SELF-SIMILARITY, CURRENT COMMUTATORS, AND VECTOR DOMINANCE IN DEEP INELASTIC LEPTON-HADRON INTERACTIONS

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A general approach based on the principle of approximate self-similarity, current algebra, and vector dominance is developed for studying inelastic lepton-hadron interactions. Since the form factors for deep inelastic electromagnetic and weak interactions are self-similar, the number of independent variables in the asymptotic region can be reduced by one. Combined with current algebra, this circumstance leads to special sum rules which can in principle be used to solve the fundamental question of the structure of the electromagnetic or weak hadron current. It is shown that the mechanism for the violation of self-similarity or invariance is related to violation of conformal symmetry up to the Poincaré symmetry group. The formation of a muon pair in a deep inelastic proton-proton collision, p + p → $\mu^+ + \mu^-$ + hadrons, is discussed in detail.

INTRODUCTION

A basic approach in the theory of elementary particles is to study the behavior of electromagnetic and weak interactions at high energies. Figure 1 illustrates the most general case of an interaction of a lepton pair with a hadron system; this interaction factors into lepton and hadron parts:

$$T_{fi} = cL^{\mu}H_{\mu}. \tag{1.1}$$

The specific forms of the "coupling constant" c and the lepton part L^{μ} are well known; the hadron part presents more difficulties. By analogy with electrodynamics, for which the local currents correctly describe the phenomena, we postulate that there exist operators corresponding to local hadron currents – the electromagnetic operator $J_{\mu}^{em}(x)$ and the weak operator $J_{\mu}^{W}(x)$. These operators have a definite experimental meaning: Their matrix elements are directly related to observables (cross sections, polarizations, etc.). These quantized currents are expressed most simply in the Bogolyubov formulation of field theory, where they arise as the response of a particle system to an unquantized external perturbation:

$$J_{\mu}(x) = (\Box - m^2) A_{\mu} = \frac{1}{i} S^{+} \frac{\delta S}{\delta A_{\mu}^{\text{ext}}} \Big|_{A_{\mu}^{\text{ext}} \equiv 0}. \tag{1.2}$$

The factors in the matrix element can thus be written

$$\overset{\text{em}}{_w c} = \begin{cases} \frac{4\pi\alpha}{q^2} \\ G/\sqrt{2} \end{cases}; \quad L^{\mu} = \begin{cases} \bar{u}\gamma^{\mu}u \\ \bar{u}\gamma^{\mu}(1-\gamma_5)u \end{cases}; \quad H_{\mu} = \begin{cases} \langle f | J_{\mu}^{\text{em}}(0) | i \rangle \\ \langle f | J_{\mu}^{w}(0) | i \rangle \end{cases}. \tag{1.3}$$

Joint Institute for Nuclear Research, Dubna. Translated from Problemy Fiziki Élementarnykh Chastits i Atomnogo Yadra, Vol. 2, No. 1, pp. 5–32, 1971.

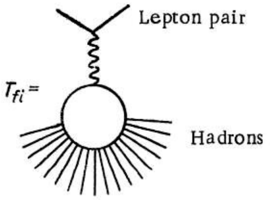

Fig. 1

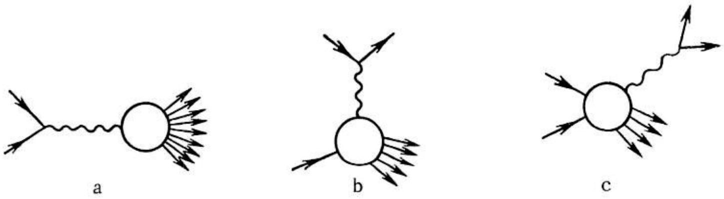

Fig. 2

Fig. 1. The matrix element T_{fi}, describing an arbitrary electromagnetic or weak interaction of a lepton pair and a hadron system.

Fig. 2. Matrix elements for processes a–c. a, c) q^2 is time-like, $q^2 > 0$; b) q^2 is space-like, $q^2 < 0$.

Although the explicit form of the hadron part of the matrix element is not yet known, we can find concrete information about the hadron part by using the requirements of the relativistic P, C, and T covariance and the selection rules which follow from the existence of internal SU(2) or SU(3) symmetry. We are left with the fundamental theoretical difficulty—the lack of a quantitative understanding of the dynamics of strong interactions. This leads to the appearance of unknown functions, the so-called structure functions or form factors, in the theory. A familiar example is presented by the electromagnetic form factors of the nucleon, which depend only on the Lorentz-invariant variable $G_E(q^2)$ or $G_M(q^2)$. In general, the form factors may depend on several Lorentz-invariant variables. The fundamental problem actually consists of a theoretical and experimental study of these form factors. Such studies will hopefully lead to a solution of such fundamental problems as particle structure and the possible existence of hadron subparticles (quarks, partons, etc.).

The importance of studying deep inelastic processes has been emphasized in several papers [1–5]. General methods have been worked out [4] for studying deep inelastic strong interactions, and rigorous calculations have been carried out for the amplitudes. These methods can also yield useful information in a study of the behavior of the form factors for deep inelastic lepton-hadron interactions. We can list some of the specific deep inelastic lepton-hadron interactions which can be studied experimentally. Depending on which particles in Fig. 1 are considered to be entering and which are considered to be leaving, these interactions can be divided into three groups: a) those involving annihilation of a lepton pair, b) those involving scattering of a lepton by a hadron, and c) those involving the formation of a lepton pair in the collision of two hadrons. The corresponding matrix elements are shown in Fig. 2.

The interactions studied most intensively in recent years are electromagnetic and weak scattering, which correspond to the diagram in Fig. 2b. Deep inelastic scattering of electrons by protons,

$$e^- + p \rightarrow e^- + \text{hadrons}, \tag{1.4}$$

has been studied experimentally at SLAC [6]. An extremely interesting "point" pattern for electron creation has emerged. The differential cross section $d\sigma/dq^2$ for large q^2 has turned out to be large, roughly equal to the Mott cross section for scattering by a structureless nucleon. Accordingly, several theoretical concepts have been advanced and checked [1–5]. An analogous point pattern was observed at CERN in experiments involving deep inelastic scattering of a neutrino by a nucleon [7, 8]:

$$\nu_\mu + N \rightarrow \mu^- + \text{hadrons}. \tag{1.5}$$

The simplest explanation for these factors is based on the assumption that, as the number of channels increases, the net contribution of the channels to the form factors depends weakly on q^2. For a point nucleon and for the case of neutrino creation, we have, on the basis of the simplest perturbation-theory diagram,

$$\sigma_{theo}(E) = 1.3 \cdot 10^{-38} E \text{ cm}^2,$$

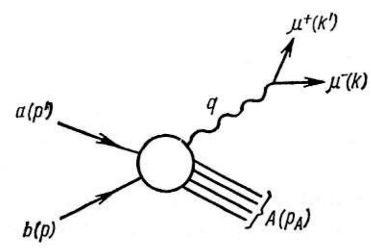

Fig. 3

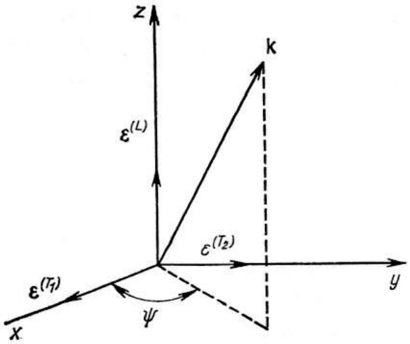

Fig. 4

Fig. 3. Kinematics of the formation of a lepton pair.

Fig. 4. The c.m. system of the muon pair. The z axis lies along the momentum **p**, while the momentum **p'** lies in the xz creation plane. The normal to the creation plane lies along the y axis.

where E is the laboratory neutrino energy (GeV). The CERN experiments yield

$$\sigma_{\text{exp}}(E) = (0.8 \pm 0.2) \cdot 10^{-38} \cdot E \ \text{cm}^2 = 0.6\sigma_{\text{theo}}(E).$$

In principle, this interaction could be studied at neutrino energies of 50 GeV on the accelerator of the Institute of High-Energy Physics.

Below we will discuss in detail the interaction corresponding to the diagram in Fig. 2c — the deep inelastic formation of a muon pair in a hadron-hadron collision:

$$p + p' \rightarrow \mu^+ + \mu^- + \text{hadrons}. \tag{1.6}$$

In §2 below we carry out a kinematic analysis of this interaction, and we treat three theoretical schemes, based on self-similarity (§3) current commutators (§4), and vector dominance (§5) in order to obtain dynamic information. This analysis is based on results recently obtained at Dubna [9-13]. An experimental study of interaction (1.6) is currently being carried out on the Brookhaven accelerator, and preliminary data have been reported [14].

There is considerable independent interest in the results obtained in a study of interaction (1.6); these results may be quite valuable in the search for the intermediate W meson, which is formed in strong interactions [15-19].

We note that the next step in the study of interactions (1.4) and (1.5) is to single out one of the hadrons in its final state. The interactions

$$e^- + p \rightarrow e^- + p' + \text{hadrons}, \tag{1.7}$$

$$\nu_\mu + p \rightarrow \mu^- + p' + \text{hadrons} \tag{1.8}$$

were studied theoretically in [20].

