LEPTONIC HADRON DECAYS

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The main properties of leptonic hadron decays are reviewed from the point of view of the modern theory of weak interactions. The first chapter is devoted to a description of the decay properties of Π and K mesons and the experimental data obtained prior to the middle of 1969 are analyzed. The second chapter is devoted to discussion of leptonic baryon decays in the framework of $SU_{(3)}$ symmetry.

1. Introduction

Leptonic and semileptonic hadron decays form a large class of reactions in elementary particle and nuclear physics; in such reaction a pair of leptons participates in the transition of a hadron A into another hadron B or to the vacuum. The most typical decays are

$$A \to B + l^- (l^+) + \tilde{\nu}_l(\nu_l); \quad (1a)$$

$$A \to l^- (l^+) + \tilde{\nu}_l(\nu_l). \quad (1b)$$

The reactions of this class include the β decay of atomic nuclei (in this case A is a nucleon bound in a nucleus) and the β (or μ) decays of baryons and mesons. The lifetimes of such processes are characterized by a gigantic spread: from 10^{-10} sec (hyperon decay) to 10^{21} years (double β decay of nuclei). These reactions also include the processes of μ and K capture:

$$l^- + A \to B + \nu_l \quad (2)$$

and reactions with a neutrino:

$$\nu_l (\tilde{\nu}_l) + A \to B + l^- (l^+). \quad (3)$$

All these reactions as well as a large class of hadron decay processes in which leptons do not participate and a whole series of other known or conjectured reactions are united in the framework of the present theory of the so-called weak interactions of elementary particles.

Turning to the theoretical aspects of the descriptions of these reactions in terms of modern field-theoretical notions [1], the weak-interaction Lagrangian must satisfy the following requirements: Lorentz invariance, Hermiticity, locality.

In the theory it is also assumed that there is linearity in the four-component Dirac spinor fields (without derivatives) and that the following quantum numbers satisfy conservation laws: the electric charge, the baryon number, the lepton number. Within the framework of these requirements, the weak-interaction Lagrangian can be expressed as a sum of two terms with opposite spatial parities:

$$L_W = \sum_n c_n L_n (B, A, l, \nu_l) + \sum_n c_n' L_n' (B, A, l, \nu_l), \quad (4)$$

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TABLE 1

Type of interaction	Invariant	Limit
Scalar S	I	1
Pseudoscalar P	γ_5	0
Axial vector A	$\gamma_5\gamma_\alpha$	$\gamma_5\gamma_i = \frac{1}{i}\sigma_i\gamma_4 = \sigma_i$; $\gamma_5\gamma_4 \approx 0$
Vector V	γ_α	$\gamma_4 \approx 1$; $\gamma \approx 0$
Tensor T	$\sigma_{\alpha\beta}$	$\sigma_{il} \simeq \varepsilon_{ikl}\sigma_l$; $\sigma_{i4} \simeq 0$

where for processes in which leptons participate

$$\left.\begin{aligned} L_n(B, A, l, \nu_l) &= [\bar{u}_B O_n u_A][\bar{u}_l O_n u_{\nu_l}]; \\ L'_n(B, A, l, \nu_l) &= [\bar{u}_B O_n u_A][\bar{u}_l O_n \gamma_5 u_{\nu_l}]; \end{aligned}\right\} \tag{5}$$

in this case, (4) is replaced by the more compact Lagrangian

$$L_W = \sum_n [\bar{u}_B O_n u_A[[\bar{u}_l O_n (c_n + c'_n \gamma_5) u_{\nu_l}], \tag{6}$$

and in the theory with a two-component neutrino ($m_\nu = 0$, which means 100% violation of spatial parity) this Lagrangian takes the form

$$L_W = \sum_n c_n [\bar{u}_B O_n u_A][\bar{u}_l O_n (1 + \varepsilon\gamma_5) u_{\nu_l}], \tag{7}$$

where ε may have the values $\varepsilon = \pm 1$. It follows from the data for $l\nu$ correlations in nuclear β decays that $\varepsilon = +1$.

In the above equations, u_i are the Dirac spinor functions of the corresponding particles and O_n are operators characterized by definite transformation properties under spatial rotations and reflections. Using the invariant quantities at our disposal, viz, the four-dimensional momenta P_i and the Dirac matrices γ_α, we can construct five relativistic invariants. These are given in Table 1 together with their nonrelativistic limits.

Assuming in the nonrelativistic limit that the baryons are at rest before and after the decay and ignoring the energy liberated in the decay, we obtain an approximation that describes the so-called allowed transitions, when the Lagrangian can be represented in the form

$$\begin{aligned} L_n = {} & \bar{u}_B u_A [\bar{u}_l (c_S + c_V \gamma_4)(1 + \varepsilon\gamma_5) u_{\nu_l}] \\ & + \bar{u}_B \sigma u_A [\bar{u}_l (c_T + c_A \gamma_4) \sigma (1 + \varepsilon\gamma_5) u_{\nu_l}]. \end{aligned} \tag{8}$$

Here, the first term describes nuclear β transitions of the Fermi type with $\Delta J = 0$; the second, transitions of the Gamow-Teller type with $\Delta J = 0, \pm 1$.

It follows from the relations (4) or (6) that the total Lagrangian in the general case contains ten complex constants c_n and c'_n or 19 real constants, since one can choose one common phase arbitrarily. If the process is invariant under time reversal, c_n and c'_n are real. If conservation of spatial parity is assumed, either all the c_n or all the c'_n vanish.

The formulation of the modern theory of weak interactions has been based in the first place on the experimental data on nuclear β decay and the decay of the μ^+ meson:

$$\mu^+ \to e^+ + \nu_e + \tilde{\nu}_\mu. \tag{9}$$

It is not our task to make a detailed analysis of these processes; such an analysis has led to our present understanding of these processes and, to a considerable extent, understanding of the general problem of weak interactions. We shall only mention some of the main results that follow from the currently available experimental data on this subject. It should be noted first that, in accordance with Eq. (8), pseudoscalar couplings do not participate in allowed β transitions. No traces of the existence of couplings of this kind have been found in the forbidden β transitions either. Secondly, an analysis of the data on the various correlations in nuclear β decays indicates that they do not contain tensor or scalar couplings, i.e.,

$$c_P = c'_P = c_T = c'_T = c_S = c'_S = 0. \tag{10}$$

It follows that the nuclear β decay is due entirely to a mixture of the vector and axial-vector variants of the weak interaction. It turns out that

$$c_A = c'_A; \quad c_V = c'_V; \tag{11}$$

these conditions mean that there is 100% violation of spatial parity in these processes. Finally, the known correlation properties of the β decay of polarized nuclei shows that the relative sign of the constants c_V and c_A is negative: $\alpha = c_A/c_V < 0$.

Thus, nuclear β decay is described by the V-A variant of the weak interaction [2].

All the available information on processes governed by the weak interaction indicate that this is a universal interaction. In its most general formulation, universality of the weak interactions must mean the equality of the coupling constants for all weak processes and identical fundamental space-time properties of the interaction. It follows that all weak interactions can be described by a single Lagrangian with one and the same constant G and that this Lagrangian contains only vector and axial-vector interactions if one ignores effects due to the presence of strongly interacting particles. The constant G is taken equal to the constant of $\mu+$ decay:

$$G_\mu = (1.4350 \pm 0.0011) \cdot 10^{-49}\ \text{erg}\cdot\text{cm}^3 \tag{12}$$

It follows that the weak-interaction Lagrangian can be constructed by analogy with electrodynamics. This means that in the case in which we are interested it can be expressed as a product of the leptonic $J_\alpha^{(l)}(x)$ and hadronic $J_\alpha(x)$ currents:

$$L_W(x) = \frac{G}{\sqrt{2}} J_\alpha^{(l)} \cdot J_\alpha + \text{h. c.} \tag{13}$$

in which G is the μ-decay constant G_μ (12). At the same time each of the currents that occurs in (13) must contain only vector and axial-vector parts, i.e., they must have the form

$$J_\alpha^{(l)}(x) = \bar{u}_l(x)\gamma_\alpha(1+\gamma_5)u_{\nu_l}(x) + \bar{u}_\mu(x)\gamma_\alpha(1+\gamma_5)u_{\nu_\mu}(x); \tag{14}$$

$$J_\alpha(x) = \bar{u}_B(x)\gamma_\alpha(c_V + c_A\gamma_5)u_A(x) = J_\alpha^{(V)}(x) + J_\alpha^{(A)}(x). \tag{15}$$

In the weak hadronic current $J_\alpha(x)$ there remain the constants of the vector and axial-vector couplings; in contrast to the leptonic current, for which one assumes $\varepsilon = +1$ in accordance with (7), it is a priori clear that $c_A/c_V \neq 1$ because of the renormalization effects due to the strong interactions. These effects are determined by the structure of the hadrons. At present they cannot be studied theoretically with sufficient accuracy and our only information about them is obtained experimentally.

If we now turn to the experimental facts and compare, for example, two pairs of hadron decays:

$$\left.\begin{array}{lll} \pi \to l\nu_l, & \text{and} & n \to pe\nu, \\ K \to l\nu_l & & \Lambda \to pe\nu, \end{array}\right\} \tag{16}$$

and take into account the phase-space for these decays, we find that in each of the pairs the matrix elements of the first decays, in which the strangeness quantum number S does not change ($\Delta S = 0$), are much greater than the matrix elements of the second decays, in which the strangeness changes. The same situation is encountered for the pair of Σ^--hyperon decays:

$$\left.\begin{array}{l}\Sigma^- \to \Lambda e^- \nu;\\ \Sigma^- \to n e^- \nu,\end{array}\right\} \tag{17}$$

and this condition is satisfied quite generally for all processes that proceed through weak interactions. Thus, if we represent the hadronic current (15) as a sum of two currents, of which one, $J_\alpha^{(0)}(x)$, is due to transitions with $\Delta S = 0$ and the other, $J_\alpha^{(1)}(x)$, to transitions with $|\Delta S| = 1$, i.e.,

$$J_\alpha^{\cdot}(x) = J_\alpha^{(0)}(x) + J_\alpha^{(1)}(x), \tag{18}$$

the constants for these two types of transition must be different. It follows that the V-A theory in its original form is not universal; even for the different processes of the same nature one must introduce separate appropriate interaction constants. Cabibbo [3] made the decisive step in restoring the universality. This is done as follows. We write down two Lagrangians: for the strangeness-conserving decay

$$L_W^{(0)} = \frac{G}{\sqrt{2}} J_\alpha^{(l)} \cdot J_\alpha^{(0)} \tag{19}$$

and for the strangeness-violating decay

$$L_W^{(1)} = \frac{G}{\sqrt{2}} J_\alpha^{(l)} \cdot J_\alpha^{(1)}. \tag{20}$$

The gist of Cabibbo's assumption is that both the hadronic currents $J_\alpha^{(0)}(x)$ and $J_\alpha^{(1)}(x)$ belong to the same octet representation of $SU_{(3)}$ symmetry and that the total hadronic current is turned through an angle θ with respect to the spin axis u_2 in the corresponding eight-dimensional unitary spin space. The upshot is that the hadronic current, which we previously wrote as the sum (18), now takes the form

$$J_\alpha(x) = \cos\theta \cdot J_\alpha^{(0)}(x) + \sin\theta\, J_\alpha^{(1)}(x), \tag{21}$$

while the leptonic current $J_\alpha^{(l)}(x)$ and the total Lagrangian of the weak interaction retain their structure.

We see that the introduction of the angle θ extends the original concept of universality in a distinctive manner: the leptonic current remains unchanged, whereas the hadronic currents acquire the additional factors $\sin\theta$ and $\cos\theta$, which maybe included among the weak-interaction constants, and the universality concept acquires a wider meaning. Now that the universality of the weak interactions has been restored by the introduction of the Cabibbo angle, these interactions are described by two parameters: the common coupling constant G and the angle θ. One could, of course, attempt to go further and make the theory more precise by introducing, for example, different angles for the vector and axial-vector parts of the current and for the components of the current that conserve and change strangeness. It is natural that an appropriate choice of such angles would lead to better agreement between the experiments and theory but the introduction of many new constants detracts from the elegance of the theory of weak interactions.

Finally, it should be emphasized that Cabibbo's assumption does not in fact violate the conception of a universal V-A interaction in its original form but merely extends and makes it more precise. Although the physical significance of the angle θ is not yet completely clear, Cabibbo's conjecture is undoubtedly a step forward in the theory of weak interactions.

We shall now proceed to a systematic exposition of the various aspects of leptonic meson and baryon decays, which, as we have seen, have a common nature and are characterized by the same constants of the theory.

1. Leptonic Meson Decays

In the meson decay processes the leptonic current in the Lagrangian (7) corresponds to the creation of two leptons from the vacuum:

$$\langle l, \nu_l | J_\alpha^{(l)} | 0 \rangle.$$

The hadronic current J_α must either annihilate the original meson A:

$$\langle 0 | J_\alpha | A \rangle,$$

or transform it into a different hadron state B:

$$\langle B | J_\alpha | A \rangle.$$

Both these currents must be constructed from the wave functions ψ_i of the particles participating in the process. The leptonic covariants which must be used to construct $J_\alpha^{(l)}$ are well known:

$$S^l = \bar{\psi}_\nu \psi_l;$$
$$V^l = \bar{\psi}_\nu \gamma_\alpha \psi_l;$$
$$T^l = \bar{\psi}_\nu \sigma_{\alpha\beta} \psi_l;$$
$$A^l = \bar{\psi}_\nu \gamma_5 \gamma_\alpha \psi_l;$$
$$P^l = \bar{\psi}_\nu \gamma_5 \psi_l.$$

To obtain a coupling with the neutrino in the form $\psi_\nu^{-}(1-\gamma_5)$ one must construct from the existing covariants the definite linear combinations

$$S^l - P^l = \bar{\psi}_\nu (1-\gamma_5) \psi_l;$$
$$V^l - A^l = \bar{\psi}_\nu (1-\gamma_5) \gamma_\alpha \psi_l = \bar{\psi}_\nu \gamma_\alpha (1+\gamma_5) \psi_l;$$
$$T^l - PT^l = \bar{\psi}_\nu (1-\gamma_5) \sigma_{\alpha\beta} \psi_l.$$

On the basis of the Lagrangian (7) we obtain the general form of the Hamiltonian:

$$H_W = S(S^l - P^l) + V(V^l - A^l) + TT^l + A(V^l - A^l) + P(S^l - P^l),$$

where S, V, T, A, and P refer to the hadronic current.

Whereas the leptonic covariants are the same for meson decays of the type $A \to 0$ and transitions of the type $A \to B$, the hadronic part of the Hamiltonian must take into account the specific form of the decay. In this connection let us consider the different transitions of π and K mesons in which leptons participate, i.e., the decays of pseudoscalar mesons.

1.1. Meson Decays that Proceed through the Channel $A \to l + \nu_l$

The leptonic decays of the charged π and K mesons

$$A^{\pm} \to e^{\pm} (\mu^{\pm}) + \nu_l (\tilde{\nu}_l) \tag{1.1}$$

are distinguished from all the other hadron decays by the absence of strongly interacting particles in the final state.

In principle, processes that are free from the effects associated with the strong interaction between particles in the final state enable one to obtain information about the decaying particle in the most unadulterated form. The properties of the decaying hadron, i.e., the π or K meson in the present case, must determine the behavior of certain functions of the kinematic variables corresponding to the given decay. Unfortunately, there are no kinematic variables in two-particle decays with fixed momenta of the secondary particles. The only parameters that characterize the hadrons in the reaction (1.1) are the coupling constants f_π and f_K, which can be compared with each other.

Let us consider this hadron decay mode for the reaction

$$K^{\pm} \rightarrow l^{\pm} + \nu_l\,(\tilde{\nu}_l).$$

The leptonic part of the Hamiltonian H_W is known; let us analyze separately each term of the hadronic part of H_W.

With a negative parity of the K meson $\langle 0\,|S|\,K\rangle = 0$, whereas the matrix element $\langle 0\,|P|\,K\rangle$ may be nonvanishing. The only invariant kinematic variable related to the hadronic part of the process is the square of the four-momentum of the K meson, $p_K^2 = m_K^2 = \text{const}$; thus,

$$\langle 0\,|\,P\,|\,K\rangle = f_p = \text{const}.$$

Arguing in the same way for the vector and axial-vector parts, we obtain

$$\langle 0\,|\,V\,|\,K\rangle = 0;$$

$$\langle 0\,|\,A\,|\,K\rangle = f_A \cdot p_K^{\alpha},$$

where f_A does not, of course, depend on p_K. For the tensor part we must write

$$\langle 0\,|\,T\,|\,K\rangle = f_T \cdot p_K^{\alpha} \cdot p_K^{\beta}.$$

The total matrix element of the decay obtained from the relation for the total Hamiltonian H_W is

$$\langle l, \nu_l\,|\,H_W\,|\,K\rangle = f_p\,\langle l, \nu_l\,|\,\bar{\psi}_\nu (1-\gamma_5)\,\psi_l\,|\,0\rangle$$
$$+ f_A \sum_\alpha p_K^{\alpha}\,\langle l, \nu_l\,|\,\bar{\psi}_\nu (1-\gamma_5)\,\gamma_\alpha\,\psi_l\,|\,0\rangle + f_T \sum_\alpha \sum_\beta p_K^{\alpha}\,p_K^{\beta}\,\langle l, \nu_l\,|\,\bar{\psi}_\nu$$
$$\times (1-\gamma_5)\,\delta_{\alpha\beta}\,\psi_l\,|\,0\rangle. \tag{1.2}$$

The last term, which describes the tensor contribution to the matrix element, must vanish, since $\sigma_{\alpha\beta}$ is antisymmetric with respect to its subscripts and the product $p_K^{\alpha} \cdot p_K^{\beta}$ is symmetric with respect to the α and β functions. It follows that this kaon decay mode cannot give any information about tensor couplings. Going over to spinors and using the Dirac equation, we obtain the final form of the decay matrix element:

$$\langle l, \nu_l\,|\,H_W\,|\,K\rangle = (f_p - m_l f_A)\,\left[\bar{u}_\nu (p_\nu)\,(1-\gamma_5)\,u_l\,(p_l)\right]. \tag{1.3}$$

For decays that proceed through the modes $K \rightarrow e\nu_e$ and $K \rightarrow \mu\nu_\mu$ there must be an important difference due to the large mass difference $m_e - m_\mu$ and the neutrino helicity. This situation arises because the helicity is determined by the ratio of the particle velocity to the velocity of light (v/c) and is therefore always equal to unity for the neutrino. Conservation of the total angular momentum necessitates an induced helicity of the charged lepton and the phase volume of the interaction is therefore proportional to $1-v/c$. Since the electron has a small mass, its velocity is nearly equal to c and the decay $K \rightarrow e\nu_e$ is suppressed compared with the transition $K \rightarrow \mu\nu_\mu$ since we obviously have $v_\mu/c \ll v_e/c$. After summation over the lepton polarizations, we immediately obtain the branching ratio of the electron and muon modes:

$$R_0 = \frac{\Gamma\,(K \rightarrow e\nu_e)}{\Gamma\,(K \rightarrow \mu\nu_\mu)} = \left[\frac{1-(m_e/m_K)^2}{1-(m_\mu/m_K)^2}\right]^2 \cdot \left[\frac{f_P - m_e f_A}{f_P - m_\mu f_A}\right]^2, \tag{1.4}$$

where the first factor is the ratio of the phase volumes.

Let us consider the limiting cases in which one of the two possible couplings predominates. If $f_A = 0$, we have pure S−P coupling and R_0 (the ratio of the phase volumes in this case) is ≈ 1.1. The other limiting case $f_P = 0$ leads to V−A coupling and in this case $R_0 \approx 1.1 \times m_e^2/m_\mu^2 \approx 2.6 \times 10^{-5}$. The values of R_0 for these

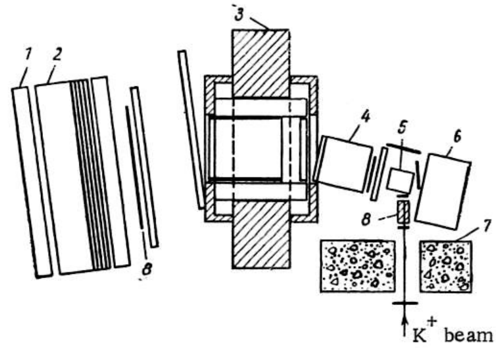


Fig. 1. Spectrometer for the investigation of the decay $K^+ \to e^+\nu e$: 1) range spark chambers; 2) aluminum plates; 3) magnet; 4) Cerenkov counter; 5) beryllium chamber; 6) lead glass; 7) concrete shield; 8) copper moderator.

two possibilities differ from each other by a factor of almost 10^5; it follows that the branching ratio of these decays is extremely sensitive to the nature of the pseudoscalar coupling.

In deriving the decay matrix element we ignored the graphs that describe the radiation corrections that arise because of the emission of a virtual photon and internal bremsstrahlung. When these corrections are taken into account, we obtain $R_1 = 0.815 \times R_0 \approx 2.1 \cdot 10^{-5}$ for the case of pure axial-vector coupling.

Experimental difficulties entailed in investigations of decays in which an electron participates have rendered it impossible until very recently to make any sort of accurate estimate of R_1. These difficulties are due to the low value of the relative rate (less than for the muon), the low accuracy in the determination of the energy of the emitting electron, and the appreciable background from the semileptonic decay $K \to \pi e \nu_e$. All these factors have meant that it has been virtually impossible to investigate the $K \to e\nu_e$ decay by means of emulsions and bubble chambers. Prior to 1967 all one knew was an upper limit, $R_1 \leq 2.6 \cdot 10^{-3}$, and it was only with the use of complicated spectrometers (with spark chambers, Cherenkov and scintillation counters) with a good momentum resolution that it proved possible [4] to measure R_1.

At the present time the mean experimental value of the $K^+ \to e^+\nu_e$ to $K^+ \to \mu^+ \nu_\mu$ branching ratio is $\langle R_1 \rangle = (2.15 \pm 0.35) \cdot 10^{-5}$, in fairly good agreement with the estimate for pure axial-vector coupling [for the processes $\pi \to e\nu_e$ and $\pi \to \mu\nu_\mu$ we have $R_1^\pi = (1.24 \pm 0.03) \cdot 10^{-4}$]. The upper limit for the pseudoscalar form factor f_p obtained from (1.4) is $|f_p| \leqslant 1.5 \cdot 10^{-3}$. One may therefore conclude that axial-vector coupling predominates in the leptonic decays of K mesons, there being no appreciable admixture of pseudoscalar coupling; however, the experimental accuracy is not yet sufficient to make a rigorous quantitative estimate of $|f_p|$. (The arrangement of the Oxford group's spectrometer and the position momentum spectrum obtained in an investigation of the K_{e2} decay are shown in Figs. 1 and 2.) Considering the parameter R_1 we have assumed that $\mu - e$ universality holds, i.e., that $f_A^e = f_A^\mu$ and $f_p^e = f_p^\mu$. A few words on $\mu - e$ universality are appropriate. In the form adopted the weak-interaction Lagrangian $\mu - e$ universality is assumed, i.e., it is assumed that the interactions are invariant under a simultaneous substitution of the form

$$\left.\begin{array}{l} \mu \rightleftarrows e; \\ \nu_\mu \rightleftarrows \nu_e. \end{array}\right\} \tag{1.5}$$

The differences in the masses m and m_e despite the otherwise complete identity of the properties of these leptons and their quantum numbers is one of the currently most important problems of high-energy physics [5]. The point is that it is quite natural to explain the differences in the masses of other particles, for example, the electron and the pion, by the fact that these particles participate in different interactions. However, this explanation breaks down for the electron and the muon, since both leptons take part in the same hitherto known interactions, i.e., the weak and the electromagnetic interactions, and, what is more, participate universally in the same manner. The experimental data on the different processes in which the electron and muon participate have hitherto confirmed this assumption.

Anticipating our later discussion, let us compare the probability of processes in which the substitution (1.5) holds. Such decays have the same absolute matrix elements and the difference in the probabilities is due solely to the difference in the phase factors, which can usually be calculated easily. For example, $\Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu) = (\frac{m_e}{m_\mu})^2 \left[\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right] 0.965 \approx 1.23 \cdot 10^{-4}$. In Table 2 we give the data for various decays that are symmetric under (1.5).

