THREE-QUASIPARTICLE STATES IN DEFORMED NUCLEI WITH MASS NUMBERS BETWEEN 150 AND 190

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We have studied the splitting and de-excitation of three-particle states and the possibility of observing these effects in deformed nuclei which have mass numbers between 150 and 190. We have computed the splitting energies for the observed three-particle states, the limiting values of W and α, which determine the interaction force, and the fraction of spin forces in the total pairing forces. An analysis of the experimental date indicates that the de-excitation of three-quasiparticle levels into single-quasiparticle levels, which is an F-forbidden process in the independent quasiparticle model, can be explained by a simple model.

INTRODUCTION

Modern nuclear models [1-4] describe the excited states of odd mass-number deformed nuclei in terms of single-quasiparticle states, three-quasiparticle states, and so on, and collective excitation states such as quadrupole β and γ oscillations, octupole oscillations, and others. Both the theoretical calculations and the experimental data show that the lowest states of the deformed nuclei are well-described by states of single-particle excitation (the level schemes of Nilsson or Saxon and Woods) and their associated rotational levels. However, noticeable amounts of admixed collection states appear in the single-particle states even at excitation energies of 200-300 keV [5]. At energies of 500 keV or higher, levels of a basically collective nature have been observed in addition to those levels which are primarily single-quasiparticle in character.

In deformed, odd mass-number nuclei three-quasiparticle states can appear in the breaking up of neutron or proton pairs. This means that the energy of a three-quasiparticle state must be greater than or near one MeV. Three-quasiparticle states can be one of two kinds: the first is a state in which all three quasiparticles are the same – three protons (3p) or three neutrons (3n); the second state has different quasiparticles such as (2p, n) or (2n,p). In the superfluid model of the nucleus each three-quasiparticle level is four-fold degenerate. Therefore the projections of the total angular momenta of the three quasiparticles onto the symmetry axis of the nucleus can be combined in four different ways*:

$$K=|-\Omega_1+\Omega_2+\Omega_3|;\ K=|\Omega_1-\Omega_2+\Omega_3|;$$
$$K=|\Omega_1+\Omega_2-\Omega_3|;\ K=|\Omega_1+\Omega_2+\Omega_3|.$$

*To describe the single-quasiparticle states we shall use the asymptotic quantum numbers $\Omega^{\pi}[Nn_Z\,\Lambda]\uparrow$, which are applied in both the collective and superfluid models. Here Ω is the projection of the angular momentum of a single-quasiparticle state onto the nuclear axis of symmetry, π is the parity of the state, N is the principal quantum number which determines the number of the main shell in the oscillator potential, n_Z is the quantum number of the oscillator along the symmetry axis of the nucleus, and Λ is the projection of the orbital angular momentum on the symmetry axis. The arrow on the right indicates the spin quantum number Σ. If the arrow points up, $\Sigma=+1/2(\Omega=\Lambda+1/2)$, if it points down, $\Sigma=-1/2(\Omega=\Lambda-1/2)$. K is the projection of the angular momentum of the three-quasiparticle state onto the symmetry axis.

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The levels of a three-quasiparticle multiplet can be split by the interaction of the quasiparticles, which was not taken into account in the independent quasiparticle model. This means that experimentally one can observe three-quasiparticle multiplets consisting of four levels.

In 1962 V. G. Solov'ev demonstrated the possibility of observing states having three-quasiparticle character [6]. They computed the energy of the center of gravity of a number of three-quasiparticle multiplets in deformed nuclei. N. I. Pyatov and A. S. Chernyshev [7] have studied the splitting of levels for (2n, p) and (2p, n)-type three-quasiparticle multiplets.

The experimental data on three-quasiparticle states was studied by one of the present authors in 1965 [8]. In this paper we shall analyze the more complete information gathered in recent years concerning three-quasiparticle states.

1. Experimental Identification of Three-Quasiparticle States

The primary source of experimental difficulty in observing and studying three-quasiparticle states is that the splitting energies are 1 MeV or greater. Here one should keep in mind the fact that in this energy range the density of levels in an odd-A nucleus increases, and that near a possible three-quasiparticle state there can be levels of a different kind but having the same spin and parity. Then states of different kinds can be strongly mixed. The amount of such mixing will of course depend on the properties of a given nucleus, such as the nature of the single-particle states in the nucleus, the locations of the collective excitation levels, and so on. But one can state immediately that the low-lying three-quasiparticle states with very high spin values (such as 21/2 or greater) will mix only slightly with other types of states. The reason is, of course, that for energies of 1-2 MeV only three-quasiparticle states can have such high spin values. Other types of states have much lower spins.