2. KINEMATIC ANALYSIS

We consider the deep inelastic collision of two hadrons, *a* and b, which gives rise to a muon pair and some hadron system A:

$$a + b \rightarrow \mu^+ + \mu^- + A. \tag{2.1}$$

In the lower-order approximation in terms of the electromagnetic interaction, this process involves the emission and decay of a virtual photon as depicted in Fig. 3, where the parentheses denote the 4-momenta of the particles. The corresponding matrix element of the T matrix is

$$T_{fi} = \frac{4\pi\alpha}{q^2} j^\mu \langle A \text{ out} | J_\mu(0) | p, p', \text{in}\rangle^c, \tag{2.2}$$

where $j^\mu = u(k)\,\gamma^\mu v(k')$ is the electromagnetic current of the muon pair, $J_\mu(x)$ is the operator corresponding to the electromagnetic hadron current, and $\alpha = e^2/4\pi = 1/137$ is the fine-structure constant. The index "c" reminds us that we are to take into account only the coupled part of the current matrix element. If the colliding particles are not polarized, and if only the muon pair is detected in the final state, the cross section for this interaction can be expressed in terms of the following second-rank tensor:

$$\rho_{\mu\nu}(p, p', q) = \sum_A (2\pi)^4 \times \delta(p + p' - q - p_A)\, \langle p, p', \text{in} | J_\mu(0) | A \text{ out}\rangle \langle A \text{ out} | J_\nu(0) | p, p', \text{in}\rangle^c. \tag{2.3}$$

Because of conservation of electromagnetic current, this tensor must satisfy the condition for gradient invariance, $q^\mu \rho_{\mu\nu} = \rho_{\mu\nu} q^\nu = 0$; from the Hermiticity of $\rho_{\mu\nu} = \rho^*_{\nu\mu}$ we see that the real part of the tensor must be symmetric, and the imaginary part must be antisymmetric, with respect to the interchange $\mu \leftrightharpoons \nu$.

It is convenient to expand tensor $\rho_{\mu\nu}$ in terms of structures [21, 9, 11, 22] corresponding to definite polarizations of the virtual photon. We determine the directions of the three-dimensional polarization vectors $\boldsymbol{\varepsilon}^{(T_1)}$, $\boldsymbol{\varepsilon}^{(T_2)}$, $\boldsymbol{\varepsilon}^{(L)}$ in the rest system $\mathbf{q} = 0$ of the virtual photon, i.e., in the c.m. system of the muon pair, as shown in Fig. 4. Then the corresponding 4-polarization vectors are

$$\varepsilon_\mu^{(T_1)} = \frac{1}{\sqrt{-\left(\mathcal{P}'^2 - \frac{(\mathcal{P}\mathcal{P}')^2}{\mathcal{P}^2}\right)}} \left(\mathcal{P}'_\mu - \frac{\mathcal{P}\mathcal{P}'}{\mathcal{P}^2}\mathcal{P}_\mu\right), \tag{2.4a}$$

$$\varepsilon_\mu^{(T_2)} = \frac{1}{\sqrt{q^2 (pp')^2 - q^2 m^2 m'^2}} \varepsilon_{\mu\alpha\beta\gamma} p^\alpha p'^\beta q^\gamma, \tag{2.4b}$$

$$\varepsilon_\mu^{(L)} = \frac{1}{\sqrt{-\mathcal{P}^2}} \mathcal{P}_\mu, \tag{2.4c}$$

where

$$\mathcal{P}_\mu = p_\mu - \frac{pq}{q^2} q_\mu; \quad \mathcal{P}'_\mu = p'_\mu - \frac{p'q}{q^2} q_\mu. \tag{2.5}$$

It is not difficult to see that the polarization vectors are orthogonal to the virtual-photon momentum q_μ and to each other; their norm is equal to -1:

$$q^\mu \varepsilon_\mu^{(i)} = 0, \quad \varepsilon_\mu^{(i)} \varepsilon^{(i)\mu} = -\delta_{ij}\,(i, j = T_1, T_2, L), \tag{2.6}$$

In addition, the completeness condition holds:

$$\sum_{i = T_1, T_2, L} \varepsilon_\mu^{(i)} \varepsilon_\nu^{(i)*} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}. \tag{2.7}$$

We use these vectors to expand the tensor $\rho_{\mu\nu}$ in terms of the five independent structures:

$$\rho_{\mu\nu} = \rho_{T_1} \varepsilon_\mu^{(T_1)} \varepsilon_\nu^{(T_1)} + \rho_{T_2} \varepsilon_\mu^{(T_2)} \varepsilon_\nu^{(T_2)} + \rho_L \varepsilon_\mu^{(L)} \varepsilon_\nu^{(L)} + \rho_{TL}^{(+)} (\varepsilon_\mu^{(T_1)} \varepsilon_\nu^{(L)} + \varepsilon_\nu^{(T_1)} \varepsilon_\mu^{(L)}) + i\rho_{TL}^{(-)} (\varepsilon_\mu^{(T_1)} \varepsilon_\nu^{(L)} - \varepsilon_\nu^{(T_1)} \varepsilon_\mu^{(L)}). \tag{2.8}$$

The structure functions* (or form factors) ρ_{T_1}, ρ_{T_2}, ρ_L, $\overset{(+)}{\rho}_{TL}$, and $\overset{(-)}{\rho}_{TL}$ are real functions which depend on four unknown Lorentz-invariant variables, which may be chosen to be, e.g., $s = (p_1 + p_2)^2$, q^2, $\nu = pq$, and $\Delta^2 = (p' - q)^2 \equiv m'^2 + q^2 - 2\nu$. Other invariant variables could be selected, e.g., $m_N^2 = (p + p' - q)^2$ (the square of the effective mass of the hadron system) or $\delta = \frac{1}{m} p(p' - q)$ (the energy transfer in the $\mathbf{p} = 0$ lab. system).

We note that in the $\mathbf{q} = 0$ system there is a simple relationship between the spatial components of the tensor ρ_{ij} and the form factors:

$$\| \rho_{ij} \| = \begin{pmatrix} \rho_{xx} & 0 & \rho_{xz} \\ 0 & \rho_{yy} & 0 \\ \rho_{zx} & 0 & \rho_{zz} \end{pmatrix} = \begin{pmatrix} \rho_{T_1} & 0 & \rho_{TL}^{(+)} + i\rho_{TL}^{(-)} \\ 0 & \rho_{T_2} & 0 \\ \rho_{TL}^{(+)} - i\rho_{TL}^{(-)} & 0 & \rho_L \end{pmatrix}. \tag{2.9}$$

Within a normalization factor, this is the density matrix for the virtual photon, specified in a linear basis.†

The angular distribution summed over spins is by definition equal to the ratio of the pentuple differential cross section $\frac{d^5\sigma(s, q^2, \Delta^2, \nu, \theta, \varphi)}{dq^2\, d\Delta^2\, d\nu\, d\Omega}$ to the triple cross section $\frac{d^3\sigma(s, q^2, \Delta^2, \nu)}{dq^2\, d\Delta^2\, d\nu}$:

$$W(\theta, \varphi) \equiv W(\theta, \varphi, s, q^2, \Delta^2, \nu) = \frac{d^5\sigma(s, q^2, \Delta^2, \nu, \theta, \varphi)}{d^3\sigma(s, q^2, \Delta^2, \nu)},$$

$$W(\theta, \varphi) = \frac{1}{4\pi\left(1 - \frac{v^2}{3}\right)\rho} [\rho_{T_1}(1 - v^2 \sin^2\theta \cos^2\varphi)$$

$$+ \rho_{T_2}(1 - v^2 \sin^2\theta \sin^2\varphi) + \rho_L(1 - v^2\cos^2\theta) - \rho_{TL}^{(+)} v^2 \sin 2\theta \cos\varphi], \tag{2.13}$$

where ρ is given by Eq. (2.12), and $v = \frac{|\vec{k}|}{E} = \sqrt{\frac{q^2 - 4m_\mu^2}{q^2}}$ is the velocity of the muons in their c.m. system. By studying this angular distribution we can determine ρ_{T_1}, ρ_{T_2}, ρ_L, and $\overset{(+)}{\rho}_{TL}$, but not $\overset{(-)}{\rho}_{TL}$. The form factor $\overset{(-)}{\rho}_{TL}$ can be found by measuring the polarization of one of the muons along the normal to the creation plane (along the y axis in Fig. 2):

$$\langle S_y \rangle \sim m_\mu E \rho_{TL}^{(-)}. \tag{2.14}$$

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*In the notation of [21], we have $\rho_{T_1} = F_3$, $\rho_{T_2} = F_2$, $\rho_L = F_1$, $\overset{(+)}{\rho}_{TL} = F_4$, $\overset{(-)}{\rho}_{TL} = F_5$.