Thus, to within the experimental errors, the measured values agree with those predicted by $\mu - e$ universality. Unless specially stipulated, we shall henceforth assume that $\mu - e$ universality is valid. Let us

TABLE 2. Comparison of Decays with Electrons and Muons

Decays	Theory	Experiment
$\Gamma(\pi^+\to e^+\nu_e)/\Gamma(\pi^+\to\mu^+\nu_\mu)$	$1{,}23\cdot10^{-4}$	$(1{,}24\pm0{,}03)\cdot10^{-4}$
$\Gamma(K^+\to e^+\nu_e)/\Gamma(K^+\to\mu^+\nu_\mu)$	$2{,}1\cdot10^{-5}$	$(2{,}15\pm0{,}35)\cdot10^{-5}$
$\Gamma(K^+\to\pi^0e^+\nu_e)/\Gamma(K^+\to\pi^0\mu^+\nu_\mu)$	0,69	$0{,}703\pm0{,}056$
$\Gamma(\Lambda\to\bar{p}e\tilde{\nu}_e)/\Gamma(\Lambda\to p\bar{\mu}\tilde{\nu}_\mu)$	5,88	$5{,}87\pm0{,}75$
$\Gamma(\overline{\Sigma}\to n\bar{\mu}\tilde{\nu}_\mu)/\Gamma(\overline{\Sigma}\to\bar{n}e\tilde{\nu}_e)$	0,45	$0{,}40\pm0{,}06$

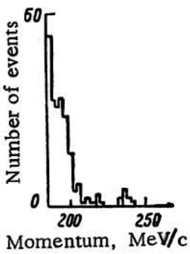


Fig. 2. Momentum spectrum of positrons from the K_{e2} decay.

now turn to the decay $K^+\to e^+\nu_e$. It is interesting to compare the relative strength of the interactions $K\to l\nu_l$ and $\pi\to l\nu_l$. If, as now seems reasonable, $f_P\approx 0$, if follows from the experiments that $f_A(K\to\mu\nu_\mu)/f_A(\pi\to\mu\nu_\mu)\approx 0.52$. There are numerous indications [6] that leptonic decays in which the strangeness changes must be at least three times less in amplitude than decays of nonstrange particles. With allowance for the mass factors in the dimensionless amplitudes, we have $f_A^{K,\pi}=Gm^{K,\pi}\cdot f_A'^{K,\pi}$ and $|f_A'^K|/|f_A'^\pi|\approx$ 0.14, which indeed yields the desired suppression.

We now ask what information these decays yield about the value of the Cabibbo angle θ. If (21) is true, the ratio of the amplitudes of the decays $K\to l_\nu l$ and $\pi\to l_\nu l$, which is proportional to the ratio of the matrix elements of the currents $J_\alpha^{(1)}$ and $J_\alpha^{(0)}$, is

$$\frac{A(K\to l\nu_l)}{A(\pi\to l\nu_l)}\sim\frac{\langle 0|J_\alpha^{(1)}|K\rangle}{\langle 0|J_\alpha^{(0)}|\pi\rangle}=\operatorname{tg}\theta\,\frac{\langle 0|j_\alpha^{(1)}|K\rangle}{\langle 0|j_\alpha^{(0)}|\pi\rangle}. \qquad (1.6)$$

For the decays $K^+\to\mu^+\nu_\mu$ and $\pi^+\to\mu^+\nu_\mu$ this means that $\theta_A=0.257$ rad. From a comparison of the decays $K(\pi)\to\pi l\nu_l$ one obtains $\theta_V\approx 0.26$ rad. We see that there is very good agreement between the two values of θ for the decays $K(\pi)\to l\nu_l$ due to the axial-vector part of the interaction and the decay $K\to\pi l\nu_l$ and $\pi^0 l\nu_l$, in which, as we shall see below, the vector part predominates. Of course, universality of the weak interactions is not restricted to universality of the pion and kaon decays; one must also make a full analysis of all the transitions that proceed through weak interactions. We shall do this later. Here, we shall only point out that the values obtained for the angle θ from the other decays do not contradict the quantities given in the present paper. The experiments yield the following values:

$\sin\theta_A=0.2655\pm0.0005$ from a comparison of $K^+_{\mu2}$ and $\pi^+_{\mu2}$;
$\sin\theta_V=0.220\pm0.003$ " " " K^+_{e3} and π^+_{e3};
$\sin\theta_V=0.207\pm0.004$ " " " K^0_{e3} and $\pi^+_{\mu3}$.

1.2. Selection Rules in Pion and Kaon Decays

Before we consider the other decay modes of pseudoscalar mesons in which leptons participate, we must obtain some general rules for all transitions, i.e., selection rules for a number of quantum numbers. It has been established that V–A coupling is present in leptonic decays in which such a meson is annihilated. However, besides a matrix element of, for example, the form $\langle 0|J_\alpha^{(1)}|K\rangle$, there exist many other matrix elements with the current $J_\alpha^{(1)}$ for which one must establish the type of the coupling and the other characteristics.

As we have seen in the previous section, the leptonic current operator $J_\alpha^{(l)}$ can lead, for example, to the creation of a neutrino and a positive lepton from the vacuum or the disappearance of a negative lepton and the appearance of a neutrino. All such transitions under the influence of the operator $J_\alpha^{(l)}$ have a common feature: the electric charge of the final state is always greater by unity than the charge of the initial state, i.e., there is a change of the charge under the influence of the current $J_\alpha^{(l)}$, $\Delta Q=+1$. Since the charge is conserved in all known interactions, this must lead to the existence of only those matrix elements with the currents $J_\alpha^{(0)+}$ and $J_\alpha^{(1)+}$ which yield a change of the charge $\Delta Q=-1$. Thus, a necessary condition for the matrix element $\langle B|J_\alpha^{(i)}|A\rangle$ to be nonvanishing is $\Delta Q=Q_B-Q_A=-1$. However, since $\langle B|J_\alpha^{(i)+}|A\rangle=\langle A|J_\alpha^{(i)}|B\rangle^*$, this rule means that transitions under the action of $J_\alpha^{(i)}$ lead to $\Delta Q=+1$. It now remains to assume that this rule is valid for all transitions under the influence of $J_\alpha^{(0)}$ and $J_\alpha^{(1)}$. Let us consider how the other hadron quantum numbers behave in different processes. In all interactions known at present the baryon number is strictly conserved. It follows that there is a further selection rule that we can introduce for the current $J_\alpha^{(i)}$, namely $\Delta N = 0$.

Let us consider a specific matrix element with $\Delta Q = 1$ which is certainly nonvanishing, $\langle 0|J_\alpha^{(1)}|K^-\rangle$. Since the strangeness of the K^- meson is $S = -1$ and $S = 0$ for the vacuum, $\Delta S = 1$ for this transition. Let us therefore assume that $\Delta S = 1$ for all transitions under the influence of $J_\alpha^{(1)}$; we immediately obtain the rule $\Delta Q = \Delta S$ for all such processes. In transitions with $\Delta S = 0$, we have the obvious consequence that $\Delta Q = 1$.

Let us now turn to the isotopic structure of our matrix elements. The K meson has isospin 1/2; the final state for a transition to the vacuum does not contain hadrons and the isospin of the vacuum is, of course, zero. If we now assume that $J_\alpha^{(1)}$ is a ΔI vector in the isotopic space, then, from the known nonvanishing matrix elements $\langle 0|J_\alpha^{(1)}|K\rangle$ one can assert that at least some of the matrix elements are such that they satisfy $|\Delta I| = 1/2$. As in the case of the strangeness, we shall again make the simplest assumption that only transitions in which the isospin changes by 1/2 are realized under the influence of the current $J_\alpha^{(1)}$. A similar treatment for transitions with $\Delta S = 0$ leads to $|\Delta I| = 1$.

1.3. Semileptonic Decays

The leptonic decay experiments discussed above made it possible to obtain information about the axial-vector and pseudoscalar coupling and the value of the Cabibbo angle θ_A. However, they do not yield any information about the scalar, vector, and tensor couplings and the corresponding angle θ_V. Such couplings can be studied by investigating the decays

$$\left.\begin{array}{ll} K \to \pi + e + \nu_e & (K_{e3}); \\ K \to \pi + \mu + \nu_\mu & (K_{\mu 3}). \end{array}\right\} \qquad (1.7)$$

Apart from the type of the interaction, semileptonic transitions make it possible to verify effects associated with the violation of T and CP invariance, to investigate the structures of the form factors that describe the contribution from the strong interactions, and also such consequences of the theory as, for example, the assumption of the local creation of a lepton pair.

Comparison of the form factors and the decay rates of neutral and charged kaons makes it possible to verify the selection rule $|\Delta I| = 1/2$, whereas equality of the corresponding form factors in the K_{e3} and $K_{\mu 3}$ decays would be confirmation of $\mu - e$ universality. The nature of the time dependence of the decays of neutral kaons depends on the extent to which the rule $\Delta Q = \Delta S$ is satisifed; in an investigation of this dependence one can estimate the value of x, the ratio of the amplitudes of decays with $\Delta Q = -\Delta S$ to the decay amplitudes in which this rule is not violated. The Hamiltonian for the semileptonic decays has the same general form as for the transitions $K(\pi) \to l\nu_l$ but, in contrast to the purely leptonic decays, the current $J_\alpha^{(i)}$ does not annihilate a pion or kaon but transforms it into a pion: $M^{(i)} = \langle \pi|J_\alpha^{(i)}|K(\pi)\rangle$. Arguing as previously and noting that the p parity of the initial and final hadrons is the same in this case, we find that three possibilities can be realized for the decays $K \to \pi l \nu_l$:

scalar interaction $M \sim f_S$;

vector interaction $M \sim 1/2 f_+ (p_K + p_\pi)_\alpha + 1/2 f_- (p_K - p_\pi)_\alpha$;

tensor interaction $M \sim f_T p_K^\alpha p_\pi^\beta$.

(Here, we have at our disposal two kinematic variables, namely, the four-momenta of the kaon p_K and the pion p_π.)

The functions f_i are dimensionless form factors that depend only on the square of the four-momentum transferred to the lepton pair; this follows from the assumption of a local creation of leptons. The form factors are relatively real if the interaction is invariant under time reversal. For the pure vector variant of the coupling, the majority of theoretical estimates indicate that $f_\pm$ must be slowly varying functions of $q^2 = (p_K - p_\pi)^2$, i.e., one is naturally led to expand them in powers of q^2:

$$f_\pm(q^2) = f_\pm(0)\left[1 + \lambda_\pm q^2/m_\pi^2\right],$$

On the other hand, the form factors $f_\pm$ can be represented in the form

$$f_\pm(q^2) = f_\pm(0)\left[X/(X - q^2/m_\pi^2)\right],$$

where $m \equiv m_\pi X^{1/2}$ is the mass of the intermediate K* state ($J^P = 1^-$, $I = 1/2$). For $|\lambda_\pm| \gtrsim 0.1$ both representations of $f_\pm$ are equivalent for $\lambda = 1/X$.

Nature of the Coupling Responsible for the Decays $K \to \pi + l + \nu_l$. Correct interpretation of the experimental data on the $K \to \pi l \nu_l$ processes due to some particular form of interaction are possible only if one allows correctly for the distortions due to the strong interactions and if the assumption of local creation of leptons is correct. Of course, the form of any energy or angular distribution of secondary particles depends on the nature of the coupling responsible for the decay but the different spectra have different sensitivities to the assumptions of the theory and the effect of the strong interactions. As we already know, all the effects from the strong interactions are included in the form factors f_i. In the simplest case of constant form factors, all the distributions are equally suited to an analysis of the nature of the interaction. However, in the general case, when the form factors depend on the energy of the strongly interacting particles, one must always investigate the parameters that are least sensitive in the given case to a change in the pion energy (or rather, to $g_2 = m_K^2 + m_\pi^2 - 2m_K E_\pi$).

Which distributions are most suited to an investigation of the nature of the interaction? The electron momentum spectrum is relatively insensitive to a change in the pion energy, being obtained by integration over the energy E_π, and it follows that the information about the nature of the interaction is not too strongly affected by an inaccuracy in the knowledge of the value and structure of the form factors. The distribution over the angle α between the momenta of the neutrino and the pion in the rest system of the dilepton (lepton plus neutrino) is independent of the change of the form factors; this is also true, of course, for all other spectra obtained for a fixed energy of the pion. (All these arguments hold if there exists pure coupling of a particular kind.) As regards the assumption of local creation of the lepton pair, this also can be most conveniently verified by analyzing the angular correlation in α. A difference in the behavior of the electron (for $m_e \approx 0$) and the neutrino would indicate either violation of the locality principle or the presence of a large contribution from the scalar-tensor term (or both). It should be noted that the asymmetry which arises if there is nonlocal creation of the leptons is manifested only in the structure of the form factor f_+ in the decay $K \to \pi e \nu_e$, whose expansion in powers of q^2 contains a term that depends on the momentum transferred to the lepton:

$$f_+(q^2) = f_+(0)\left[1 + \lambda_+ q^2/m_\pi^2 + \lambda_e q_e^2/m_\pi^2\right],$$

where $q_e^2 = m_K^2 + m_e^2 - 2m_K E_e$.

Attempts to distinguish a term proportional to λ_e in the structure of $f_+(q^2)$ clearly smack of sophistry but, approaching the Milan group's result [7] purely formally, we may remark that it does not contradict the principle of local ($\lambda_e = 0$) lepton creation $\lambda_e = 0.11^{+0.013}_{-0.011}$. For more than a decade unceasing investigations of semileptonic decays have been made. The accuracy of the measurements and the estimate of the background have been improved and an ever greater body of statistical material has been accumulated. The recent experiments [8, 9] of the groups at Saclay (K_{e3}^0) and Princeton (K_{e3}^+), which were carried out by means of spark chambers and counters (Fig. 3), were based on more events than all the foregoing experiments together. As in the earlier investigations, it was again found that the vector coupling is predominant and that the contribution to the amplitude from the scalar and tensor interactions satisfies the following inequalities: for $K_{e3}^0 : A_S/A_V^0 \leq 0.11$, $A_T/A_V \leq 0.06$ with a 68% confidence limit and for $K_{e3}^+ : A_S/A_V \leq 0.15$, $A_T/A_V \leq 0.04$ with a 90% confidence limit.

The Form Factors $f_\pm$ and ξ in Kaon Decays. If vector coupling is predominant in semileptonic decays, the total matrix element must have the form

$$M = \frac{G}{\sqrt{2}} \sum_\alpha \left[f_+(q^2)(p_K + p_\pi)_\alpha + f_-(q^2)(p_K - p_\pi)_\alpha\right] \bar{u}_e \gamma_\alpha \times (1 + \gamma_5) u_\nu, \tag{1.8}$$

where, as we have indicated above, the form factors $f_\pm(q^2)$ can be represented by expansions in q^2. These form factors are directly related to virtual strong interaction in which kaons and pions participate. To

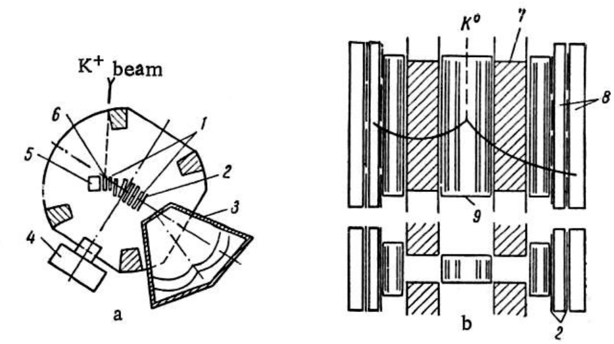


Fig. 3. Apparatus of the groups at Princeton (a) and Saclay (b) for the investigation of the K^{+}_{e3} and K^{0}_{e3} decays: 1) spark chambers; 2) scintillation counters; 3) Cerenkov counters; 4) photographic system; 5) detector of π^0 mesons; 6) kaon detector; 7) magnet; 8) range spark chambers; 9) momentum spark chambers.

analyze their energy structure one can apply dispersion relations under the most varied hypotheses, for example, under the assumption [10] that the virtual strong interactions dominate with an intermediate K* state present. In addition, numerous predictions based on current algebra [11] have recently been made concerning the structure of the form factors and their mutual relationships for different decay modes. Of particular interest from the point of view of comparison with the predictions is the form factor $\xi(q^2)$, the ratio of the form factors f_- and f_+:

$$\xi(q^2) = f_-(q^2)/f_+(q^2).$$

The solution of several problems depends on the investigation of ξ. The violation of T invariance in semileptonic decays leads to the appearance of a phase of ξ that is not equal to 0 or 180°. The selection rule $\Delta I = 1/2$ gives a quantitative relationship between the form factors ξ in decays of neutral and charged kaons. If only the amplitude with $\Delta I = 1/2$ makes a contribution to the K_{l3} decays, we must have $\xi_0 = \xi_{\pm}$. On the other hand, if ξ is known, a comparison of the form factors f_+^e and f_+^{μ} from the K_{e3} and $K_{\mu 3}$ decays (by analogy with the leptonic K_{l2} decays) can be used to verify $\mu - e$ universality, which predicts $f_+^e = f_+^{\mu}$.

Some methods of determining ξ are based on measurements that can only be interpreted correctly when the answers to the questions listed above are known. For example, in many experiments ξ has been determined by measuring the ratio of the partial rates of K-meson decay through the $K_{\mu 3}$ and K_{e3} modes:

$$\eta = \frac{\Gamma(K \to \pi\mu\nu_{\mu})}{\Gamma(K \to \pi e \nu_e)} \approx 0.65 + 0.13\,\xi + 0.02\xi^2.$$

The branching ratio η has this form because the terms of the matrix element with ξ are proportional to the mass of the leptons and the denominator of the fraction does not contain ξ for $m_e = 0$. In this method of estimating ξ one must assume, first, that f_+ and f_- are constants and secondly, that $\mu - e$ universality holds. For example, if the first assumption is false, η takes the form

$$\eta \approx 0.649 + 0.127\,\mathrm{Re}\,\xi + 0.019\,|\xi|^2 + 1.34\lambda_+ + 0.008\lambda_+\,\mathrm{Re}\,\xi + 0.459\lambda_-\,\mathrm{Re}\,\xi + 0.163\lambda_-\,|\xi|^2 - 0.068\lambda_+\,|\xi|^2, \quad (1.9)$$

and to estimate ξ correctly one must know the energy structure of the form factors $f_{\pm}(q^2)$, i.e., the parameters λ_+ and λ_-. A further possibility of estimating ξ arises in an investigation of the energy spectra of pions and muons and also from a measurement of the muon polarization in the decay $K \to \pi\mu\nu_{\mu}$. The interpretation of such experiments does not depend on the extent to which $\mu - e$ universality holds but additional assumptions must be used. The only method that does not depend on any of these assumptions, i.e., $\mu - e$ universality, invariance under time reversal, and the nature of the energy structure of $f_{\pm}$, is to make measurements for fixed energy of the muons and pions, for then necessarily ξ = const. This can be done by measuring the polarization direction of the muon at each point of the Dalitz plot (E_{π}, E_{μ}). The polarization direction of the muon at a given point (100% polarization) depends in this case only on the corresponding value of ξ: $\hat{p} = f(\xi)$. The angular distribution over the directions of the momenta of the electrons $\hat{e}$ from the muon decay relative to the direction of the magnetic field B employed is a simple function of ξ:

$$dN/d(\hat{l}\,\hat{B}) \sim [1 + \alpha \hat{p}_{\mu}(\xi)\cdot\hat{B}]\,(\hat{l}\,\hat{B}),$$

where α is the asymmetry parameter of the muon decay.

Many experiments have now been performed in which ξ has been determined by different methods [12]. The weighted mean of the branching ratio for the $K_{\mu 3}$ and K_{e3} modes are $\eta^+ = 0.73 \pm 0.03$ for K^+ mesons and

$\eta_0 = 0.78 \pm 0.05$ for K^0 mesons. If we assume that $f_\pm$ are constant, i.e., $\lambda_\pm = 0$, and that there is invariance under time reversal, Im $\xi = 0$, then for $\langle\eta\rangle = 0.75$ we obtain $\xi = 0.6 \pm 0.2$. On the other hand, measurements of the muon polarization in decays of charged and neutral kaons yield Re $\xi = -1.0 \pm 0.2$. Thus, $\xi_{br \cdot rat} - \xi_{polar} \approx 1.6 \pm 0.3$, which differs from zero by more than five standard deviations.

It is true that results have recently been obtained which indicate that $\eta_+ = 0.596 \pm 0.025$ and $\xi = -0.72 \pm 0.21$. The last result agrees with the data found from the polarization experiments. However, if such a difference is present, how could one to attempt to explain it? First of all, it would be natural to try an energy dependence of the form factors, i.e., to introduce nonvanishing values of the parameters λ_+ into the expresion for the branching ratio. We must now explain why we take λ_+ and not both λ_+ and λ_-, which would enable one to allow for an energy structure of both f_+ and f_-. The point is that many theoretical estimates show that $|f_-| < |f_+|$ [13] or $\xi \approx 0$, and in the limit of strict $SU_{(3)}$ symmetry we would exact vanishing of ξ. One could introduce a nonvanishing λ_+ into (1.9) without regard to the experimental polarization data since the latter are much less sensitive to λ_+ than the $K_{\mu3}$ and K_{e3} branching ratio. However, to change the value of ξ from the experimental value $\xi = +0.6$ to $\xi = -1.0$ without changing the branching ratio $\eta = 0.75$ one would have to take $\lambda_+ \approx 0.15$. Such a value of λ_+ contradicts not only many theoretical predictions ($|\lambda_+| \le 0.02$) but also the majority of experimental data obtained from the study of K_{e3} decays. At the present time nothing is known definitely about the values of λ_+ in the K^+_{e3} and K^0_{e3} decays. It follow from the previous experimental data that $\langle\lambda_+\rangle_{K^+} = 0.024 \pm 0.009$ and $\langle\lambda_+\rangle_{K^0} = 0.018 \pm 0.009$. However, there have been recent indications that the values of λ_+ may lie in the interval 0.06-0.09. If $\xi = -1$ and we assume $\lambda_- = 0$, then for $\lambda_+ = 0.02$ the branching ratio η must be equal to 0.6, which differs from the previously found mean value (0.75 ± 0.03) by more than five standard deviations but corresponds to the value of η_+ obtained in recent experiments with K^+ mesons. On the other hand, if $\lambda_+ = 0.09$, then $\eta \approx 0.75$; this agrees with the conclusions drawn from an analysis of the results of the earlier experiments but contradicts the more recent data.

There remains the possibility of reconciling the data by taking $\lambda_- \neq 0$. Although theoretical estimates indicate $|f_-| < |f_+|$, the majority of them do, on the other hand, predict [14] a stronger energy dependence of f_- than that of f_+, i.e., $|\lambda_-| > |\lambda_+|$. Let us consider what values of λ_- could reconcile the experimental data. For $\xi = -1.0$, $\lambda_+ = 0.02$, and $\eta = 0.75$ one must take $|\lambda_-| = 0.6$. Some theoretical models indicate $|\lambda_-| \ge 0.2$ if $|\lambda_+| \le 0.02$, but if we have such a large value, $\lambda_- = -0.6$, the very expansion of f_- in powers of q^2 in the form $1 + \lambda_- \frac{q^2}{m_\pi^2}$ may be incorrect. We still have $\mu - e$ universality at our disposal; suppose we suddenly find $f_+^e \neq f_+^\mu$; then, for example, for $\lambda_\pm = 0$ and Im $\xi = 0$ we have $\eta = (f_+^\mu/f_+^e)^2 \times (0.065 + 0.13\,\xi + 0.02\ \ \xi^2)$ and for $|f_+^\mu|\ \ |f_+^e| > 1$ we then have $\xi_{br.rat} > \xi_{polar}$. To reconcile the existing experimental values of ξ_{polar} and $\xi_{br.rat}$ it is necessary to assume that $|f_+^\mu/f_+^e| \approx 1.16$ for $\xi = -1.0$ and $\eta = 0.73$. However, the violation of $\mu - e$ universality goes against the grain more than, for example, the introduction of a large λ to reconcile the results for ξ obtained by different methods.

<u>The Selection Rules $|\Delta I| = 1/2$ and $\Delta Q = \Delta S$ for Semileptonic Kaon Decays.</u> In Sec. 1.3 we have adduced simple arguments to derive the selection rules $|\Delta I| = 1/2$ and $\Delta Q = \Delta S$. These arguments were based on the assumption that the strangeness-changing part of the weak-interaction Lagrangian transforms as a isospinor.

A more fundamental basis for the introduction of these selection rules was proposed by Cabibbo and others; it reduces to the assumption that the strangeness-changing current $J_\alpha^{(1)}$ is composed of the components of an $SU_{(3)}$-symmetry octet. Within the framework of such an assumption the selection rules $|\Delta I| = 1/2$ and $\Delta Q = \Delta S$ are obtained for all processes with $\Delta S = \pm 1$. For semileptonic decays the rule $|\Delta I| = 1/2$ predicts the branching ratio:

$$R = \Gamma(K^0 \to \pi l \nu_l / \Gamma(K^+ \to \pi l \nu_l) = 2.024,$$

where the differences of the pair arises because of the mass difference of the particles. The experimental value is $R_{exp} = 1.9 \pm 0.08$ and therefore $R_{exp}/R_{theor} = 0.94 \pm 0.04$, which does not differ from unity by more than one standard deviation [15].

