relation based on quantization of the projection of the angular momentum of the electromagnetic field produced by a static pairs of electric and magnetic charges.

The following value was found for the projection of the angular momentum of the field on the line connecting the charges:[22)]

$$\mathbf{s} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{H})\, dV = eg\hat{\mathbf{n}}/4\pi, \quad (37)$$

where $\hat{\mathbf{n}} = \mathbf{r}/r$ is the unit vector along the line connecting the charges. According to the rules for quantization of the angular momentum, this quantity can take on only integral and half-integral values, so we immediately find the charge-quantization condition. In an analogous manner we could obtain a generalized Dirac–Schwinger condition by treating two dually charged particles (see, e.g., ref. 166). Using the usual Newtonian equations of motion with a dually symmetric Lorentz force, we can easily see that the role of conserved angular momentum in this case is played by the quantity

$$\mathbf{J} = \mathbf{r} \times \mathbf{p} - [(e_1g_2 - e_2g_1)/4\pi r]\, \mathbf{r}, \quad (38)$$

where $\mathbf{r}$ is the vector distance between the particles. Projecting $\mathbf{J}$ on $\hat{\mathbf{r}} = \mathbf{r}/r$, we find $\mathbf{J}\hat{\mathbf{r}} = (e_1g_2 - e_2g_1)/4\pi$; setting $\hat{J}\hat{r} = n\hbar$ (where n is an integer or half-integer), we find condition (33).

Nevertheless, procedures of this sort cannot be treated as rigorous derivations of the quantization condition. Quantization of the angular momentum necessarily implies that the projections of the angular momentum are generators of the rotation group (or its representations). If we assume $\mathbf{r}$ and $\mathbf{p}$ to be the operators corresponding to the canonical coordinates and momenta, respectively, we find that the components of vector (37) commute with each other while the components of vector (38) do not form an O(3) algebra even though they do not commute.

The Dirac–Schwinger condition has also been derived[150–152] on the basis of quantization of magnetic flux. Of particular interest is the derivation[152] of the Dirac condition as a consequence of the requirement that the singularity line be unobservable in the Aharonov–Bohm interference experiment.[D6] More formal and essentially semiclassical is a derivation of the charge-quantization condition through an analysis of the motion of an electric charge in the field of an infinite magnetically charged plane and an analysis of the motion of a monopole in the field of a plane capacitor.[150,151]

Difference between the Dirac and Schwinger quantization conditions. According to Dirac the charge-quantization condition can obtain both integers and half-integers, while Schwinger rules out the half-integers. In field theory this difference results formally from a different choice of singularity line. It would be desirable to find further evidence in favor of one of these versions. Such attempts have been undertaken on the semiclassical level, but none have been rigorous attempt has been made to relate this difference to the requirement that the theory be rotationally invariant.[147,166] On the other hand, analysis[146,149] has shown that considerations of O(3) invariance necessarily single out sets of integers and half-integers for the quantity $\mu = eg/4\pi$ without discriminating between these two sets.

In principle, the Schwinger condition could also be derived on the basis of a semiinfinite singularity line,[8] through an analysis of the limiting values of curvilinear integrals of the type $\int \mathbf{A} \cdot d\mathbf{l}$, for the case in which the integration contour crosses the singularity line. As Zumino[203] has pointed out, however, the values of integrals in such exceptional situations are poorly defined.

Working on the basis of a dually symmetric theory, Zwanziger[16,21] found half-integral charge quantization. Although he believed this result to be a consequence of the dual symmetry, it seems instead to be due to the use of the Schwinger formalism.

The Dirac condition can be supported by at least the following physical considerations.[148] The monopole theory is noncontradictory if there are no physical effects traceable to the singular field of the Dirac string. However, the cross section for the scattering of an electron wave by a semiinfinite thin solenoid representing the magnetic charge with the string vanishes if the magnetic flux along the string satisfies the condition $e\Phi = 1/2n\hbar c$ (see also ref. D7), which corresponds to the Dirac charge-quantization condition. This result means that the Schwinger condition is not necessary for physical consistency of the theory.

Discussion. The charge-quantization condition can be reached by at least three different paths: 1) from the requirement that phase transformations be single-valued; 2) from the condition of invariance with respect to the spatial-rotation group, and 3) from the quantization of magnetic flux. Although objections can be raised to the rigor of the derivation procedure in each of these approaches, the very fact that there are several logically independent methods for deriving the Dirac–Schwinger relation is a strong argument that this relation is a fundamental element of magnetic-charge theory.