†Transforming from the linear basis to the helical basis,

$$\varepsilon_\mu^{(\pm 1)} = \mp \frac{1}{\sqrt{2}} (\varepsilon_\mu^{(T_1)} \pm i\varepsilon_\mu^{(T_2)}), \quad \varepsilon_\mu^{(0)} = \varepsilon_\mu^{(L)}, \tag{2.10}$$

we find, following Oakes [21], a relationship between the form factors and the orthonormal matrix elements of the density matrix in the helical basis:

$$\left.\begin{aligned} &\rho^{11} = \rho^{-1-1} = \frac{1}{2\rho}(\rho_{T_2} + \rho_L); \\ &\rho^{00} = \frac{1}{\rho}\rho_L; \\ &\rho^{1-1} = \rho^{-11} = \frac{1}{2\rho}(\rho_{T_2} - \rho_L); \\ &\rho^{10} = \rho^{01*} = -\rho^{-10} = \rho^{0-1*} = -\frac{1}{\sqrt{2}\rho}(\rho_{TL}^{(+)} + i\rho_{TL}^{(-)}), \end{aligned}\right\} \tag{2.11}$$

where

$$\rho = \rho_{xx} + \rho_{yy} + \rho_{zz} = \rho_{T_1} + \rho_{T_2} + \rho_L. \tag{2.12}$$

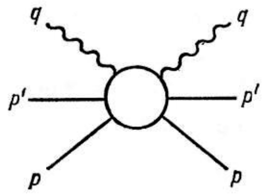


Fig. 5. Amplitude of the Compton effect with two hadrons in the forward direction.

Integrating (2.13) over $d\varphi$ or over $d\cos\theta$, we find the distributions only with respect to θ or φ, respectively:

$$W(\theta)=\frac{1}{2\rho\left(1-\frac{v^2}{3}\right)}\left[(\rho_{T_1}+\rho_{T_2})\left(1-\frac{v^2}{2}\sin^2\theta\right)+\rho_L(1-v^2\cos^2\theta)\right]; \quad (2.15a)$$

$$W(\varphi)=\frac{1}{2\pi\rho\left(1-\frac{v^2}{3}\right)}\left[\rho_{T_1}\left(1-\frac{2}{3}v^2\cos^2\varphi\right)+\rho_{T_2}\left(1-\frac{2}{3}v^2\sin^2\varphi\right)+\rho_L\left(1-\frac{v^2}{3}\right)\right]. \quad (2.15b)$$

The form factors $\rho_{T_1}, \ldots, \rho_{TL}^{(-)}$ have kinematic singularities. The form factors ρ_1, ρ_2, ρ_3, ρ_4, and ρ_5 can be determined from [9]

$$\rho_{\mu\nu}=\rho_1\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\rho_2\mathcal{P}_\mu\mathcal{P}_\nu+\rho_3\mathcal{P}'_\mu\mathcal{P}'_\nu+\rho_4(\mathcal{P}_\mu\mathcal{P}'_\nu+\mathcal{P}_\nu\mathcal{P}'_\mu)+i\rho_5(\mathcal{P}_\mu\mathcal{P}'_\nu-\mathcal{P}_\nu\mathcal{P}'_\mu), \quad (2.16)$$

where $\mathcal{P}_\mu$ and $\mathcal{P}_\nu$ are defined above, in Eqs. (2.5). It is not difficult to find a relationship between these two sets of form factors:

$$\rho_L=\rho_1-\mathcal{P}^2\rho_2-\frac{(\mathcal{P}\mathcal{P}')^2}{\mathcal{P}^2}\rho_3-2\mathcal{P}\mathcal{P}'\rho_4; \quad (2.17a)$$

$$\rho_{T_1}=\rho_1-\frac{(\mathcal{P}\mathcal{P}')^2-\mathcal{P}^2\mathcal{P}'^2}{\mathcal{P}^2}\rho_3; \quad (2.17b)$$

$$\rho_{T_2}=\rho_1; \quad (2.17c)$$

$$\rho_{TL}^{(+)}\pm i\rho_{TL}^{(-)}=\frac{\mathcal{P}\mathcal{P}'}{\mathcal{P}^2}\left[\mathcal{P}^2\mathcal{P}'^2-(\mathcal{P}\mathcal{P}')^2\right]^{1/2}\rho_3+\left[\mathcal{P}^2\mathcal{P}'^2-(\mathcal{P}\mathcal{P}')^2\right]^{1/2}(\rho_4\pm i\rho_5). \quad (2.17d)$$

The triple differential cross section for interaction (2.1) for the case in which only the muon pair with definite q^2, Δ^2, and δ is detected in the final state, and in which the summation has been carried out over all possible hadron states, is

$$\frac{d^3\sigma(s,q^2,\Delta^2,\delta)}{dq^2\,d\Delta^2\,d\delta}=-\frac{\alpha^2}{8\pi^2}\left(1-\frac{q^2-4m_\mu^2}{3q^2}\right)\sqrt{\frac{q^2-4m_\mu^2}{q^2}}\,\frac{1}{\sqrt{s-(m+m')^2}\,\sqrt{s-(m-m')^2}}\,\rho(s,q^2,\Delta^2,\delta), \quad (2.18)$$

where $\alpha = e^2/4\pi = 1/137$; m, m', and m_μ are the masses of hadron b, hadron *a*, and the muon, respectively; and

$$\rho(s,q^2,\Delta^2,\delta)=\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)\rho_{\mu\nu}(p,p',q)=\rho_{T_1}+\rho_{T_2}+\rho_L. \quad (2.19)$$

We note that tensor $\rho_{\mu\nu}$(p, p', q) describes the contents of the hadron "black box" for the Compton effect with two hadrons, depicted in Fig. 5.

The distribution with respect to the square effective mass of the muon pair is found from (2.2) by integration over $d\Delta^2$ and $d\delta$ within the physical region. Neglecting the muon mass, we find the following equation for the muon-pair mass spectrum:*

$$\frac{d\sigma}{dq^2}=-\frac{\alpha}{12\pi^2}\cdot\frac{m}{\sqrt{s-(m+m')^2}\,\sqrt{s-(m-m')^2}}\,\frac{1}{q^2}\int\limits_{\Delta^2_{min}}^{\Delta^2_{max}}d\Delta^2\int\limits_{\delta_{min}}^{\delta_{max}}d\delta\rho(s,q^2,\Delta^2,\delta). \quad (2.20)$$

*The definition of the boundaries of the physical region is discussed in the Appendix.

For use of the vector-dominance hypothesis (see §5), it is convenient to write the mass spectrum as

$$\frac{d\sigma}{dq^2}=\frac{\alpha}{3\pi}\cdot\frac{1}{q^2}\sigma^{\gamma*}(s,\ q^2). \tag{2.21}$$

where

$$\sigma^{\gamma*}(s,\ q^2)=\sigma_{T_2}^{\gamma*}+\sigma_{T_1}^{\gamma*}+\sigma_L^{\gamma*} \tag{2.22}$$

is the total cross section for the creation of a virtual γ^* photon having mass q^2 in the interaction

$$a+b\longrightarrow\gamma^*+\text{hadrons}. \tag{2.23}$$

As the last item in this section, we note that there is an interesting kinematic analogy between the reaction considered here and inelastic neutrino creation: If we replace the square lepton mass m_l^2 by q^2 in the Appendix of Adler's paper [2], and if we replace Adler's q^2 by our Δ^2, we are actually defining the boundaries of the physical region for interaction (2.1). The Appendix below takes up in detail the definition of the boundaries of the physical region. In order to obtain dynamic information about the form factors, we consider below three theoretical schemes, based on: 1) self-similarity or scale invariance, 2) vector dominance, and 3) current commutators.

3. THE SELF-SIMILARITY PRINCIPLE

As we mentioned above, the SLAC and CERN experiments indicate a point nature for deep inelastic interactions of leptons with hadrons. This behavior could be understood on the basis of the hypothesis of approximate self-similarity or scale invariance. We assume that in the description of deep inelastic lepton-hadron interactions, for which the energies and momentum transfers are large, no dimensional quantities, such as the mass, "elementary length," etc. can play important roles, so the form factors can depend only on the kinematic-invariant variables.

N. N. Bogolyubov drew our attention to the possible self-similar behavior of the form factors in these problems. He emphasized that this behavior of the form factors for inelastic weak and electromagnetic interactions might be extremely similar in nature to the so-called self-similar solutions for several problems of classical hydrodynamics, e.g., the problem of a powerful point explosion [24, 25]. In the search for self-similar solutions for hydrodynamic problems, it turns out to be quite useful to use the methods of similarity and dimensionality theory in combination with certain qualitative arguments about the nature of the physical processes. It is well known that electromagnetic and weak interactions can be described quite successfully by means of local electromagnetic and weak currents. The strong interactions, on the other hand, are taken into account through the introduction of form factors. At low energies, the need to take into account the particle masses will presumably distort the strong-interaction picture, while at high energies (and at high values of the other invariant variables), in which case the masses of the particles created can be neglected, the situation becomes much simpler and, in a certain sense, "hydrodynamic." This hypothesis is supported qualitatively by the fact that the principal singularities of the singular functions of field theory are mass-independent (see, e.g., [26]).

We will attempt to discuss the principle of approximate self-similarity or scale invariance with regard to lepton-hadron interactions at high energies and high momentum transfers, and we will derive several consequences which can be checked experimentally. We will assume that the asymptotic behavior of the form factors for interactions involving leptons at high energies and high momentum transfers is governed by dimensionality considerations and by the requirement of approximate invariance under scale transformations $q\rightarrow\lambda q$ and $p_i\rightarrow\lambda p_i$, where q is the momentum transferred from the lepton to the hadron, and p_i are the momenta of the hadrons participating in the reaction.