One can investigate a possible violation of $\Delta Q = \Delta S$ by considering the four decay amplitudes of the neutral kaons:

$$f = A(K^0 \to \pi^- + l^+ + \nu_l);\ q = A(K^0 \to \pi^+ + l^- + \tilde{\nu}_l);\ f^* = A(\overline{K}^0 \to \pi^+ + l^- + \tilde{\nu}_l);\ q^* = A(\overline{K}^0 \to \pi^- + l^+ + \nu_l),$$

where CPT invariance is assumed. Here, $f(f^*)$ is the amplitude of the process that satisfies $\Delta Q = \Delta S$ and $g(g^*)$ is the amplitude that violates this rule ($\Delta Q = -\Delta S$). The ratio of these two amplitudes, $x = g/f$, must vanish if the weak interaction satisfies $\Delta Q = \Delta S$ exactly. Violation of this rule must affect the nature of the time distribution of decays with positively and negatively charged leptons and also, if there is a CP-invariance violating amplitude in semileptonic decays, the charge asymmetry

$$\Delta = (\Gamma_+ - \Gamma_-)/(\Gamma_+ - \Gamma_-),$$

where $\Gamma_\pm = \Gamma(K^0 \to \pi l \pm \nu_l)$ are the partial rates of the processes.

Let us consider what possibilities there are for obtaining information about the value of x from an experimental study of the time dependence for the decays $K_L^0 \to \pi l \nu_l$. If at the initial instant there is, for example, a pure $\overline{K}^0$ state, then after a certain τ_S (τ_S is the lifetime of the short-lived K_S^0 state) there will be a mixture of the $\overline{K}^0$ and K^0 states. For small t, negatively charged leptons will arise because of the amplitude f^* that satisfies $\Delta Q = \Delta S$ and positively charged leptons will arise from the amplitude which satisfies $\Delta Q = -\Delta S$. If both f and g have approximately the same energy dependence in the kinematically allowed region, the time distribution of the electronic K_{e3} from the initial $\overline{K}^0$ state is

$$\widetilde{N}^{\pm}(t) \sim |1+x|^2 e^{-\lambda_S t} + |1-x|^2 e^{-\lambda_L t} - 2e^{\frac{1}{2}(\lambda_S+\lambda_L)t} [2 \mathrm{Im} x \sin \delta t \pm (1-|x|^2) \cos \delta t], \qquad (1.10)$$

where the sign ± refers to the electron charge; λ_S and λ_L are the total rates of the short-lived K_S^0 and the long-lived K_L^0 states and δ is the mass difference $m(K_S) - m(K_L)$. The necessary condition for CP invariance of the semileptonic decays is $\mathrm{Im}\, x = 0$, whereas the simultaneous fulfillment of the conditions $\mathrm{Im}\, x = \mathrm{Re}\, x = 0$ is equivalent to $\Delta Q = \Delta S$. Violation of CP invariance in this case necessarily entails nonfulfilment of $\Delta Q = \Delta S$ but the condition $\Delta Q = \Delta S$ and hence $\mathrm{Im}\, x = 0$ is not sufficient to deduce CP invariance since the latter may be violated in the allowed channel with $\Delta Q = \Delta S$. If CP invariance holds, the sum $N^+(t) + N^-(t)$ is independent of δ. However, this obvious advantage is in reality lost because of the indeterminacy of x due to the quadratic dependence and the large indeterminacy in x resulting from the loss of information about the sign of the lepton charge. If one assumes that the form factor f_- (g_-) does not dominate in the hadron currents, the time distribution of the $K_{\mu 3}$ decays can also be expressed by Eq. (1.10) with the same value of x. We recall that violation of $\Delta Q = \Delta S$ leads to a transition with $|\Delta I| = 3/2$ (the amplitudes g and g^*), i.e., in this case the selection rule $|\Delta I| = 1/2$ must necessarily be violated for semileptonic decays.

The time dependence for the K_{e3}^0 decays has been investigated [16] for both the $K_{\mu 3}$ and K_{e3} modes and for the initial states K^0 and $\overline{K}^0$. The combined data from all the experiments yield

$$\mathrm{Re}\, x = 0.021 \pm 0.036;$$
$$\mathrm{Im}\, x = -0.10 \pm 0.005.$$

We see that whereas $\mathrm{Im}\, x$ vanishes to within the experimental errors, $\mathrm{Re}\, x > 0$ by more than four standard deviations. However, the value of the error $\Delta \mathrm{Re}\, x$ should not be taken too seriously. The combined data of the various experiments do not take into account the almost unavoidable systematic errors that could greatly change the result. In addition, the experiments discussed above do not yet indicate a violation of $|\Delta I| = 1/2$, which would be necessary if there is a contribution to the amplitude with $\Delta Q = -\Delta S$.

Summing up, our very tentative interpretation of the measurements of the time dependence of semileptonic decays is that the selection rule $\Delta Q = \Delta S$ may be violated with CP invariance nevertheless holding.

Let us consider the possible effects that could arise if the selection rule $\Delta Q = \Delta S$ is violated in the charge asymmetry of the decays $K_L^0 \to \pi l \pm \nu_l$. It is difficult to determine the contribution to the K_{l3} decays from the $\Delta Q = \Delta S$ violating amplitude by measuring the charge asymmetry $\Delta = (\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-)$ because x in this case is a function of many experimental variables:

$$\mathrm{Re}\, x = f(|\eta_{+-}|, |\eta_{00}|, \theta_{+-}, \theta_{00}, \Delta).$$

Here, η_{+-} and η_{00} are the ratios of the amplitudes of two-pion decays: $\eta_{+-} = \langle \pi^+ \pi^- | J_\alpha^{(1)} | K_L^{(0)} \rangle | \langle \pi^+\pi^- | J_\alpha^{(1)} | K_S^0 \rangle$ and $\eta_{00} = \langle \pi^0 \pi^0 | J_\alpha^{(1)} | K_L^0 \rangle / \langle \pi^0 \pi^0 | J_\alpha^{(1)} | K_S^0 \rangle$ with corresponding phases θ_{+-} and θ_{00}. A phenomenological analysis of the majority of the experimental data leads to the value $\theta_{00} = (51 \pm 30)°$. In accordance with many experiments* we have $\theta_{+-} = (42.5 \pm 3)°$. For $|\eta_{+-}|$ and $|\eta_{00}|$ the weighted mean values are

$$\langle |\eta_{+-}| \rangle = (1.92 \pm 0.03)\, 10^{-3};$$

$$\langle |\eta_{00}| \rangle = (2.06 \pm 0.17) \cdot 10^{-3};$$

however, different values were found for $|\eta_{00}|$ in the different experiments. The recent measurements [17] of the charge asymmetry Δ in the K^0_{l3} decays gave

$$\Delta_e = (2.24 \pm 0.36) \cdot 10^{-3};$$

$$\Delta_\mu = (4.00 \pm 1.40) \cdot 10^{-3}.$$

Using all these values and the relationship $2\mathrm{Re}\,x = 3\Delta/(2|\eta_{+-}| \times \cos\theta_{+-} + |\eta_{00}| \cos\theta_{00})$, we obtain $\mathrm{Re}\,x = +0.11 \pm 0.09$ for K^0_{Le} and $\mathrm{Re}\,x = -0.20 \pm 0.26$ for $K^0_{L\mu}$. Within the limits of the errors these values agree and their weighted mean is

$$\mathrm{Re}\,x_\Delta = +0.080 + 0.100.$$

In addition, data on the K^0_{e3} decay yield

$$\mathrm{Im}\,x = -0.12 \pm 0.15;$$

$$\frac{1 - |x|^2}{|1 + x|^2} = 1.06 \pm 0.06.$$

For the relative probability of the transitions with $\Delta Q = -\Delta S$ we therefore obtain a value less than 2×10^{-2}.

The Problem of T Invariance Violation. The need to verify T invariance arose with the discovery of CP-noninvariant transitions in decays of K^0 mesons. If CP invariance is violated, it immediately follows from the fundamental CPT theorem that T invariance must necessarily be violated.

Experiments with weak decays enable one to verify directly the extent to which invariance under time reversal holds since they are sensitive to terms of the form $\sigma \cdot (p_1 \times p_2)$, where σ is the spin of one of the particles and p_1 and p_2 are the momenta of the two secondary particles that participate in the decay. Under time reversal the signs of both the momentum and the angular momentum are changed; it follows that invariance under time reversal requires the vanishing of $\sigma \cdot (p_1 \times p_2)$ if one can neglect the CP-violating interaction in the final state.

Such an effect can be sought in the decays of neutral and charged kaons through the $K_{\mu 3}$ mode. The parity violating decay of the muon depends on its polarization; with allowance for this fact one can measure the normal component of the polarization vector, which is proportional to $\sigma_\mu \cdot (\bar{p}_\pi \times \bar{p}_\mu)$.

At the present time the most complete data have been obtained in the investigation of neutral kaons [12]. The coefficient of the term $\sigma_\mu \cdot (\bar{p}_\mu \times \bar{p}_\pi)$ is proportional to $\mathrm{Im}\,\xi$, the value of the form-factor ratio taking into account the possible presence of the $\Delta Q = \Delta S$ violating amplitude g: $\xi = (f_- - g_-)/(f_+ - g_+)$. The experimental value is $\mathrm{Im}\,\xi = -0.014 \pm 0.066$. (Similar investigations with charged kaons lead to $\mathrm{Im}\,\xi \approx 0$.) Since both $\mathrm{Im}\,\xi$ and $\mathrm{Re}\,\xi$ have hitherto been determined with a very poor accuracy, the individual measurements are characterized by large fluctuations. Assuming the value $\mathrm{Re}\,\xi = -1.0 \pm 0.2$, which is the weighted mean of the polarization measurements, we obtain $\varphi = 0.8 \pm 3.0°$ for the phase φ ($\xi = |\xi| e^{i\varphi}$). The electromagnetic interaction in the final state may give rise to a phase $\varphi \approx 0.3°$; it follows that the result obtained for $\mathrm{Im}\,\xi$ is in complete agreement with the assumption of T invariance.

Verification of CP Invariance. There has recently been an experimental discovery of charge asymmetry in the semileptonic decays of neutral kaons [17]. The magnitude of this asymmetry is an important parameter in the phenomenological study of the violation of CP invariance. The Californian group, who studied the decay $K^0_L \to \pi\mu\nu$, obtained

*The individual measurements are characterized by a very large spread of the θ_{+-} values.

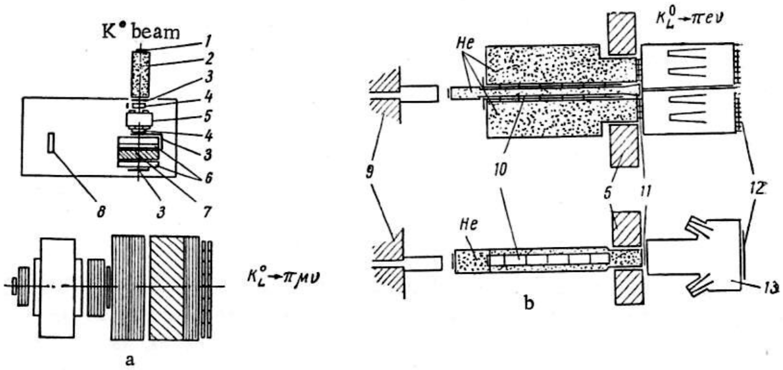


Fig. 4. Experimental arrangement for the investigation of the charge asymmetry in the decays $K_L^0 \to \pi\mu\nu$ (a) and $K_L^0 \to \pi e\nu$ (b): 1) anticoincidence counters; 2) helium bag; 3) scintillation counters; 4) thin-plate spark chambers; 5) magnet; 6) thick-plate spark chambers; 7) pion stopper; 8) photographic system; 9) collimator; 10) hodoscope A; 11) hodoscope B; 12) hodoscope C; 13) Cerenkov counter.

$$R \equiv R_{\mu^+}/R_{\mu^-} = 1.0081 \pm 0.0027, \qquad (1.11)$$

and the group at Columbia University found

$$\Delta = (\Gamma_{e^+} - \Gamma_{e^-})/(\Gamma_{e^+} + \Gamma_{e^-}) = (2.24 \pm 0.36) \cdot 10^{-3} \qquad (1.12)$$

for the $K_L^0 \to \pi e\nu$ decay.

Both these results gave the first indications that CP invariance may be violated in not only the $K^0 \to 2\pi$ transitions but also in other processes that take place because of the weak interaction; this is because one must have $R = 1$ and $\Delta = 0$ if CP invariançe is satisfied (the arrangements of the experiments are shown in Fig. 4). The charge asymmetry data can also be attributed to other properties of the neutral kaon decays. If we denote by α the transition $\alpha = |\langle L|S\rangle|$, where L and S are the long- and short-lived states, then

$$\Delta = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} = \alpha \frac{(1 - |x|^2)}{|1 + x|^2}.$$

Here, the parameter x is again the ratio of the amplitudes g/f; on the other hand, the parameter Δ can be expressed in terms of parameters known from phenomenological analysis of the decays $K_L^0 \to 2\pi^0$ and $K_L^0 \to \pi^+\pi^-$, that proceed with CP violation, namely, η_{+-}, η_{00}, ε, and ε', which are related to one another by the equations $\eta_{+-} = \varepsilon + \varepsilon'$ and $\eta_{00} = \varepsilon - 2\varepsilon'$. From the definition of ε and α (with allowance for the fact that they are small) we obtain $\mathrm{Re}\,\varepsilon \approx \alpha/2$. Hence, $x = 0$ implies $\Delta = 2\mathrm{Re}\varepsilon$ or $R \approx 1 + 4\mathrm{Re}\varepsilon$ and it follows from (1.11) and (1.12) that $\mathrm{Re}\varepsilon_\mu = 0.0020 \pm 0.0007$ and $\mathrm{Re}\varepsilon_e = 0.0011 \pm 0.0002$.

As can be seen, the results obtained from measurements of the charge asymmetry in the decays K_L^0 $\pi\mu\nu$ and $K_L^0 \to \pi e\nu$ agree well. We must emphasize once more that the observed charge asymmetry is associated with the CP-violating amplitudes of the 2π decays of the K^0 mesons and not with the CP-noninvariant amplitudes of the semileptonic decays, which we have neglected in the analysis. It should be noted that values found for the charge asymmetry in the semileptonic decays agree with the predictions [18] obtained for violation of CP invariance in the two-pion decays, $\Delta \lesssim 4 \cdot 10^{-3}$.

1.4. The Decays $K \to \pi + \pi + l + \nu_l$

Among all the kaon decay processes in which leptons are included among the secondary particles, the K_{l4} decays are distinguished from all decays for which one can expect the accumulation of extensive experimental data by the saturation of their kinematic structure. In these decays one can investigate almost all the consequences of the theory of weak interactions, for example, the validity of the selection rules $|\Delta I| = 1/2$ and $\Delta Q = \Delta S$, and T invariance; one can also verify the conclusions obtained from current algebra, etc. Let us consider the reactions:

$$K^+ \to \pi^+ \pi^- e^+ \nu_e; \tag{1.13a}$$

$$K^+ \to \pi^+ \pi^+ e^- \tilde{\nu}_e. \tag{1.13b}$$

Let us compare the possibility of investigating the degree of violation of the selection rule $\Delta Q = \Delta S$ from a study of the reactions (1.13) and in the K_{e3} decay. The latter is due to the vector current, whereas the reaction (1.13b), which violates $\Delta Q = \Delta S$, is evidently described by an axial-vector current. The decay (1.13a) can contain an admixture of axial-vector and vector currents although it is expected that the axial-vector current predominates. Thus, the decays K_{e4} and K_{e3}, which violate $\Delta Q = \Delta S$, if this violation exists at all, are due to different currents and a comparison can test the notion of the universality of the currents. However, the most important aspect of the $K \to \pi\pi l \nu_l$ transitions is the possibility of obtaining direct information about the $\pi\pi$ interaction in the final state. The fact that two strongly interacting particles appear together with two leptons that participate in weak interactions enables one to study the interaction in the final state of these two pions in a pure form without the influence of other strong interactions. The $\pi\pi$ interaction in the final state must affect the following characteristics of the K_{l4} decay: 1) the decay rate; 2) the form of the dipion mass distribution; 3) the decay symmetry with respect to π^+ and π^- in the dipion rest system, which, in turn, must lead to differences in the π^+ and π^- spectra in the laboratory system; 4) the angular correlation between the dipion and dilepton planes.

The following feature of the decays (1.13) should also be noted. These processes differ from one another not only with regard to the rule $\Delta Q = \Delta S$ but also in the isospin states of the two pions [19, 20]. Whereas a final state with $I = 2$ is realized in (1.13b), states with $I = 0$, 1, or 2 may be present in the reaction (1.13a) providing appropriate angular momenta are present. If both states with $I = 0$ and $I = 1$ are present, the s and p angular momentum states may interfere if they have amplitudes of comparable magnitude. This interference could lead to a forward-backward asymmetry in the emission of one of the pions relative to the dipion direction.

The intensity and form of the K_{l4} decay spectra are functions of five variables. If these variables are chosen appropriately and only the single assumption of an effective local coupling of the lepton pair to the hadron currents is made, the structure of the decay can be expressed in terms of two of the five variables and will be independent of the hadron interactions. All the dynamic effects will be contained in the form factors, which, in the general case, are functions of the three remaining variables. This 2 + 3 separation appreciably simplifies the kinematic situation and can be used to decompose the general structure of the decays into parts that can be internally even more interesting. One such set of variables is: the angles θ_π and θ_l, which describe the "decay" of the dipion and dilepton in their rest systems; the angle φ between the normals to the planes containing the dilepton and dipion; and the invariant masses $m_{2\pi}$ and m_{2l} of the dipion and dilepton. The angles θ_l and φ are "simple" variables and the dependence on them can be expressed exactly. In general, all the form factors depend only on the remaining variables θ_π, $m_{2\pi}$, and m_{2l}.

We shall consider the decay

$$K^+ \to \pi^+ + \pi^- + e^+ + \nu_e, \tag{1.14}$$

since it is only for only this process that more or less sufficient experimental material has so far been accumulated [21-23]. We already know how to construct the matrix elements, so we shall not go into this question in detail. For the decay (1.14)

$$M \sim \frac{G}{\sqrt{2}} \bar{u}_\nu (p_\nu) \gamma_\alpha (1 + \gamma_5) u_e (p_e) \langle \pi^+ \pi^- | J_\alpha^V + J_\alpha^A | K^+ \rangle, \tag{1.15}$$

where J_α^V and J_α^A are the weak vector and axial hadron currents, whose matrix elements can be written in the form [20]

$$\langle \pi^+ \pi^- | J_\alpha^V | K^+ \rangle = \frac{ih}{m_K^3} \varepsilon_{\alpha\mu\nu\gamma} p_K^\mu (p_+ + p_-)^\nu (p_+ - p_-)^\gamma; \tag{1.16}$$

$$\langle \pi^+\pi^- | J_\alpha^A | K^+ \rangle = \frac{f}{m_K}(p_+ + p_-)_\alpha + \frac{g}{m_K}(p_+ - p_-)_\alpha, \tag{1.17}$$

where we have omitted the term in (1.17) that makes a contribution to the matrix element proportional to the square of the lepton mass. The matrix element (1.15) includes the three form factors h, g, and f, which, in the general case, are functions of $(p_K + p_{\pi+})^2$, $(p_K + p_{\pi-})^2$ and $(p_{\pi+} + p_{\pi-})^2$. If we make the assumption that the dependence on $(p_K + p_{\pi+})^2$ and $(p_K + p_{\pi-})^2$ can be neglected, the calculation of the decay amplitudes can be greatly simplified. In the case when the form factors depend only on $(p_{\pi+} + p_{\pi-})$, one can readily see that the f term in Eq. (1.17) is symmetric under transposition of π^+ and π^-, whereas g in (1.17) and h in (1.16) are antisymmetric under this transposition. This fact, together with the selection rule $|\Delta I| = 1/2$, indicates that f corresponds to the amplitude for the emission of two pions in a state with I = 0 and that the g and h terms are the amplitudes for the emission of pions in a state with I = 1.

If invariance under time reversal is assumed, f, g, and h can be represented in the form

$$f = f_0 e^{i\delta_0}, \quad g = g_0 e^{i\delta_1}, \quad h = h_0 e^{i\delta_1},$$

where δ_0 is the s-wave phase shift of $\pi\pi$ scattering for I = 0 and δ_1 is the p-wave phase shift of $\pi\pi$ scattering for I = 1, and g_0, f_0, and h_0 are real quantities.

At the present time international statistics provides about 300 decays for which a sufficiently detailed analysis has been made and individual cases of decay through the mode $K^+ \to \pi^+\pi^-\mu^+\nu_\mu$. The data obtained from $K^+ \to \pi^+\pi^- e^+\nu_e$ have been considered under various assumptions about the behavior of the s-wave of $\pi\pi$ scattering [24]. In the analysis all the form factors were assumed constant with the exception of the possible presence of a factor leading to a monotonic increase of the form factors. The s-wave phase shift δ_0 can be expressed in terms of definite parameters by two methods:

1) By Chew-Mandelstam parametrization with variable scattering length a_0:

$$\operatorname{ctg} \delta_0 = \frac{1}{\beta a_0} + \frac{2}{\pi} \ln\left[\frac{m_K}{2m_\pi} x (1 + \beta)\right], \tag{1.18}$$

where $\beta = (1 - 4m_\pi^2/x^2\, m_K^2)^{1/2}$ and $x^2 = (p_{\pi+} + p_{\pi-})^2/m_K^2$.

2) By Breit-Wigner resonance with variable width γ and energy E_R:

$$\operatorname{ctg} \delta_0 (x^2) = 2\,(x^2 m_K^2 - E_R^2)/(\gamma m_K x \beta). \tag{1.19}$$

In the analysis the p-wave scattering phase δ_1 was ignored, an assumption that is perfectly justified in the energy range of the dipion in the given decay. The analysis was made for three cases, which are as follows.

A. All form factors constant; this is the simplest model.

B. The s-wave form factor f depends on the energy through a possible strong $\pi\pi$ and s-wave interaction at low energies. This can be taken into account by introducing, for example, a relativistic increasing Watson factor $f_0 = f_0 \sin \delta_0(x^2)/a_0\beta$.

C. All the form factors depend on the energy. In this case one introduces a so-called increasing ρ factor for g_0 and h_0:

$$g_0 = g_0' (m_\rho^2 - 4m_\pi^2)/(m_\rho^2 - x^2 m_K^2);$$
$$h_0 = h_0' (m_\rho^2 - 4m_\pi^2)/(m_\rho^2 - x^2 m_K^2).$$

The best agreement with the experimental data is obtained in all three cases if the Chew-Mandelstam $\pi\pi$ scattering phase is used in the calculations; this is especially true for the first case. Parametrization in the form of a Breit-Wigner resonance does not greatly affect the values of the form factors. The values of the form factors and the scattering length a_0 for the above cases are

$$a_0 = 1.04 \pm 0.50;\; a_0 = 0.89 \pm 0.44;\; a_0 = 0.84 \pm 0.43;$$
$$f_0 = 1.19 \pm 0.13;\; f_0' = 1.45 \pm 0.16;\; f_0' = 1.42 \pm 0.15;$$
$$g_0 = 1.34 \pm 0.30;\; g_0 = 1.36 \pm 0.30;\; g_0' = 1.25 \pm 0.28;$$
$$h_0 = -4.84 \pm 1.77;\; h_0 = -4.89 \pm 1.73;\; h_0' = -4.57 \pm 1.63.$$

Note that the main contribution to the error of the form factors arises because of the large error in the determination of the decay rate, which was found for the minority of the cases [19]:

$$\Gamma(K^+ \to \pi^+\pi^- e^+ \nu_e) = (2.9 \pm 0.6)\cdot 10^3 \text{ sec}^{-1}.$$

The values found for f_0 and g_0 are in good agreement with the result $|f_0| = |g_0| = 0.97 \pm 0.03$ obtained with the help of current algebra under the assumption that the form factors in the K_{l3} and K_{l4} decays are constant. The value $a_0 = 0.2$, which is also found in [19], does not appear too seriously different from the experimentally determined value, firstly, because of the large error and, secondly, because of the possible effects of unitarity restrictions on the $\pi\pi$ scattering amplitude (which may increase the value of a_0 deduced from current algebra requirements).

The most interesting result of the investigation is the large value of the vector form factor h, which differs from zero by almost three standard deviations. Unfortunately, the statistical reliability is so low and the errors in the values of the form factors are so large that one cannot draw any definite conclusions on this subject. We shall only mention that the form factor h_0 obtained in the framework of $SU_{(3)}$ symmetry (with the assumption of vector dominance) by using the relationship between the K_{l4} decay and the transition $\eta \to \pi\pi\gamma$ is much less (by approximately a factor of four) than the value obtained experimentally.

The accumulation of data on the K_{l4} decay is very desirable. If the conclusions given here are confirmed, it will mean that the K_{l4} decay is even more interesting than is at present assumed.

The Decays $K \to \pi\pi l \nu_l$ and the Selection Rules $\Delta Q = \Delta S$ and $|\Delta I| = 1/2$. The selection rule $\Delta Q = \Delta S$ allows the decays $K^+ \to \pi^+ \pi^- l^\pm \nu_l$ and forbids the transitions $K^\pm \to \pi^\pm \pi^\pm l^\mp \nu_l$ with similarly charged pions. Up to now approximately 300 allowed decays $K^\pm \to \pi^+ \pi^- l^+ \nu_l$ and some cases of $K^+ \to \pi^+ \pi^- \mu^+ \nu_\mu$ have been found but not a single transition that is forbbiden by $\Delta Q = \Delta S$. Thus, the upper limit for K_{l4} decays with $\Delta Q = -\Delta S$ is at present $\Gamma(K^+ \to \pi^+ \pi^+ l^- \nu)/(\Gamma(K^+ \to \text{all})) \leq 0.5 \cdot 10^{-6}$.