Comparison of the various methods for deriving the charge-quantization condition allows us to distinguish a characteristic tendency: The more rigorous the derivation, the greater the need for mathematical anomalies. In the Dirac–Schwinger formalism the singularity line is such an anomaly. In an alternative approach not explicitly using the singularity line we are confronted with violation of the Jacobi identity for the kinetic momenta of the charged particles: Starting from the standard commutation relations for canonical coordinates and momenta, we find that the commutation rules for the kinetic momenta of charged particles in an electromagnetic field become

$$[p_\mu,\ p_\nu] = -ieF_{\mu\nu}, \tag{39}$$

from which we find

$$\varepsilon^{\alpha\beta\mu\nu}[p_\beta,\ [p_\mu,\ p_\nu]] = -e\partial_\beta \tilde{F}^{\alpha\beta}. \tag{40}$$

The left side of this expression always vanishes by virtue of the Jacobi identity. Whether the right side vanishes, however, depends on the nature of the field equations used. In standard single-charge electrodynamics ($d_\beta \tilde{F}^{\alpha\beta} = 0$) the Jacobi identity is satisfied automatically. When there are magnetic sources ($d_\beta F^{\alpha\beta} = J_g^\alpha$) the use of commutation relations (39) always leads to a violation of the Jacobi identity. But if we are to find the correct expression for the Lorentz force we must impose relations (39).

This discussion clearly shows that the appearance of "pathological" features in electrodynamics with the two types of sources is inescapable if the structure of the equations of motion for charged particles is fixed. Characteristically, use of the charge-quantization condition both eliminates the effect of the singularity line on physical consequences of the theory and eliminates the contradiction with the Jacobi identity.[59,149]

It therefore seems that internally consistent electrodynamics with two types of sources, if possible at all, must contain the Dirac–Schwinger relation.

5. APPLICATIONS OF THE THEORY OF MAGNETIC CHARGE

Discrete symmetries in the theory of magnetic charge. If we assume that the vectors E and H retain their behavior with respect to P and T inversions, we find from the Maxwell equations that the magnetic charge must behave like a pseudoscalar, while the magnetic current density must behave like an axial vector with respect to P and T inversions.

Assuming the magnetic current to be a polar vector, we find violation of P, T parity in electromagnetic processes involving monopoles. The requirement that parity be conserved in electromagnetic interactions, e.g., could serve as a prohibition of monopoles or could significantly reduce the allowed value of the magnetic charge.[155]

Treating the magnetic current as an axial vector in quantum theory implies the following definition for it:[160,161]

$$J_\mu^{(g)} = g\bar{\psi}\gamma_\mu\gamma_5\psi. \tag{41}$$

However, the field ψ in this case corresponds to a massless particle, and there is no classical limit for the theory of magnetic charge an unsatisfactory result.

From (41) for the magnetic current was used in refs. 160 and 161 to analyze the possible violation of charge conjugation in electromagnetic interactions.

In the case of purely electrically and purely magnetically charged particles one can find conserved P- and T-inversion operators without assuming the magnetic charge to be a pseudoscalar. As was shown first by Ramsey,[154] in the presence of magnetic charges the CPT theorem can be generalized by introducing operation M, magnetic charge conjugation (the CPTM theorem).

Using the Majorana representation, we can write

$$C'\colon \mathbf{E},\ \mathbf{H},\ \psi_e,\ \psi_g \to -\mathbf{E},\ \ -\mathbf{H},\ \psi_e^+,\ \psi_g^+;$$
$$P'\colon \mathbf{E}(\mathbf{x}),\ \mathbf{H}(\mathbf{x}),\ \psi_e(\mathbf{x}),\ \psi_g(\mathbf{x})$$
$$\to -\mathbf{E}(-\mathbf{x}),\ \mathbf{H}(\mathbf{x}),\ \gamma^0\psi_e(-\mathbf{x}),\ \gamma^0\psi_g^+(-\mathbf{x});$$
$$T'\colon \mathbf{E}(t),\ \mathbf{H}(t),\ \psi_e(t),\ \psi_g(t)$$
$$\to \mathbf{E}(-t),\ \ -\mathbf{H}(-t),\ \gamma^0\gamma^5\psi_e(-t),\ \gamma^e\gamma^5(\psi_g(-t).$$

