COLLECTIVE ACCELERATION OF IONS

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The collective method in the acceleration of ions is described. Questions relating to the formation of an electron ring charged with ions are considered, together with problems of stability, focusing, and acceleration.

1. Introduction

The rapid development of high-energy physics has already led to a number of discoveries of fundamental significance. Nevertheless, in order to establish the ultimate laws governing the world of elementary particles and the structure of matter, the creation of accelerators producing particles with energies in the range of hundreds and thousands of GeV is absolutely vital.

Work is even now taking place in a number of countries on the manufacture of accelerators with energies of hundreds of gigaelectron volts; preliminary design exercises indicate that enormous amounts of equipment will be required for such accelerators. The weight of the electromagnets and the size and cost of the equipment involved already exceed all reasonable limits, being comparable with the total resources of whole nations. The reasons underlying this rapid rise in the size and cost of accelerators with increasing particle energy are the following.

In linear accelerators the effective field strength acting on the particles is comparatively low, so that a machine with an energy of the order of tens of gigaelectron volts or over must be made extremely long. In cyclical accelerators, which are at present capable of producing the highest energies of all such installations (strong-focusing synchrophasotrons), higher and higher energies can be achieved only by increasing the particle trajectory radius, since the greatest magnetic field now used for retaining particles in orbit is no greater than 12-15 kOe. The size of the electromagnet, its weight, power supply, and the net cost of the whole installation increase accordingly.

It is therefore quite obvious that the creation of accelerators of extremely high energies (order of 1000 GeV) will require the development of fundamentally new methods of acceleration, so that the effective fields acting on the particles (accelerating or retaining) may be far greater than the effective fields employed in present-day automatic-phasing accelerators.

In 1956 V. I. Veksler [1] indicated the possibility of achieving new acceleration mechanisms using collective interactions. The basic idea of these methods is that the field accelerating a particle is created not only by external sources but also by the interaction of the group of particles being accelerated with another group of charges, a stream of electrons, a flow of plasma, or electromagnetic radiation.

Essentially coherent methods of acceleration (all Veksler's new ideas were originally lumped together under this heading) are characterized by the fact that under certain conditions the field acting on an individual particle is proportional to the number of particles actually being aceelerated.

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In collective methods [2] the accelerating field is created by another group (assembly, ring, etc.) of charges. The field strength is proportional to the number of charges in the group; the number of accelerated particles may be quite arbitrary. It is not difficult to show that a substantial gain in accelerating field compared with the ordinary linear accelerator can be achieved only if there is a large number of particles (order of 10^{13}–10^{14}) in the accelerating group.

At the present time it is technically feasible to produce electron groups or assemblies containing 10^{14} particles. Such groups may be used for accelerating ions by the collective method. The essence of the method is that a small number of ions captured by the electron group is accelerated, under certain conditions, by its intrinsic field; the electron group itself may in turn be accelerated by external fields of moderate intensity:

$$\mathscr{E} = \mathscr{E}_{\text{K}} \cdot \frac{m_{\perp}}{M},$$

where $m_{\perp}$ is the effective mass of the electron,* M is the mass of an ion, $\mathscr{E}_{\text{K}}$ is the field acting on the ion by virtue of the electrons. In view of the great difference between the masses of the ions and electrons, the final energy of the ions is considerably greater than the energy of the electrons (by a factor of $M/m_{\perp}$ times). The present review is primarily concerned with the physical bases of the collective method of acceleration.

II. Formation of an Electron-Ion Group

1. Principal Requirements Imposed upon a Group and Method of Creating the Latter. One of the main problems arising in the creation of a collective accelerator is the formation of a charged electron-ion group. In this group the number of electrons has to be much greater than the number of ions, and the electron density has to be as high as possible. In any event it is essential that the fields acting on the ions in the group be of the order of 10^6–10^7 V/cm.

The collective method may then constitute an extremely promising technique for producing particles of extremely high energies (hundreds and thousands of gigaelectron volts) and also for making compact accelerators for multiply charged ions with a fair particle density and relatively high energies.