Of course, to be able to say anything about the ratio of the currents corresponding to $\Delta Q = \Delta S$ and $\Delta Q = -\Delta S$ one must take into account fully the interaction effects in final states possessing different isospins. If these effects are negligibly small, the existing experimental data indicate that

$$R = \frac{A(K_{l4}^+,\ \Delta Q = -\Delta S)}{A(K_{l4}^+,\ \Delta Q = \Delta S\ )} \lesssim 0.13.$$

As we have seen in the foregoing section, the experimental results on the verification of $\Delta Q = \Delta S$ for vector currents in neutral kaon decays are compatible with the above value of R. Thus, as far as this question is concerned there has not yet been observed any difference in the behavior of transitions under the influence of the vector and axial-vector currents. Unfortunately, there is still no experimental data on the decay $K^+ \to \pi^0 \pi^0 l^+ \nu_l$ and on the K_{l4} decays of neutral kaons; this means that we cannot verify the consequence of the selection rule $|\Delta I| = 1/2$ which relates the relative decay rates of charged and neutral kaons.

1.5. Weakly Electromagnetic Leptonic Meson Decays

The description "weakly electromagnetic" has now been accepted for the meson decays through the modes

$$K \to \pi + \pi + \gamma;$$
$$K \to \pi + \pi + \pi + \gamma;$$
$$K \to l + \nu + \gamma;$$
$$\pi \to l + \nu + \gamma.$$

In this review we shall consider only the last two decays.

The emission of a photon in any hadron decay

$$A \to a + b + \ldots + \gamma$$

can occur in the general case in two ways: by means of internal bremsstrahlung which necessarily accompanies all nonradiative processes and the possible mechanism of direct emission of a photon on the transition from the initial A state to the final state (a +b +. . .).

The process of internal bremsstrahlung is well known; its amplitude is proportional to the product eG of the constants of the electromagnetic and weak interactions and the bremsstrahlung γ ray is always emitted in a C- and P-invariant manner. The so-called direct (or structural) emission of a photon has not yet been detected experimentally but it happens that a number of interesting problems are related to this mechanism γ-ray creation. Indeed, as follows from the very name of the mechanism, the simple confirmation of the existence of direct processes would enable one to draw certain conclusions about the structure of the decaying meson. For this purpose it is clearly most expedient to investigate the weakly electromagnetic leptonic decays of kaons, for which there is no interfering influence of strongly interacting particles in the final state and the possible intermediate states (see below) have a mass that is closer to the kaon mass than to m_π in the decay $\pi \to l\nu_l\gamma$. If the direct emission of a photon exists and its amplitude is at least comparable in magnitude with the amplitude of the internal bremsstrahlung, interest in the weakly electromagnetic decay of kaons would greatly increase. In contrast to the CP-invariant amplitude of the internal bremsstrahlung process (for which the parent decay without γ ray, $A \to a + b + \ldots$, is CP invariant) the amplitude of the direct processes can, in general, be complex, i.e., noninvariant under time reversal. In such a case, violation of T orCP invariance could lead to experimentally observable effects.

The possibility of detecting T-noninvariant effects in weakly electromagnetic kaon decays appears in an investigation of the leptonic reaction

$$K^{\pm} \to \mu^{\pm} + \nu_\mu (\tilde{\nu}_\mu) + \gamma,$$

in which, in contrast to transitions of the type $K \to \pi\pi\gamma$, there is a new variable, the nonvanishing spin of the muon. This enables one to investigate a correlation of the form $\sigma_\mu \cdot (\overline{p}_K \times \overline{p}_\mu)$, which changes its sign under time reversal. In the absence of strong interactions in the final state the appearance of a nonvanishing normal component of the muon polarization $\hat{p}_\perp \sim \sigma_\mu \cdot (\overline{p}_K \times \overline{p}_\pi)$ would be a direct indication of noninvariance under time reversal in the given process. As we have seen in Sec. 1.3, a similar effect must obtain in the decay $K^{\pm} \to \mu^{\pm}\pi^0\nu_\mu$, which is due to the weak interaction. Careful searches for a normal component of the muon polarization have yielded a negative result, whereas this component of the polarization must attain values of ~20% if CP invariance were violated in the weak interactions.

The decay $K \to \mu\nu\gamma$ differs quantitatively from the $K_{\mu 3}$ process in that it takes place because of both the weak and electromagnetic interactions. If CP invariance is violated in the electromagnetic interactions, the expected effect must, of course, be much larger in the $K_{\mu\nu\nu}$ decay than in the $K_{\mu\pi\nu}$ decay, in which the electromagnetic interaction appears as a correction to the weak interaction. As in the case of the decay $K \to \pi\pi\gamma$, such violation can occur only for the amplitude for the direct emission of a γ ray.

If one assumes maximal violation, i.e., $\mathrm{Im} f_i = \mathrm{Re} f_i$ for the form factors corresponding to the direct transitions, the maximal value of the normal component of the muon polarization may attain values of ~ 50% of the total polarization for certain points of the Dalitz plot [25, 26]. However, the integral effect leads to an experimentally observable asymmetry in the muon decay electron distribution of only 1-2% so the search for T-invariance violation for this decay is neither a simple nor a reliable undertaking.

However, as we have already mentioned, these effects can be appreciable only if the direct emission amplitude is at least comparable in magnitude with the internal bremsstrahlung amplitude. Hitherto the weakly electromagnetic $K_{e\nu\gamma}$ leptonic decays have not been investigated experimentally and the search for the direct emission of a photon in the nonleptonic processes $K \to \pi\pi\gamma$ has shown that the relative rate of transitions with direct emission, if the latter occur, does not exceed $\sim 10^{-4}$.

In this connection it would seem worthwhile to investigate in detail the decay $K \to l\nu_l\gamma$ since, as we have already mentioned in Sec. 1.1, the leptonic transition $K \to e\nu_e$ is suppressed because of the neutrino and the small mass of the charged lepton. Thus, in the decay $K \to e\nu_e\gamma$ the mode that competes with internal bremsstrahlung must be suppressed. For the putative direct-emission decay mode the exclusion rule is lifted by the presence of a third particle and we can reasonably expect that the rate of the process $K \rightarrow e\nu_e\gamma$ may be even greater than that of the decay without γ ray provided, of course, there is no special suppression of direct transitions generally.

We may also mention another characteristic feature of the decay $K \rightarrow e\nu_e\gamma$ that distinguishes it from the other weakly electromagnetic processes: strongly interacting particles are not present in the final state and the energy and angular correlations between the decay products depend on the properties of a single hadron – the kaon; in principle, this enables one to obtain information about the latter in the purest form. In the decay $K \rightarrow \mu\nu_\mu\gamma$ there are also no strongly interacting particles in the final state but this is offset by the internal bremsstrahlung which cannot be suppressed in any manner and is an undesirable background that interferes with the search for direct processes.

Despite the undoubted advantages of the decay $K \rightarrow e\nu_e\gamma$ for an investigation of the direct emission of a photon, experiments of this kind are beyond our possibilities at the present time. The trouble is that one must simultaneously and with sufficient accuracy measure the electron and photon energies in order to be able to distinguish the background process $K \rightarrow e\nu_e\pi^0$ when one of the γ rays from the decay of the π^0 meson is not detected by the instrument. These factors and the low probability of the process $K \rightarrow e + \nu_e$ have meant that hitherto experiments with the decay $K \rightarrow \mu\nu\gamma$ have seemed more realistic. In this connection it is natural to ask whether one can separate direct processes if their amplitude is less than or only comparable in magnitude with the bremsstrahlung amplitude. Such a possibility was investigated in [27], which was devoted to an analysis of the angular and energy correlations between the decay products in this process $K \rightarrow \mu\nu\gamma$, and in [28], in which an investigation was made of the possibility of polarization measurements in the decay $K \rightarrow \mu\nu\gamma \rightarrow \mu\nu e^+e^-$. These investigations indicate that the direct-transition effects could (for a certain choice of the kinematic regions of variation of the variables) be appreciable even if the direct-emission amplitude is only ~ 0.5 of the bremsstrahlung amplitude.

Now a few words about the possible structure of the decay $K \rightarrow l\nu\gamma$, which proceeds through the mode with the direct emission of a photon. Since two leptons can only arise in the final state as a result of a strangeness-changing weak interaction, the intermediate state required for the emission of the photon must have the same strangeness as the kaon. In the general case the direct transitions may have vector and (or) axial-vector nature. The main contribution to the vector transitions, which are characterized by the form factor f_V, must arise from two-meson $K\pi$ states with threshold $m_K + m_\pi$ and the resonance K^*. For axial-vector direct transitions with the corresponding form factor f_A the intermediate states $K\pi\pi$, $K\rho$, $K\omega$, and $K^*\pi$ are allowed. They are all more distant in the mass than in the case of the vector transition. Therefore, in general, $|f_A| < |f_V|$. In addition, the possible intermediate states have a mass that is greater than the kaon mass in both cases; one must therefore expect that the dependence of the form factors on the momentum transferred to the leptons must be smooth in the physical region and that it can be completely ignored in a first approximation. The total matrix element of the decay $K = l\nu_l\gamma$ in this case has the form

$$\langle l\nu\gamma | J | K \rangle = ie\frac{G}{2}\cdot\frac{1}{(2\pi)^4}\cdot\frac{1}{2\sqrt{E_K E_\gamma}}\,\delta^4(p_K - p_l - p_\nu - p_\gamma)$$

$$\times u_l\left\{f_K m_l\left[\frac{p_K\varepsilon}{p_K p_\gamma} - \frac{p_l\varepsilon}{p_l p_\gamma} - \frac{\hat{\varepsilon}\hat{p}_\gamma}{2p_l p_\gamma}\right] + \frac{1}{m_K^2}\gamma^\alpha p_K^\beta \varepsilon^\rho\right.$$

$$\left.\times [f_A(\delta_{\alpha\rho}\delta_{\beta\delta} - \delta_{\delta\alpha}\delta_{\beta\rho}) + if_V\varepsilon^{\alpha\beta\delta\rho}]\right\}(1+\gamma_5)u_\nu.$$

This expression shows that the internal bremsstrahlung contribution (the term proportional to the lepton mass) depends on the kaon structure only to the extent that the two-particle decay $K \rightarrow l\nu_l$ depends on this structure; this is because the same factor f_K, which depends on the kaon properties, is present in both channels. Finally, we must emphasize once more that at the present time only this range of leptonic transitions remains uninvestigated among the ensemble of weakly electromagnetic decays $K \rightarrow a + b + \ldots + \gamma$. The nonleptonic decays have hitherto failed to reveal any appreciable direct transition effects [29] and although one can hardly hope for any unexpected properties of the $K \rightarrow l\nu_l\gamma$ processes, their analysis would nevertheless enable one to complete the initial stage of the search for direct transitions.

1.6. Rare Leptonic Kaon Decays

One of the groups of rare leptonic kaon decays consists of the processes which can exist if the so-called neutral currents are present. Before we discuss this problem we should like to make a small digression.

We should first like to emphasize how important it is to make systematic investigations in weak interactions of effects of higher orders in small quantities. Such investigations are very important since they indicate whether the existing weak-interaction Lagrangian plays a purely phenomological role, i.e., is an effective Lagrangian, or whether it can be interpreted as the primary Lagrangian needed to construct a field theory of weak interactions. All the hitherto observed weak processes for the range of momentum tranfers $\lesssim 2$ GeV can be well described by an effective local Lagrangian L_{eff} which can be represented as a sum of three terms [30]:

$$L_{\mathrm{eff}} = L_{ll} + L_{Ll} + L_{LL}, \tag{1.20}$$

where L_{ll} describes processes in which only leptons participate; L_{Ll}, the semileptonic decays; and L_{LL}, the processes in which leptons do not participate. The matrix elements of lowest order in the interaction described by this Lagrangian at once determine the amplitudes of the observable processes. In the general case the matrix elements of the higher orders are divergent and are rejected. In this sense the Lagrangian (1) is a purely effective Lagrangian.*

If the effects of higher orders are investigated comprehensively, one of the first problems that must be faced is undoubtedly the striking absence of neutral leptonic currents in the first-order of the weak interactions. Indeed, as we have already pointed out in the introduction, the interaction responsible for the majority of reactions can be expressed phenomenologically in the form of the coupling of two currents; now such a formalism of the theory of weak interactions allows the presence of both charged and neutral currents and the absence of the latter is rather mysterious.

In the current-current representation one can rewrite (1.20) in the form

$$L_{\mathrm{eff}} = \frac{G}{\sqrt{2}} \left(l_\alpha l_\alpha^* + L_\alpha l_\alpha^* + L_\alpha L_\alpha^* + \ldots \right), \tag{1.21}$$

where l_α is the current that includes only leptons and L_α is the corresponding current containing hadrons. The interaction $l_\alpha l_\alpha^* \equiv L_{ll}$ describes, for example, a purely leptonic process, the decay of the muon. In the framework of the V-A interaction, we must choose the leptonic current for this decay in the form

$$l_\alpha = \bar{u}_l \gamma_\alpha (1 + \gamma_5) u_\nu, \tag{1.22}$$

which leads to the existence of at least two components of this current:

$$l_\alpha (e\nu_e) + l_\alpha (\bar{\mu}\nu_\mu).$$

We know only a single purely leptonic decay and we therefore know nothing about the other possible components of the current l_α or, conversely, about other leptonic processes which could be described by such a current. Our knowledge of the form of the hadron currents is even more meager. All the processes in which hadrons participate can be split into three groups: 1) semileptonic; 2) nonleptonic, containing only bosons in the initial and final states; 3) nonleptonic, with fermions in the initial and final states.

The existing experimental data indicate that the semileptonic processes are indeed described by an interaction of the current-current form. As regards the nonleptonic processes, the question remains

*It should be noted that at high energies the Lagrangian (1) ceases to be correct even as an effective Lagrangian. It must be modified at high momentum transfers, i.e., at short distances, by the introduction of a certain "effective" nonlocality to eliminate the divergences that appear, for example, in the cross sections for the scattering $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$.

open since conclusions of this nature are prevented in these processes by the interfering influence of the strong interactions in the final state.

Thus, we have at our disposal at least one group of reactions for which the proposed form of the corresponding interaction is of the current-current nature; in principle, this enables one to use the reactions to establish the nature of the currents and, in particular, to look for the neutral currents allowed by such a structure.

Form of the Leptonic and Hadronic Neutral Currents. The most general form of the leptonic current that satisfies lepton conservation (see below) is

$$l_\alpha = l_\alpha(\bar{e}\nu_e) + l_\alpha(\bar{\mu}\nu_\mu) + l_\alpha(\bar{\mu}\mu) + l_\alpha(\bar{\nu}_e\nu_e) + l_\alpha(\bar{\nu}_\mu\nu_\mu) + l_\alpha(\bar{e}e) + \ldots \tag{1.23}$$

The general form is the hadronic currents that contains fermions or bosons is

$$L_\alpha^F = L_\alpha(\bar{p}n) + L_\alpha(\bar{p}\Lambda) + L_\alpha(\bar{p}p) + L_\alpha(\bar{n}n) + L_\alpha(\bar{n}\Lambda) + \ldots; \tag{1.24}$$

$$L_\alpha^B = L_\alpha(\overline{K}^+\pi^\circ) + L_\alpha(\overline{K}^+\pi^-) + L_\alpha(\overline{K}^\circ K^\circ) + L_\alpha(\bar{\pi}^+\pi^\circ) + L_\alpha(\overline{K}^+, \pi^+\pi^-) + \ldots \tag{1.25}$$

If the all the components of l_α and L_α^l existed, a very large number of decays and scattering processes would be allowed. In Table 3 we give all such possible kaon decay processes with the exception of those that could be due to currents with $\Delta Q = -\Delta S$. We mention here that in the framework of the V−A theory of weak interactions the components of the current (1.24) must have vector and axial-vector parts.

At present we have no experimental proof of the presence of neutral currents in the purely leptonic interactions. However, their absence cannot be regarded as established either. Whereas earlier the absence of decays of the type

$$\mu \to e + \gamma;$$
$$\mu \to e + e + e$$

could be adduced to prove the absence of neutral leptonic currents, this absence can now be attributed to the separate conservation of the electron L_e and muon L_μ numbers ($L_e = +1$ for e^- and ν_e; -1 for e^+ and $\bar{\nu}_e$; 0 for all other particles; $L_\mu = +1$ for μ^- and ν_μ; -1 for μ^+ and $\bar{\nu}_\mu$; 0 for all other particles).

The most complete experimental data have been obtained in the study of semileptonic processes in which kaons participate. As regards the processes with $\Delta S = 0$, there are no experimental proofs applicable to neutral currents since the predominant electromagnetic interaction competes with such transitions if $\Delta S = 0$. (We shall discuss the question of the electromagnetic "competition" in more detail below.) Since the interfering (from this point of view) electromagnetic interaction conserves strangeness, processes with $\Delta S \neq 0$ should be more sensitive in a search for neutral currents than the processes with $\Delta S = 0$ provided, of course, the corresponding coupling constant is not infinitesimally small. If both the leptonic and the hadronic current contain vector and axial-vector parts, we have at our disposal ten coupling constants that completely characterize these currents:

$$g^{A,V}(\mu\mu),\ g^{A,V}(ee),\ g^{A,V}(\nu\nu),\ g^{A,V}(\mu\nu_\mu),\ g^{A,V}(e\nu_e).$$

These constants are related to the processes

$$\begin{array}{ll} g^V(\mu\nu_\mu) - K \to \pi\mu\nu_\mu; & g^A(\mu\nu_\mu) - K \to \mu\nu_\mu; \\ g^V(e\nu_e) - K \to \pi e\nu_e; & g^A(e\nu_e) - K \to e\nu_e; \\ g^V(ee) - K \to \pi ee; & g^A(\mu\mu) - K_2^\circ \to \mu\mu; \\ g^V(\mu\mu) - K \to \pi\mu\mu; & g^A(ee) - K_2^\circ \to ee; \\ g^V(\nu\nu) - K \to \pi\nu\nu; & g^A(\nu\nu) - K_2^\circ \to \nu\nu. \end{array}$$

Apart from the above processes, conservation of the lepton number allows the decays [31]

$$K_1^0 \to \mu^+\mu^-,\ K_1^0 \to \pi^0 e^+e^-.$$

These two decays violate CP invariance if the weak interaction is strictly local. The transition $K_2^0 \to \bar{\nu}_\mu \nu_\mu$ is absolutely forbidden because of helicity conservation for $m_\nu = 0$ (provided, of course, ν and $\tilde{\nu}$ are not identical particles). Similarly, the decay $K_2^0 \to e^+e^-$ is also suppressed because of the small electron mass even if the corresponding constant $g^A(ee)$ is very large. In addition, the transitions $K \to \pi \nu_e \bar{\nu}_e$ and $K \to \pi \nu_\mu \bar{\nu}_\mu$ cannot be identified separately; we therefore have eight possible semileptonic transitions in which kaons participate.

Hitherto only the processes

$$K \to \pi + l + \nu_l \text{ and } K \to l + \nu_l$$

have been observed experimentally. Numerous searches have been made for the other decays but no single sufficiently reliable case has yet been reported. All experiments with decays of K^+ mesons have been made in bubble chambers and a search for neutral currents in the decays of K^0 mesons has been made as an additional task in the investigation of the decays $K_L^0 \to 2\pi$. The most complete data on the $K^0 \to ll$ decays have been obtained recently at CERN [32]:

$$K_L^0 \to \mu^+\mu^-;\ K_L^0 \to \mu^\pm e^\mp;$$
$$K_L^0 \to e^+e^-;\ K_S^0 \to \mu^+\mu^-$$

and at Princeton [33]:

$$K_L^0 \to \mu^+\mu^-;$$
$$K_L^0 \to \mu^\pm e^\mp.$$

In both cases the experimental set-up was designed to investigate interference effects in the decays $K^0 \to 2\pi$. This naturally meant that the experimental conditions were not optimal for the detection of the decays $K \to ll$ although some necessary additions to the apparatus (for example, pion stoppers that allow the passage of muons etc.) were made. For the upper limit of the branching ratio the Princeton group obtained

$$\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma(K_L^0 \to \text{all}) \leqslant 3.5 \cdot 10^{-5}$$

with a 90% confidence level.

An estimate of the upper limit of the relative rate of the decay $K_L^0 \to \mu^\pm e^\mp$ (with identification of the $e^\pm$ track) gave

$$\Gamma(K_L^0 \to \mu^\pm e^\mp)/\Gamma(K_L^0 \to \text{all}) \leqslant 6 \cdot 10^{-6}.$$

The group at CERN found

$$\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma(K_L^0 \to \text{all}) \leqslant 1.6 \cdot 10^{-6};$$
$$\Gamma(K_L^0 \to e^+e^-)/\Gamma(K_L^0 \to \text{all}) \leqslant 1.8 \cdot 10^{-5};$$
$$\Gamma(K_L^0 \to \mu^\pm e^\mp)/\Gamma(K_L^0 \to \text{all}) \leqslant 9 \cdot 10^{-6}.$$

(We emphasize once more that the transition $K_L^0 \to \mu^\pm e^\mp$ is forbidden by the law of separate conservation of the lepton numbers.) In the CERN investigation the exact number of decays of the short-lived K^0 mesons was known; this made it possible to estimate the upper limit of the branching ratio of the CP-violating decay $K_L^0 \to \mu^+\mu^-$:

$$\Gamma(K_S^0 \to \mu^+\mu^-)/\Gamma(K_S^0 \to \text{all}) \leqslant 7.3 \cdot 10^{-5}.$$

As yet no experimental data are available for the decays

$$K_L^0 \to \pi^0\mu^+\mu^- \text{ and } K_L^0 \to \pi^0 e^+e^-,$$

TABLE 3. Allowed Kaon Decays

ΔQ	$\Delta S=0$	$\Delta S=1$
1	$K^0 \to K^+ + e^- + \bar{\nu}_e$	$K^+ \to \pi^0 + e^+ + \nu_e$ $K^0 \to \pi^\pm + e^\mp + \nu_e$ $K^+ \to \pi + \pi + e^+ + \nu_e$
0	—	$K^+ \to \pi^+ + l + \bar{l}$ $K_L^0 \to \mu^+ + \mu^-$ $K_L^0 \to \pi^0 + \mu^+ + \mu^-$ $K^+ \to \pi^+ + \pi^0 + e^+ + e^-$ $K^0 \to \pi^+ + \pi^- + e^+ + e^-$

and for the transitions of charged kaons to $\pi l\bar{l}$ we have the following data:

$$\Gamma(K^+ \to \pi^+ e^+ e^-)/\Gamma(K^+ \to \text{all}) \leqslant 8.8 \cdot 10^{-7} \text{ [34, 35]};$$

$$\Gamma(K^+ \to \pi^+ \mu^+ \mu^-)/\Gamma(K^+ \to \text{all}) \leqslant 3 \cdot 10^{-6} \text{ [35]};$$

$$\Gamma(K^+ \to \pi^+ \nu\bar{\nu})/\Gamma(K^+ \to \text{all}) \leqslant 1.1 \cdot 10^{-4} \text{ [36]}^*;$$

$$\Gamma(K^+ \to \pi^+ \mu^+ e^-)/\Gamma(K^+ \to \text{all}) \leqslant 3 \cdot 10^{-5} \text{ [37]}.$$

The last process violates the conservation of the lepton numbers.

One can relate the rates of the decays due to the neutral and charged currents and the corresponding coupling constants.

For the charged K-meson decays [37]

$$\Gamma(K^+ \to \pi^+ \mu^+ \mu^-) = 5.5\,[g^V(\mu^+\mu^-)/g^V(\mu^+\nu_\mu)]^2 \cdot 10^7 \text{ sec}^{-1};$$

$$\Gamma(K^+ \to \pi^+ e^+ e^-) = 1.2\,[g^V(e^+e^-)/g^V(e^+\nu_e)]^2 \cdot 10^8 \text{ sec}^{-1}.$$

and for the K^0-meson decays [38]

$$\frac{\Gamma(\bar{K}_L^0 \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu_\mu)} = 4\,[g^A(\mu^+\mu^-)/g^A(\mu^+\nu_\mu)]^2 \frac{[m_K^3(m_K^2 - 4m_\mu^2)]^{1/2}}{(m_K^2 - m_\mu^2)^2}.$$

If the above estimates for the upper limit of the branching ratios are used, the following limits for the ratios of the corresponding coupling constants are obtained†:

$$g^V(e^+e^-)/g^V(e^+\nu_e) \approx 7 \cdot 10^{-4};$$

$$g^V(\mu^+\mu^-)/g^V(\mu^+\nu_\mu) \approx 1.5 \cdot 10^{-2};$$

$$g^V(\nu\bar{\nu})/g^V(e^+\nu_e) \approx 6 \cdot 10^{-2};$$

$$g^A(\mu^+\mu^-)/g^A(\mu^+\nu_\mu) \approx 7 \cdot 10^{-4}.$$

Since the processes due to neutral currents have been sought in different reactions including vector and axial-vector strongly interacting currents and for all lepton combinations, the absence of such processes (or, at least, the small values of the corresponding coupling constants) is one of the fundamental properties

*In the most recent experiments the limit was lowered to $1.4 \cdot 10^{-6}$ for this ratio.
†In a recent experiment at Berkeley new data have been obtained on the decays $K_L^0 \to l^+ l^-$, from which it follows that $\Gamma(K_L^0 \to \mu^+\mu^-)/(\Gamma(K_L^0 \to \text{all}) < 8 \cdot 10^{-9}$ and there are similar estimates for the other transitions of this type; this considerably changes the estimates given here for the ratios of the coupling constants.

of the weak interactions. The existing data indicate that if the primary neutral currents do exist, the strength of their coupling is at least three-orders of magnitude less than that of the charged currents. Reaction with four particles in the final state such as

$$K^0 \to \pi^+\pi^-e^+e^- \text{ and } K^0 \to \pi^0\pi^0e^+e^-$$

differ slightly from the decays already considered. Whereas decays into two or three particles are due to vector or axial-vector transitions, processes with four particles are characterized by an interference term between the vector and axial-vector transitions. In addition, such processes cannot proceed through electromagnetic transitions with a Dalitz pair for the 0^+ state of the $\pi\pi$ system (the 0-0 transitions), i.e., the $\pi\pi ee$ final state must be a good subject for the search for primary neutral currents.