With respect to the C'P'T' − Θ transformation we have

$$\Theta\colon \mathbf{E}(t,\mathbf{x}),\ \mathbf{H}(t,\ \mathbf{x}),\ \psi_e(t,\mathbf{x}),\ \psi_g(t,\ \mathbf{x})$$
$$\to \mathbf{E}(-t,-\mathbf{x}),\ \mathbf{H}(-t,\ \ -\mathbf{x}),\ \gamma^5\psi_e^+(-t,\ -\mathbf{x}),\ \gamma^5\psi_g(-t,\ \mathbf{x}).$$

This analysis corresponds to the use of the generalized operations C', P', T' (C' = CM, P' = PM, T' = TM) in the theory including magnetic charge.[16,154,204]

In the theory with electrically and magnetically charged particles there are no conserved P- and T-inversion operators. The extents to which P, T parity is violated can be characterized by the dually invariant quantity $\xi \sim (e_i e_j + g_i g_j) \times (e_i g_j - e_j g_i)$.

Dynamics of dually charged particles. The nonrelativistic quantum Hamiltonian for the relative motion in the x representation, which describes the interaction of two dually charged particles having charges e_1, g_1 and e_2, g_2, can be written in the following form, in terms of potentials (11):

$$H = (\mathbf{p} - \mu\mathbf{D})^2/2m - \alpha/r, \tag{42}$$

where $\mathbf{r}$ is the relative distance, $\mathbf{p} = -i\partial/\partial\mathbf{r}$, $\mathbf{D} = (\mathbf{r}\times\hat{\mathbf{n}}) \times \mathbf{r}\hat{\mathbf{n}}/r\{r^2 - (\mathbf{r}\cdot\hat{\mathbf{n}})^2\}$, and $\alpha = -(e_1e_2 + g_1g_2)/4\pi$, and $\mu = (g_1e_2 - g_2e_1)/4\pi$ are the electric and magnetic interaction parameters, respectively.

The conserved angular momentum operator is

$$\mathbf{J} = \mathbf{r}\times(\mathbf{p} - \mu\mathbf{D}) - \mu\hat{\mathbf{r}}.$$

It was shown in ref. 62 that a state of the discrete spectrum of the Hamiltonian corresponds to the so-called oscillator representations of algebra O(4,2) $\mu = 0, \pm 1, \pm 2, \ldots$. The energy levels are

$$E = (-mc^2\alpha^2/2)\{s + [(j+1/2)^2 - \mu^2]^{1/2}\}^{-2},\quad s = 0,\ 1,\ 2\ldots$$

It was also shown in ref. 62 that group O(4,2) is a dynamic-symmetry group in the relativistic case also. Using the radial-displacement operator $\pi_r = (\hat{\mathbf{r}}\pi + \pi\hat{\mathbf{r}})/2$, we can rewrite Hamiltonian (42) as

$$H = \pi_r^2/2m + (\mathbf{J}^2 - \mu^2)/2mr^2 - \alpha/r,\quad \text{where}\quad \pi = \mathbf{p} - \mu\mathbf{D}. \tag{43}$$

It, following refs. 59 and 61, we supplement (43) with a centrifugal potential $V = \mu^2/2mr^2$, we can completely reconstruct the symmetry characteristic of the purely Coulomb interaction. The bound states will then correspond to representations of the four-dimensional rotation group, and the scattering states will correspond to representations of the Lorentz group.

The states of the discrete spectrum have the interesting property that if μ runs over all allowed values, all irreducible representations $D(n_1, n_2)$ of group O(4) are found, not only the diagonal representations ($n_1 = n_2$) as in the ordinary Kepler problem ($n_1 + n_2 = |\mu|, |\mu|+1, |\mu|+2, \ldots; n_1 - n_2 = \mu$).