Simple analysis shows that an annular grouping with rotating electrons (an "electron ring") is the most suitable for these purposes. In such a ring the forces of Coulomb repulsion between the electrons are weakened by a factor of $\gamma_{\perp}^2$ as a result of magnetic attraction ($\gamma_{\perp}$ is the energy of the transverse motion of the electrons, referred to a unit of mc^2); fairly compact groupings with large numbers of electrons may accordingly be achieved. The more compact the electron ring, the greater will its Coulomb field be, and hence the stronger the forces acting on the ions. When it is the formation of a quiescent grouping which is in question, then these forces will retain the ions in the grouping. If, however, the duly formed ring is moving forward in the course of acceleration (Sec. III), then the Coulomb forces will determine the acceleration of the ions. The ring will in this case be polarized: the center of the ion formation will lie slightly behind the center of the electron formation, and the mean ion-accelerating force will coincide with the Coulomb force acting on the "central" ion by virtue of the electron ring.

At the present time a ring with the required parameters is in fact being created in a time-increasing magnetic field [3]. The electrons are injected from a heavy-current linear induction accelerator, over a period of one or several turns, into the large-radius orbit of a device called an "adhesor." A soft-focusing, barrel-shaped magnetic field is created in this installation by a series of iron-free coils. In the region of the injection radius $n = -(\partial B_z/\partial r \cdot r/B_z) \approx 0.5$; in the center $n = 0$ (B_z is the z component of the magnetic field).

The electron ring so formed is compressed in an adiabatically increasing, azimuthally uniform magnetic field. This compression is accompanied by the azimuthal acceleration of the electrons and an adiabatic reduction in the cross section of the ring. As a result of this, the intensity of the intrinsic electric field rises to the desired value. At the final stage of compression, ions are injected into the ring. The configuration of the coils creating the field enables the ring to be drawn out along the axis.

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*The electron may execute a finite transverse motion; then $m_{\perp} = m\gamma_{\perp}$ is the mass of the electron with due allowance for this motion.

2. Equations of Motion of the Particles and Adiabatic Change in the Beam Parameters. The adhesor constitutes a betatron in which the ordinary condition, specifying the constancy of the radius of the equilibrium orbit (the 2 :1 condition), is not satisfied, so that during the acceleration the electron trajectory takes the form of a deflected spiral. In contrast to the ordinary betatron, in which the accelerated currents are small and the intrinsic fields of the beam may be neglected, in the adhesor the intrinsic fields of the electron ring are comparable with the external fields.

It is convenient to conduct our analysis of particle motion using the approximation of the self-consistent field. Earlier [4, 5] quasistationary adiabatic beam models corresponding to linear intrinsic fields were considered. Models corresponding to two possible cases were chosen:

a) a beam without any energy spread (in the linear approximation), but with radial and axial oscillations ("symmetrical beam");

b) a beam having an energy spread and axial oscillations, but no radial betatron oscillations. Both models correspond to a beam of toroidal shape with a sharp boundary. The cross section of the beam is elliptical.

Apart from the external forces, the equilibrium electron moving along the axis of the ring is acted upon by a constant force due to the mutual influence of the intrinsic fields of different parts of the ring (the electrostatic repulsion of the ring along the major radius). This force introduces a correction into the formula relating the energy of the equilibrium particle to the magnetic field B_s in its orbit (radius R):

$$B_s = -\frac{mc^2 \beta_\theta \gamma_\perp}{eR}(1+\mu P). \tag{II.1}$$

Here e is the charge on the electron, β_θ is the transverse velocity of the electrons relative to the velocity of light c, $\gamma_\perp = 1/\sqrt{1-\beta_\theta^2}$; $\mu = \nu/\gamma_\perp$, where $\nu = r^*(N_e/2\pi R) = (r^*/\beta_\theta c)\cdot(I/e)$; N_e is the number of electrons in the ring, $r^* = 2.8\times10^{-13}$ cm is the classical radius of an electron, I is the beam current, P = 2 ln (b + g)]; b and g are the semiaxes of the elliptical cross section of the torus, referred to the radius of the orbit R (b along the z axis, g along the r axis).