Electromagnetic Competition (Induced Neutral Currents). Even if primary neutral leptonic currents are absent in the first-order in the weak interaction, the presence of certain strangeness-conserving neutral hadron currents in the primary Lagrangian is sufficient to generate neutral leptonic currents (the so-called induced neutral leptonic currents). This process may proceed through the intermediate electromagnetic field; for example, a current of the form $\overline{p}\gamma_\mu p$ will generate the current $\overline{l}\gamma_\mu l$ through the process $\overline{p}+p\to\gamma\to\overline{l}l$.

Thus, the combined action of the weak and electromagnetic interactions may lead to processes in which the final state contains leptons with a vanishing total charge. Since usual conservation of the electric charge of the particles always leads to $\Delta Q = 0$ for the hadrons for such processes, one may conjecture that the existence of neutral hadron currents is due to the presence of such processes. The converse assertion is not always valid because of the possible contribution from strong interactions, whose effects require a detailed theoretical analysis. Such induced neutral leptonic currents are quite capable of competing with the primary currents and are an interfering background in this sense.

Numerical estimates of the branching ratios of the different induced processes obtained by different methods are given in Table 4. Comparing the experimental and theoretical estimates for the upper limits of the branching ratios, we see that they do not contradict one another, at least not in their order of magnitude. It should be noted that the smallest calculated branching ratio ($\sim 10^{-8}$) corresponds to the process $K_L^0\to\mu^+\mu^-$, whereas the experimental limit is also $\sim 10^{-8}$. Thus, for all the remaining processes the estimates of the limits of the branching ratios lie in practice at the level of the theoretical estimates for the induced processes.

Possible Violation of CP Invariance in Processes with Neutral Currents. Suppose that the observed violation of CP invariance can be attributed to the existence of neutral leptonic currents and that the smallness of CP-violating effects is due to the weakness of the coupling of the neutral currents. Such an assumption can be immediately verified by investigating the decay $K^+\to\pi^+\mu^+\mu^-$, since it must reveal T-noninvariant correlations of the form

$$\sigma_{\mu^+}(\overline{p}_\mu \times \overline{p}_\pi)\ \sigma_{\mu^-}(\overline{\sigma}_{\mu^+} \times \overline{p}_\pi).$$

In addition, one could observe a number of CP-noninvariant decay modes such as $K_S^0\to\mu^+\mu^-$.

It is shown in [31] that in the case of strict locality of the weak interaction this decay, if it occurs in the first-order in the weak interaction and proceeds without interference from the electromagnetic field (i.e., the $\mu^+\mu^-$ system is formed in the $^1s_0$ state) is CP violating; for the system of two fermions ll we have $CP = (-1)^{S+1}$. A similar situation obtains for the decay modes

$$K_S^0 \to \begin{matrix} \pi^0\nu\overline{\nu} \\ \pi^0\mu^+\mu^- \\ \pi^0 e^+e^-, \end{matrix}$$

which are vector transitions, i.e., the $\overline{l}l$ state must be in the ground $^3s_1$ state and the system must have CP = -1; it follows that such modes also violate CP invariance.

TABLE 4. Predictions for Induced Neutral Currents

Decay	Branching ratio	Literature	Decay	Branching ratio	Literature
$K^+ \to \pi^+ + e^+ + e^-$	10^{-7}	[39[	—	$(1,8-4)10^{-7}$	[43]
			$K_L^0 \to \pi^0 + e^+ + e^-$	10^{-8}	[40]
—	10^{-6}	[40]	—	$4 \cdot 10^{-8}$	[41]
—	10^{-6}	[41]	$K_L^0 \to \mu^+ + \mu^-$	$10^{-8} - 10^{-9}$	[44]
—	10^{-6}	[42]			

2. Leptonic Baryon Decays

Virtually all the known baryons that are stable against strong interactions (apart from the Ω^- hyperon) are unified in accordance with the $SU_{(3)}$-symmetry classification into a baryon octet with the quantum numbers of the spin J and the parity P satisfying $J^P = 1/2^+$. Therefore, in the semileptonic decay of a baryon A into a baryon B in accordance with the scheme

$$A \to B + l^- + \tilde{\nu}_l \tag{2.1}$$

both baryons are regarded as having the set of quantum numbers $J^P = 1/2^+$.

In the introduction we formulated the general form of the Lagrangian in the universal four-fermion V−A theory for the process (2.1) [the expressions (13)-(15)].

It should be noted that one can, in principle, extend the analogy with electrodynamics and introduce a vector particle (the intermediate boson W, the analog of the photon in electrodynamics) and go over from the formalism of a contact interaction to a nonlocal formulation of the theory of the weak interactions. The nonlocality effects depend on the mass of the intermediate W boson and decrease with increasing mass of this particle. The present-day experimental data indicate that if an intermediate vector W meson exists, its mass is at least greater than 3.5 GeV. If this is its mass, its influence on the phenomena considered below is very small. In what follows we shall therefore content ourselves with a Lagrange formalism of the form (13).

2.1. Structure of Matrix Elements

The semileptonic hadron decays can be described by a matrix element of the form

$$M = l^{\alpha} X_{\alpha}, \tag{2.2}$$

where l_α and X_α are the parts of the matrix element due to the leptonic and hadronic currents, respectively. In the presence of strong interactions the form of the hadronic currents [15] changes. In a general form, we therefore have

$$X_{\alpha} = \langle f | J_{\alpha}(x) | i \rangle. \tag{2.3}$$

Thus, the expression of the matrix element in the form (2.2) and (2.3) presupposes that the weak interaction is taken into account in the first order of perturbation theory but that the strong interaction has been taken into account fully.

The part X_α can be represented as the sum of a vector V_α and an axial-vector A_α part:

$$X_{\alpha} = V_{\alpha} + A_{\alpha}; \tag{2.4}$$

if the baryons A and B have positive relative parity ($P_{AB} = +1$), each of these parts can be expressed in the most complete form as follows:

$$V_{\alpha} = \bar{u}(p_B)\left[f_1(q^2)\gamma_{\alpha} + \frac{f_2(q^2)}{m_A + m_B}\sigma_{\alpha\beta} q_{\beta} + \frac{f_3(q^2)}{m_A + m_B} q_{\alpha}\right] u(p_A);$$

$$A_\alpha = \bar{u}(p_B)\left[g_1(q^2)\gamma_\alpha + \frac{g_2(q^2)}{m_A+m_B}\sigma_{\alpha\beta}q_\beta + \frac{g_3(q^2)}{m_A+m_B}q_\alpha\right]\gamma_5 u(p_A). \quad (2.5)$$

Here, p_A and p_B are the four-momenta of the baryons A and B and $q = p_A - p_B$ is the momentum transferred to the leptons. In the case of a negative relative AB parity ($P_{AB} = -1$) these expressions swap their positions. In the general case we therefore have six interactions; because of the renormalization effects from the strong interactions the two original V and A interactions with the form factors $f_1(q^2)$ and $g_1(q^2)$ are augmented by induced interactions which have received the following names: weak magnetism [the form factor $f_2(q^2)$], weak electricity [the form factor $g_2(q^2)$]; the scalar interaction [the form factor $f_3(q^2)$], and the pseudoscalar interaction [the form factor $g_3(q^2)$]. The hadronic part X_α of the matrix element (2.2) is determined by six form factors, which are functions of the square of the four-momentum q^2 transferred to the leptons. One can replace (2.4) and (2.5) by the compact expression

$$X_\alpha = u(p_B)\left[\sum_{k=1}^{6} f_k(q^2)\, O_\alpha^{(k)}\right] u(p_A),$$

where summation is extended over k for all possible variants of the interaction. Here, we have the same five invariants as are listed in Table 1 and these augmented by a pseudotensor (weak electricity) of the form $\sigma_{\alpha\beta}q_\beta\gamma_5$.

In semileptonic baryon decays a small amount of energy is usually liberated. It follows that the range of variation of q^2 is small and one can assume that the form factors change little. As a rule, their q^2 dependence is represented by the approximation

$$f_i(q^2) = f_i(0)\left[1 + \lambda_i q^2/m_\pi^2\right], \quad (2.7)$$

2.2. Expressions for Observables

Thus, the semileptonic baryon decays are described by expressions that depend on 12 real constants in the complex form factors $f_i(0)$ and $g_i(0)$ and also on the six parameters λ_i in the q^2 dependence of the form factors $f_i(q^2)$ and $g_i(q^2)$. If T invariance holds, there remain only six form-factor constants. Estimates show that, generally speaking, the dominant form factors are still the form factors of the vector and axial-vector interactions. The remainder lead to corrections to the main terms. These corrections are $\sim (m_A - m_B)/m_A$ or 1% and less. For example, if one neglects the recoil terms and the lepton mass, the differential probability of the decay $A \to Bl\nu_l$ with the emission of a lepton with energy in the interval from $\eta = E_l/E_l^{max}$ to $\eta + d\eta$ and acccordingly a baryon with kinetic energy in the interval from $\xi = \frac{T(\eta)_{max} - T}{T(\eta)_{max} - T(\eta)_{min}}$ to $\xi + d\xi$ takes the form

$$\frac{d^2W}{d\eta \cdot d\xi} = \eta^2(1-\eta)^2\left[|f_1|^2(1-\xi) + |g_1|^2(1+\xi)\right]. \quad (2.8)$$

This expression in the same approximation also determines the $l\nu$ correlations since $\xi \approx 1/2\,(1 - \cos\theta_{l\nu})$, which yields

$$\frac{dW}{d(\cos\theta_{l\nu})} \sim 1 - \frac{1 - |g_1/f_1|^2}{1 + 3|g_1/f_1|^2} \cdot \cos\theta_{l\nu}. \quad (2.9)$$

It can be seen from (2.9) that the electron energy spectrum in such an approximation is quite independent of the form factors and that the spectrum of the secondary baryons or the $l\nu$ correlations are determined solely by the modulus of the form-factor ratio g_1/f_1. The electron spectrum depends on the form factors in the order $(m_A - m_B)/m_A$. This term is determined by the interference effect of the form factors f_1 and g_1 in the presence of also the interference term of the form factors f_1 and f_2. It has the form

$$(f_1 + 2f_2)\, g_1 \frac{m_A - m_B}{m_A} \cdot \xi(2\eta - 1) \quad (2.10)$$

and is due to the influence of three form factors at once or two of their ratios, for example, f_2/f_1 and g_1/f_1. Thus, the ensemble of decays $A \to B + l + \nu_l$ can be represented on a Dalitz plot in the variables of the kinetic energies of the charged lepton and the secondary baryon T_B. One can then analyze such diagrams in the

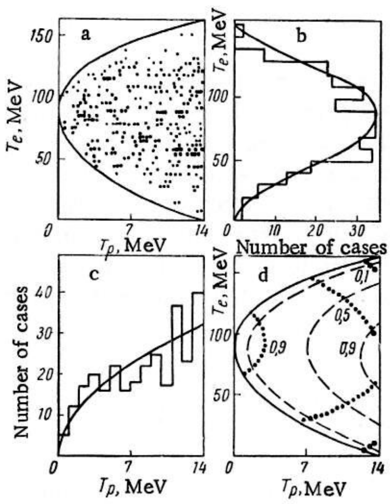


Fig. 5

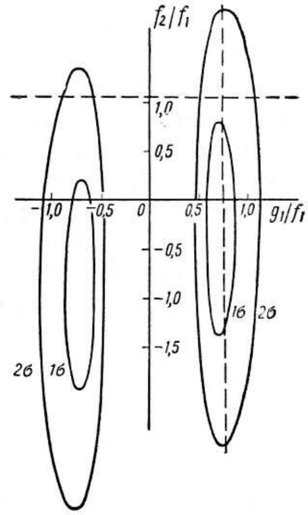


Fig. 6

Fig. 5. Investigation of the properties of the decay A $\rightarrow$ pe$^-$ $\bar{\nu}_e$: a) Dalitz plot; b) spectrum of the electron kinetic energies; c) spectrum of the proton kinetic energies in the rest system of the Λ hyperon; d) contours of the relative probability in the decay for the purely vector (dashed curves) and purely axial-vector (dotted curves) interaction variants and different values of the ratio g_1/f_1.

Fig. 6. Function of maximal likelihood for the form-factor ratios f_2/f_1 and q_1/f_1 in the decay $\Lambda \rightarrow pe\nu$ represented by the contours for one (1σ) and two (2σ) standard deviations. The horizontal dashed line is the ratio f_2/f_1 predicted by hypothesis of a conserved vector current (CVC), and the vertical dashed line is the ratio q_1/f_1 predicted by the Cabibbo theory (see Table 6).

variables g_1/f_1 and f_2/f_1. An example of the distribution of decay events $\Lambda \rightarrow pe\nu_e$ on the Dalitz plot is shown in Fig. 5a. An analysis by the method of the function of maximal likelihood in the variables g_1/f_1 and f_2/f_1 for this decay is shown in Fig. 6. This figure shows that the accuracy of the present-day data on the decay $\Lambda \rightarrow pe\nu_e$ is not yet sufficient to draw an unambiguous conclusion about the parameters of such a description of the semileptonic decay [45]. A similar conclusion is reached in an analysis of the corresponding data for the decay $\Sigma^- \rightarrow ne^-\tilde{\nu}_e$ [46] (Fig. 7).

Let us now consider in more detail the expressions for the observable quantities. In deriving expressions for observable quantities (spectra, correlations etc.) one does not usually use the expressions (2.5) for the hadronic part of the matrix element but more convenient expressions. To obtain these we replace the dimensionless form factors $f_i(q_i^2)$ and $g_i(q^2)$ by the linear combinations [47]

$$\left.\begin{aligned} F_1 &= f_1 + \left(1 + \frac{m_A}{m_B}\right) f_2; \\ F_2 &= -2f_2; \\ F_3 &= f_2 + f_3; \\ G_1 &= g_1 - \left(1 - \frac{m_A}{m_B}\right) g_2; \\ G_2 &= -2g_2; \\ G_3 &= g_2 + g_3. \end{aligned}\right\} \tag{2.11}$$

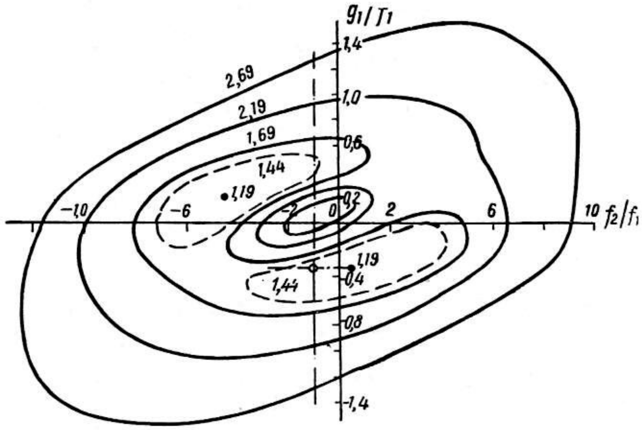


Fig. 7. Function of maximal likelihood for the form-factor ratios f_2/f_1 and g_1/f_1 in the decay $\Sigma^- \to ne\nu$ represented by the contours for one, two, and three standard deviations. The figure also includes lines corresponding to the values of these ratios for the hypothesis of a conserved vector current (CVC) (the vertical dash-dot-dash line) and the Cabibbo theory (the horizontal dash-dot-dash line) with the parameters from Table 6.

Then Eqs. (2.5) are replaced by the following expressions for the calculation of the matrix elements:

$$V_\alpha = \bar{u}(p_B)\left[F_1(q^2)\gamma_\alpha + \frac{F_2(q^2)}{m_A+m_B}(p_B)_\alpha + \frac{F_3(q^2)}{m_A+m_B} q_\alpha\right] u(p_A); \tag{2.12}$$

$$A_\alpha = \bar{u}(p_B)\left[G_1(q^2)\gamma_\alpha\gamma_5 + \frac{G_2(q^2)}{m_A+m_B}(p_B)_\alpha\gamma_5 + \frac{G_3(q^2)}{m_A+m_B} q_\alpha\right] u(p_A). \tag{2.13}$$

Let us determine the q^2 dependence of the new form factors $F_i(q^2)$ and $G_i(q^2)$, again using expressions of the form (2.7). For the products of the form factors we use the approximations

$$F_i(q^2)F_j^*(q^2) = F_i(0)F_j^*(0)\left[1+(\lambda_i+\lambda_j)q^2/m_\pi^2\right]; \tag{2.14}$$

$$G_i(q^2)G_j^*(q^2) = G_i(0)G_j^*(0)\left[1+(\mu_i+\mu_j)q^2/m_\pi^2\right]. \tag{2.15}$$

We now determine the different quadratic expressions in $F_i(q^2)$ and $G_i(q^2)$ as follows:

$$\left.\begin{aligned}
a_{ij} &= \mathrm{Re}\left[F_i(0)F_j^*(0)\right];\\
c_{ij} &= \mathrm{Re}\left[G_i(0)G_j^*(0)\right];\\
b_{ij} &= \mathrm{Re}\left[F_i(0)F_j^*(0)\frac{\lambda_i+\lambda_j}{m_\pi^2}\right];\\
d_{ij} &= \mathrm{Re}\left[G_i(0)G_j^*(0)\frac{\mu_i+\mu_j}{m_\pi^2}\right];\\
e_{ij} &= \mathrm{Re}\left[F_i(0)G_j^*(0)\right];\\
h_{ij} &= \mathrm{Re}\left[F_i(0)G_j(0)\frac{\lambda_i+\mu_j}{m_\pi^2}\right].
\end{aligned}\right\} \tag{2.16}$$

In this terminology the expressions for the energy spectra of the baryons (W_B), leptons (W_l), and also for the angular correlation of the charged lepton and the neutrino ($W_{l\nu}$) have the same form [48]:

$$W(R) = \frac{dW}{dR} = \left[\sum_{i,j=1}^{3} \left(a_{ij} A_k^{ij} + b_{ij} B_k^{ij} + c_{ij} C_k^{ij} + d_{ij} D_k^{ij} + l_{ij} E_k^{ij} + h_{ij} H_k^{ij} \right) \right] \Phi_k(x), \tag{2.17}$$

where the functions $\Phi_k(x)$ in the formulas for the energy spectra of the baryons and leptons have the simple form

$$\Phi_k(x) = \sqrt{x^2 + bx + a} \left(\frac{x+c}{x^2} \right)^2 x^k \frac{G^2}{(4\pi)^3} \cdot 2 \left(\frac{m_B}{m_A} \right)^2;$$
$$x_{B(l)} = -\frac{m_A^2 - m_{B(l)}^2}{m_B^2} + 2 \frac{m_A}{m_B^2} E_{B(l)}, \tag{2.18}$$

where $k = 1, 2, 3, \ldots, 6$; and a, b, and c are constants.

For the $l\nu$ correlations, $\Phi(x)$ has a more complicated form. However, one can use an approximation in which one ignores the terms from the recoil baryon proportional to $(m_A - m_B)/m_A$ and the lepton mass. For the $l\nu$ correlations we then obtain the simple formula

$$W(\cos\theta_{l\nu}) = \frac{1}{2} \left(1 + \frac{|z|^2 - 1}{|z|^2 + 3} \cos\theta_{l\nu} \right), \tag{2.19}$$

where now

$$z = f_1/q_1. \tag{2.20}$$

It is also helpful to give the expression for the angular distribution of the polarization of the baryon B from the decay $A \to Bl\nu_l$. In the frame of reference defined by the vectors

$$\boldsymbol{\alpha} = \frac{\mathbf{q}_l + \mathbf{q}_\nu}{|q_l + q_\nu|}; \quad \boldsymbol{\beta} = \frac{\mathbf{q}_l - \mathbf{q}_\nu}{|q_l + q_\nu|}; \quad \boldsymbol{\gamma} = \frac{\mathbf{q}_l \cdot \mathbf{q}_\nu}{|q_l + q_\nu|}, \tag{2.21}$$

this distribution becomes

$$W(\mathbf{S}_B) = 1 + \frac{8}{3} \mathbf{S}_B \left[\frac{\operatorname{Re} z}{|z|^2 + 3} \boldsymbol{\alpha} + \frac{1}{|z|^2 + 3} \boldsymbol{\beta} + \frac{3\pi}{16} \cdot \frac{\operatorname{Im} z}{|z|^2 + 3} \boldsymbol{\gamma} \right]. \tag{2.22}$$

If we now consider the following decay of the baryon B in accordance with a scheme of the nonleptonic type

$$B \to N\pi, \tag{2.23}$$

then the angular distribution for the decay nucleon in the same frame of reference is

$$W(\mathbf{N} \cdot \boldsymbol{\alpha}) = 1 + \alpha_B \frac{8}{3} \cdot \frac{\operatorname{Re} z}{|z|^2 + 3} \mathbf{N} \cdot \boldsymbol{\alpha}; \tag{2.24}$$

$$W(\mathbf{N} \cdot \boldsymbol{\beta}) = 1 + \alpha_B \frac{8}{3} \cdot \frac{1}{|z|^2 + 3} \mathbf{N} \cdot \boldsymbol{\beta}; \tag{2.25}$$

$$W(\mathbf{N} \cdot \boldsymbol{\gamma}) = 1 + \alpha_B \frac{\pi}{2} \cdot \frac{\operatorname{Im} z}{|z|^2 + 3} \mathbf{N} \cdot \boldsymbol{\gamma}. \tag{2.26}$$

Here, $\mathbf{N}$ is a unit vector in the direction of the nucleon momentum and α_B is the asymmetry coefficient in the corresponding decay of the baryon B in accordance with (2.23). Note that since the total lepton momentum is equal to the momentum of the baryon B, (2.24) is obviously the angular distribution of the decay nucleons in formula (2.23) with respect to the direction of the momentum of the baryon B.

Let us now consider the decay of a polarized baryon A at rest with polarization vector $\mathbf{p}_A$. The probability of its decay into the state $dE_l d\Omega_l d\Omega_\nu$ is

$$W(E_l, \Omega_l, \Omega_\nu, \mathbf{p}_A)\, dE_l\, d\Omega_l\, d\Omega_\nu = [c_1 + c_2 \mathbf{p}_l \mathbf{p}_A + c_3 \mathbf{p}_\nu \mathbf{p}_A + c_4 \mathbf{p}_A (\mathbf{p}_l \times \mathbf{p}_\nu)] \times \frac{|\mathbf{p}_l||\mathbf{p}_\nu|\, dE_l\, d\Omega_l\, d\Omega_\nu}{m_A - E_l + |\mathbf{p}_l| \cos\theta_{l\nu}}, \tag{2.27}$$

where the coefficients c_i depend on the form factors $F_i(q^2)$ and $G_i(q^2)$ and the kinematic characteristics of the decay (for example, E_l and $\cos\theta_{l\nu}$). For the angular distributions of the recoil baryons $W(\cos\theta_B)$ and the charged leptons $W(\cos\theta_l)$ with respect to the polarization vector of the original baryon $\mathbf{p}_A$ we hence obtain the expressions [48]:

$$W(\cos\theta_{B\,(l)}) = \frac{dW}{d\cos\theta_{B\,(l)}} = \frac{W_0}{2}\left[1 + |\mathbf{p}_A| \frac{W_a^{B\,(l)}}{W_0} \cos\theta_{B\,(l)}\right]. \tag{2.28}$$

Here, W_a^l again has the form (2.17) with $\Phi(x)$ = const and W_a^B is given by

$$W_a^B = \frac{G^2}{2\pi^3}\left[e_{ij} J_e^{ij} + h_{ij} J_h^{ij}\right],$$

i.e., the sum contains terms that depend only on the interference of the form factors of the vector and axial-vector parts of the interaction.