The problem of the interaction of two dually charged particles can also be solved exactly[60,62] in the relativistic case.[23)] In particular, for the eigenvalues of the Dirac Hamiltonian we find[60]

$$E_{nj} = mc^2\{1+\alpha^2[n+\{(j+1/2)^2-\mu^2-\alpha^2\}^{1/2}]^{-2}\}^{-1/2}. \quad (44)$$

It is interesting to compare (44) with the expression for the energy spectrum in the ordinary Coulomb problem: $E_{nj} = mc^2\{1+(\alpha_0 z)^2[n+\{(j+1/2)^2-(\alpha_0 z)^2\}^{1/2}]^{-2}\}^{-1/2}$, where $\alpha_0 = 1/137$, and z is the nuclear charge. The parameter γ, which governs the behavior of the radial functions at the origin, is found from the relation $\gamma^2 = (j+1/2)^2 - \mu^2 - \alpha^2$ in this case, so when the Coulomb center contains magnetic charge as well as electric charge, and when the charge-quantization condition holds, the solutions of the Dirac equation will be singular not only for $z \sim 137$, but for any values of z starting with unity.

Zwanziger[59] found a closed expression for the S matrix for the nonrelativistic scattering of two dually charged particles. A specific feature of this problem is that during rotations the amplitude for the spinless particles behaves like a "helicity-flip" amplitude, and the $a|\mathrm{in}>$ and $|\mathrm{out}>$ states do not transform like a product of free-particle states.

This behavior arises because the quantity μ appears in all the expressions as the projection of some additional angular momentum which, however, cannot be assigned to any one of the particles, but is nonvanishing[45] even when the particles are infinitely separated.[24)]

In earlier studies of the nonrelativistic[1,53,54,56,140] and relativistic[56-58] cases of the e−g interaction, it was in fact the states of the continuum which were being studied, since bound states do not arise in a system consisting of a pure electric charge and a pure magnetic charge with any combination of the signs of e and g.

The noncentral nature of the e−g interaction leads to unusual features in the nonrelativistic scattering behavior.

At small scattering angles the amplitude has the quasi-Rutherford form: (see, e.g., ref. 58)

$$f(\theta, \varphi) = i(-1)(eg/4\pi)\exp\{(2ieg/4\pi)\varphi\} \times (eg/(2mvc\cdot\sin^2\theta/2).$$

At large scattering angles the amplitude behavior is very irregular; the irregularities can be seen best from the classical expression for the cross section,[47,58]

$$d\sigma/d\Omega = (eg/mvc)^2\sin\psi/\cos^4\psi\,|\,2\sin\psi\{1-\cos(\pi/\sin\psi)\} - \pi\sin(\pi/\sin\psi)|^{-1}, \quad (45)$$

where $\psi = \mathrm{arcctg}(eg/mvcb)$, b is the impact parameter, L = vb is the magnitude of the orbital angular momentum, and the scattering angle is given by

$$\cos\theta = \sin\psi\sin(\pi/2\sin\psi).$$

It is easy to see that cross section (45) oscillates rapidly as $\theta \to \pi$ and has an integrable singularity at $\theta = \pi$; an infinite set of impact parameters satisfying the condition $\psi_k = \arcsin(1/2k)$, $k = \pm1, \pm2, \ldots$ corresponds to back scattering. The reason for this unusual behavior of the cross section becomes clear when we recall that the classical trajectory of an electric charge in the field of a monopole is not planar, but lies on the surface of a circular cone having a half vertex angle of ψ and whose axis is parallel to the vector $\mathbf{J} = \mathbf{L} - eg\hat{\mathbf{r}}/c$. For each given L (except for L = 0) there is a certain closest-approach distance between the charge and the plane perpendicular to J and passing through the center (the "magnetic-mirror effect"[48]).

<u>Magnetic charge and composite hadron models.</u> According to the approach recently outlined by Schwinger[15,166] and Barut,[167] hadrons should be thought of as magnetically neutral formations consisting of dually charged particles:[25)]

As we mentioned above, these approaches are based on a generalized charge-quantization condition which allows the existence of a new unit of electric charge, e', which differs from the usual unit e ($e^2/\hbar c \approx 1/137$). This can be seen most simply by treating the electric and magnetic charges of a particle as the components of a vector q = (e, g) in a real two-dimensional linear vector space. The vector product of two such vectors satisfies the charge-quantization condition: $q_1 \times q_2 = e_1 g_2 - e_2 g_1 = n\hbar c$. There are only two linearly independent vectors which satisfy the relation $q_1 \times q_2 = \hbar c$, and it is simple to show that any vector can be written as $q_n = Z_{n1}q_1 + Z_{n2}q_2$, $Z_{ni} = 0, \pm1, \pm2 \ldots$. Using q_1 as the purely electric effective charge, i.e., setting $q_1 = (e, 0)$ ($e^2/\hbar c \approx 1/137$), we find $q_2 = (e_2, e^{-1})$, where e_2 is the second quantum of electric charge. Analogously, in addition to magnetic charge g = 137e there must exist a second quantum of magnetic charge.

