NEUTRON ACCUMULATION IN A BOTTLE AND MEASURING THE PHASE OF STRUCTURE AMPLITUDES IN NEUTRON DIFFRACTION

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There is an analysis of the possibility of determining the phases of structure amplitudes during the diffraction of polarized neutrons by a crystal whose elementary cell contains a paramagnetic center. There is a discussion of the limiting density of neutrons in a bottle.

1. MEASURING THE PHASES OF STRUCTURE AMPLITUDES

In a diffraction experiment, the quantity which is measured is the scattering intensity, which is proportional to the square modulus $|F(\varkappa)|^2$ of the structure amplitude

$$F(\varkappa)=\sum_j b_j e^{i\varkappa r}, \tag{1}$$

where b_j and r_j are the coherent-scattering amplitudes and radius vectors of the scattering centers, and $\varkappa$ is the change in the neutron wave vector resulting from the scattering. In order to determine the crystal structure, i.e., to determine all the r_j one must know phases as well as the moduli of the structure amplitudes. If these phases are not known, it is an extremely difficult problem to decipher complex crystal structures. Although there are methods available for obtaining partial information about the phases [1, 2], no general solution of the phase problem has yet appeared.

In this connection it seems worthwhile to consider some other possibilities which arise during the interaction of neutrons with matter.

We consider a crystal whose elementary cell contains one or several paramagnetic centers, e.g., rare earth atoms or atoms from the iron group. In a sufficiently intense magnetic field H and at a low temperature, the atomic magnetic moments are aligned, and coherent magnetic scattering of neutrons occurs with an amplitude $\pm b_m$, where the sign depends on the sign of the projection of the neutron spin on the field direction. In this case the structure amplitude can be written as

$$F=F_N e^{i\varphi_N} \pm F_M e^{i\varphi_M}, \tag{2}$$

where F_N and F_M are positive numbers, the $F_N \exp(i\varphi_N)$ term results from the nuclear scattering by all the atoms, and the $F_M \exp(i\varphi_M)$ term results from magnetic scattering by the paramagnetic centers. The reflection intensity is proportional to

$$|F|^2=F_N^2+F_M^2 \pm 2F_N F_M \cos(\varphi_N-\varphi_M). \tag{3}$$

Carrying out measurements with polarized neutron beams with each polarization, we find each of the quantities $F_N F_M \cos(\varphi_N - \varphi_M)$ and $F_N^2 + F_M^2$. Coherent magnetic scattering does not occur if the sample is not

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magnetized (H = 0) or if it is magnetized in the direction of the scattering vector $\varkappa$. The quantity F_N^2 is determined from the scattering intensity under these conditions. Combining this result with the data of the previous measurements, we find F_M^2 and $\cos(\varphi_N - \varphi_M)$; i.e., we determine within a sign the phase shift of the nuclear and magnetic components of the structure amplitude: $\varphi_N - \varphi_M = \pm\delta$. Since there are, by assumption, few magnetic atoms in the elementary cell, we can find the positions of these magnetic atoms and calculate the phases from the measured F_M^2 values for a sufficient number of reflections. In the case of a crystal which does not have a symmetry center, the nuclear scattering phase is found ambiguously: $\varphi_N = \varphi_M \pm \delta$. Additional considerations are required to resolve this ambiguity (see, e.g., [2]). If, on the other hand, the crystal has a symmetry center, the phase can assume only one of two values: $\delta = 0$ or $\delta = \pi$, and there is no ambiguity in the determination of φ_N.

This method of determining the phases is based on the ability to control the amplitude of the coherent magnetic scattering of neutrons. A "controllable" scattering amplitude is also found in the scattering of polarized neutrons by polarized nuclei during the so-called Schwinger scattering of polarized neutrons or during the diffraction of neutrons which are resonant for one of the isotopes in the crystal. The amplitude of the scattering resonance, of course, changes sign as the neutron energy passes through the exact resonance. An analogous effect occurs during the elastic resonant scattering of γ rays, i.e., in the Mossbauer effect.*

In principle, the phases of the structure amplitudes can be determined by any of these methods for controlling the scattering amplitude, but in each method certain difficulties arise. It would seem worthwhile to determine whether these methods are useful in practice for creating intense neutron sources for neutron spectroscopy, with an account of the progress which has been made in polarization techniques for atomic electrons and nuclei.

2. ACCUMULATION OF NEUTRONS

Neutrons can be accumulated in a closed bottle at a velocity lower than the limiting velocity [3]

$$v_{li} = 2\hbar(\pi N b)^{1/2}/m, \tag{4}$$

where m is the neutron mass, N is the number of nuclei per cubic centimeter, and b is the coherent scattering length. For beryllium, e.g., we have $v_{li} = 6.8$ m/sec.

The basic limitation on the number of neutrons which can be accumulated in a bottle is imposed by Fermi statistics: the neutron velocity at the boundary of the Fermi distribution must be smaller than the limiting containment velocity v_{li}. This limit is unachievably high; for a beryllium model, e.g., the limiting density is $4 \cdot 10^{16}$ neutrons/cm^3.

What limit is imposed on neutron accumulation by the apparatus and techniques currently available?