Two cases can be cited in which the experimental observation of three-quasiparticle states is simplified. V. G. Solov'ev et al. [6] have shown it to be possible to see three-quasiparticle states in β decay. As mentioned earlier, three-quasiparticle states can be divided into two groups: states like (3n) or (3p) where all three particles are the same, and states like (2n, p) and (2p, n). β decay from the ground state of an odd nucleus (a single-quasiparticle state) to either a (3n) or (3p) three-quasiparticle excited state of the daughter nucleus will be strongly forbidden, because a transition like (p)→(3n) or (n)→(3p) requires a number of quasiparticles to change state (F-forbidden [9]). But β decay from either a (2n, p) or (2p, n) three-quasiparticle state is quite another thing. Such β decays can be depicted in this manner:

$$p_1 \rightarrow p_2\, p_3\, n_4 \quad \text{(states of the type } 2p, n\text{)}$$

or

$$n_1 \rightarrow n_2\, n_3\, p_4 \quad \text{(states of the type } 2n, p\text{)}.$$

Here the indices 1, 2, 3, 4 are a set of asymptotic quantum numbers which characterize the single-quasiparticle states. If the state 1 is identical to state 2 (or 3) then only one other particle must appear in such a transition in the proton and neutron schemes, and the transition will not be F-forbidden. But if states 1, 2, 3 are different, then in one of the systems two quasiparticles appear, and the transition is F-forbidden. Therefore, in the case where state 1 is identical to state 2 β decay to three-quasiparticle states can be observed. The probability of β decay in this case is determined by the probability of a β transition between the single-quasiparticle proton and neutron states labeled 3 and 4. Of special interest then are the neutron and proton states between which unhindered β transitions are allowed:

$$p\,7/2-[523] \rightleftarrows n\,5/2-[523];$$

$$p\,9/2-[514] \rightleftarrows n\,7/2-[514].$$

It is known that rather fast β transitions take place between these states [8], with log ft ranging between 4.6 and 4.8. It seems likely that β transitions to three-quasiparticle states of the type indicated will also be allowed and unhindered, and one would expect that log ft for these transitions will lie in the same range. The systematics of the β decay matrix elements for deformed odd-A nuclei indicate that the allowed unhindered β transitions are rather clearly isolated from the rest of the group; log ft for the allowed hindred β transitions are greater than 5.5. Thus, if β decay to an excited state of an odd-A nucleus is observed, and log ft for the transition is smaller than 5.2, and one can eliminate the possibility of a β transition to a single-quasiparticle state, it can then be concluded that we are dealing with a β transition to a three-quasiparticle state.

It was determined in studies of ^{165}Tm [10] that the 1428-keV level of the daughter nucleus ^{165}Er is populated in 12% of the cases. Preibisz et al [11] measured the ^{165}Tm-^{165}Er decay energy and found the value of log ft for K-capture in the 1428 keV level to be 5.1±0.2. The spin of the ^{165}Tm ground state has been measured to be 1/2 [12]. Thus the ground state of ^{165}Tm is characterized by the asymptotic quantum numbers $1/2^+$ [411]. One cannot have an allowed unhindered β transition from this state to the single-quasiparticle state of ^{165}Er. Other kinds of β transitions must have larger values of log ft. It was therefore shown that the only remaining possibility was to interpret the 1428-keV level as a three-quasiparticle state of the type $\rho_1\ 1/2^+$ [411] + $\rho_2\ 7/2^-$ [523] − n $5/2^-$ [523]. Measurements of the multipolarities of the γ transitions from the 1428-keV level showed that this level has spin and parity $3/2^+$, confirming the conclusion reached relative to the type of state [10].

Three-quasiparticle states were found in additional nuclei, based on the identification of allowed unhindered β decay. The data for these states are collected in Table 1. The first column of the table gives the nucleus in which the three-quasiparticle state was observed. The second column contains the configuration of the three-quasiparticle state. The third gives the experimental spin and parity, and the fourth presents the experimental energy for the level. Column five contains the theoretical estimate of the energy of the center of gravity for the three-quasiparticle multiplet. Also shown are the basic experimental data which enabled us to identify the level as a three-quasiparticle state. Column six contains the value of log ft for this particular case (three-quasiparticle states excited by an allowed undelayed β decay of the nucleus).

Another possibility for observing three-quasiparticle states are the three-quasiparticle isomer states. High-spin three-quasiparticle states are excited in various nuclear reactions, among them being the β decay of three-quasiparticle isomer states. As an example, let us consider the isomer state in ^{177}Lu. By irradiating the natural mixture of lutecium isotopes with thermal neutrons, Jorgensen et al [13] observed that in addition to the already well-known radioactive ^{177}Lu nucleus ($T_{1/2}$=6.8 days) there was a new activity having a half-life of 155 days. Studying the gamma-ray and conversion-electron spectra they showed that this new activity is connected with the 969 keV level of ^{177}Lu. It was also determined that the spin and parity of this level is $23/2^-$. It is impossible to explain a level with such high spin on the basis of a single-quasiparticle; in Nilsson's level scheme there is no level with spin greater than 13/2 at this energy. The possibility that this is a collective level is also excluded. The only possible interpretation is that the level is a three-quasiparticle state:

$$n_1\,9/2^+\,[624]+n_2\,7/2^-\,[514]+p\,7/2^+\,[404].$$

It was shown in the same paper [13] that a three-quasiparticle state of the type β is excited in ^{177}Hf at 1315 keV during the $23/2^+$ {$p_1\ 7/2^+$ [404], $p_2\ 9/2^-$ [514], n $7/2^-$ [514], n $7/2^-$ [514]} decay of the isomer state in ^{177}Lu.