If the dyons obey Fermi−Dirac statistics, the construction of baryons requires at least two allowed values of the magnetic charge. Otherwise magnetically neutral formations can be constructed only by combining dyons and antidyons, i.e., in this case there will be only mesons.

According to Schwinger, dyons carry magnetic charges $2g_0, -g_0$ and electric charges $2e_0, -e_0, -e_0$. Here $e_0 = e/3$ ($e^2/\hbar c \approx 1/137$) is the new unit of electric charge, and $g_0 = g/3$ ($g^2/\hbar c \approx 36\cdot137$) is the second unit of magnetic charge.

Using nonrelativistic Hamiltonian (42), we can find an estimate for the dyon mass M_D,[166] which, although rough, is completely plausible ($M_D \sim 6$ GeV). Han and Biedenharn[168] showed that this approach could be interpreted within the framework of SU(3) ⊗ SU(3) symmetry for strong interactions.[26)]

This model has several attractive features: There is no difficulty with Fermi–Dirac statistics, and there is an explanation for the fractional electric charges. For mesons the model fits in naturally with the 0^- and 1^- multiplets, predicts the electromagnetic splitting of the K-meson masses, and in principle leads to a qualitative explanation for the violation of CP parity.

This latter circumstance is critical to the model. If we treat the electric dipole moment of the nucleons as in a calculation of magnetic moments in the quark model, we find (E1) of the nucleon to be $\sim g\hbar/mc = 10^{-12} e \cdot cm$,[168] in contradiction of the experimental upper limit of 10^{-22} $e \cdot cm$ for the (E1) of the neutron. In addition, the sign of the electromagnetic mass splitting differs from the experimental sign. According to Schwinger,[166] this discrepancy can be eliminated by introducing an exchange mechanism involving an intermediate magnetic boson, which should be related to the ordinary neutrino field and should decay into a magnetic lepton and a neutrino. The Schwinger approach was analyzed in refs. 168-170, and modifications were proposed in refs. 171 and 172.

Barut[167] based his discussion on a system of two spinless dyons. However, since the Dirac charge-quantization condition was used, the minimum nonvanishing projection of angular momentum $\mathbf{J} = \mathbf{r} \times \pi - \mu\hat{\mathbf{r}}$ in the ground state of the system of two spinless dyons turned out to be 1/2; i.e., this approach leads to a half-integral spin for the composite particle. The dyons are assumed to have charges e_0, g_0 and $e_0 - g_0$, so that we have $e_0 = e/2$ $(e^2/\hbar c \approx 1/137)$, $g_0 \approx 68.5e$. The excited states of this hydrogen-like system (dyonium) form a representation of group O(4,2).[62] In this approach one can explain the dipole magnetic form factor of the proton, obtain a mass spectrum corresponding to linearly increasing Regge trajectories,[167] and obtain parity degeneracy for baryon trajectories. Evidence in favor of the dyon model is the fact that it leads to a quite broad agreement with other approaches in hadron physics (quarks, Regge poles, and higher symmetries).

At present, however, it is difficult to think of the Schwinger–Barut approach as constituting a well-established direction in the field of composite models of hadrons, since many of the arguments on which it is based are semiintuitive and extremely nonrigorous. The basic problem is that there are no reliable methods available for calculating observable effects within the framework or a model which uses magnetic charges having the magnitudes following from the charge-quantization condition.

Magnetic charge and other problems. In 1960, Porter[173] advanced the hypothesis that cosmic rays having energies above 10^{17} eV consist primarily of monopoles accelerated to very high energies by galactic magnetic fields (see also ref. 174). However, the results of refs. 97, 101, 105, and 108 imply that monopoles do not make up any appreciable number of the cosmic rays having energies above 10^{17} eV (see also ref. 180).