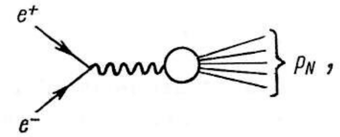
In essence, this assumption means that in the asymptotic limit in which $q^2\rightarrow\infty$ and $qp_i\rightarrow\infty$, the form factors for lepton interactions are governed by functions of the dimensionless ratios $\omega_i = q^2/qp_i$ and are approximately independent of the particle masses and of other dimensional parameters, such as the interaction radius, etc. We emphasize that this principle does not hold for purely strong interactions, since in this case the processes apparently depend strongly on the constant dimensional quantities.

Below we derive several consequences of the principle of scale invariance for interactions involving the annihilation of electron-positron pairs into hadrons, electron creation, and the formation of lepton pairs in two-hadron collisions.

We first consider the simplest deep inelastic process involving leptons—the annihilation of a lepton pair resulting in the production of hadrons:

$$e^+ + e^- \longrightarrow \text{hadrons}.$$

In the one-photon approximation, this interaction corresponds to the diagram



and the total cross section is (for $m_e \equiv 0$)

$$\sigma_{tot} = \frac{8\pi^2\alpha^2}{q^2}\rho(q^2). \tag{3.1}$$

All the information about the dynamics of the process is incorporated in the unknown spectral function (or form factor) $\rho(q^2)$, related by definition to the tensor $\rho_{\mu\nu}(q^2)$:

$$\rho_{\mu\nu}(q) = \int dx\, e^{iq.x} \langle 0 | J_\mu(x) J_\nu(0) | 0\rangle = \sum_N (2\pi)^4 \delta(q - p_N) \langle 0 | J_\mu(0) | N\rangle \langle N | J_\nu(0) | 0\rangle = (-g_{\mu\nu}q^2 + q_\mu q_\nu)\rho(q^2) \tag{3.2}$$

It is easy to calculate the dimensionality of the tensor* $\rho_{\mu\nu}$:

$$[\rho_{\mu\nu}(q)] = [m^2]. \tag{3.3}$$

We see that $\rho(q^2)$ is, as expected, dimensionless:

$$[\rho(q^2)] = 1. \tag{3.4}$$

For scale transformations of the momentum scale,

$$q \longrightarrow \lambda q, \tag{3.5}$$

it follows from an account of the self-similarity principle that

$$\rho_{\mu\nu}(\lambda q) = \lambda^2 \rho_{\mu\nu}(q); \quad \rho(\lambda^2 q^2) = \rho(q^2) = \text{const}. \tag{3.6}$$

At large q^2 the total cross section must thus asymptotically display "point" behavior, analogous to the case of the annihilation of an electron-positron pair resulting in the production a muon pair:

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*We use a system of units here in which action and velocity are dimensionless and in which mass is a dimensional quantity. We recall that in this system the current dimensionality is $[J_\mu] = [m^3]$, and $|n\rangle$, the partial state vector for relativistic invariant normalization, has the dimensionality

$$[|p_1, p_2, p_3, \ldots p_n\rangle] = [m^{-n}].$$

$$e^+ + e^- \to \mu^+ + \mu^-. \tag{3.7}$$

This behavior is the same as that predicted by quark-current algebra [27, 28]. Using an inverse Fourier transformation, we can construct the space-time picture, finding that the commutator of the electromagnetic currents between the vacuum states is

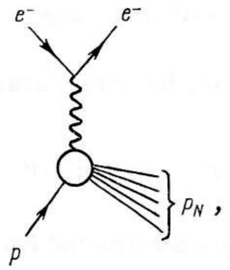
$$\langle 0 \,|\, [J_\mu(x),\ J_\nu(0)] \,|\, 0\rangle = \frac{ic}{\pi}\left(g_{\mu\nu}\Box - \partial_\mu\partial_\nu\right)\delta(\mathbf{x})\,\mathcal{P}\left(\frac{1}{t}\right), \tag{3.8}$$

where $c = \rho(q^2) = \text{const}$; $\Box$ is the d'Alembertian, and $\mathcal{P}$ represents the principal value. In particular, it follows that the simultaneous commutator between the time and space components is

$$\langle 0 \,|\, [J_0(\mathbf{x},\ 0),\ J_i(0)] \,|\, 0\rangle = \lim_{\tau\to 0}\frac{1}{\tau^2}\cdot\frac{ic}{\pi}\ \nabla_i\delta(\mathbf{x}), \tag{3.9}$$

i.e., it is equal to the Schwinger term with a numerical coefficient which diverges as c^2 [29].

In the one-photon approximation, electron creation is described by



and the cross section is expressed in the standard manner (see, e.g., [30]) in terms of the tensor

$$W_{\mu\nu}(p,\ q) = \sum_N (2\pi)^4 \langle p \,|\, J_\mu(0) \,|\, N\rangle^c \langle N \,|\, J_\nu(0) \,|\, p\rangle^c \,\delta(p + q - p_N)$$
$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(pq,\ q^2) + \left(p_\mu - \frac{pq}{q^2}q_\mu\right)\left(p_\nu - \frac{pq}{q^2}q_\nu\right) W_2(pq,\ q^2). \tag{3.10}$$

It is easy to see that this tensor is dimensionless:

$$[W_{\mu\nu}(p,\ q)] = 1; \tag{3.11}$$

it follows that

$$[W_1(q^2,\ pq)] = 1; \quad [W_2(q^2,\ pq)] = [m^{-2}]. \tag{3.12}$$

It follows from self-similarity that for scale transformations

$$q \to \lambda q; \quad p \to \lambda p \tag{3.13}$$

the form factors W_1 and W_2 must satisfy

$$\left.\begin{aligned} W_1(\lambda^2 q^2,\ \lambda^2 pq) &= W_1(q^2,\ pq); \\ \lambda^2 W_2(\lambda^2 q^2,\ \lambda^2 pq) &= W_2(q^2,\ pq). \end{aligned}\right\} \tag{3.14}$$

These requirements can be satisfied by setting

$$W_1(q^2, pq) = F_1\left(\frac{q^2}{pq}\right), \quad W_2(q^2, pq) = \frac{1}{q^2} F_2\left(\frac{q^2}{pq}\right). \tag{3.15}$$

This universal dependence on the single dimensionless variable q^2/pq for the form factor W_2 has actually been observed experimentally at SLAC [4]. Some theoretical arguments in favor of this dependence were advanced in [23]. We note that many attempts are currently being made to find a qualitative explanation for this "point" behavior of the form factors for electron creation for high momentum transfers through the construction of specific models (see, e.g., [31-33]).

We turn now to the formation of a muon pair in a strong interaction. The dimensionality of the tensor $\rho_{\mu\nu}$ and thus of the form factors ρ_i is

$$[\rho_{\mu\nu}] = [\rho_i] = [m^{-2}]. \tag{3.16}$$

Using the requirement of self-similarity, and taking account of (3.2), we find

$$\left.\begin{aligned} \rho_{\mu\nu}(\lambda p, \lambda p', \lambda q) &= \lambda^{-2}\rho_{\mu\nu}(p, p', q); \\ \rho_i(\lambda^2 s, \lambda^2 q^2, \lambda^2\Delta^2, \lambda^2\delta) &= \lambda^{-2}\rho_i(s, q^2, \Delta^2, \delta), \end{aligned}\right\} \tag{3.17}$$

from which it follows that the form factors are, for large invariants,

$$\rho_i(s, q^2, \Delta^2, \delta) = \frac{1}{q^2} F_i(\alpha, \beta, \omega), \tag{3.18}$$

where α, β, and ω are dimensionless variables constructed from ratios of the invariants s, q^2, Δ, and δ.

The self-similarity principle can also be used to analyze the behavior of the form factors for weak interactions. There is considerable interest in an experimental check of the consequences of the principle of approximate self-similarity for deep inelastic scattering of neutrinos by nucleons. We note that the self-similar nature of the form factors for the electromagnetic and weak interactions permits the number of independent variables in the asymptotic region to be reduced by one; moreover, knowing the form factors for one set of invariants, we can predict them for another set if certain ratios remain fixed.

We believe it would be very interesting to experimentally check the behavior predicted by the self-similarity principle up to certain large values of the invariants. Deviations from these predictions could be interpreted as evidence that some dimensional factor, e.g., the "elementary length," etc. comes into play here and violates self-similarity over supersmall distances.

We have thus far treated the possible existence of "maximum self-similarity," i.e., self-similarity with respect to all variables. The possibility is not ruled out that "partial self-similarity" may occur, in which case the self-similarity would not hold for all variables, but only for some of them. In particular, it would of course be extremely interesting to understand the mechanism for the violation of the self-similarity principle and to work out methods for calculating the corrections to the self-similar approximations. This question is apparently closely associated with the concept of the spontaneous violation of conformal symmetry up to the symmetry of the Poincaré group.