Peker has shown [14] that three-quasiparticle isomer states should be sought in odd-A nuclei for which the neighboring even-even nuclei reveal two-quasiparticle isomers. Thus, in ^{178}Hf, which is a neighbor of ^{177}Lu, an isomer state is observed at 1148 keV ($T_{1/2}$=4.8 sec) of type 8^-. Gallagher and Solov'ev explained it as a two-quasiparticle state $n_1\ 9/2^+$ [624] + $n_2\ 7/2^-$ [514]. One would expect that the lowest-energy three-quasiparticle state in ^{177}Lu is obtained by adding to this state a third particle whose state is the same as that of the proton in the ground state of ^{177}Lu; i.e., p $7/2^+$ [404].

The experimental data which are now available on three-quasiparticle isomer states are also shown in Table 1. The seventh column gives the measured values of the half-lives for the isomer states.

It is clear from Table 1 that all the more or less reliable levels showing a three-quasiparticle nature belong to the (2p, n) or (2n, p) types. The excitation of a three-proton or three-neutron state during β decay of the ground state of a nucleus is highly unlikely. (3p) and (3n) states can be observed in nuclear reactions (such as $n\gamma$ reactions) or during β decay of three-quasiparticle isomer states.

2. Splitting of Three-Quasiparticle States

The energy difference between levels of the (2n, p) or (2p, n) three-quasiparticle types can be computed using the equations obtained in [7]:

$$E_{(K=|-\Omega_1+\Omega_2+\Omega_3|)}-E_{(K=\Omega_1+\Omega_2+\Omega_3)}=(1-4\alpha)\,w\,[A_{12}+B_{12}]-2\alpha w\,[A_{13}+B_{13}];$$

$$E_{(K=|\Omega_1-\Omega_2+\Omega_3|)}-E_{(K=\Omega_1+\Omega_2+\Omega_3)}=(1-4\alpha)\,w\,[A_{12}+B_{12}]-2\alpha w\,[A_{23}+B_{23}];$$

$$E_{(K=|\Omega_1+\Omega_2-\Omega_3|)}-E_{(K=\Omega_1+\Omega_2+\Omega_3)}=-2\alpha w\,[A_{23}+B_{23}]-2\alpha w\,[A_{13}+B_{13}].$$

TABLE 1. Three-Quasiparticle States in Deformed Nuclei with Mass Number A between 150 and 190

Nucleus	Configuration	K^{π}	E_{exp}, keV	E^{*}_{theo}, keV	lg** ft	$T_{1/2}$	Reference
^{163}Dy	$p_1\,7/2^-$ [523] $p_2\,3/2^+$ [411] $n\,5/2^-$ [523]	$(5/2^+)$ $1/2^+$	935 884	≈1200	5,2 4,9	— —	[16]
^{161}Er	$p_1\,7/2^-$ [523] $p_2\,7/2^+$ [404] $n\,5/2^-$ [523]	$(9/2^+, 5/2^+)$	1838	≈3200	5,0	— —	[21]
^{163}Er	$p_1\,7/2^-$ [523] $p_2\,1/2^+$ [411] $n\,5/2^-$ [523]	$1/2^+$ $3/2^+$	1802,0 1538,4	≈1400	4,9 5,3	— —	[19][22]
^{165}Er	$p_1\,7/2^-$ [523] $p_2\,1/2^+$ [411] $n\,5/2^-$ [523]	$3/2^+$	1428	≈1400	5,1	—	[10] [23] [24]
^{175}Yb	$p_1\,7/2^-$ [523] $p_2\,1/2^+$ [411]*** $n\,5/2^-$ [523]	$1/2^+$ $3/2^+$	2113 1792	≈3000	5,2 5,5	— —	[25]
^{175}Yb	$p_1\,9/2^-$ [514] $p_2\,1/2^+$ [411] $n\,7/2^-$]514]	$(1/2^+)$ $3/2^+$	(1891) 1497	≈1600	4,8 5,0	— —	[25]
^{177}Hf	$p_1\,9/2^-$ [514] $p_2\,7/2^+$ [404] $n\,7/2^-$ [514[	$23/2^+$	1315	≈1400	—	1,12 sec	[26] [27]
^{179}W	$p_1\,9/2^-$ [514] $p_2\,5/2^+$ [402] $n\,7/2^-$ [514]	$7/2^+$ $3/2^+$	1680,1 720,5	≈1400	5,2 5,2	— —	[18]
^{161}Ho	$n_1\,5/2^-$ [523] $n_2\,3/2^-$ [521] $p\,7/2^-$ [523]	$5/2^-$ $1/2^-$	1943 1897	≈1800	5,1 4,8	— —	[28]
^{177}Lu	$n_1\,9/2^+$ [624] $n_2\,7/2^-$ [514] $p\,7/2^+$ [404]	$23/2^-$	970	≈1000	—	155 days	[13] [29]