In the formulas, W_0 is the total probability of the given type of semileptonic decay of the baryon A:

$$W_0 = \frac{G^2}{(2\pi)^3} \sum_{i,\,j=1}^{3} \left(a_{ij} J_a^{ij} + b_{ij} J_b^{ij} + c_{ij} J_c^{ij} + d_{ij} J_d^{ij}\right). \tag{2.29}$$

The structure of the coefficients a_{ij}, b_{ij}, c_{ij}, and d_{ij} [see (2.16)] shows that in this expression the summation is extended only over terms that contain form factors of one parity type, i.e., the VV, AA, A, and PT types but not the types VA, TA, etc.

In all these expressions the coefficients A^{ij} and J^{ij} are determined by the dynamic characteristics of the decay and, ultimately, are functions of only the masses of the particles that participate in the process. The values of these coefficients have been calculated by different authors and they can be found in the corresponding tables.

The expression (2.28) for the angular distribution of the leptons relative to the polarization vector of the original baryon in the above approximation takes the form

$$W(\cos\theta_l) \sim 1 + \alpha_l \cos\theta_l. \tag{2.30}$$

The asymmetry coefficient α_l in this expression is given by

$$\alpha_l = -2\beta_l \frac{g_1/f_1 + (g_1/f_1)^2}{1 + 3(g_1/f_1)^2}, \tag{2.31}$$

where β_l is the lepton velocity.

The expression (2.29) for the total probability can be written down by separating the principal part that depends on the kinematics:

$$W_0 = \frac{G^2}{60\pi^3} \cdot \frac{\Delta^5}{(1+\xi)^3} H_l, \tag{2.32}$$

where $\Delta = m_A - m_B$; $\xi = (m_A - m_B)/(m_A + m_B)$; and H_l is a bilinear function of the form factors:

$$H_l = \sum_{M,\,N} f_M f_N S^{MN}. \tag{2.33}$$

In this representation we obtain the following relationships for decays with the emission of an electron [49]:

$$\left.\begin{aligned} S^{VV} &= 1; \\ S^{AA} &= 3; \\ S^{A,PT} = S^{PT,A} &= -\varepsilon; \\ S^{VT} = S^{TV} &= \varepsilon^2; \\ S^{VA} = S^{AV} = S^{TA} = S^{AT} &\approx \frac{1}{2}\varepsilon^2. \end{aligned}\right\} \tag{2.34}$$

Here, $\varepsilon = W/m_A$; $W = (m_A^2 - m_B^2 + m_C^2)/2m_A$.

Thus, if the form factors f_i and g_i have similar orders of magnitude

$$f_1 \approx f_2 \approx f_3 = g_1 \approx g_2 \approx g_3, \tag{2.35}$$

then the total probablity of an electronic decay of a baryon (and also of a decay with the emission of a muon) is determined primarily, as one would expect, by the form factors of the vector f_1 and axial-vector g_1 interactions. The contribution of the induced interactions is determined by the interference terms with the form factors g_1 and g_2 and also f_1 and f_2, whose numerical coefficients have the values $\varepsilon \approx 10^{-2}$ and $\varepsilon^2 \approx 10^{-4}$. One can write down the relationship

$$H_l = f_1^2 + 3g_1^2 - \varepsilon g_1 g_2 + \varepsilon^2 f_1 f_2. \tag{2.36}$$

In the order ε^3 one would obtain interference terms from the form factors with different parities, namely, terms of the form $f_1 g_1$ and $f_2 g_1$.

In the case of a decay with the emission of a muon, calculations show [5] that one may have terms with different form factors as well as terms that arise from the q^2 dependence of the form factors. If one writes down the ratio of the probabilities of μ decay and β decay of a baryon in the form

$$\frac{\Gamma_\mu}{\Gamma_l} = \sigma_1 \left(1 + \frac{\Delta}{H_l}\right), \tag{2.37}$$

where

$$\Delta = (H_\mu - \sigma_1 H_l)/\sigma_1; \tag{2.38}$$

σ_1 is a constant, and H_l is already defined by (2.36), a comparison of the decays $\Sigma^- \to n\mu^-\tilde{\nu}_\mu$ and $\Sigma^- \to ne^-\tilde{\nu}_e$ yields

$$\Gamma_\mu/\Gamma_e = 0.45\,(1 + 1.33\,\gamma), \tag{2.39}$$

where, if one does not assume the V−A variant of the theory, the quantity γ is given by

$$\gamma = \frac{\mathrm{Re}\, f_3 f_1^* + 6\,\mathrm{Re}\, g_1 f_2^* + 6\,\delta\,\mathrm{Re}\, f_2 f_2^* - \delta\,\mathrm{Re}\, g_3 g_1^*}{|f_1|^2 + 3\,|g_1|^2 + 12\,|f_2|^2 + |f_3|^2}, \tag{2.40}$$

in which

$$\delta = \frac{m_{\Sigma^-} + m_n}{m_{\Sigma^-} + m_n}. \tag{2.41}$$

It follows from the presently available experimental data on the semileptonic Σ^- decays [51] that

$$\Gamma_\mu/\Gamma_e = 0.42 \pm 0.06. \tag{2.42}$$

In the framework of $\mu-e$ universality this corresponds to the following interval for γ:

$$-0.18<\gamma<0.02. \tag{2.43}$$

In a comparison of the corresponding Λ-hyperon decays it is found that $(\Gamma_\mu/\Gamma_e)_{theor}=0.17$ and $(\Gamma_\mu/\Gamma_e)_{exp}=0.17\pm0.02$.. This again suggests that a vanishing value of γ would be compatible.

The influence of the induced-electricity form factor g_2 can be observed from the interference effect in a comparison of the decays $\Sigma^-\to\Lambda e^-\tilde{\nu}_e$ and $\Sigma^+\to\Lambda e^+\nu_e$. Here, Eq. (2.32) of [50] gives

$$\frac{\Gamma_-}{\Gamma_+}=\left(\frac{m_{\Sigma^-}-m_\Lambda}{m_{\Sigma^+}-m_\Lambda}\right)^5\left(\frac{1+\xi_+}{1+\xi_-}\right)^3\frac{H_-}{H_+}=1.64\left(1+\frac{\Delta}{H_+}\right), \tag{2.44}$$

where

$$\Delta=H_--H_+\approx\frac{m_--m_+}{4m_\pi}g_1g_2=0.015\,g_1g_2. \tag{2.45}$$

At the same time, since $f_1=0$ (see below) in these decays,

$$H_+=3g_1^2+0.52\,g_1g_2. \tag{2.46}$$

The experimental value of branching ratio (2.44) is at present

$$\frac{\Gamma_-}{\Gamma_+}=1.60\pm0.27, \tag{2.47}$$

which corresponds to the following interval of Δ/H_+:

$$-0.19<\frac{\Delta}{H_+}<0.05. \tag{2.48}$$

Using Eq. (2.45) and (2.46), we obtain

$$\frac{\Delta}{H_+}=\frac{0.005\,g_2/g_1}{1+0.17\,g_2/g_1}, \tag{2.49}$$

which amounts to ~0.5% for $g_2/q_1\approx1$. Thus, to resolve the question of the possible contribution of the form factor g_2 to the decays $\Sigma^\pm\to\Lambda e^\pm\nu_e$ it will also be necessary to increase the accuracy of the experimental data considerably.

The experimental situation can be illustrated most clearly for the problem of semileptonic baryon decays by the example of the now relatively well studied β decay of the neutron:

$$n\to pe^-\tilde{\nu}_e. \tag{2.50}$$

The most recent experiments have given the following result for its half-decay period:

$$\tau_{n\to pe\tilde{\nu}_e}=(10.80\pm0.16)\text{ min}, \tag{2.51}$$

i.e., the value of τ for the neutron is known with an accuracy of ~1.5%. Since $\varepsilon\approx10^{-3}$ in this decay, the present-day accuracy in the determination of τ is clearly insufficient to reveal the influence of the induced form factors on the value of τ. The accuracy would have to be increased by at least a further order of magnitude. Recalling that the conservation of the G parity of the nucleon current implies

$$g_2=0, \tag{2.52}$$

we see from (2.36) that the detection of a contribution from the weak magnetism form factor f_2 necessitates an experimental value of τ for the neutron with an accuracy not worse than 10^{-6}.

Thus, using the information about the neutron lifetime, one can obtain only the modulus of the ratio of the axial-vector to the vector form factors $|g_1/f_1|$. It is found to be

$$|g_1/f_1| = 1.23 \pm 0{,}01 \text{ (from } n \to pe\nu). \tag{2.53}$$

Measurements of the asymmetry in the emission of electrons in the β decay of polarized neutrons in accordance with Eqs. (2.30) and (2.31) enables one to determine both the value and the sign of the ratio g_1/f_1. Recent experiments gave the following result [52]*:

$$g_1/f_1 = -1.25 \pm 0.05 \text{ (from } n \to pe\nu). \tag{2.54}$$

Here, the corrections from the induced weak magnetism form factor again have the order of magnitude

$$\frac{m_n - m_p}{3m_n}, \tag{2.55}$$

i.e., they are $\sim 10^{-3}$.

Finally, measurements of the electron-neutrino correlation in the β decay of the neutron [67] yield [from (2.9)]

$$\alpha_{l\nu} = \frac{1 - |\alpha|^2}{1 + 3|\alpha|^2} = -0.091 \pm 0.039,$$

from which

$$|\alpha| = |g_1/f_1| = 1.33 \pm 0.15. \tag{2.56}$$

Thus, these three experiments give the same results (to within the errors of the measurements) for the ratio g_1/f_1 for β decay. The ratio is negative and is at present assumed to be 1.23 ± 0.01. At the present stage there is no point in considering a possible contribution of the induced form factors to the experimentally observed characteristics of neutron β decay.

The formalism expounded above for the description of the weak interactions is in excellent agreement with the vast experimental material on the β decay of nuclei. However, the β (μ) decay probabilities calculated for hyperons by this theory are in serious contradiction with the experimentally found values. This was a serious problem of the universal theory of weak interactions. Its solution was found in the framework of the idea of $SU_{(3)}$ symmetry in the world of elementary particles.

2.3. Relevant Aspects of $SU_{(3)}$ Symmetry

There are now available a number of excellent reviews of $SU_{(3)}$ symmetry and its development. We shall therefore only discuss the aspects that have a bearing on our problem. The concept of $SU_{(3)}$ symmetry developed out of isospin symmetry, the nucleon spinor

$$N = \begin{pmatrix} p \\ n \end{pmatrix}; \quad \overline{N} = (p, n)$$

being replaced by the three-component spinor

$$b = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}; \quad \overline{b} = (p, n, \Lambda),$$

by means of which the isospin I is augmented by the hypercharge Y as a group characteristic.

*The most recent results give -1.26 ± 0.02 for this ratio.

The states p, n, and Λ are the states of a single particle with the same mass, spin, and parity. They are distinguished by the quantum numbers of the isospin and hypercharge. Then any interaction in which they participate must be invariant under rotations in a certain formal space. This is equivalent to the assertion that under such rotations under the influence of a transformation Ω a bilinear form remains invariant:

$$\bar{b}\Omega^{+}\Omega b=\bar{b}b.$$

It follows that Ω is represented by a 3×3 matrix such that $\Omega^{+}=\Omega^{-1}$, i.e., the Hermitian-conjugate matrix Ω^{+} is equal to the inverse matrix Ω^{-1} and the transformation Ω is therefore unitary. There are nine linearly independent 3×3 matrices which can be taken as the basis matrices in an investigation of a unitary transformation $U_{(3)}$ in three dimensions. One of them is the identity matrix I and the remaining eight matrices $\lambda_1, \lambda_2, \ldots, \lambda_8$ are Hermitian with vanishing traces. A general transformation Ω can be expressed as a product $\Omega=e^{iI\alpha}\cdot e^{i\lambda\theta}$. This corresponds to a representation of the unitary transformation $U_{(3)}$ as a product of a unitary transformation $U_{(1)}$ of dimension 1 and a unitary unimodular $SU_{(3)}$ transformation of rank 2 and dimension 3 since there are three basis vectors p, n, and Λ in the latter case and two of the eight generators can be reduced to diagonal form. This last fact corresponds to the presence in the group of two conserved quantities, namely, the isospin I and the hypercharge Y. Thus, instead of a three-dimensional isotopic space we now deal with an eight-dimensional unitary space. Invariance under $U_{(1)}$ corresponds to conservation of a baryon current of the form $n_\alpha=i\bar{b}\gamma_\alpha b$. Invariance under $SU_{(3)}$ corresponds to conservation of the eight component current of the so-called unitary spin:

$$\vec{\mathscr{F}}_\alpha=i\bar{b}\gamma_\alpha\frac{\lambda}{2}b. \tag{2.57}$$

Like the isospin current, the unitary spin current is conserved in the approximation of strict $SU_{(3)}$ symmetry. The baryon number n and the eight components of the unitary spin are given by the expressions

$$n=-i\int n_4 d^3x;$$
$$F_k=-i\int\vec{\mathscr{F}}_{k4}\,d^3x.$$

The λ_i can be taken as the following eight matrices [63]:

$$\lambda_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix};\quad \lambda_2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix};\quad \lambda_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix};$$

$$\lambda_4=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix};\quad \lambda_5=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix};\quad \lambda_6=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix};$$

$$\lambda_7=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix};\quad \lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}.$$

These matrices satisfy the relations

$$\mathrm{Sp}\,\lambda_k\lambda_l=2\delta_{kl};$$
$$[\lambda_k,\widetilde{\lambda}_l]=2if_{klm}\lambda_m;$$
$$[\lambda_k,\lambda_l]=2d_{klm}\lambda_m,$$

where f_{klm} is a completely antisymmetric tensor and d_{klm} is a completely symmetric tensor (under permutations of the subscripts). We have the following nonvanishing values of these tensors:

$$f_{123}=1;$$
$$f_{147}=f_{246}=f_{257}=f_{345}=-f_{156}=-f_{367}=\frac{1}{2};$$
$$f_{458}=f_{678}=\frac{\sqrt{3}}{2};$$
$$d_{118}=d_{228}=d_{338}=-d_{888}=\frac{1}{\sqrt{3}};$$
$$d_{448}=d_{558}=d_{668}=d_{778}=-\frac{1}{2\sqrt{3}};$$
$$d_{146}=d_{157}=d_{256}=d_{344}=d_{355}=-d_{247}=-d_{366}=-d_{377}=\frac{1}{2}.$$

It follows from the structure of the λ_i matrices that the first three matrices λ_1, λ_2, and λ_3 determine the isospin subgroup $SU_{(2)}$. The operators of its projections are the matrices $I_1={}^1/_2\lambda_1$, $I_2={}^1/_2\lambda_2$, $I_3={}^1/_2\lambda_3$. The diagonality of λ_3 corresponds to conservation of the third component of the isospin. As we have already mentioned, there is a further diagonal matrix λ_8. It is associated with the hypercharge, whose operator can be expressed in the form

$$Y=\frac{1}{\sqrt{3}}\lambda_8=\frac{1}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}.$$

It commutes with the matrices of the projection operators of the isospin. Using the Gell-Mann–Nishijima formula, we obtain the following expression for the electric charge operator Q:

$$Q=e\left(I_3+\frac{1}{2}Y\right)=\frac{1}{2}e\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)$$

or

$$Q=\frac{1}{3}e\begin{pmatrix}2&0&0\\0&-1&0\\0&0&-1\end{pmatrix}.$$

In the same normalization we have an expression for λ_0, the matrix of the baryon number operator (it corresponds to $U_{(3)}$ and not to $SU_{(3)}$):

$$\lambda_0=\sqrt{\frac{2}{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}.$$

This yields an expression for the baryon number operator in the form

$$B=\frac{1}{\sqrt{6}}\lambda_0=\frac{1}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}.$$

The group $SU_{(3)}$ is characterized by a set of irreducible representation, which are taken as the basis for the classification of particles, sets of particles being associated with irreducible representations of the group. The tensors of such representations $\varphi^{\alpha\beta\gamma\cdots}_{\lambda\mu\nu\cdots}$ with q superscripts and p subscripts that take the values from one to three are characterized by definite sets of quantum numbers of the hypercharge and isospin. The number of dimensions of the representation is given by

$$N(p,q)=\frac{1}{2}(p+1)(q+1)(p+q+2).$$

The hypercharge of the tensor component is given by

$$Y=p(3)-q(3)-\frac{p-q}{3}, \tag{2.58}$$

and I_3, the isospin component, by

$$I_3 = \frac{1}{2}\,[p(2) - q(2) - p(1) + q(1)].$$

In these expressions q(i) and p(i) are the superscripts and subscripts, i taking the values 1, 2, and 3. If real particles are characterized by only integral values of Y, it follows from (2.58) that the families of particles can only be associated with representations for which the numbers of the superscripts and subscripts are such that (p−q)/3 is an integer.

The lowest of these irreducible representations of $SU_{(3)}$ have the following dimensions: singlet D(0,0), octet D(1,1), decuplet D(3,0), 27-plet D(2,2) etc. In what follows, we shall be interested in the octet representation D(1,1). We write the tensor of this representation in the form of a 3 × 3 matrix $\varphi_\alpha^\beta(I_3, Y)$, indicating the quantum numbers I_3, the isospin projection, and the hypercharge Y. This matrix has the form

$$\varphi_\alpha^\beta = \begin{pmatrix} \varphi_1^1(0,0) & \varphi_1^2(1,0) & \varphi_1^3\left(\frac{1}{2}, +1\right) \\ \varphi_2^1(-1,0) & \varphi_2^2(0,0) & \varphi_2^3\left(-\frac{1}{2}, 1\right) \\ \varphi_3^1\left(-\frac{1}{2}, -1\right) & \varphi_3^2\left(\frac{1}{2}, -1\right) & \varphi_3^3(0,0) \end{pmatrix} \tag{2.59}$$

All the presently known baryons that are stable against strong interactions and have spin-parity quantum numbers $J^P = 1/2^+$ are united in the unitary octet. We write the matrix for these baryons in the form

$$B = \begin{pmatrix} \frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix} \tag{2.60}$$

Similarly, one can write the matrix for the pseudoscalar mesons with $J^P = 0^-$:

$$P = \begin{pmatrix} \frac{\eta^0}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta^0}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^\circ \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{pmatrix} \tag{2.61}$$

and the other particles that are united in $SU_{(3)}$-symmetry octets.

Finally, forming the scalar products of the eight-dimensional vectors F and λ, we obtain a representation of the unitary spin in the form of a mixed tensor of second rank, which again can be written as a 3 × 3 matrix with vanishing trace:

$$U = \begin{pmatrix} F_3 + \frac{1}{\sqrt{3}}F_8 & F_1 + iF_2 & F_4 + iF_5 \\ F_1 - iF_2 & -F_3 + \frac{1}{\sqrt{3}}F_8 & F_6 + iF_7 \\ F_4 - iF_5 & F_6 - iF_7 & -\frac{2}{\sqrt{3}}F_8 \end{pmatrix}$$

We have already discussed the main properties of the unitary spin components F_1, F_2, and F_3. They are always conserved. Here, we should like to mention that, in accordance with (2.59), the components F_4 and F_5 and also F_6 and F_7 are characterized by a strangeness equal to unity and isospin equal to 1/2.

2.4. Semileptonic Baryon Decays in the Framework of $SU_{(3)}$ Symmetry

In the framework of this formalism the vector hadronic current in the weak-interaction Lagrangian (18) has the form

$$J_\alpha^{(V)} = F_{1\alpha} + iF_{2\alpha} + F_{4\alpha} + iF_{5\alpha}. \tag{2.62}$$

Here, the first two terms correspond to transitions with $\Delta S = 0$ and the third and fourth terms to transitions with $|\Delta S| = 1$. A comparison of the matrices (2.59) and (2.61) indicates that these two pairs of terms correspond to the π^+ and K^+ components in the matrix for the pseudoscalar mesons. The vector hadronic current is therefore frequently written as the sum of the two corresponding terms:

$$J_\alpha^{(V)} = V_\alpha^{\pi+} + V_\alpha^{K+}. \tag{2.63}$$

By analogy with (2.57), we shall now define an eight-component axial-vector quantity:

$$A_\alpha = i\bar{b}\frac{\lambda}{2}\gamma_\alpha\gamma_5 b.$$

Then the axial-vector part of the hadronic current in the Lagrangian (18) has the form

$$J_\alpha^{(A)} = A_{1\alpha} + iA_{2\alpha} + A_{4\alpha} + iA_{5\alpha}, \tag{2.64}$$

the terms of this expression having the same properties as those of Eq. (2.62) with respect to the strangeness and isospin quantum numbers. By analogy with (2.63), one can also write

$$J_\alpha^A = A_\alpha^{\pi+} + A_\alpha^{K+}.$$

The total hadronic current is the sum of the currents (2.62) and (2.64). It contains terms corresponding to transitions with $\Delta S = 0$ and $|\Delta S| = 1$:

$$J_\alpha = J_\alpha^{(0)} + J_\alpha^{(1)}, \tag{2.65}$$

where

$$J_\alpha^{(0)} = V_\alpha^{\pi+} + A_\alpha^{\pi+};$$
$$J_\alpha^{(1)} = V_\alpha^{K+} + A_\alpha^{K+}.$$

We now make the decisive step proposed by Cabibbo [3], namely, the hypothesis that the hadronic weak current J_α is characterized like the vector current (2.65) by a unit length but is obtained from the latter by means of a certain rotation in the unitary space through an angle θ. Then the weak hadronic current J_α is not represented by (2.65) but by

$$J_\alpha = \cos\theta J_\alpha^{(0)} + \sin\theta J_\alpha^{(1)}. \tag{2.66}$$

In principle the parameter θ may be different for the vector and axial-vector parts in the currents $J_\alpha^{(0)}$ and $J_\alpha^{(1)}$. We introduce the angles θ_V and θ_A for the vector and axial-vector parts, respectively. We then obtain

$$J_\alpha = \cos\theta_V V_\alpha^{\pi+} + \cos\theta_A A_\alpha^{\pi+} + \sin\theta_V V_\alpha^{K+} + \sin\theta_A A_\alpha^{K+}.$$

Later we shall return to this question but, as a rule, we shall generally use the variant of the theory with a single Cabibbo angle θ in which the hadronic current is described by the expression (2.66).

Thus, we have a set of baryons with $J^P = 1/2^+$ whose wave functions transform in accordance with an octet representation of $SU_{(3)}$ and make up the matrix (2.60). We also have two sets of currents $J_\alpha^{(V)}$ and

$J_\alpha^{(A)}$ defined by (2.62) and (2.64), respectively. The components of these currents also transform in accordance with octet representations of $SU_{(3)}$. For the weak-interaction Lagrangian we obtain

$$L = \frac{G}{\sqrt{2}} \left\{ \bar{u}_e \gamma_\alpha (1+\gamma_5) u_{\nu_e} + \bar{u}_\mu \gamma_5 (1+\gamma_5) u_{\nu_\mu} \right\} \times \left\{ \cos\theta_V V_\alpha^{\pi+} + \cos\theta_A A_\alpha^{\pi+} + \sin\theta_V V_\alpha^{K+} + \sin\theta_A A_\alpha^{K+} \right\}.$$

For the hadronic part of the matrix element due to the action of the n-th component of the current $F_{m\alpha} + A_{m\alpha}$, we have an expression of the form

$$\langle \bar{B} | F_{m\alpha} + A_{m\alpha} | A \rangle = i f_{ABm} \Phi_\alpha + d_{ABm} \Delta_\alpha, \tag{2.67}$$

This is a generalization of the well-known Wigner-Echart theorem in the theory of spatial rotations. The quantities Φ_α and Δ_α play the role of reduced matrix elements and the tensors f_{ABm} and d_{ABm} the role of the Clebsch-Gordan coefficients in the $SU_{(3)}$ formalism. The presence of two reduced matrix elements corresponds to the existence of f and d type couplings in the interactions of the baryon octet with $J^P = 1/2^+$ and the octet of pseudoscalar mesons.

On the other hand, one can write down matrix elements for the vector and axial-vector parts of the hadronic current in the form

$$\begin{aligned} \langle \bar{B} | F_{m\alpha} | A \rangle &= \bar{u}_B \left[i f_{ABm} V^\Phi + d_{ABm} V^\Delta \right] \gamma_\alpha u_A; \\ \langle \bar{B} | A_{m\alpha} | A \rangle &= \bar{u}_B \left[i f_{ABm} A^\Phi + d_{ABm} A^\Delta \right] \gamma_\alpha \gamma_5 u_A, \end{aligned} \tag{2.68}$$

where V^Φ, V^Δ, A^Φ, and A^Δ determine the contributions of the f and d type couplings to the vector and axial-vector interactions. Comparing Eqs. (2.67) and (2.68), we obtain the structure of the reduced matrix elements in the form

$$\left. \begin{aligned} \Phi_\alpha &= \bar{u}_B (V^\Phi \gamma_\alpha + A^\Phi \gamma_\alpha \gamma_5) u_A; \\ \Delta_\alpha &= \bar{u}_B (V^\Delta \gamma_\alpha + A^\Delta \gamma_\alpha \gamma_5) u_A. \end{aligned} \right\} \tag{2.69}$$

In this terminology the hadronic part X_α of the matrix element for the semileptonic baryon decay process has the form

$$X_\alpha = T(\theta, \Delta S)\, \bar{u}_B (f_m^V \gamma_\alpha + g_m^A \gamma_\alpha \gamma_5) u_A.$$

Here, the vector f_m^V and axial-vector g_m^A form factors are given by the expressions

$$\begin{aligned} f_m^V &= (i f_{ABm} - f_{ABm+1}) V^\Phi + (d_{ABm} + i d_{ABm+1}) V^\Delta; \\ g_m^A &= (i f_{ABm} - f_{ABm+1}) A^\Phi + (d_{ABm} + i d_{ABm+1}) A^\Delta \end{aligned}$$

and

$$T(\theta, \Delta S) = \begin{cases} \cos\theta & \text{for transitions with } \Delta S = 0, \\ \sin\theta & \text{for transitions with } |\Delta S| = 1, \end{cases}$$

and the subscript m is equal to unity in the case $\Delta S = 0$ and 4 in the case $|\Delta S| = 1$.