Conformal symmetry is one of the possible generalizations of Poincaré symmetry which is of physical interest. We briefly review the basic information about the conformal group, which contains as one transformation the space-time scale transformation. This 15-parameter group includes the following transformations [34-37]:

1. Space-time translations (four parameters),

$$x'^{\mu} = x^{\mu} + \alpha^{\mu}. \tag{3.19}$$

2. Homogeneous Lorentz transformations (six parameters),

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}, \ \Lambda^{\mu}_{\nu} \in 0 \ (3.1). \tag{3.20}$$

3. Special conformal transformations (four parameters),

$$x'^{\mu} = \frac{x^{\mu} + \beta^{\mu} x^2}{1 + 2\beta x + \beta^2 x^2}. \tag{3.21}$$

4. Scale transformations (one parameter),

$$x'^{\mu} = \rho x^{\mu}, \quad \rho > 0. \tag{3.22}$$

According to the Noether theorem, local currents correspond to these transformations; in particular, the currents of the special conformal transformations $C^{\mu\nu}$ and scale transformations are expressed in the following manner in terms of the moments of the energy-momentum tensor (the gravitational current):

$$C^{\mu\nu} = \theta^{\mu\alpha}(2x^{\nu}x_{\alpha} - g^{\nu}_{\alpha}x^2); \tag{3.23}$$

$$S^{\mu} = \theta^{\mu\alpha}x_{\alpha}. \tag{3.24}$$

The generators for the transformations are expressed in terms of the spatial integrals of the zeroth current components:

$$C^{\mu} = \int (2\theta^{\mu\alpha}x^0 x_{\alpha} - \theta^{\mu 0}x^2)\, d\mathbf{x}; \tag{3.25}$$

$$S = \int \theta^{0\alpha} x_{\alpha}\, d\mathbf{x}. \tag{3.26}$$

It can be shown [34, 38, 39] that the divergence fields of currents (3.23) and (3.24) are related in the following manner over a broad class of Lagrangian theories:

$$\partial_{\mu}C^{\mu\nu} = 2x^{\nu}\partial_{\mu}S^{\mu} = x^{\nu}\theta^{\mu}_{\mu}. \tag{3.27}$$

The vanishing of the current divergences corresponds to conservation of "charges" (3.25) and (3.26). Equations (3.27) show that invariance with respect to the total conformal group follows in this case from scale invariance, so conformal symmetry is violated "minimally" because of the violation of scale invariance. If the Lagrangian is independent of the mass and other dimensional constants, we have $\theta^{\mu}_{\mu} = 0$, and we thus find, as suggested above, scale invariance. The question of the possible spontaneous violation of this symmetry is currently being discussed widely in the literature in connection with the violation of chiral symmetry [35, 38–40].

4. CURRENT COMMUTATORS AND ASYMPTOTIC SUM RULES

We consider the Fourier transform of the matrix element of the electromagnetic-current commutator between the two-particle in-states [9, 11]:

$$R_{\mu\nu}(p, p', q) = \int dx\, e^{-iqx} \langle p, p', \mathrm{in} | [J_{\mu}(x), J_{\nu}(0)] | p, p', \mathrm{in}\rangle^c = r_{\mu\nu}(p, p', q) - r_{\nu\mu}(p, p' - q). \tag{4.1}$$

We assume here that the enclosed particles are not polarized; the "c" indicates that we take that part of the matrix element which is coupled as a whole. Let us consider the quantity $r_{\mu\nu}$ in more detail. Using the completeness condition for the out-state vectors, we find

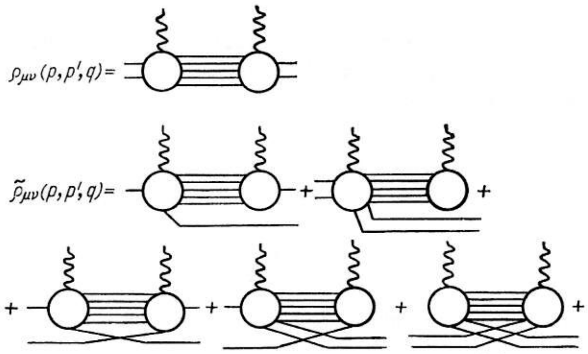
$$r_{\mu\nu}(p, p', q) = \int dx\, e^{-iqx} \langle p, p', \mathrm{in} | J_{\mu}(x) J_{\nu}(0) | p, p', \mathrm{in}\rangle^c$$

$$= \sum_A^c (2\pi)^4 \delta(p + p' - q - p_A) \langle p, p', \mathrm{in} | J_{\mu}(0) | A\ \mathrm{out}\rangle \langle A\ \mathrm{out} | J_{\nu}(0) | p, p', \mathrm{in}\rangle, \tag{4.2}$$

where the "c" on the summation symbol indicates that we take only the matrix elements of the product of two currents which are coupled as a whole. We extract from this sum the completely coupled part corresponding to the quantity*

$$r_{\mu\nu}(p, p', q) = \rho_{\mu\nu}(p, p', q) + \tilde{\rho}_{\mu\nu}(p, p', q), \tag{4.3}$$

where $\tilde{\rho}_{\mu\nu}$ denotes the contribution of 15 z diagrams. This separation can be depicted graphically as



\+ Diagrams obtained from the symmetrization of the initial and final states.

From momentum conservation and the spectrality condition it follows that, for $q^2 > 0$, we have

$$\rho_{\mu\nu}(p, p', q) = \theta(\nu)\,\theta\left((\sqrt{s} - \sqrt{q^2})^2 - m_N^2\right)\rho_{\mu\nu}(p, p', q); \tag{4.4}$$

$$\tilde{\rho}_{\mu\nu}(p, p', q) = \theta(-\nu)\,\theta\left(m_N^2 - (\sqrt{s} + \sqrt{q^2})^2\right)\rho_{\mu\nu}(p, p', q). \tag{4.5}$$

In the physical region the $\rho_{\mu\nu}$(p, p', q) contribution thus vanishes exactly.

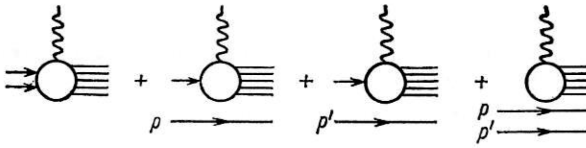
In the derivation of the sum rules (see below), however, we use the entire region—both the physical and the nonphysical parts—so the z diagrams from the second part of the ρ(p, p', −q) commutator may give a nonvanishing contribution. We show below that, with the standard assumptions used in a current-algebra derivation of sum rules, the contribution of these diagrams tends toward zero as s → ∞.

We will show that the problem of determining the behavior of the form factors for the creation of a muon pair at high energies of the colliding hadrons and at high energies and masses of the virtual photon, for which s, q^2, $\nu \to \infty$ (i.e., such that the ratios

$$\alpha = \frac{p'q}{pq} = \frac{m'^2 + q^2 - \Delta^2}{2\nu}, \quad \omega = \frac{q^2}{2\nu} \tag{4.6}$$

---

*If the < A out | state contains particle p or p', the current matrix element < A out | $J_\mu(0)$ | p, p' in > will contain uncoupled parts corresponding to free propagation of these particles. Graphically, the matrix element can be divided into coupled and uncoupled parts in the following manner:



23б.

The first term here, the completely coupled part, is involved in the determination of the cross section for the physical process; the other three terms are uncoupled parts which lead to the appearance of so-called semicoupled z diagrams.

remain fixed), can be reduced to a study of the simultaneous commutation relations between the spatial components of the operator for the electromagnetic hadron current and for its time derivative.

The use of simultaneous commutation relations is much simpler in the c.m. system of the muon pair, where we have $q = \{q_0, 0\}$. In this system the expansion of the tensor $\rho_{ij}(\mathbf{p}, \mathbf{p}', q_0)$, $(i, j = x, y, z)$ becomes

$$\rho_{ij}(\mathbf{p}, \mathbf{p}', q_0) = \rho_{T_1}\delta_{ix}\delta_{jx} + \rho_{T_2}\delta_{iy}\delta_{jy} + \rho_i\delta_{iz}\delta_{jz} + \rho_{TL}^{(+)}(\delta_{ix}\delta_{jz} + \delta_{jx}\delta_{iz}) + i\rho_{TL}^{(-)}(\delta_{ix}\delta_{jz} - \delta_{jx}\delta_{iz}). \qquad (4.7)$$

We see that R_{ij}, r_{ij} and ρ_{ij} can be expanded in a similar manner in terms of the five structures:

$$R_{ij}(\mathbf{p}, \mathbf{p}', q_0) = R_{T_1}\delta_{ix}\delta_{jx} + \ldots\ldots\ldots \qquad (4.8a)$$

$$r_{ij}(\mathbf{p}, \mathbf{p}', q_0) = r_{T_1}\delta_{ix}\delta_{jx} + \ldots\ldots\ldots \qquad (4.8b)$$

$$\hat{\rho}_{ij}(\mathbf{p}, \mathbf{p}', q_0) = \rho_{T_1}\delta_{ix}\delta_{jx} + \ldots\ldots\ldots, \qquad (4.8c)$$

where

$$R_i(\mathbf{p}, \mathbf{p}', q_0) = r_i(\mathbf{p}, \mathbf{p}', q_0) - r_i(\mathbf{p}, \mathbf{p}', -q_0) = \varepsilon(q_0)\rho_i(\mathbf{p}, \mathbf{p}', |q_0|) + \varepsilon(-q_0)\tilde{\rho}_i(\mathbf{p}, \mathbf{p}', -|q_0|), \qquad (4.9)$$

where $\varepsilon(q_0) = \pm 1$, $q_0 \gtrless 0$; $i = T_1, T_2, L$, and $TL^{(+)}$;

$$R_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', q_0) = r_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', q_0) + r_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', -q_0) = \rho_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', |q_0|) + \rho_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', -|q_0|). \qquad (4.10)$$