TABLE 1, continued

Nucleus	Configuration	K^{π}	E_{exp}, keV	E^{*}_{theo}, keV	lg ft†	$T_{1/2}$	Reference
^{177}Lu	n_1 7/2$^-$ [514] n_2 9/2$^+$ [624] p 9/2$^-$ [514]	7/2$^+$ 11/2$^+$	1241 1230	≈1200	4,4 4,4	— —	[30]
	n_1 1/2$^-$ [510] n_2 7/2$^-$ [514] p 7/2$^+$ [404]	13/2$^+$ 15/2$^+$	1503 1357	≈1900	— —	— —	[31]
^{183}Re	n_1 9/2$^+$ [624] n_2 11/2$^+$[615]‡ p 5/2$^+$ [402]	25/2$^+$	1907	≈2300	—	1,02 μsec	[32]

*As in [6], E_{theor} was calculated roughly using the superfluid model without including the interactions of quasiparticles.
†Only those levels whose β decay corresponds to log ft < 5.5 were considered.
‡The interpretation is not unique.

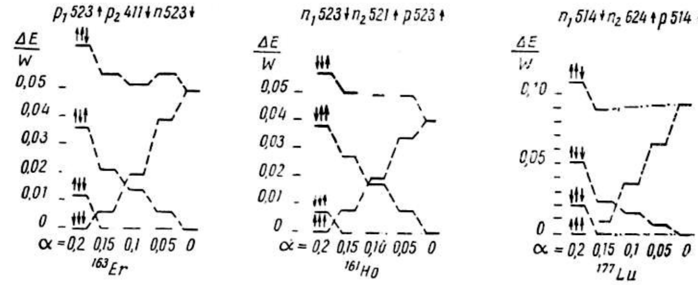

Fig. 1. Splittings of three-quasiparticle multiplets for various states.

The parameter ω determines the nucleon-nucleon interaction force, and is spin dependent. The parameter α determines the contribution of spin forces to the total pairing forces ($0 \leq \alpha \leq 1$). A_{ij} and B_{ij} are coefficients which depend on the asymptotic quantum numbers of the quasiparticles. Both A_{ij} and B_{ij} have been calculated in [7] for a number of single-quasiparticle pairs states (p, n), (p, p) and (n, n).

It is clear from the equation that the character of the splitting – the order of the levels in the multiplet – is determined by α. The splitting energy depends on ω. Figure 1 shows the splitting as a function of α for certain three-quasiparticle states. By examining all possible combinations of asymptotic spin quantum numbers Σ_1, Σ_2 and Σ_3 Pyatov and Chernyshev[7] have concluded that in states like (2p, n) or (2n, p) the highest level in the multiplet will always be the level in which the aymptotic spins of the nucleons in the disrupted pair (p_1, p_2) or (n_1, n_2) are parallel, while the spin of the third particle is antiparallel to them (↑↑, ↓). The three other possible levels are always lower (see Fig. 1). We point out here that we will arrive at the same conclusion if the Gallagher-Moshkovskii rule is extended to three-quasiparticle states. For according to their rule a state of the type (↑↑, ↓) is the most undesirable: the asymptotic spins of the identical particles are parallel while the spin of the third particle is antiparallel to the first two.

The experimental data on the splitting of three-quasiparticle states is meager at present. All available data are collected in Table 2. The first column gives the nucleus in which the three-quasiparticle levels are observed, the second contains the asymptotic quantum numbers of the single-quasiparticle states

TABLE 2. Splittings of Three-Quasiparticle States in Deformed Nuclei with A between 150 and 190