Ruderman and Zwanziger[177] suggested a mechanism for the production of high-energy photon showers based on the annihilation of monopoles produced as a result of the interaction of high-energy cosmic rays with the atmosphere.

Parker[179] analyzed various hypotheses for the appearance of galactic magnetic fields to see if they were consistent with the existence of monopoles. Domogatskii and Zheleznykh[178] analyzed the allowed monopole masses and production cross sections on the basis of the hot-universe model.

The consequences of the neutrino's having a magnetic charge were analyzed in refs. 175 and 176. Attempts have been made to construct analogs of magnetic charge in geometrodynamics (refs. 185 and 189; see also ref. 112); the possibility of a relation between monopoles and heavy tachyons has been discussed;[190] and an elementary-particle model has been proposed on the basis of quantization of magnetic flux.[192] Modifications of macroscopic electrodynamics in the presence of a magnetic charge were analyzed in refs. 195-200.

CONCLUSION

Further progress in the study of the magnetic-charge problem, dealing primarily with the outlook for experimental detection of this charge, requires a clarification of the validity of current theoretical descriptions of the Dirac monopole. Although opinion is divided on this question, we can apparently assert that the "first nonvanishing approximation" of the theory of magnetic charge has been found, and the difficulties remaining are related to the inadequacy of the formalism which has been used.

We are immediately confronted with the question of shether we can avoid introducing anomalous elements such as Dirac strings in magnetic-charge theory. At present the answer is that we can avoid the explicit use of singular potentials only by accepting violation of the Jacobi identity for the kinetic-momentum operators of the charged particles, i.e., equivalent mathematical anomalies in fact arise even if singular potentials are not used. We are left with the impression that these singularities cannot be avoided within the framework of the Lagrangian or Hamiltonian formalism. It would be extremely interesting to either seek a formulation of the theory free of such anomalies or find a general proof that such a formulation cannot be found.

Certain "innate" difficulties of electrodynamics with electric charges [that of eliminating the effect of the singularity line on the physical consequences of the theory; the violation of the Jacobi identity; the absence of O(3) and Lorentz invariance; etc.] can be handled successfully on the basis of charge quantization. This condition therefore seems to be a necessary element of any physically plausible magnetic-charge theory. As Goldhaber[58] has correctly noted, the detection of monopoles not satisfying the Dirac–Schwinger relation would cast doubt on even the standard postulates of quantum mechanics.

Nevertheless, we need a further analysis of the corresponding versions of the derivation of this relation and of a possible relation among these versions, and we need to seek new methods for establishing this relation. In fact there are two charge-quantization conditions, one of which does not allow half-integers.[27)] What is the actual meaning of this restriction ? If we believe in the possibility of an experimental detection of the Dirac monopole, this question is far from academic, since such a procedure could lift the threshold for the monopole by a factor of 4 under otherwise equal conditions.

Although an internally consistent quantum field theory can be constructed with the Dirac–Schwinger condition, and a Lorentz-invariant S matrix can be constructed, we still lack a satisfactory calculation scheme. The problem is due not only to the lack of calculation methods suitable for a large coupling constant;[28)] it is no less important that the mechanism for the electromagnetic interaction between electric and magnetic charges is not completely clear. At any rate, this mechanism differs from the ordinary picture of photon exchange in single-charge quantum electrodynamics. In the phenomenological theory of e–g scattering this difference is manifested in the presence of an additional angular momentum (see Section 5), which should apparently lead to fundamental difficulties in the construction of a relativistic S matrix (difficulties in describing free two-particle states,[58,59] violation of crossing symmetry,[58] etc.). All these difficulties are present in the theory of dually charged particles.

The requirement that electrodynamics have dual symmetry is not, paradoxically, the basic argument in favor of the Dirac monopole. If, e.g., we were able to show that the universality of the ratio g/e for dually charged particles is a necessary condition, dual symmetry would be retained in electrodynamics even in the presence of sources. Nevertheless, there would be no formal basis whatsoever for introducing monopoles.

We also note that there would be no need for an atomic magnetic charge if we were able to find a theoretical basis for quantization of electric charge independent of the monopole concept.[29)] The magnetic-charge concept has a broad range of applications, but they remain extremely problematical. The most promising application – the dyon model of hadrons – is particularly primitive.