We see that the quantities R_{T_1}, R_{T_2}, R_L, and $R_{TL}^{(+)}$ are odd functions of q_0, and $R_{TL}^{(-)}$ is an even function. Integrating Eq. (4.1) over dq_0 and $q_0 dq_0$, we find several relations:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dq_0 R_{ij}(\mathbf{p}, \mathbf{p}', q_0) = iB_{ij}(\mathbf{p}, \mathbf{p}'), \qquad (4.11)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} q_0\, dq_0 R_{ij}(\mathbf{p}, \mathbf{p}', q_0) = C_{ij}(\mathbf{p}, \mathbf{p}'), \qquad (4.12)$$

etc. Here we have

$$B_{ij}(\mathbf{p}, \mathbf{p}') = -i\int d\mathbf{x}\,\langle p, p' \text{ in}|[J_i(\mathbf{x}, 0), J_j(0)]|p, p', \text{in}\rangle^c; \qquad (4.13)$$

$$C_{ij}(\mathbf{p}, \mathbf{p}') = -i\int d\mathbf{x}\,\langle p, p', \text{in}|[\dot{J}_i(\mathbf{x}, 0), J_j(0)]|p, p' \text{ in}\rangle^c. \qquad (4.14)$$

Of these relations we retain only those which are nontrivial in terms of parity:

$$\frac{1}{\pi}\int_0^{\infty} dq_0 R_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', q_0) = B_{xz}(\mathbf{p}, \mathbf{p}') - B_{zx}(\mathbf{p}, \mathbf{p}'); \qquad (4.15a)$$

$$\frac{1}{\pi}\int_0^{\infty} dq_0 q_0 R_{T_1}(\mathbf{p}, \mathbf{p}', q_0) = C_{xx}(\mathbf{p}, \mathbf{p}'); \qquad (4.15b)$$

$$\frac{1}{\pi}\int_0^{\infty} dq_0 q_0 R_{T_2}(\mathbf{p}, \mathbf{p}', q_0) = C_{yy}(\mathbf{p}, \mathbf{p}'); \qquad (4.15c)$$

$$\frac{1}{\pi}\int_0^\infty dq_0 q_0 R_L(\mathbf{p}, \mathbf{p}', q_0) = C_{zz}(\mathbf{p}, \mathbf{p}'); \tag{4.15d}$$

$$\frac{1}{\pi}\int_0^\infty dq_0 q_0 R_{TL}^{(+)}(\mathbf{p}, \mathbf{p}', q_0) = C_{xz}(\mathbf{p}, \mathbf{p}') + C_{zx}(\mathbf{p}, \mathbf{p}'). \tag{4.15e}$$

We note that in the $\mathbf{q} = 0$ system selected, the invariant variables on which the form factors depend are

$$s = m^2 + m'^2 + 2(p_0 p_0' - \mathbf{p}\mathbf{p}'); \quad q^2 = q_0^2; \quad \nu = p_0 q_0; \quad \alpha = \frac{p_0'}{p_0}. \tag{4.16}$$

It follows that the variables s and α are fixed in the integration over dq_0, while $q^2 = \nu^2/p_0^2$; i.e., in the (q^2, ν) plane, the integration in (4.15) is carried out along a parabola.

Such sum rules for arbitrary fixed momenta **p** and **p'** contain contributions from the spectral functions $\tilde{\rho}$ of the corresponding z diagrams. As condition (4.5) shows, the contributions of the z diagrams in the limit $s \to \infty$ are governed by the intermediate states of hadrons A with infinitely heavy effective masses m_A. Following the convention of the current-algebra method, we assume that the z-diagram contributions vanish as $s \to \infty$; this assumption is valid when the order of the integration and the transition to the $s \to \infty$ limit in Eqs. (4.15) can be changed. For sum rule, e.g., (4.15a), the z-diagram contribution is given by

$$\frac{1}{\pi}\int_0^\infty \rho_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', -|q_0|)\, dq_0 = -\frac{1}{\pi}\int_s^\infty \frac{dm_N^2}{2E_N}\rho_{TL}^{(-)}(\mathbf{p}, \mathbf{p}', -|q_0|). \tag{4.17}$$

Taking the limit $s \to \infty$ under the integral for fixed m_A^2, and using (4.5), we find that the contribution of the z diagrams to the sum rules vanishes in this limit.

In the c.m. system of the lepton pair, the limiting transition $s \to \infty$ is carried out under the conditions $p_0 \to \infty$, $p_0' \to \infty$. We assume that

$$\left.\begin{array}{llll} \alpha = \dfrac{p_0'}{p_0} & \text{is fixed,} & \beta = \dfrac{p_z'}{p_z} & \text{is fixed.} \\ \omega = \dfrac{q_0}{2p_0} & \text{is fixed,} & & \end{array}\right\} \tag{4.18}$$

The constancy of β in the invariant form means that

$$\frac{s}{\nu} = \frac{\alpha(1-\beta)}{\omega} \quad \text{is fixed.} \tag{4.19}$$

We now assume that there exist limits for fixed α and β:

$$B_{ij}(\alpha, \beta) = \lim_{p_0,\, p_0' \to \infty} p_0 B_{ij}(\mathbf{p}, \mathbf{p}'); \tag{4.20}$$

$$C_{ij}(\alpha, \beta) = \lim_{p_0,\, p_0' \to \infty} C_{ij}(\mathbf{p}, \mathbf{p}'), \tag{4.21}$$

where the tensors on the left side are dimensionless.

We now take the limits $s \to \infty$, $\nu \to \infty$, $q^2 \to \infty$ in the sum rules under the condition that α, β, and ω are constant. In this limit, as was mentioned above, the z-diagram contributions drop out, and the form factors $\rho_i(s, q^2, \alpha, \nu)$ have the following self-similar behavior:

$$\rho_i(s, q^2, \alpha, \nu) = \frac{\omega^2}{q^2} F_i(\alpha, \beta, \omega), \quad i = T_1, T_2, L, TL^{(+)}, TL^{(-)}. \tag{4.22}$$

Converting to an integration over $d\omega$ in Eq. (4.15), we find the following final sum rules relating the limiting self-similar form factors to the current matrix elements:

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega F_{TL}^{(-)}(\alpha, \beta, \omega) = B_{xz}(\alpha, \beta) - B_{zx}(\alpha, \beta); \tag{4.23a}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{T_1}(\alpha, \beta, \omega) = C_{xx}(\alpha, \beta); \tag{4.23b}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{T_2}(\alpha, \beta, \omega) = C_{yy}(\alpha, \beta); \tag{4.23c}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{L}(\alpha, \beta, \omega) = C_{zz}(\alpha, \beta); \tag{4.23d}$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\omega F_{TL}^{(+)}(\alpha, \beta, \omega) = C_{xz}(\alpha, \beta) + C_{zx}(\alpha, \beta), \tag{4.23e}$$

where

$$\omega_0 = \frac{p_0 + p_0'}{2p_0} = \frac{1}{2}(1+\alpha). \tag{4.24}$$

The right sides of these equations depend on the specific model chosen for the current, so they may be used as a criterion for selecting some model or other.

In the model in which quarks interact by exchanging a neutral vector meson (the "gluon" model) and in the model of vector commutator fields, we have

$$[J_i(\mathbf{x}, 0), J_j(0)] = \begin{cases} 2i\delta(\mathbf{x})\,\varepsilon_{ijk}\psi^+(0)\,\sigma_k Q^2\psi(0) & \text{(quarks)}, \quad (4.25) \\ 0 & \text{(fields)}; \quad (4.26) \end{cases}$$

$$[\dot{J}_i(\mathbf{x}, 0), J_j(0)] = \begin{cases} -\delta(\mathbf{x})\,\psi^+(0)\{i(\alpha_i\partial_j + \alpha_j\partial_i - 2\boldsymbol{\alpha}\mathbf{d}\delta_{ij}) \\ \quad -2g(\alpha_i B_j + \alpha_j B_i - 2\boldsymbol{\alpha}\mathbf{B}\delta_{ij}) \\ \quad + 4M\delta_{ij}\}\,Q^2\psi(0) \quad \text{(quarks)}; & (4.27) \\ \delta(\mathbf{x})\,C_{ab}J_i^a(0)\,J_j^b(0) + C - \text{a number (fields)}, & (4.28) \end{cases}$$

where

$$Q^2 = \frac{2}{9} + \frac{1}{3}Q, \quad Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}. \tag{4.29}$$

Using (4.27), we find from the sum rule for the polarization form factor that

$$\int_0^{\omega_0} d\omega F_{TL}^{(-)}(\alpha, \beta, \omega) = \begin{cases} \text{const} & \text{(quark model)}; \\ 0 & \text{(field algebra)}. \end{cases} \tag{4.30}$$

It can also be shown from the sum rules that the quark model predicts greater values for the transverse form factors F_{T_1} and F_{T_2} than for the longitudinal form factor F_L.

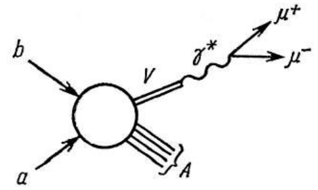


Fig. 6. Diagram illustrating the vector-dominance model.