Nucleus	Configuration	K^π	$(\Sigma_1, \Sigma_2, \Sigma_3)$	E_{exp}, kev	ΔE_{exp}, kev	ΔE_{theo}, kev	α	ω, MeV
^{163}Dy	$p_1\,7/2^-$ [523] $p_2\,3/2^+$ [411] $n\,5/2^-$ [523]	$5/2^+$ $1/2^+$	(↓↓, ↓) (↓↑, ↓)	935 884	51	—	—	—
^{163}Er	$p_1\,7/2^-$ [523] $p_2\,1/2^+$ [411] $n\,5/2^-$ [523]	$1/2^+$ $3/2^+$	(↓↓, ↓) (↓↑, ↓)	1802,0 1538,4	264	380	less than 0,17	greater than 5,4
^{175}Yb	$p_1\,7/2^-$ [523] $p_2\,1/2^+$ [411] $n\,5/2^-$ [523]	$1/2^+$ $3/2^+$	(↓↓, ↓) (↓↑, ↓)	2113 1792	321	380	less than 0,17	greater than 6,4
	$p_1\,9/2^-$ [514] $p_2\,1/2^+$ [411] $n\,7/2^-$ [514]	$1/2^+$ $3/2^+$	(↓↓, ↓) (↓↑, ↓)	1891 1497	394	—	—	—
^{179}W	$p_1\,9/2^-$ [514] $p_2\,5/2^+$ [402] $n\,7/2^-$ [514]	$7/2^+$ $3/2^+$	(↓↓, ↓) (↓↑, ↓)	1680,1 720,5	960	about 600	—	—
^{161}Ho	$n_1\,5/2^-$ [523] $n_2\,3/2^-$ [521] $p\,7/2^-$ [523]	$5/2^-$ $1/2^-$	(↓↓, ↓) (↓↑, ↓)	1943 1897	46	330	less than 0,19	greater than 1,1
^{177}Lu	$n_1\,7/2^-$ [514] $n_2\,9/2^+$ [624] $p\,9/2^-$ [514]	$7/2^+$ $11/2^+$	(↓↑, ↓) (↓↓, ↓)	241 1230	—11	740	less than 0,17	greater than 0,1
	$n_1\,1/2^-$ [510] $n_2\,7/2^-$ [514] $p\,7/2^+$ [404]	$13/2^+$ $15/2^+$	(↓↓, ↓) (↓↑, ↓)	1503 1357	146	—	—	—

from which the observed three-quasiparticle state arises, and column three shows the spins and parities of the three-quasiparticle levels. The fourth column gives the relative orientations of the asymptotic spin quantum numbers and column five cites the experimental values for the three-quasiparticle levels. The table indicates a few cases where two levels of the multiplet have been observed, but there are no cases in which a greater number of multiplet levels have been observed experimentally. This makes it impossible to calculate α and w simultaneously from the data.

The parameters α and w, necessary in order to compute the splitting energies of the three-quasiparticle states, were determined in [7] from the experimental energy splittings of two-quasiparticle states in even-even and odd-odd nuclei. By using the values picked in [7] for $(1-4\alpha)$ w = 8.45 MeV and αw =0.314 MeV, we have computed the energy differences between the corresponding three-quasiparticle levels. These

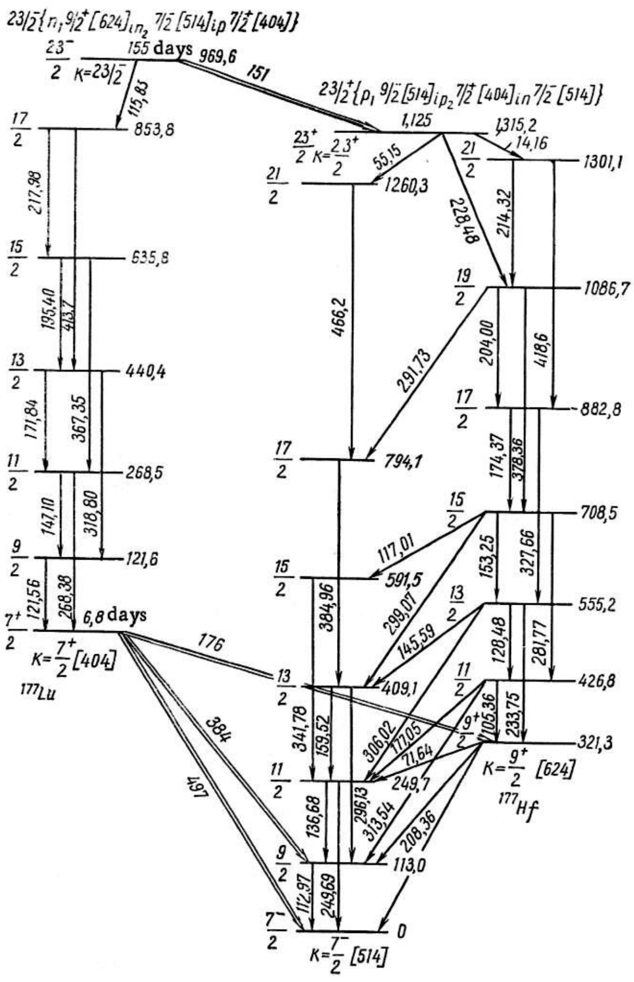

Fig. 2. Discharge of three-quasiparticle states in ^{177}Lu and ^{177}Hf.

are presented in colum seven of Table 2. The experimental values for these splittings are shown in the sixth column. The last two columns give estimates for the limits on α and w obtained on the basis of the experimental energy differences for the three-quasiparticle levels. These estimates were obtained in the following fashion. Using the equation for the corresponding energy difference, we found α when $\Delta E = 0$. The experimental value of ΔE (its sign) enabled us to determine if the value of α (for $\Delta E = 0$) is an upper or lower limit. The values of w shown in the last column were obtained from the experimental values of ΔE under the assumption that $\alpha = 0$. The forms of the expressions for ΔE and the values of $(A_{ij} + B_{ij})$ let us determine how w changes when α is increased and when ΔE = const; we were again able to determine whether we were dealing with an upper or a lower limit. The blanks in the last three columns indicate that A_{ij} and B_{ij} where not computed in [7] for the appropriate pairs of single-quasiparticle states.