For small beam currents ($\mu P \ll 1$) Eq. (II.1) transforms into the ordinary formula for the equilibrium value of the field in a betatron.

The momentum integral gives the following relation between $B_s(R, t)$ and $R(t)$:

$$B_s(R, t) R^2 [1-\delta(R)] = \text{const}, \tag{II.2}$$

where $\delta(R) = \frac{1}{B_s R^2}\int_0^R n(\xi) B_z(\xi) d\xi$ characterizes the nonuniformity of the magnetic field in the adhesor. In the case of a uniform field $\delta(R) \equiv 0$.

The change in the energy of the equilibrium relativistic electron is determined by the equation

$$\gamma_\perp R = \frac{\text{const}}{[1-\delta(R)](1+\mu P)} \approx \gamma_{\perp 0} R_0 \frac{[1-\delta(R_0)]}{[1-\delta(R)]} \approx \text{const}. \tag{II.3}$$

Here, $\mu \approx \mu_0 \frac{[1-\delta(R)]}{[1-\delta(R_0)]} \approx \text{const}$; the zero index relates to the initial values of the quantities.

We see from Eq. (II.3) that a reduction in the radius of the ring is accompanied by an increase in the energy of the electrons, resulting from the accelerating effect of the electric eddy field.

The behavior of the nonequilibrium particles is described by

$$\left.\begin{aligned} &\frac{d}{dt}(\gamma_\perp \dot{\rho}) + \gamma_\perp \omega_s^2 \nu_r^2 \rho = \frac{\omega_s \Delta\widetilde{M}}{Rm}; \\ &\frac{d}{dt}(\gamma_\perp \dot{z}) + \gamma_\perp \omega_s^2 \nu_z^2 z = 0. \end{aligned}\right\} \tag{II.4}$$

Here, $\omega_s = (c\beta_\theta/R)$; $\rho = r-R$; $\Delta\tilde{M} = \tilde{M}-\tilde{M}_s$; $\tilde{M}$ is the generalized momentum. The index s relates to the equilibrium particle. It follows from Eqs. (II.4) that the nonequilibrium particles execute betatron oscillations around the equilibrium position with dimensionless frequencies

$$\left.\begin{aligned} \nu_r^2 &= (1-n)(1+\mu P) - \left[\frac{4\mu}{g(g+b)\gamma_\perp^2\beta_\theta^2} + \frac{\mu P}{2}\right]; \\ \nu_z^2 &= n(1+\mu P) - \left[\frac{4\mu}{b(g+b)\gamma_\perp^2\beta_\theta^2} + \frac{\mu P}{2}\right]. \end{aligned}\right\} \quad \text{(II.5)}$$

The first terms in expressions (II.5) are the ordinary frequencies of betatron oscillations in a weakly-focusing magnetic field, but with a correction due to the repulsion of the ring along the major radius. The terms in square brackets characterize the displacement of the frequency arising from the intrinsic field of the ring.

On slowly changing the magnetic field, the following quantities constitute adiabatic invariants of Eqs. (II.4) in the case of a "symmetrical" beam:

$$J_{r,z} = \gamma_\perp R\beta_\theta \frac{a_{r,z}^2}{R^2} \nu_{r,z}. \quad \text{(II.6a)}$$

In the case of a beam with an energy spread, instead of the invariant J_r we have

$$\gamma_\perp R\beta_\theta \frac{a_r}{R} \nu_r^2 = \frac{\Delta\tilde{M}}{mc} = \text{const}, \quad \text{(II.6b)}$$

where $a_{r,z}$ is the amplitude of the betatron oscillations.

Analysis of the adiabatic invariants enables us to find the cross section of the ring in its final state in relation to its initial parameters:

$$b/b_0 = \psi_b; \quad g/g_0 = \psi_g. \quad \text{(II.7)}$$

The functions ψ_b and ψ_g characterizing the change in the cross section of the ring are close to unity. It should be remembered that b and g are relative quantities, and hence, the small dimensions of the ring vary approximately as R.