These sum rules can thus be used to select a model. Analogous sum rules were found in [41] (see also 22, 42) for the case of ordinary electron creation. The analogous sum rules were treated in [20] for electron creation with a single hadron distinguished in the final state.

5. VECTOR DOMINANCE AND THE MUON-PAIR MASS SPECTRUM

According to the vector-dominance hypothesis (see, e.g., reviews [43, 44]) muon-pair formation involves the emission of a virtual vector meson, which converts into a virtual photon, which then decays into a muon pair, as shown in Fig. 6.

It can be shown that the vector-dominance hypothesis leads to a correct description of this process, since here q^2 is time-like.

Let us determine the matrix density for the virtual vector meson V ($V = \rho_0$, ω, or φ) formed in the reaction

$$a+b \to V + \text{hadrons}, \tag{5.1}$$

according to

$$W_{\mu\nu}(p, p', q) = \sum_A (2\pi)^4 \, \delta(p+p'-q-p_A)\langle p, p', \text{in} | J_\mu^{(v)}(0) \; | A \text{ out}\rangle^c \langle A \text{ out} | J_\nu^{(v)}(0) | p, p' \text{ in}\rangle^c, \tag{5.2}$$

where $J_\mu^{(v)}(x) = (\Box^2 - m_v^2) V_\mu(x)$ is the density of the V-meson current. Using the current-field identity,

$$J_\mu(x) = -\sum_v \frac{m_v^2}{2\gamma_v} V_\mu(x) = -\left(\frac{m_\rho^2}{2\gamma_\rho}\rho_\mu^0(x) + \frac{m_\omega^2}{2\gamma_\omega}\omega_\mu(x) + \frac{m_\varphi^2}{2\gamma_\varphi}\varphi_\mu(x)\right), \tag{5.3}$$

we find the following relation between the density matrices of the virtual photon and the vector mesons:

$$\rho_{\mu\nu}(p, p', q) = \sum_v \left(\frac{m_v^2}{2\gamma_v}\right)^2 \frac{1}{(m_v^2 - q^2)^2} W_{\mu\nu}^{(v)}(p, p', q) + \text{interference terms.} \tag{5.4}$$

Equation (5.4) can be used to express the five form factors ρ_{T_1}, ρ_{T_2}, ρ_L, $\rho_{TL}^{(\pm)}$, which completely describe the muon-pair formation in terms of the corresponding V-meson form factors.*

For use of the vector-dominance hypothesis it is convenient to write the equation for the mass spectrum in the form [12]

$$\frac{d\sigma}{dq^2} = \frac{\alpha}{2\pi} \cdot \frac{1}{q^2}\left(1 - \frac{q^2 - 4m_\mu^2}{3q^2}\right)\sqrt{\frac{q^2 - 4m_\mu^2}{q^2}}\, \sigma^{\gamma^*}(s, q^2), \tag{5.5}$$

where

$$\sigma^{\gamma^*}(s, q^2) = \sigma_{T_1}^{\gamma^*} + \sigma_{T_2}^{\gamma^*} + \sigma_L^{\gamma^*} \tag{5.6}$$

is the total cross section for the creation of a virtual γ^* photon of mass q^2 in the process

$$a+b \to \gamma^* + \text{hadrons.} \tag{5.7}$$

*We recall that only the form factors ρ_{T_1}, ρ_{T_2}, and ρ_L contribute to the cross section; the form factor $\rho_{TL}^{(+)}$ can be determined from the angular distribution of the muon pair, while $\rho_{TL}^{(-)}$ can be determined by measuring the polarization of one of the muons (see §2).

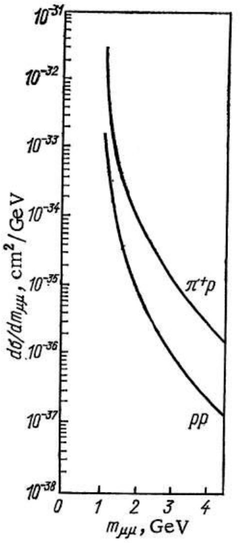


Fig. 7. Mass spectra of the muon pair formed in pp collisions (with $P_{lab.}$ = 28.5 GeV/c and in π^+-p collisions (with $P_{lab.}$ = 8.5 GeV/c) predicted by the vector-dominance model [Eqs. (5.13) and (5.16)].

According to the vector-dominance hypothesis, this cross section is related to the total cross section for the formation of real vector mesons in process (5.1) by

$$\sigma^{\gamma^*}(s, q^2) = \frac{\alpha}{4}\left[\left(\frac{m_\rho^2}{m_\rho^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\rho^2}\sigma^\rho(s) + \left(\frac{m_\omega^2}{m_\omega^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\omega^2}\sigma^\omega(s) + \left(\frac{m_\varphi^2}{m_\varphi^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\varphi^2}\sigma^\varphi(s)\right] + \text{interference terms.} \qquad (5.8)$$

Substituting this approximate expression for $\sigma\gamma^*$ into Eq. (5.5), neglecting the muon mass ($m_\mu = 0$), and assuming the contribution of the interference terms to be small, we find the following expression for the mass spectrum of the muon pair:

$$\frac{d\sigma}{dq^2} = \frac{\alpha^2}{12\pi} \sum_{v=\rho^0,\,\omega,\,\varphi} \left(\frac{m_v^2}{m_v^2 - q^2}\right)^2 \frac{4\pi}{\gamma_v^2}\sigma^v(s). \qquad (5.9)$$

The φ meson is weakly created in hadron-hadron collisions, so by retaining only the contributions of the ρ^0 and ω mesons, and assuming $m_\rho \approx m_\omega$, $\rho_\rho^2 : \gamma_\omega^2 = 1:9$, and $\gamma_\rho^2/4\pi = 0.5$, we convert Eq. (5.9) to $(m_{\mu\mu} \equiv \sqrt{q^2})$

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{2\cdot 10^{-6}}{m_{\mu\mu}(m_{\mu\mu}^2 - 0.6)^2}\left[\sigma^\rho(s) + \frac{1}{9}\sigma^\omega(s)\right]\frac{\text{cm}^2}{\text{GeV}} \qquad (5.10)$$

or, for large $m_{\mu\mu}$,

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{2\cdot 10^{-6}}{m_{\mu\mu}^5}\left[\sigma^\rho(s) + \frac{1}{9}\sigma^\omega(s)\right]\frac{\text{cm}^2}{\text{GeV}} \qquad (5.11)$$

We will use Eq. (5.10) [or (5.11)] to analyze muon-pair formation in specific hadron-hadron collisions.

a. Proton–Proton Collisions (a = b = p). The formation of the ρ^0 meson in a $p + p \to p + p + \rho^0$ reaction has not been observed at any energy up to $P_{lab.}$ = 28.5 GeV/c. In this range, the cross sections for ω-meson formation in the $p + p \to p + p + \omega$ reaction are [45]

$P_{lab.}$, GeV/c	5	10	28.5
σ^ω μb	140±20	60	50±10

This result is in agreement with the results of an analysis based on the double Regge-pole model [46]. Analysis of the six-ray reaction

$$pp \to pp\pi^+\pi^+\pi^-\pi^-$$

shows that about 24% of the events involve the formation of a ρ^0 meson; the corresponding cross section is 90 μb [47]. The cross section for the right-ray process $pp \to pp\pi^+\pi^+\pi^+\pi^-\pi^-\pi^-$ is 20 μb. Assuming that here also about one-fourth of the events involve the formation of the ρ^0 meson, we can estimate the corresponding cross section to be about 5 μb. We can thus assume that the total cross section for ρ^0 formation in pp collisions at $P_{lab.}$ = 28.5 GeV/c is equal to roughly 100 μb:

$$\sigma^{pp\to\rho+\cdots} = 100\ \mu\text{b}. \qquad (5.12)$$

The ω contribution in (5.10) can be neglected because of the factor of 1/9. If, on the other hand, we assume that $\sigma^\omega \approx \sigma^\rho = 100$ μb, we find the following expression for the mass of the muon pair formed in pp collisions with $P_{lab.}$ = 28.5 GeV/c, from Eqs. (5.10) or (5.11):

$$\frac{d\sigma^{pp}}{dm_{\mu\mu}} = \frac{2.2\cdot 10^{-34}}{m_{\mu\mu}\,(m_{\mu\mu}^2 - 0.6)}\ \text{cm}^2/\text{GeV} \approx \frac{2.2\cdot 10^{-34}}{m_{\mu\mu}^5}\ \text{cm}^2/\text{GeV}. \tag{5.13}$$

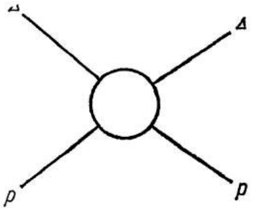


Fig. 8 Amplitude for the scattering of hadron a having 4-momentum Δ and a nonphysical mass Δ^2 by hadron b having mass $p^2 = m^2$. In the lab. system ($\mathbf{p} = 0$), the energy of the nonphysical hadron is $\delta = (1/m)\, p\Delta$.