By comparing the experimental and theoretical energy splittings (ΔE_{exp} and ΔE_{theo}) we see that in all cases except one we have obtained the right sign for the energy splittings: in a number of cases the agreement in magnitudes is not bad, but for ^{161}Ho ΔE_{exp} and ΔE_{theo} differ by a factor of six or seven. The limiting values for α and w in Table 2 are not inconsistent with each other.

3. De-Excitation of Three-Quasiparticle States

When treating γ transitions from three-quasiparticle states one must keep in mind that for these transitions there is a limitation on the number of particles which can participate in the transition (F-limitation), which is a restriction added to the selection rules on the asymptotic quantum numbers K^{π}_{for} [$Nn_z\lambda$]

usually used in the extended nuclear model. In the de-excitation of (2p, n) or (2n, p) three-quasiparticle states there is no F-limitation if, in the γ transition, one of the identical particles goes into the state occupied by the other particle, forming a pair. The γ transition takes place to the single-quasiparticle state occupied by the unpaired particle. In the independent-quasiparticle model γ transitions to the other single-quasiparticle states are forbidden because they take place with a change in the state of more than one particle.

From analysis of the de-excitation of states showing three-quasiparticle character, it is known that three-quasiparticle states can be classified into two groups. The first group contains three-quasiparticle isomer states; the second group is for states showing three-quasiparticle character and having small spins.

The isomerism of low-lying three-quasiparticle states with high angular momentum is explained very satisfactorily by the selection rules on the angular momentum and its projection on the nuclear symmetry axis (the asymptotic quantum number K). For example, consider the three-quasiparticle isomer state with spin $23/2^-$ in ^{177}Lu (Fig. 2). This state is discharged primarily (78%) by a β transition to the $23/2^+$ three-quasiparticle state in ^{177}Hf. The structure of these three-quasiparticle states in ^{177}Lu and ^{177}Hf is shown in Fig. 2. During the β transition the following conversion takes place n$9/2^+$ [624] $\xrightarrow{\beta}$ p $9/2^-$ [514]; the two other quasiparticles do not alter their state. This β transition belongs to the class of first forbidden unhindered β transitions. The measured matrix element for this transition (log ft=6.1) agrees with the commonly observed values of log ft for first forbidden unhindered β transitions [8]. A type E3 γ transition with energy 115.8 keV from the 970.2 keV three-quasiparticle level of ^{177}Lu to the 17/2 $7/2^+$ [404] – 854.2 keV rotational level of the ground state band is forbidden by the asymptotic quantum number K. The degree of forbiddenness is $\nu = \Delta K - L = 8 - 3 = 5$. The Weisskopf hindrance factor for the 115.8 keV γ transition is $F_W = T_{exp}/T_W \approx 2 \cdot 10^8$.

Another situation is observed in the discharge of states showing three-quasiparticle character (of the type (2p, n) or (2n, p)) and having small angular momenta. In the independent quasiparticle model γ transitions from these levels to all lower single-quasiparticle levels except the basis state* are forbidden, for all such γ transitions involve changes in the states of more than one particle (F-forbidden). Therefore one might expect that the life times for these levels ought to be determined by the probability of γ transitions to the basis state. But it turns out that γ transitions to single-quasiparticle states other than the basis state are observed and the γ transitions to the basis states are relatively weak and generally not observed. This indicates that these states are not pure three-quasiparticle states, but instead are mixtures with other states. It appears that the study of the nature of the de-excitation of levels showing three-quasiparticle character and having small angular momenta permits one to analyze the structure of these states in more detail.

By analyzing the experimental material we have concluded that the discharge of three-quasiparticle states to single-quasiparticle states can be explained by a simple model in which these three-quasiparticle states are mixed with single-quasiparticle states and states of the "quasiparticle+phonon" type. States of the latter kind appear as the result of the interaction of phonons in the even-even nuclear core with the quasiparticles of the odd nucleus. We shall investigate a number of examples.