The monopole is undoubtedly an "exotic" particle, but the magnetic-charge problem itself is hardly exotic, since it touches on one of the fundamental elements of modern theoretical physics: the equations of electrodynamics. Analysis of possible modifications of contemporary electrodynamics could lead to a deeper understanding of its structure and could stimulate a search for new mathematical formulations of the theory and perhaps methods to generalize it.

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1)Here we use the metric $g^{\mu\nu} = \{-1, 1, 1, 1,\}$ and the Heaviside unit system, and wherever it causes no confusion we set $\hbar = c = 1$.

2)For a relativistically covariant generalization of (6) see refs. 2 and 34.

3)It was shown in ref. 203 how this circumstance must be taken into account in the theory of the Dirac monopole.

4)The behavior of a monopole in matter was analyzed in detail in refs. 68, 90, and 204.

5)Its polarization differs from that of the Cerenkov radiation of an electrically charged particle ($\mathbf{E} \to \mathbf{H}$, $\mathbf{H} \to -\mathbf{E}$ 75, 76), and its intensity is roughly 4700 times that of an electron moving at the same velocity.

6)Experimental efforts to find the monopole have been reviewed in detail in refs. 206 and 207; see also refs. 104, 109, 110, 202, and 204.

7)An upper limit was found in ref. 194 for the cross section for the production of monopoles by cosmic neutrinos.

8)The results of the search for particles having a magnetic charge of g = (1-7)e were reported in ref. 102.

9)According to various estimates, the mass of the magnetically charged particles could be 6 GeV,[166] 10 GeV,[177] 17 GeV,[168] 25 GeV,[83] or higher.[83,178,191]

10)In ref. 110 the experimental studies were classified on the basis of the assumed properties of the monopole.

11)The relationship between the dual symmetry of the Maxwell equations and the conservation of the difference between the numbers of left-hand and right-hand polarized photons was apparently first pointed out in ref. 115.

12)This approach can be used to solve the familiar problem of the discrepancy between the number of independent elements of the energy–momentum tensor (5) and those of the field tensor (6).[111,112,122,127,136]

13)The question of observables in classical electrodynamics was first analyzed sufficiently thoroughly in ref. 121.

14)To check the universality of the ratio g/e for all known particles it would be sufficient to measure the electric and magnetic charges of the proton, neutron, and muon with respect to the electron.[205]

15)The approach of Cabibbo and Ferrari[3] can be used to generalize the Dirac charge-quantization condition.[20,138]

16)These latter two circumstances have been used profoundly in the dyonium models of Schwinger[15,166] and Barut[167] (see Section 5).

17)Interestingly, in the construction of the S matrix the condition g/e = univ is not an a priori assumption, but follows from the requirement that the theory be Lorentz-invariant [under the condition of explicit O(3) invariance].

18)In refs. 35 and 39 the equivalence of the two formulations of electrodynamics was analyzed on the basis of a Lagrangian approach on the classical level (see also refs. 43, 132, 133, and 137)

19)This formalism was used in ref. 10 for the case g/e = univ.

20)We will see below that violation of the Jacobi identity is characteristic of quantum theories with two types of sources.

21)The same commutation relations could be obtained formally by starting from the canonical momentum $\mathbf{p} = \boldsymbol{\pi} + e\mathbf{A}$, which includes a potential.

22)This equation was first derived by Thomson.[45]

23)The case treated in ref. 60 corresponds to the motion of a "pure" electric charge –e in the field of an electrically charged monopole $e_1 = (ze, g)$.

24)This result follows from the fact that the angular momentum of field (37), formed by a static e–g pair, does not depend on the separation of charges.

25)Pertinent in this regard is Dirac's 1948 assertion that elementary particles with poles can be assumed to form an important constituent part of protons.

26)The antibaryon (the magnetic charges of the dyons are $-2g_0$, g_0, g_0) in this case is a particle radically different from the baryon (for which case the magnetic charges of the dyons are $2g_0$, $-g_0$); this circumstance can in principle be used, according to Schwinger, to interpret the empirical properties of the baryon charge.

27)According to Schwinger, only integers are allowed for dually charged particles.[8,166]

28)Barut[167] has taken a definite step toward the derivation of a suitable method.

29)Curiously, Dirac[183] attempted to explain the quantization of electric charge within the framework of single-charge electrodynamics (see, however, ref. 186). Yang[193] dealt with the same problem.

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