The density of ultracold neutrons within a moderator is

$$n = \frac{4}{3\sqrt{\pi}} n_T (E_{li}/T)^{3/2}, \tag{5}$$

where $E_{li} = mv_{li}^2/2$; T is the temperature of the neutron gas (the thermal-neutron spectrum is assumed Maxwellian), and n_T is the density of thermal neutrons.

A thermal-neutron flux of $\Phi_T = 10^{15}$ neutrons/(cm^2 · sec) corresponds to a density of $n_T = 4 \cdot 10^9$ neutrons/cm^3. As the moderator is cooled, the density remains constant if the thermal-neutron diffusion length is small in comparison with the moderation length and with the size of the moderator; otherwise, the flux remains constant. Let us adopt the first, less favorable assumption. Substituting T = 20°K and $E_{li} = 2.4 \cdot 10^{-7}$ eV (for beryllium) into Eq. (5), we find $n = 5 \cdot 10^3$ neutrons/cm^3.

The density of ultracold neutrons achievable cannot exceed that in the moderator, regardless of the accumulation method used – direct extraction, as in [4-6], or the method of slowing the faster neutrons by means of moving mirrors, as suggested in [7-9].

* The use of resonant (anomalous) scattering of neutrons was discussed in [2]. According to Meyer-Leibnitz, Mossbauer attempted to use resonant scattering of γ rays to determine phases.

This assertion is a consequence of the familiar Liouville theorem regarding constancy of the distribution function in phase space. We assume $d\nu = \rho(p, q, t)dpdq$ to be the number of particles in a phase-volume element dpdq at time t. According to the Liouville theorem, we have ρ = const along a phase trajectory, i.e., $\rho(p', q', t') = \rho(p, q, t)$, where p', q' is the point in phase space which the particles pass at time t'. The theorem holds if the forces acting on a particle are governed by a potential, which may be time-dependent. Mirror reflection of neutrons can be thought of as the effect of a potential $U = 2\pi\hbar^2 N(x)b/m$, i.e., the Liouville theorem remains valid in the case of moving mirrors and in the case of alternating magnetic fields.

The integral density of ultracold neutrons is

$$n = \int \frac{d\nu}{dq'} dp' = \int \rho(p', q')\, dp' = \langle \rho(p', q') \rangle\, 4\pi/3p_{1i}^3,$$

where $p_{1i} = mv_{1i}$, and $4\pi/3p_{1i}^3$ is the volume occupied by the ultracold neutrons in momentum space.

For a gas of thermal neutrons we have $\rho(p, q) = \text{const}\, e^{-E/T}$. Since we have $\rho(p', q') = \rho(p, q) = \text{const}\, e^{-E/T}$, the density of ultracold neutrons does not depend on the initial energy E of the neutrons which are moderated to the ultracold region if $E \ll T$; if $E \gtrsim T$, this density decreases with increase in E.

Although mechanical neutron moderators in principle offer no advantage over the use of "prepared" ultracold neutrons from a reactor, in practice these moderators may turn out to be useful. Because the absorption cross section increases in proportion to 1/v, direct extraction of ultracold neutrons from a thick-walled bottle with a liquid-hydrogen moderator is hardly possible; on the other hand, cooling of the moderator may increase the ultracold-neutron yield by up to two orders of magnitude.

According to Meyer-Leibniz, the construction of a "Steyerl turbine" [7] has been planned for the powerful new reactor at Grenoble for precisely this purpose.

The magnetic bottles proposed by Vladimirskii [10] are interesting for use in increasing the density of accumulated neutrons. In a toroidal bottle only the transverse (radial) component of the neutron momentum is limited ($p_r^2/2m \lesssim \mu H$): the longitudinal (tangential) component is limited only by the diameter of the orbit.

An increase in the volume in momentum space leads to an increase in neutron density; an increase on the order of ten seems possible.

The density of accumulated neutrons can be increased through the use of a pulsed reactor.* If we assume n_T in Eq. (5) to represent the average thermal-neutron density (governed by the average reactor power), the density corresponding to a reactor pulse is

$$n_{pul} = n\frac{\theta}{\tau}, \qquad (6)$$

where τ is the average neutron lifetime in the moderator, and θ is the time interval between pulses.

If the bottle is rapidly isolated after the reactor pulse, the density of accumulated neutrons has a limiting value corresponding to the pulsed power of the reactor. Losses apparently occur because of, e.g., the stretching of the neutron pulse at the entrance into the bottle (due to the velocity scatter of the neutrons), absorption of neutrons in the bottle walls, etc. Without going into detail, we state that an optimistic estimate of the gain is $\theta/10\tau$.

The IBR-30 reactor of the Neutron Physics Laboratory, Joint Institute for Nuclear Research, Dubna, operates at an average power of 30 kW and a maximum time of θ = 10 sec between pulses; the average thermal-neutron density in the moderator is $n_T = 6 \cdot 10^6$ neutrons/cm^3. As techniques are improved, each of the quantities θ and n_T can probably be increased by a factor of 10. In this case, assuming $10\tau = 2 \cdot 10^{-3}$ sec and using Eq. (5) at T = 20°K, we find an ultracold-neutron density of $n = 4 \cdot 10^6$ neutrons/cm^3.

With this possibility in mind, it would be interesting to analyze the possible use of ultracold-neutron bottles as neutron targets for various types of particle beams and as a neutron source for neutron microscopy.

* An extremely simple method for extracting ultracold neutrons from a pulsed reactor was discussed in [11].

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