γ Transitions from the 884-keV level of ^{163}Dy (Fig. 3), interpreted in Table 1 as three-quasiparticle states of the type $1/2^+$ $\{p_1\ 7/2^-\ [523],\ p_2\ 3/2^+\ [411],\ n\ 5/2^-\ [523]\}$, to the rotational band of the ground state have not been observed [16]. But there have been observations of intense E1 transitions to the levels 351 keV – 1/2 $1/2^-$ [521], 390 keV – 3/2 $1/2^-$ [521] and 422 keV 3/2 $3/2^-$ [521]. Calculations [5] show that the states $1/2^-$ [521] and $3/2^-$ [521] in ^{163}Dy are, to a high degree, purely single-quasiparticle states, and that the possible mixtures to these states cannot remove the F-forbiddenness of the γ transitions to them from a three-quasiparticle state. Thus, the observed γ transitions can be explained only by assuming that the 884 keV level has a more complex structure. According to Solov'ev et al. [17] the octupole state $Q_1(32)$ with $K^\pi = 2^-$ in ^{162}Dy at 1148 keV contains these basic two-quasiparticle components:

$$p\,[523]\uparrow\ p\,[411]\uparrow\ 51.4\%;$$

$$n\,[651]\uparrow\ n\,[521]\downarrow\ \ 0.9\%;$$

$$n\,[642]\uparrow\ n\,[521]\downarrow\ \ 1.7\%;$$

$$n\,[633]\uparrow\ n\,[521]\uparrow\ 24.2\%.$$

*A single-quasiparticle state is called a basis state if it has the same characteristics as the state of the unpaired quasiparticle in a three-quasiparticle state.

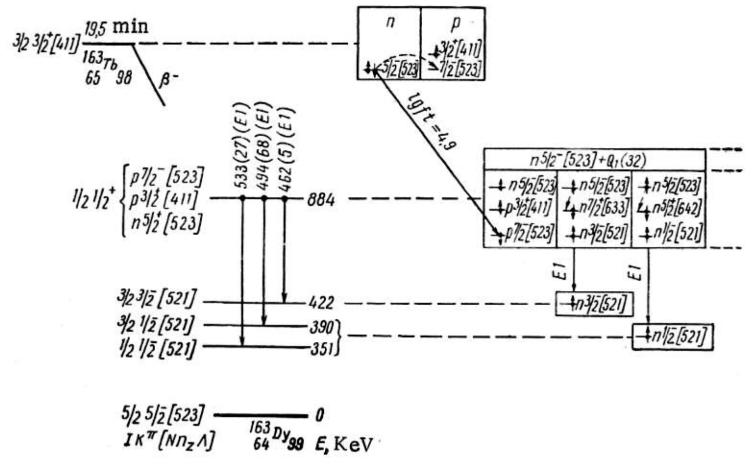

Fig. 3. Discharge of a three-quasiparticle-like state in ^{163}Dy.

Since ^{162}Dy is the even-even core of ^{163}Dy, it can be assumed that the 884 keV level, spin 1/2, is related to the octupole excitation of the even-even core; i.e., this level is n [523] $\downarrow + Q_1(32)$. In fact, according to [5], this component makes the major contribution to the 884 keV state with $K^\pi = 1/2^+$. The odd particle in ^{163}Dy will only slightly alter the contribution of the phonon components with $K^\pi = 2^-$ from ^{162}Dy nucleus [5]. One might therefore expect that the largest phonon components with $K^\pi = 2^-$ in ^{162}Dy will also be the largest in ^{163}Dy. The strongest three-quasiparticle component of this state will be n [523] $- p_1$ [523] $\uparrow$ $- p_2$ [411] $\uparrow$ ($\approx 50\%$). This component explains the allowed unhindered β transition from the ground state of ^{163}Tb. Other components explain the γ transitions from the 884 keV level (Fig. 3). Similar conclusions are reached when considering the de-excitation of the 720.5 keV level in ^{179}W [18].

Now let us study the discharge of the 1538.4 and 1802.0 keV levels in ^{163}Er (Fig. 4). These levels were interpreted in Table 1 as being $3/2^+$ $\{p_1 7/2^-[523]$, $p_2 1/2^+[411]$, n $5/2^-[523]\}$ and $1/2^+$ $\{p_1 7/2^-[523]$, $p_2 1/2^+$ [411], n $5/2^-[523]\}$ three-quasiparticle states, respectively.

In order to provide an example, we shall discuss just the de-excitation of the 1538.4 keV level because the discharge mechanism for the 1802.0 keV level is of the same character as that which de-excites the 1538.4 keV level (Fig. 4). γ transitions from the 1538.4 keV level to a level of the rotational band of the ground state are not observed [19]. The 1538.4 keV level is discharged through strong γ transitions to the $3/2^-$ [521] level of the rotational band and the 5/2 $5/2^+$[642] level (see Fig. 4), for which good agreement is found between the experimental and theoretical values of the reduced transition probabilities. In the independent quasiparticle model these γ transitions are F-forbidden. The presence of these γ transitions can be explained by taking into account the interaction phonons in the even-even core of the ^{162}Er nucleus with the quasiparticles of the odd ^{163}Er nucleus. The result of this interaction between the ^{162}Er phonons and the ^{163}Er quasiparticles is the formation of quasiparticle + phonon states. Additions of these states to the 1538.4 keV level of the ^{163}Er nucleus can allow it to discharge to the single-quasiparticle states. Thus, in our case the experimental data indicate that only a mixture of the type $3/2^-$ [521] + Q_1 (30) can explain the discharge of the 1538.4 keV level to the $3/2^-$ [521] level of the rotational band (see Fig. 4). In this case we have γ transitions of type E1 with $\Delta K = 0$. For transitions of this kind the coupling of particle and rotational motions has little effect on the transition probabilities. We therefore observe good agreement between the experimental and theoretical values of the reduced probabilities for these transitions [20].