Allowance for the induced interactions naturally complicates the reduced matrix elements. Instead of (2.69), we shall now have expressions for the reduced matrix elements obtained by the following substitutions:

$$V^{\Phi(\Delta)} \gamma_\alpha \to f_1^{\Phi(\Delta)}(q^2) \gamma_\alpha + \frac{f_2^{\Phi(\Delta)}(q^2)}{m_A + m_B} \sigma_{\alpha\beta} q_\beta + \frac{f_2^{\Phi(\Delta)}(q^2)}{m_A + m_B} q_\alpha; \tag{2.70}$$

$$A^{\Phi(\Delta)}\gamma_\alpha\gamma_5 \rightarrow g_1^{\Phi(\Delta)}(q^2)\gamma_\alpha\gamma_5 + \frac{g_2^{\Phi(\Delta)}(q^2)}{m_A+m_B}\sigma_{\alpha\beta}q_\beta\gamma_5 + \frac{g_3^{\Phi(\Delta)}(q^2)}{m_A+m_B}q_\alpha\gamma_5. \quad (2.71)$$

In these expressions, as in (2.69), $f_i^{\Phi(\Delta)}$ and $g_i^{\Phi(\Delta)}$ denote the contributions from the forces of f and d type to the corresponding form factors. Arguing as in the derivation of (2.7), we obtain an expression for the q^2 dependence of the form factors $f_i^{\Phi(\Delta)}(q^2)$:

$$f_i^{\Phi(\Delta)}(q^2) = f_i^{\Phi(\Delta)}(0)\left[1+\lambda_i^{\Phi(\Delta)}q^2/m_\pi^2\right]$$

and similar expressions for the form factors $g_i^{\Phi(\Delta)}(q^2)$.

2.5. Selection Rules in Leptonic Baryon Decays

In accordance with this formalism for the description of the weak interactions the following selection rules must be satisfied.

For the strangeness $|\Delta S| \leq 1$ and if $|\Delta S| = 1$ then $\Delta Q = \Delta S$. For the isospin $|\Delta I| = 1$ for transitions with $\Delta S = 0$ and $|\Delta I| = 1/2$ for transitions with $|\Delta S| = 1$.

The selection rule $|\Delta S| \leq 1$ can be verified by attempting to discover direct decays of Ξ^0 hyperons into nucleons:

$$\Xi \rightarrow Ne\nu_e;$$
$$\Xi \rightarrow N\pi.$$

As yet the upper limit for the branching ratios of these decays is small, the combined results giving [64]

$$\frac{\Gamma_\Xi(\Delta S=2)}{\Gamma_\Xi(\text{all})} \lesssim 10^{-3}.$$

The selection rule $\Delta Q = \Delta S$ can be verified by searching for the forbidden β and μ decays of the Σ^+ hyperon. The complete data indicate that

$$\frac{\Gamma(\Sigma^+ \rightarrow ne^+\nu_e)}{\Gamma(\Sigma^+ \rightarrow \text{all})} \lesssim 10^{-4}$$

and

$$\frac{\Gamma(\Sigma^+ \rightarrow n\mu^+\nu_\mu)}{\Gamma(\Sigma^+ \rightarrow \text{all})} \lesssim 5 \cdot 10^{-2}.$$

These estimates take into account the following data. Altogether in experiments with bubble chambers and photoemulsions a total of about $2.3 \cdot 10^6$ Σ^+ hyperons have been detected. Three events have been regarded as possible candidates for the decay $\Sigma^+ \rightarrow ne^+\nu_e$ and one event as a candidate for the decay $\Sigma^+ \rightarrow ne^+\nu_e$. For Σ^- hyperons there have been detected 174 of the decays $\Sigma^- \rightarrow n\mu^-\tilde{\nu}_\mu$ and 881 of the decays $\Sigma^- \rightarrow ne^-\tilde{\nu}_e$.

Finally, these decays should not exhibit effects due to neutral leptonic currents. To verify this assertion searches have been made for decays of the form

$$\Sigma^+ \rightarrow pe^+e^-.$$

Three events corresponding to these decays have been found with small effective masses of the e^+e^- system. It follows that

$$\frac{\Gamma(\Sigma^+ \rightarrow pe^+e^-)}{\Gamma(\Sigma^+ \rightarrow \text{all})} \approx 10^{-5}.$$

However, it is known that there is a radiative Σ^+ decay with branching ratio

$$\frac{\Gamma(\Sigma^+ \to p\gamma)}{\Gamma(\Sigma^+ \to \text{all})} = (1.6 \pm 0.3)\cdot 10^{-3}.$$

Then with allowance for the internal conversion coefficient, which is approximately equal to 1/130, and eliminating the contribution from this effect, we have

$$\frac{\Gamma(\Sigma^+ \to pe^+e^-)}{\Gamma(\Sigma^+ \to \text{all})} \lesssim 10^{-5}.$$

We can thus assume with reasonable accuracy that the above selection rules hold and are satisfied.

2.6. Conserved Vector Current Hypothesis and Semileptonic Baryon Decays

The expressions (2.70) and (2.71) show that in the most general case a semileptonic baryon decay is now described by 24 real constants in the form factors $f_i^{\Phi}(q^2)$, $f_i^{\Delta}(q^2)$, $g_i^{\Phi}(q^2)$ and $g_i^{\Delta}(q^2)$ and also the 12 λ parameters in their q^2 dependence and the two values of the Cabibbo angle θ_V and θ_A for the vector the axial-vector parts of the weak interaction. The universal constant of the weak interaction is taken from the muon decay data. If the semileptonic hadron processes are T invariant, there remain 12 form-factor constants. This is, of course, very many for a comparison with experimental data. It is therefore necessary to invoke additional hypotheses to obtain equations between these constants and thus reduce the number of constants to be determined experimentally.

In the understanding of effects due to the weak interaction great importance also attaches to the consequences of the hypothesis of a conserved vector current (CVC). This hypothesis was first formulated by Zel'dovich and Gershtein [53] and also Marshak and Sudarshan [54] and was then reformulated by Feynman and Gell-Mann [55]. Its original content reflects attempts to explain the rather good agreement between the constant of the vector coupling g_β^V measured in nuclear Fermi transitions with the Fermi coupling constant G obtained from the muon lifetime. Since it is known that

$$\delta G^2 = \frac{G^2 - (g_\beta^V)^2}{G^2} \approx 0.05,$$

this coincidence can be explained by assuming that: 1) the constant of the vector coupling g^V in the weak hadronic interactions is not renormalized by the strong interactions; 2) the value of this unrenormalized constant g^V is equal to the value of the muon decay constant G^μ.

The constant of the axial-vector coupling g^A is renormalized by strong interactions and for the neutron β decay the ratio of the two constants is $g^A/g^V = -1.23 \pm 0.01$.

The universality of the weak interactions follows from the second assertion. The first assertion yields an important consequence for the properties of the vector part of the hadronic current $J_\alpha^V(x)$ as an operator acting in the world of strong interactions. Here, one can employ an analogy with electrodynamics, in which, under the assumption that the unrenormalized electric charges of the electron and proton are equal, the equality of the renormalized charges is a consequence of the conservation of the electric current. By analogy with electrodynamics it also follows that g^V is not renormalized because of the conservation of the vector part of the weak hadronic current, $\partial_\alpha V_\alpha(x) = 0$. In the most general form we have the hypothesis of the isovector nature of the hadronic current, according to which the three operators V_α, $\left(\frac{g_V^\beta}{e}\right)(J_\alpha^{\text{e.m.}})_{I=1}$, and V_α^* form three components of a single isotopic vector. These ideas enable one to relate a number of electromagnetic effects and weak-interaction effects. Important conclusions are drawn about the form factors for $q^2 = 0$ and also about their q^2 dependence. We shall mention only the well-known consequences for the neutron β decay, according to which the form factor of the effective scalar $f_3(0)$ must vanish and $f_1(0) = 1$, $f_2(0) = \mu_p - \mu_n \approx 3.7$.

In addition

$$f_1(q^2) = f_1(0)\, G_Q^V(q^2);$$

$$f_2(q^2) = f_2(0)\, G_M^V(q^2),$$

where $G_Q^V(q^2)$ and $G_M^V(q^2)$ are the isovector parts, respectively, of the charge and magnetic form factors of the nucleons with the normalization $G_Q^V(0) = G_M^V(0) = 1$. We obtain an estimate of the q^2 dependence from the known data on this dependence for the Sachs nucleon form factors found from experiments on elastic ep scattering. It is known [55] that

$$\frac{G_{MP}}{1+\mu_p} = \frac{G_{Mn}}{\mu_n} = G_{EP} = \left[\frac{1}{1+1.25\,q^2/m_p^2}\right]^2,$$

$$\left(\frac{dG_{En}}{dq^2}\right)_{q^2=0} = \frac{(0.563 \pm 0.001)}{m_p^2}.$$

Hence, we find that the form factors of the vector and weak magnetism satisfy

$$\lambda_1 \approx 2; \quad \lambda_2 \approx 3. \tag{2.72}$$

The CVC hypothesis yields important information about other decays of baryons with $\Delta S = 0$. Consider the decays

$$\Sigma^{\pm} \to \Lambda e^{\pm} \nu_e. \tag{2.73}$$

For the matrix elements of the isovector part of the hadronic current we have the expressions

$$\left.\begin{aligned} \langle \Lambda | J_\alpha^+ | \Sigma^+ \rangle &= -\bar{u}_\Lambda \sum_k g_k O_\alpha^{(k)} u_{\Sigma^+}; \\ \langle \Lambda | J_\alpha^+ | \Sigma^- \rangle &= \bar{u}_\Lambda \sum_k g_k O_\alpha^{(k)} u_{\Sigma^-}. \end{aligned}\right\} \tag{2.74}$$

These decays are due to two different isotopic components of one and the same weak baryon current and one can therefore immediately conclude that their matrix elements are the same and that the branching ratio of these decays is determined solely by the difference of the phase volumes due to difference of the masses of the Σ^- and Σ^+ hyperons. Consequently,

$$\Gamma_+/\Gamma_- = 0.61. \tag{2.75}$$

The experimentally found relative probabilities for the decays (2.73) R_+ and R_- and the hyperon lifetime τ_+ and τ_- yield the following value for this ratio:

$$\frac{\Gamma_+}{\Gamma_-} = \frac{R_+ \tau_-}{R_- \tau_+} = 0.62 \pm 0.15, \tag{2.76}$$

which does not contradict the calculated value (2.75) and confirms this general assertion of the hypothesis of a conserved vector current.

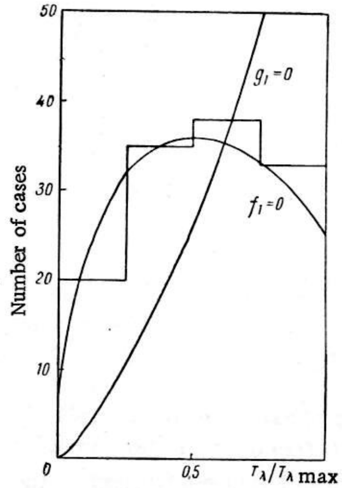
The form factors g_k in (2.74) are again related to the form factors of the electromagnetic transition $\Sigma^0 \to \Lambda^0\gamma$:

$$\langle \Lambda | J_\alpha^{e.m} | \Sigma^0 \rangle = \bar{u}_\Lambda \sum_{k=V,T,S} F_k O_\alpha^{(k)} u_{\Sigma^0}. \tag{2.77}$$

This relationship has the form

$$\left.\begin{aligned} g_V(q^2) &= \sqrt{2} F_V(q^2) = q^2 \sqrt{2} F'_V \to 0; \\ g_S(q^2) &= \sqrt{2} F_S(q^2) = -\sqrt{2}\,(m_\Sigma^2 - m_\Lambda^2)\, F'_V; \\ g_T(q^2) &= \sqrt{2} F_T(q^2). \end{aligned}\right\} \tag{2.78}$$

Here, F'_V is the analog of R_e^2/G in the nucleon electromagnetic form factors when R_e is regarded as the electromagnetic radius of the particle. The tensor form factor $F_T(0)$ determines the magnetic moment of the radiative transition $\Sigma^0 \to \Lambda^0\gamma$- and can be found from the probability of this transition:



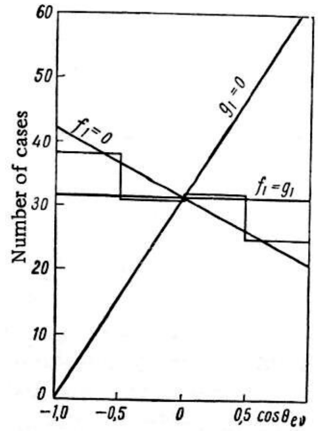


Fig. 8 Fig. 9

Fig. 8. Energy spectrum of the Λ hyperons in the decay $\Sigma \rightarrow \Lambda e\nu$. The predictions for the case of purely vector ($q_1 = 0$) and purely axial-vector ($f_1 = 0$) interactions are indicated.

Fig. 9. The eν correlation in the decay $\Sigma \rightarrow \Lambda e\nu$. The predictions for the purely vector ($q_1 = 0$) and purely axial-vector ($f_1 = 0$) variants and their equal-probability mixture ($q_1 = f_1$) are also shown.

$$\Gamma(\Sigma^0 \rightarrow \Lambda^0 \cdot \gamma) = \frac{e^2}{4\pi} \cdot \frac{4E_\gamma^3}{(m_\Sigma + m_\Lambda)^2} F_T^2(0). \tag{2.79}$$

It follows from the relations (2.78) that, within the framework of the CVC hypothesis, the following limiting relation is satisfied in the decays $\Sigma^\pm \rightarrow \Lambda e^\pm \nu_e$:

$$g_V(0) = f_1(0) = 0. \tag{2.80}$$

Thus, the form factor of the vector interaction here vanishes and the properties of this decay are primarily determined by the axial-vector interaction. This consequence is confirmed by the experimentally measured [57] energy spectra of the Λ hyperons (Figs. 8 and 9) and the eν correlation in this decay. From the combined data obtained from the experiments one can conclude that in the decays (2.73) $|f_1/g_1| = 0.26 \pm 0.20$.

Above [see (2.44)-(2.49)] we have discussed the possible contribution to the Σ^+ decay from the form factor g_2. It follows from theoretical considerations that this form factor vanishes. It is therefore necessary to consider what one can expect here from the weak magnetism effect [58]. In fact, the value of the form factor $f_2(0)$ can, in accordance with (2.79), be obtained from measurements of the lifetime of the Σ^0 hyperon. However, such results are not yet available and one can therefore use the theoretical estimates of τ_{Σ^0}, which give values of $\sim 7 \cdot 10^{-20}$ sec. Then, using (2.79), we obtain $f_2|_{\Sigma \rightarrow \Lambda} = 2.98 \cos\theta_V$.

From a combined analysis of the data on the semileptonic baryon decays it follows that $g_1|_{\Sigma \rightarrow \Lambda} \approx 0.618 \cos\theta_A$. Setting $\theta_A = \theta_V$, we find $|f_2/g_1|_{\Sigma \rightarrow \Lambda} \approx 4.83$. This means there is a contribution of ~0.5% to the total probability of the decay $\Sigma \rightarrow \Lambda e\nu$ from the weak magnetism effect. Another way to estimate the value of this form factor is to attempt to detect a longitudinal polarization of the Λ^0 hyperons. If $f_2(0) \neq 0$, then, taking into account the recoil terms, we find that (2.24) is replaced by the following expression for the angular distribution of the protons from the decay $\Lambda^0 \rightarrow p e^- \tilde{\nu}_e$:

$$W(\mathbf{p} \cdot \boldsymbol{\alpha}) = 1 + \alpha_\Lambda \frac{2.68 \, \mathrm{Re}\, z + 0.08 \, \mathrm{Re}\, z'}{|z|^2 + 3} \mathbf{p} \cdot \boldsymbol{\alpha},$$

where $z' = f_2/g_1$ and $\alpha_\Lambda = 0.62$.

Since we assume z = 0, this distribution will be slightly anisotropic because of the term 0.08 Re z'. It can be seen that the asymmetry coefficient in this distribution is very small: for Re z' ≈ 1 we have

$$\alpha_\Lambda \frac{0.08 \operatorname{Re} z'}{3} = 6.2 \cdot 10^{-3}.$$

The distribution of the longitudinal polarization of the Λ hyperon in the decay $\Sigma \to \Lambda e\nu$ is shown in Fig. 10.

2.7. Parameters of the Cabibbo Theory from the Experimental Data on Semileptonic Baryon Decays

First a few words on the T invariance of β-decay processes. If this invariance holds, the form factor constants must be real. Restricting ourselves now to the form factors of the vector and axial-vector interactions, we can assert that in this case too the constants c_V and c_A must be real, i.e., if one writes $\alpha = c_A/c_V = |\alpha| e^{i\varphi}$ in the V−A variant of the theory, the phase must be $\varphi = 180°$. One can verify this consequence experimentally in several ways. For example, there are T-odd correlations of the form $\overline{\sigma}_R \cdot (\overline{p}_e \times \overline{p}_\nu)$ with coefficient equal to $\frac{2 \operatorname{Im} \alpha}{1+3|\alpha|^2}$. Measurements of this coefficient in the decay of polarized neutrons yielded the result [59]: $\varphi - \pi = (1.3 \pm 1.3)°$. It also follows from the data [60] of the β decay of Ne^{19} that $\varphi - \pi = (0.2 \pm 1.6)°$.

One can also attempt to detect T-odd effects by making precise measurements of the energy dependence of the electron polarization,in, for example, the β decay of the nucleus RaE (Bi^{210}). This dependence is sensitive to the value of $\operatorname{Re} \alpha/\alpha$. It follows from the measurements [61] that $\varphi - \pi = (2.0 \pm 2.5)°$. Thus if an admixture of a T-odd weak interaction is present in β decay processes, it is small. In what follows we shall, as a rule, assume that the semileptonic baryon decay processes are described by T-invariant interactions.

In the situation that arises with contributions from induced interactions it is natural to see what the $SU_{(3)}$ and $SU_{(6)}$ symmetries and the hypothesis of a conserved isovector current yield. In this case only f type forces make a contribution to the form factor of the vector interaction. It follows that $V^{\Phi} = 1$ and $V^{\Delta} = 0$ in the expressions (2.69). At the same time we find that the weak-magnetism form factor f_m^M is independent of q^2 and satisfies the formula

$$f_m^M = (\mu_A - \mu_B)\left[i f_{ABm}(1-\alpha_m) + d_{ABm}\alpha_m\right] \frac{2m_A}{m_A + m_B},$$

where $\alpha = \frac{3}{2} \cdot \frac{\mu_A}{\mu_A + \mu_B}$ are the magnetic moments of the corresponding baryons that participate in the process $A \to Bl\nu_l$. Their values are known for the Λ^0 and Σ^+ hyperons as well as for the proton and neutron. They do not differ strongly form the values predicted by $SU_{(6)}$ symmetry. We can now therefore use their theoretical expression [62] in terms of μ_p and μ_n. As a result we obtain the form factors f_1, f_2, and g_1, which are given in Table 5 for different semileptonic decays. They have a particularly simple form for the decays (2.73). Using them in the expression (2.32) for the probability, we find that

$$R^{-} = 1.82 \cdot 10^{-4} \cos^2\theta_A \frac{2}{3} (A^{\Delta})^2;$$

$$R^{+} = 0.54 \cdot 10^{-4} \cos^2\theta_A \frac{2}{3} (A^{\Delta})^2.$$

Using the tabulated data for R^- and R^+, we obtain

$$\cos\theta_A A^{\Delta} \approx 0.75.$$

It follows form the neutron β decay that

$$\cos\theta_A (A^{\Phi} + A^{\Delta}) = 1.23,$$

and then

$$\cos\theta_A \cdot A^{\Phi} \approx 0.5,$$

TABLE 5. Form Factors of the Vector f_V, Weak Magnetism f_M, and Axial-Vector f_A Interactions in $SU_{(3)}$ Symmetry*

Reaction	ΔS	f_V	f_A	f_M
$n\to pe\nu$	0	1	$A^\Phi + A^\Delta$	$\frac{1}{2} k_{np} (\mu_p - \mu_n)$
$\Sigma^-\to\Lambda e\nu$	0	0	$-\sqrt{2/3}\, A^\Delta$	—
$\Sigma^+\to\Lambda e\nu$	0	0	$-\sqrt{2/3}\, A^\Delta$	$-\frac{1}{2} k_{\Sigma\Lambda}\mu_n$
$\Sigma^-\to\Sigma^0 e\nu$	0	$\sqrt{2}$	$\sqrt{2}\, A^\Phi$	—
$\Xi^-\to\Xi^0 e\nu$	0	-1	$A^\Phi - A^\Delta$	—
$\Lambda\to pe\nu$	1	$\sqrt{3/2}$	$\sqrt{3/2}\left(A^\Phi + \frac{1}{3} A^\Delta\right)$	$\frac{1}{2}\sqrt{\frac{3}{2}}\, k_{\Lambda p}\mu_p$
$\Sigma^-\to ne\nu$	1	-1	$A^\Phi - A^\Delta$	$\frac{1}{2} k_{\Sigma n}(\mu_p + 2\mu_n)$
$\Xi^-\to\Lambda e\nu$	1	$-\sqrt{3/2}$	$\sqrt{3/2}\left(A^\Phi - \frac{1}{3} A^\Delta\right)$	$\frac{1}{2} k_{\Xi\Lambda}(\mu_p + \mu_n)$
$\Xi^-\to\Sigma^0 e\nu$	1	$\sqrt{1/2}$	$\sqrt{1/2}(A^\Phi + A^\Delta)$	—
$\Xi^0\to\Sigma^+ e\nu$	1	1	$A^\Phi + A^\Delta$	—

* $k_{AB} = \begin{cases} 1 \text{ for exact } SU_{(3)} \text{ symmetry,} \\ \frac{2m_A}{m_A + m_B} \text{ for broken } SU_{(3)} \text{ symmetry,} \end{cases}$

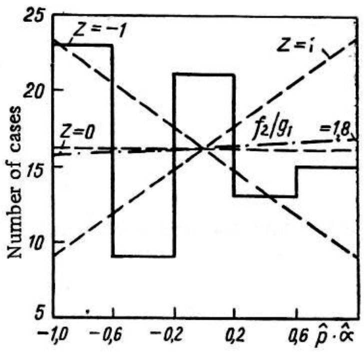


Fig. 10. Distribution of the longitudinal polarization of the Λ hyperons in the decay $\Sigma \to \Lambda e\nu$. The influence of the weak magnetism form factor is shown (the straight line $f_2/q_1 = 1.8$).

and hence

$$\alpha = \frac{A^\Delta}{A^\Phi + A^\Delta} \approx 0.61.$$

This corresponds to the results of the analysis [63] of strong interactions: $\alpha = 0.67 \pm 0.06$ and also the predictions of $SU_{(6)}$ symmetry, for which $\alpha = 2/3$.