The corresponding curve is shown in Fig. 7.

b. Pion-Proton Collisions. We consider the case of π^+p collisions ($a = \pi^+$, b = p). We can conclude from the analysis in [48] that the cross section for ρ^0 meson formation in the reaction $\pi^+ + p \rightarrow \rho^0$ hadrons is greater than or approximately equal to 1840 μb at $P_{lab.} = 8.5$ GeV/c,

$$\sigma^{\pi^+ p \rightarrow \rho^0 + \ldots} \geqslant 1840\ \mu\text{b}, \tag{5.14}$$

and the cross section for ω formation in the reaction $\pi^+ p \rightarrow \omega$ + hadrons is

$$\sigma^{\pi^+ p \rightarrow \omega + \ldots} \geqslant 200\ \mu\text{b}. \tag{5.15}$$

From Eqs. (5.10) or (5.11) we thus find the following approximate (lower) estimate for the mass spectrum of the muon pair formed in π^+p collisions with a momentum of $P_{lab.} = 8.5$ GeV/c:

$$\frac{d\sigma^{\pi^+ p}}{dm_{\mu\mu}} = \frac{3.7\cdot 10^{-33}}{m_{\mu\mu}\,(m_{\mu\mu}^2 - 0{,}6)^2}\ \text{cm}^2/\text{GeV} \approx \frac{3.7\cdot 10^{-33}}{m_{\mu\mu}^5}\ \text{cm}^2/\text{GeV}. \tag{5.16}$$

6. LOWER LIMIT FOR THE MASS SPECTRUM

To find an asymptotic expression for the muon-pair mass spectrum, we consider the hadron part of the matrix element for the formation of a muon pair as $|\mathbf{p}'| \rightarrow \infty$. With an accuracy to within terms $O(1/|\mathbf{p}'|)$, the matrix element is

$$\langle A\,\text{out}\,|\,J_\mu^{\text{em}}(0)\,|\,p,\,p',\,\text{in}\rangle^c \xrightarrow[|\mathbf{p}'|\rightarrow\infty]{} \frac{p'_\mu}{E'}\,\langle A\,\text{out}\,|\,J_0^{\text{em}}(0)\,|\,p,\,p',\,\text{in}\rangle^c + O\left(\frac{1}{|\mathbf{p}'|}\right). \tag{6.1}$$

This means that the muon-pair formation is governed primarily by the $J_0(0)$ component of the electromagnetic field; i.e., it is a "Coulomb" process.

Using the Bjorken limit, i.e., expanding the T product in a series of simultaneous commutators, and retaining only the first term of this asymptotic series, we find the following approximate relation with the matrix element for hadron-hadron scattering away from the energy surface:

$$\begin{aligned} \langle A\,\text{out}\,|\,J_0^{\text{em}}(0)\,|\,p,\,p',\,\text{in}\rangle^c &= -i\int dx e^{iqx}\,\langle A\,\text{out}\,|\,T\,(J_0^{\text{em}}(x)\,J^{(a)}(0)\,|\,p\rangle^c \\ &\xrightarrow[q^2\rightarrow\infty]{} \frac{1}{\sqrt{q^2}}\int dx e^{-iqx}\,\langle A\,\text{out}\,|\,[J_0^{\text{em}}(\mathrm{x},\,0),\,J^{(a)}(0)]\,|\,p\rangle^c \\ &= \frac{1}{\sqrt{q^2}}\,\langle A\,\text{out}\,|\,J^{(a)}(0)\,|\,p\rangle^c + \text{contribution of quasilocal terms}, \end{aligned} \tag{6.2}$$

where $J^{(a)}(x)$ is the current of the hadron carrying a 4-momentum Δ.

Using Eqs. (6.1) and (6.2), we can estimate the form factor ρ governing muon-pair creaction:

$$\rho\,(s,\,q^2,\,\Delta^2,\,\delta) \approx \frac{4m\sqrt{\delta^2 - \Delta^2}}{q^2}\,\sigma_{ab}\,(\delta,\,\Delta^2). \tag{6.3}$$

The quantity $\sigma_{ab}(\delta, \Delta^2)$ which appears here is the analytic continuation of the total cross section for the interaction of a and b hadrons into the nonphysical region, where the square mass of hadron a is negative and equal to Δ^2; δ is the lab. energy of the nonphysical hadron (Fig. 8). In this approximation we find the following triple differential cross section (neglecting m' and m_μ):

$$\frac{d^3\sigma}{dq^2\,d\Delta^2\,d\delta} = \frac{\alpha^2}{3\pi}\cdot\frac{m^2}{s^2 q^4}\sqrt{\delta^2 - \Delta^2}\,\sigma_{ab}\,(\delta,\,\Delta^2). \tag{6.4}$$

The corresponding mass spectrum is

$$\frac{d\sigma}{dq^2} = \frac{\alpha^2}{3\pi^2} \cdot \frac{m^2}{s^2 q^4} \int\limits_{q^2-s}^{0} d\Delta^2 \int\limits_{-\frac{\Delta^2}{2m}}^{\varepsilon\frac{\Delta^2}{\Delta^2-q^2}+\frac{\Delta^2-q^2}{4\varepsilon}} d\delta \sqrt{\delta^2-\Delta^2}\, \sigma_{ab}(\delta, \Delta^2). \tag{6.5}$$

If axiomatic field theory or analytic S-matrix theory [22, 49, 50] imposes a restriction on $\sigma_{ab}(\delta, \Delta^2)$ away from the mass surface, (6.5) imposes a restriction on the mass spectrum. For the simpler case of the electromagnetic form factor F(t), field theory and S-matrix theory predict an exponential restriction for the lower limit of the form factor. By analogy, we can expect

$$\sigma_{ab}(\delta, \Delta^2) \geqslant \sigma_{ab}^{ph} e^{-a\sqrt{-\Delta^2}}, \tag{6.6}$$

where σ_{ab} is the total cross section for the interaction of real particles, and a is some constant. Then we find the following lower estimate for the mass spectrum from (6.5) under the conditions $s \gg q^2 \gg 1/a^2$:

$$\frac{d\sigma}{dq^2} \geqslant 20\alpha^2 \frac{\sigma_{ab}}{q^8 a^6}. \tag{6.7}$$

Another method for estimating the mass spectrum was discussed in [51].*

APPENDIX

Determination of the Boundaries of the Physical Region for Muon-Pair Formation. Conservation of 4-momentum is described by

$$p' + p = q + p_N. \tag{A.1}$$

Introducing the vector $\Delta = p' - q$, we obtain

$$p + \Delta = p_N. \tag{A.2}$$

Then we have $\Delta^2 = m_N^2 - m^2 - 2m\delta$, where $\delta = 1/m\, p\Delta = (\varepsilon - q^0)$.

The identity $m_N \equiv m$ corresponds to elastic scattering. Then Δ^2 and δ are unambiguously related, i.e., are not independent variables $\delta = -\Delta^2/2m$.

The δ_{min} value is at a minimum, since q^0 here is at a maximum. We consider the case in which the virtual photon moves backward in the lab. system. For fixed invariants, this virtual photon clearly acquires a minimum energy $(q_0)_{min}$; this means that

$$\delta_{max} = \varepsilon - (q_0)_{min}. \tag{A.3}$$

We find $(q_0)_{min}$ from

$$\Delta^2 = m'^2 + q^2 - 2\varepsilon (q_0)_{min} - 2\sqrt{\varepsilon^2 - m'^2}\sqrt{(q_0)^2_{min} - q^2}. \tag{A.4}$$

Setting $m' \equiv 0$ and solving this equation, we find

$$(q_0)_{min} = \frac{q^2 - \Delta^2}{4\varepsilon} + \frac{\varepsilon q^2}{q^2 - \Delta^2}; \tag{A.5}$$

*By analogy with the dynamics of a planar explosion in hydrodynamics, we have formulated a principle for approximate self-similarity for high-energy hadron-hadron collisions [52].

$$\delta_{max} = \varepsilon - (q_0)_{min} = \varepsilon\left(1 - \frac{q^2}{q^2 - \Delta^2} - \frac{q^2 - \Delta^2}{4\varepsilon}\right) = \varepsilon^* + \frac{\Delta^2}{\varepsilon^*}, \tag{A.6}$$

where

$$\varepsilon^* = \varepsilon\Delta^2/(\Delta^2 - q^2).$$

In the physical region we thus have

$$-\frac{\Delta^2}{2m} \leqslant \delta \leqslant \varepsilon^* + \frac{\Delta^2}{4\varepsilon^*}. \tag{A.7}$$

We now find the physical region Δ^2 for fixed s and q^2, from the condition

$$\delta_{min} = \delta_{max}. \tag{A.8}$$

The result is

$$\Delta^{2(-)} \leqslant \Delta^2 \leqslant \Delta^{2(+)}, \tag{A.9}$$

where

$$\Delta^{2(\pm)} = \frac{q^2\varepsilon + q^2 m - 2m\varepsilon^2 \pm \varepsilon\sqrt{4m^2\varepsilon^2 + q^4 - 4q^2\varepsilon m - 4q^2 m^2}}{2\varepsilon + m}.$$

We note that there is an interesting analogy between the reaction under consideration here and inelastic neutron formation: If we replace the square of the lepton mass in the Appendix of Adler's paper [2] by q^2 and replace Adler's q^2 by our Δ^2, we essentially reduce Adler's problem to ours, and vice versa.

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