The presence of a 1469.0-keV γ transition between the 1538.4-keV level and the 69.21 keV level, ($5/2^+$ [642]), allowing for the theoretical and experimental energies of the single-quasiparticle states, can be explained only by an addition of the $3/2^+$[651] single-quasiparticle state and a state of the $3/2^+$ [651] + Q (20) type to the 1538.4-keV level.

According to the calculations of [5], at 1200 keV the ^{163}Er nucleus ought to have a state of spin $3/2^+$ and configuration $3/2^+$ [651] 2%; $3/2^-$ [521] + Q_1 (30) 72%, $3/2^+$[651] + Q_1 (20) 3%, ..., which can mix with the three-quasiparticle state $3/2^+$ $\{p_1$ 7/2 [523], p_2 $1/2^+$ [411], n $5/2^-$ [523]$\}$ [5]. Thus, rather good agreement is obtained between our interpretation, which follows from the experimental data and the above-

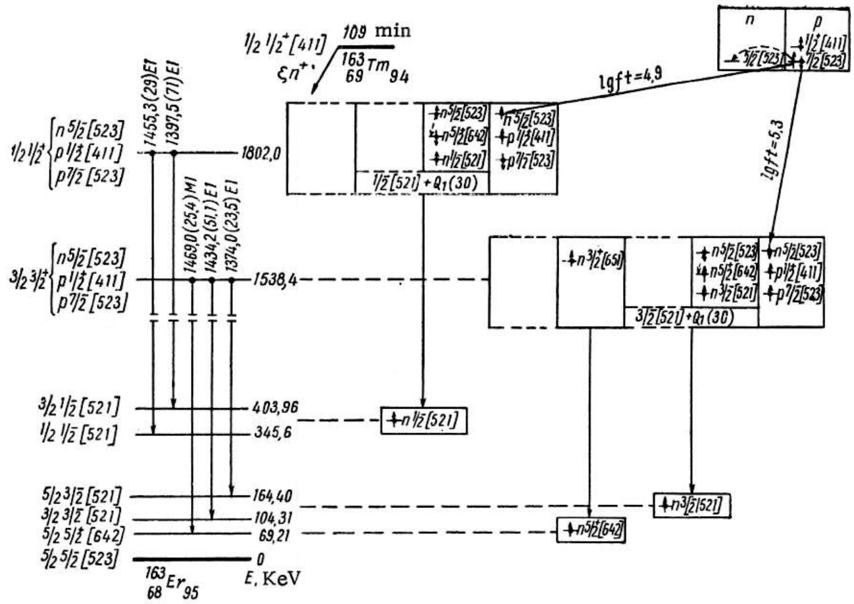

Fig. 4. Discharge of a three-quasiparticle state in ^{163}Er.

described simple model, and the assumption of Solov'ev et al. [5] about the nature of the 1538.4 keV level in ^{163}Er. An analysis of the experimental data on the properties of the levels considered here indicates that use of this simple model will make it possible to explain the discharge of all presently known three-quasiparticle states.

CONCLUSIONS

Among the excited states of odd-A deformed nuclei with mass numbers between 150 and 190 are observed a number of levels having a three-quasiparticle character. All the three-quasiparticle type levels observed until now are of the type (2n, p) or (2p, n). There are as yet no three-quasiparticle levels of the type (3n) or (3p). The energies of the three-quasiparticle levels show satisfactory agreement with the energies calculated using the superfluid model for the center of gravity of the three-quasiparticle states.

The available experimental data on the splitting of three-quasiparticle states is satisfactorily explained qualitatively by the calculations of [7] which include the interaction between quasiparticles. However, quantitative agreement has not yet been achieved.

The experimental data on the discharge of three-quasiparticle-like states enables one to make a more detailed analysis of the structure of these states. To a large extent the large-spin isomer three-quasiparticle states are pure three-quasiparticle states. The state with smaller spins contain significant mixtures of vibrational states.

It is of considerable interest to obtain new experimental data on the three-quasiparticle states. There would be great value in detecting new three-quasiparticle states in nuclear reaction studies, especially in observing (3p) and (3n) type states in (γ, n) reactions. In β decay studies of nuclei far from the stability line one might observe a large number of three-quasiparticle states populated by allowed unhindered decay.

The study of three-quasiparticle states is important in that it can help to explain the degree to which quasiparticle character is preserved in nuclear levels at high excitation energies.

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