A comprehensive statistical analysis [64] of the data on the probabilities of semileptonic baryon decays confirms these estimates. Fitting of data with two Cabibbo angles, the consequences of the CVC hypothesis, and the ideas of current algebra on the relationship between the axial-vector and pseudoscalar form factors yield the results

$$\sin\theta_V = 0.190 \pm 0.035;$$
$$\sin\theta_A = 0.280 \pm 0.030;$$
$$\alpha = 0.66 \pm 0.03.$$

Hence, we obtain the following estimates for the limiting values of the renormalized effects in the axial-vector form factors ($\beta = g_1/f_1$):

$$\beta_{\Sigma\Lambda} \approx \beta_{np} = 1.23;$$
$$(1.38 \pm 0.13) \leqslant \beta_{\Lambda p} \leqslant (1.64 \pm 0.16);$$
$$(1.1 \pm 0.2) \leqslant \beta_{\Sigma^- n} \leqslant (4.4 \pm 0.7);$$
$$(2.3 \pm 1.1) \leqslant \beta_{\Xi^- \Lambda} \leqslant (9.1 \pm 4.2).$$

We have already mentioned on several occasions that the present-day data on the quantities $\Gamma(A\to B l\nu_l)$ are characterized by large experimental errors. Attempts are therefore made to estimate the parameters of the theory on the basis of the complete set of experimental data using the maximum of resonable theoretical arguments. A typical approach of this kind has been developed by the group at Heidelberg University [65]. In this approach the strongly interacting part of the matrix element is used in the form (2.70) and one assumes a q^2 dependence of the form factors:

$$\left.\begin{array}{l} f_i(q^2)=f_i(0)\left[1+\lambda_1 q^2/m_\pi^2\right]; \\ f_4=g_1;\ f_5=g_2;\ f_6=g_3, \end{array}\right\} \quad (2.81)$$

where $f_i(0)$ is represented in accordance with (2.70) as a sum of reduced matrix elements $f_i^{\Phi(\Delta)}(0)$ with corresponding Clebsch-Gordan coefficients of the group $SU_{(3)}$:

$$f_i^{(k)}(0)=\left[C_F^{(k)} f_i^{(F)}(0)+C_D^{(k)} f_i^{(D)}(0)\right] T_i^{(k)}. \quad (2.82)$$

Here, we again have

$$T_i^{(k)}=\begin{cases} \cos\theta_i & \text{for } \Delta S=0, \\ \sin\theta_i & \text{for } |\Delta S|=1, \end{cases} \quad (2.83)$$

and the Cabibbo angle

$$\theta_i=\begin{cases} \theta_V & \text{for } i=1, 2, 3, \\ \theta_A & \text{for } i=4, 5, 6. \end{cases} \quad (2.84)$$

The factor $\eta_i^{(k)}$ takes into account the corrections for the difference between the baryon masses of the $SU_{(3)}$ octet with $J^P=1/2^+$:

$$\eta_i^{(k)}=\begin{cases} 1 & \text{for } i=1,4 \\ \dfrac{2m_A^{(k)}}{m_A^{(k)}+m_B^{(k)}} & \text{for } i=2, 3, 5, 6. \end{cases} \quad (2.85)$$

Exact $SU_{(3)}$ symmetry yields

$$f_3^{\Phi}(0)=f_5^{\Delta}(0)=0, \quad (2.86)$$

and the CVC hypothesis yields the following restrictions on the vector form factor and the weak-magnetism form factor:

$$\begin{array}{c} f_1^{\Phi}(0)=1;\ f_1^{\Delta}(0)=0; \\ f_2^{\Phi}(0)=\dfrac{1}{2}\mu_p+\dfrac{1}{2}\mu_n;\ f_2^{\Delta}(0)=-\dfrac{3}{4}\mu_n; \\ \lambda_1=2{,}0\,\dfrac{m_A^2}{m_p^2};\ \ \lambda_2=2{,}6\,\dfrac{m_A^2}{m_p^2}. \end{array} \quad (2.87)$$

Use of the PCAC hypothesis relates the form factors $f_6^{(k)}$ and $f_4^{(k)}$:

$$\left.\begin{array}{l} f_6(0)=\dfrac{m_A(m_A+m_B)}{m_{\Delta S}^2} f_4(0); \\ \lambda_6=\lambda_4-\dfrac{m_p^2}{m_{\Delta S}^2}, \end{array}\right\} \quad (2.88)$$

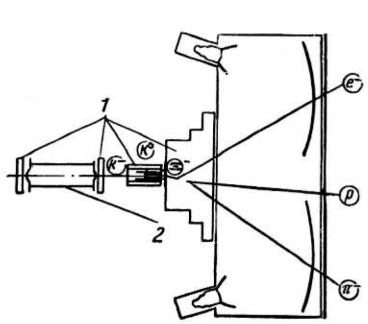


Fig. 11. Arrangement of an experimental electronic apparatus for studying the semileptonic decays of Ξ^- hyperons. The apparatus includes spark chambers (1), a large Cerenkov counter (2), and a large system of scintillation counters.

where

$$m_{\Delta S} = \begin{cases} m_\pi \text{ for transitions with c } \Delta S = 0, \\ m_K \text{ for transitions with c } |\Delta S| = 1; \end{cases} \quad (2.89)$$

it is also assumed that $\lambda_4 = \lambda_1$. Thus, only the following parameters are free:

$$\theta_V;\ \theta_A;\ f_4^\Phi(0);\ f_4^\Delta(0);\ f_5^\Phi(0);\ f_5^\Delta(0). \quad (2.90)$$

If we lift the restrictions (2.86), there are a further two parameters $f_3^\Delta(0)$ and $f_3^\Delta(0)$, and one can assume that $\lambda_5 = \frac{1}{2}\lambda_2$.

The statistical analysis is based on the requirement of a minimum of the function $\chi^2(h_i)$ of the free parameters h_i defined by the formula

$$\chi^2(h_i) = \sum_k \left[\frac{x_{\text{theor}}^{(k)}(h_i) - x_{\text{exp}}^{(k)}}{\Delta x_{\text{exp}}^{(k)}} \right]^2, \quad (2.91)$$

where $x^{(k)}$ is either $\Gamma^{(k)}$ or $f_4^{(k)}/f_1^{(k)}$.

Establishment of these experimental quantities entails a very substantial amount of experimental work. Because of the small branching ratio $\Gamma(A \to Bl\nu_l)/\Gamma\ (A \to \text{all})$ of the semileptonic processes the experimental groups must investigate ensembles of Λ^0 or $\Sigma^\pm$ decays with $\sim 10^5 - 10^6$ events with basically nonleptonic modes of the type $N\pi$ recorded in experiments with bubble chambers. About 1×10^2 events of the requisite type with the creation of leptons are detected. The analysis of the selected events yields information about the branching ratios and also about the spectra and correlations in these processes. A recent innovation is the use of Cerenkov and scintillation counters in conjunction with spark chambers in experiments on the semileptonic decays of the Λ^0 and Ξ^- hyperons. The arrangement of one such experiment is shown in Fig. 11.

More detailed information is now available on the characteristics of the decays $\Lambda^0 \to pe\nu_e$ and $\Sigma^- \to ne\nu_e$. A typical electron spectrum obtained for the decay $\Sigma^- \to ne\nu_e$ is shown in Fig. 12. An analysis of the Dalitz plot for the decay $A \to Bl\nu_l$ in the variables of the electron and proton kinetic energies T_e and T_p or the spectra of these quantities does not at present enable one to estimate the contributions of the induced form factors. It is therefore assumed that the weak-magnetism form factor can be taken from $SU_{(3)}$ symmetry and the CVC hypothesis, as is done in Table 5. From these data we then find [51] that in the decay $\Lambda \to pe\nu_e$

$$|g_1/f_1| = 0.72^{+0.19}_{-0.14}.$$

Measurements of the asymmetry in the emission of the electrons on the decay of polarized Λ^0 hyperons yield, in accordance with (2.30) and (2.31), not only the value but also the sign of the ratio g_1/f_1. The combined data of different experiments yield the following result:

$$g_1/f_1 = -0.63 \pm 0.08.$$

Finally, an analysis of the νl correlations yields [57]

$$|g_1/f_1|_{\Lambda \to pe\nu} = 0.77^{+0.25}_{-0.17}.$$

Thus, in the decay $\Lambda \to pe\nu_e$ these measurements indicate that the ratio g_1/f_1 is negative and has the value

$$\left.\frac{g_1}{f_1}\right|_{\Lambda \to pe\nu} = -0.61 \pm 0.065. \quad (2.92)$$

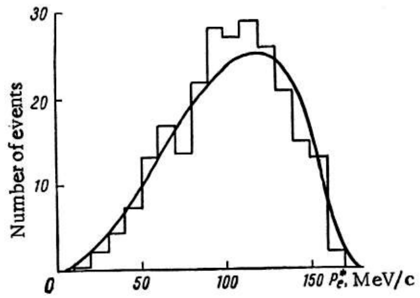


Fig. 12

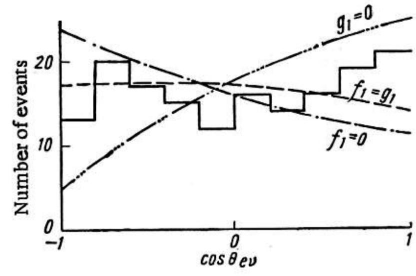


Fig. 13

Fig. 12. Electron energy spectrum in the decay $\Sigma^- \to ne\nu$.

Fig. 13. Electron-neutrino correlations in the decay $\Sigma^- \to ne\nu$. The solid line gives the expected spectrum.

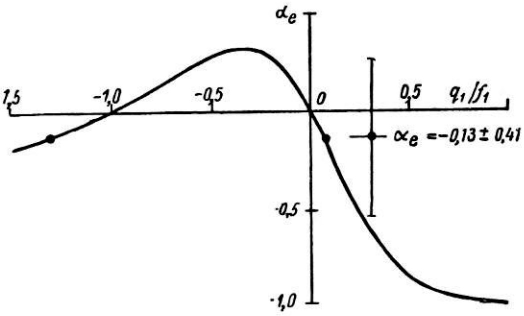


Fig. 14. Asymmetry coefficient for the emission of electrons in the decay of polarized Σ^- hyperons as a function of g_1/f_1. The measured value of α_e is shown.

Now let us consider this ratio in the decay $\Sigma^- \to ne^-\tilde{\nu}_e$. Here the experimental situation is much more complicated since one of the three decay products is charged and the remaining two are neutral. It follows that the direction of emission of the neutron and its energy can be measured only by detecting the recoil proton from np scattering. Approximately 11% of the neutrons from Σ^- decays generate recoil protons in a hydrogen bubble chamber with a track length greater than 2 mm at distances of 20 cm from the decay point. It is therefore very difficult to allow for the background, which account for up to 30% of the orginally chosen candidates for the decay $\Sigma^- \to ne^-\tilde{\nu}_e$.

The combined data at present available on the $e\nu$ correlation are given in Fig. 13. These data indicate that

$$|g_1/f_1|_{\Sigma^-} = 0.36 {+0.18 \atop -0.15}. \qquad (2.93)$$

Results have been published of one attempt to measure the asymmetry parameter in the emission of electrons in the decay of polarized Σ^- hyperons. From a total of 49 events the value for this parameter α_e in (2.30) was found to be $\alpha_e = -0.13 \pm 0.41$. Since α_e is a quadratic function of g_1/f_1, two solutions are obtained (this is well shown in Fig. 14):

$$g_1/f_1|_{\Sigma^-} = -0.05 {+0.23 \atop -0.32}$$

or

$$g_1/f_1|_{\Sigma^-} = -1.3 {+0.9 \atop -1.0}.$$

Thus, for $|g_1/f_1|$ in the decay $\Sigma^- \to ne^-\tilde{\nu}_e$ one should now take the value (2.93), but nothing can be said about the sign of the ratio.

The examples we have given characterize the present state of the study of semileptonic baryon decays. The experimental data at present known on this subject are given in Table 6.

It follows from [65] that at the present level of accuracy of the experimental data the contribution from the form factors f_3 and f_5 cannot be determined. It was therefore assumed that $f_3 = f_5 = 0$ and the set of parameters was determined from Eq. (2.90). For $\theta_V = \theta_A = \theta$ [57] the following results were obtained [64]: $\theta = 0.235 \pm 0.06$; $f_4^{\Phi}(0) = 0.49 \pm 0.02$; $f_4^{\Delta} = 0.74 \pm 0.02$; $\alpha = 0.60 \pm 0.02$. Fitting for $\theta_V \neq \theta_A$ gives the same result for $f_4^{\Phi}(0)$ and $f_4^{\Delta}(0)$ and also $\theta_V = 0.233 \pm 0.012$ and $\theta_A = 0.238 \pm 0.018$.

TABLE 6. Comparison of Experimental Data on Leptonic Baryon Decays and Predictions of the Cabibbo Theory

Decay	Branching ratio $[\Gamma(A\to B+l+\nu_e)/\Gamma(A\to \mathrm{all})]\times 10^4$		Form-factor ratio g_1/f_1	
	Experiment	Cabibbo theory*	Experiment	Cabibbo theory*
$n\to pe\nu$	—	—	$-1,23\pm0,01$	$-1,227$
$\Sigma^-\Lambda e\nu$	$0,604\pm0,06$	0,62	$f_1/g_1=+0,35\pm\pm0,18$	$f_1/g_1=0$
$\Sigma^+\to\Lambda e\nu$	$0,202\pm0,047$	0,19	—	$f_1/g_1=0$
$\Lambda\to pe\nu$	$8,60\pm0,45$	8,74	$-0,77^{+0,13}_{-0,09}$	0,72
$\Lambda\to p\mu\nu$	$1,35\pm0,60$	1,44	—	0,72
$\Sigma^-\to ne\nu$	$10,92\pm0,43$	10,6	$\lvert g_1/f_1\rvert=0,21\pm\pm0,21$	$-0,31$
$\Sigma^-\to n\mu\nu$	$4,5\pm0,5$	5,0	—	$-0,31$
$\Xi^-\to\Lambda e\nu$	$15,0^{+9,0}_{-6,0}$	5,4	—	0,20
$\Xi^-\to\Sigma^0 e\nu$	$6,2^{+2,0}_{-3,0}$	0,09	—	1,23

*The constants of the Cabibbo theory are: $\theta = 0.242 \pm 0.004$; $g_1^{\Phi}(0) = 0.46 \pm 0.02$; $g_1^{\Delta}(0) = 0.77 \pm 0.02$.

The theoretical predictions that follow from this analysis are given in Table 6 and compared with the experimentally measured parameters. The results of a graphical analysis are given in Fig. 15.

It can be seen that there is basically good agreement between the experimental data and the predictions of the Cabibbo theory. However, a great improvement is required in the experimental data on semileptonic baryon decays.

To analyze this question one can also invoke the more far reaching additional theoretical argument based on the ideas of current algebra. An example of such an investigation can be found in [66].

We define in the following manner the constants

$$(G_A)_{pn}=g_{pn}(0)\cos\theta_A;$$
$$(G_V)_{pn}=f_{pn}(0)\cos\theta_V;$$
$$(G_A)_{p\Lambda}=g_{p\Lambda}(0)\sin\theta_A;$$
$$(G_V)_{p\Lambda}=f_{p\Lambda}(0)\sin\theta_V$$

and in an obvious manner all the remaining constants $(G_{A(V)})_{BA}$. Here, the form factors $g_{BA}(0)$ and $f_{BA}(0)$ are taken from the observable constants, for example,

$$\frac{G_\mu}{\sqrt{2}}g_{p\Lambda}(0)\sin\theta_A,$$

$$\frac{G_\mu}{\sqrt{2}}f_{p\Lambda}(0)\sin\theta_V.$$

Using the CVC hypothesis, we obtain $f_{pn}(0)=1$, $f_{p\Lambda}(0)=-\sqrt{\frac{3}{2}}$, etc. (see Table 5). Using current algebra arguments one obtains the following sum rules for the axial-vector constants:

$$g_{\Lambda\Sigma}=\sqrt{\frac{3}{2}}g_{pn}+g_{p\Lambda};\; g_{\Lambda\Xi}=g_{\Lambda\Sigma}-\sqrt{\frac{3}{2}}g_{\Xi^0\Xi^-};$$
$$g_{\Xi^0\Xi^-}=\sqrt{\frac{3}{2}}g_{\Lambda\Sigma}+\sqrt{\frac{1}{2}}g_{\Sigma^+\Sigma^0};\; g_{n\Sigma^-}=\sqrt{6}g_{\Lambda\Sigma}-g_{pn};$$
$$g_{\Sigma^+\Sigma^0}=\sqrt{6}g_{\Lambda\Sigma}-g_{\Xi^0\Xi^-};\; g_{\Sigma^\pm\Sigma^0}=\pm(\sqrt{3}g_{\Lambda\Sigma}-\sqrt{2}g_{pn});$$

where $g_{\Lambda\Sigma}=g_{\Sigma^+\Lambda}=g_{\Sigma^-\Lambda}$.

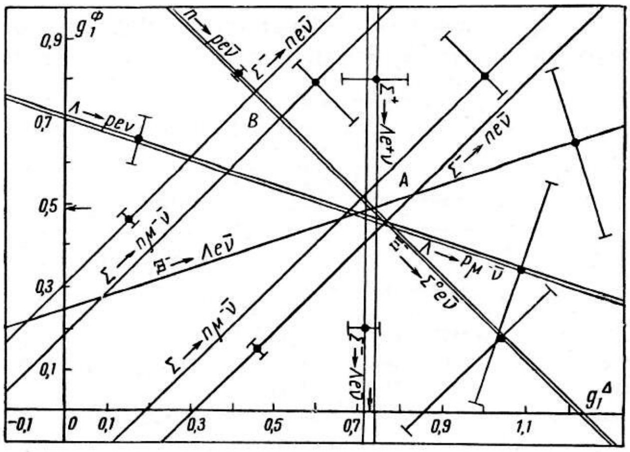


Fig. 15. Graphical representation of the data on the probabilities of semileptonic baryon decays $A \to B + l + \nu_l$ in the plane of the parameters $q_1^{\Phi}(0)$ and $q_1^{\Delta}(0)$ for the Cabibbo angle $\theta = 0.235$ (the arrows indicate their most probable values, which were used to obtain the predictions given in Table 6).

We shall now write down an expression for the partial probability of the process $A \to Bl\nu_l$ as follows:

$$\Gamma_{BA} = \frac{G_\mu^2 f\Delta}{60\pi^3} 3\,(G_A)^2_{BA}\left[\left(1 - \frac{3}{2}\beta + \frac{4}{7}\beta^2\right)(1+a) + \frac{1}{3}x^2_{BA}\left(1 - \frac{3}{2}\beta + \frac{6}{7}\beta^2\right)(1+b)\right],$$

where

$$\beta = \frac{m_A - m_B}{m_A};\ \Delta = m_A - m_B;\ x_{BA} = \frac{(G_V)_{AB}}{(G_A)_{BA}}$$

and

$$f = \begin{cases} 0.47 & \text{for the decay } n \to pe\nu, \\ 1 & \text{for decays with } |\Delta S| = 1. \end{cases}$$

The factors a and b take into the account the finite mass of the charged lepton in m_l. In what follows we shall assume for the case of electronic decays that a = b = 0. The sum rule

$$g_{p\Lambda} = g_{\Sigma^+\Lambda} - \sqrt{\frac{3}{2}}\, g_{pn}$$

yields

$$\mathrm{tg}\,\theta = \frac{(G_A)_{p\Lambda}}{(G_A)_{\Lambda\Sigma^+} - \sqrt{\frac{3}{2}}\,(G_A)_{pn}}.$$

On the other hand,

$$\mathrm{tg}\,\theta_V = \frac{x_{p\Lambda}\,(G_A)_{p\Lambda}}{f_{p\Lambda}},$$

where

$$f_{p\Lambda} = -\sqrt{\frac{3}{2}}.$$

Thus, knowledge of the values of Γ_{fi} and x_{fi} enables one to determine the angles θ_V and θ_A. For example, using the experimental data on the branching ratios of the decays $\Lambda \to pe\nu_e$ and $\Sigma^- \to \Lambda e\nu_e$, and also the value of g_1/f_1 for the decay $\Lambda \to pe\nu_e$, we obtain $\theta_V = 0.225 \pm 0.02$.

From the neutron lifetime (2.51) we now have $|x_{pn}|^{-1} = 1.23 \pm 0.02$, which also corresponds to the data from the asymmetry measurement, from which it follows that $|x_{pn}|^{-1} = 1.25 \pm 0.04$. We then obtain

$$\sin\theta_A = 0.225 \begin{matrix} +0.055 \\ -0.05. \end{matrix}$$

Thus, in this analysis the values of the angles θ_V and θ_A are equal and for the corresponding constants we obtain the values $g_{pn} = 1.27$, $g_{p\Lambda} = 0.945$, and $g_{\Lambda\Sigma} = 0.618$. In addition, we find $\alpha = (A^{\Delta})/A^{\Delta} + A^{\Phi} = 0.58$. There are also predictions for the other axial-vector constants:

$$g_{\Sigma^+\Xi^0} = 1.27; \quad g_{n\Sigma^-} = 0.244; \quad g_{\Lambda\Xi^-} = 0.319; \quad g_{\Sigma^\pm\Sigma^0} = \mp 0.726;$$

$$g_{\Xi^0\Xi^-} = 0.244.$$

Allowance for the electromagnetic corrections slightly changes the values of the above constants. For example, if one takes into account only $\pi^0\eta^0$ and $\Sigma^0\Lambda^0$ mixing, the above constants are replaced by modified sum rules from which it then follows that

$$\sin\theta_V = 0.229 \pm 0.02,$$

$$\sin\theta_A = 0.230 \begin{matrix} +0.055 \\ -0.05, \end{matrix}$$

$$|x_{n\Sigma^-}|^{-1} = \begin{cases} \text{either} & -0.38 \pm 0.2, \\ \text{or} & -0.28 \pm 0.2. \end{cases}$$

Similar but less reliable results can be obtained by using other sum rules and corresponding experimental data whose accuracy is less than that of those used above. In this approach the values of the Cabibbo angles θ_V and θ_A are also found to be almost equal.

Since a comparison of the data on mesonic decays with $|\Delta S| = 1$ and $\Delta S = 0$ indicate that the values of θ_A and θ_V may differ, this question must continue to be investigated. Here we should only like to draw attention to the fact that a comparison of mesonic decays yields information about the quantity

$$\mathrm{tg}^2\,\theta_i \frac{f_k^{(i)}(q^2)}{f_\pi^{(i)}(q^2)}$$

and not $\tan^2\theta_i$ and that the value of the Cabibbo angle can be affected both by possible q^2 dependences of the different form factors and, in general, a departure from unity of ratios of the type $f_k^i(q^2)/f_\pi^i(q^2)$. As yet this question remains open.

As regards the Cabibbo angle θ itself, it was introduced into the theory as a parameter needed to save the hypothesis that the weak hadronic current has unit length. Its numerical value, found experimentally, is very close to the ratio of the π- and K-meson masses:

$$\mathrm{tg}\,\theta \approx \frac{m_\pi}{m_K} = 0.28.$$

It is therefore very attractive to try and explain this fact by the idea that the splitting of the particle masses in the unitary multiplets and the orientation of the weak hadronic current in the unitary spin space have a common origin, i.e., that they are due to the common $SU_{(3)}$-symmetry breaking interaction.

Thus, the further development of the universal four-fermion theory of weak interactions based on the inclusion in the theory of the unitary symmetry of the strong interactions has made it possible to achieve further progress in the problem of the weak interactions of elementary particles. No other theories leading to equivalent consequence have as yet been proposed. The predictions obtained on the basis of $SU_{(3)}$-symmetry agree well with the experimental facts. In addition, the numerical values of a number of general parameters of the $SU_{(3)}$-symmetry and its developments, which determine the properties of the strong, electromagnetic, and weak processes, are found to be similar when they are extracted from data on the various processes. This confirms the generality of the effects in the world of elementary particles. More precise experimental data and new developments of the theoretical ideas will enable us to judge the correctness of these tendencies in our construction of a theory of the microscopic world.

2.8. Achievements of the Investigation of Leptonic Hadron Decays

At the present level of our knowledge of the properties of semileptonic hadron decays we have a characteristic accuracy of ~10% for the values of the most important parameters. The main conclusions obtained after almost a decade of investigations of these processes can be briefly summarized as follows.

1. The general properties of semileptonic hadron decays agree well with predictions of the universal four-fermion theory of the weak interaction in the variant proposed by Cabibbo. Possible admixtures from other couplings have hitherto remained undetected.

2. Effects indicating a violation of T invariance of the weak interactions have not been found; agreement has been obtained with the predictions for the semileptonic decays of K^0 mesons induced by violation of CP invariance in the two-pion decays of K_2^0 mesons.

3. The principle of muon-electron universality is satisfied with an accuracy not worse than 15%.

4. There are no appreciable departures from the consequences of the isotopic and also $SU_{(3)}$-symmetry properties of the theory; the selection rule $|\Delta I| = 1/2$ is satisfied with an accuracy of ~5% and the selection rule $\Delta Q = \Delta S$ with an accuracy of ~10%.

5. In the range of momentum transfers from 0 to 0.3 GeV^2 the interaction is local; this excludes the possibility of a mass of the hypothetical intermediate W meson less than 1 GeV.

6. The effects associated with the strong interactions do not depend very strongly on the energy.

7. The values of the Cabibbo angle obtained from an analysis of meson and baryon decays are almost equal. The existing indications that the angles θ_V and θ_A may be different must be made more precise both experimentally and theoretically.

8. The relative influence of f and d type forces in the axial-vector form factor are found to be the same as in the strong interactions of the octet of pseudoscalar mesons and baryons.

Of the most important problems that await solution we would mention the following.

1. Verification of the consequences of CPT and CP invariance for processes due to weak interactions. In this connection, the most promising experiments are those in which the properties of the K^+ and K^- mesons are compared.

2. Improvement in the accuracy of data that cast light on $\mu - e$ universality in the weak interaction.

3. Advances in the question of neutral currents.

4. Searches for contributions from couplings distinct from the vector and axial-vector couplings. This would test the consequences of the CVC hypothesis and some deductions of other models at present popular.

5. Improvement in the accuracy of the determination of the parameters of the Cabibbo theory, the angles θ_V and θ_A, and also the mixing parameter of the f and d type forces and the discovery of ways to explain the nature of this phenomenon.

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