A real beam is more complicated than the foregoing two "extreme" models, and will constitute an intermediate case between them. For one-turn injection the initial state corresponds more to the model of the "symmetrical" beam. For several-turn injection it is useful to vary the injection energy from turn to turn, since the radial betatron oscillations will be small, while the radial dimension associated with the energy spread will fall off more strongly. In this case the beam will be closer to the second model.

3. Intrinsic Field of the Electron Ring. The duly-shaped electron ring is subsequently accelerated by the external fields in the accelerating system, while the ions lying within it are accelerated by the intrinsic Coulomb field of the ring. The greater the intrinsic field of the ring, the greater is the efficiency with which the ions are accelerated in the collective accelerator.

If an ion lies at the edge of the potential well, then the accelerating field acting upon it is

$$\mathcal{E}_\text{к} = \frac{2|e|N_e}{\pi R^2(b+g)}. \quad \text{(II.8)}$$

We see from Eq. (II.8) that the greater the current in the ring and the greater its degree of compression the greater will the field accelerating the ions be. If we express all the parameters of the ring in terms of the initial parameters specified on injection, we obtain [3]

$$\mathcal{E}_\text{к} \approx \sqrt{N_e} \approx \sqrt{I_0}. \quad \text{(II.9)}$$

The fact that $\mathcal{E}_\text{к}$ depends on the square root of the current is associated with the fact that, although, according to Eq. (II.8), $\mathcal{E}_\text{к} \approx N_e$, the quantities b_0 and g_0 defining $b = b_0\psi_b$ and $g = g_0\psi_g$ in the denominator of this equation are nevertheless proportional to $\sqrt{N_e}$. The calculations carried out in the earlier paper [3] show that for a ring radius of R = 5 cm and an external magnetic field of $B_s = 2 \cdot 10^4$ G.

$$\mathcal{E}_K \approx 2.4 \sqrt{\frac{N_e}{10^{13}}} \text{ [MV/cm]}, \qquad \text{(II.10)}$$

which for $N_e = 10^{13}$ gives $\mathcal{E}_K = 2.4$ MV/cm, and for $N_e = 10^{14}$ $\mathcal{E}_K = 7.6$ MV/cm.

4. Effect of Screens. In considering the motion of the electrons in a ring we have not so far taken any account of the effect of the image of the ring in the walls of the chamber. This influence is substantial when the ring lies quite close to the walls. The wall limiting the beam radially thus has the greatest effect. The influence of the beam image in this wall is considerable at the initial stage of deflection. Subsequently the beam moves away from the wall, and the effect of the image forces may be neglected.

Allowance for the image forces slightly displaces the initial frequencies of the betatron oscillations and affects the adiabatic change in the parameters of the ring. Consideration shows that the wall focuses the ring in the z direction (increases the frequency of the z oscillations) and defocuses it in the r direction (reduces the frequency of the r oscillations). In the adhesor these changes are not terribly vital. However, in the accelerating system the image forces are used to sustain the z dimensions of the ring (Sec. IV). A certain defocusing along the r direction is not particularly dangerous, since the r dimensions is fully maintained by the external magnetic field.

The effect of the image may also lead to instability of the ring with respect to the major radius, when the ring undergoes a random deviation from the axis of the adhesor and is drawn to the wall. In order to prevent this from happening, the following relation should be satisfied:

$$n < 1 - \frac{\xi(1+\xi)}{2} \cdot \frac{1}{(1-\xi)^2} \mu, \qquad \text{(II.11)}$$

where $\xi = R/R_w$; R_w is the radius of the wall. For $\xi = 0.6$–0.8; n = 0.5; R = 40 cm, and $\gamma_\perp = 7$, inequality (II.11) is satisfied if $N_e < 2 \cdot 10^{14}$.

5. Part Played by Resonances in the Compression of the Ring. In the course of ring compression the frequencies of the betatron oscillations ν_r and ν_z change and may pass through a series of resonance values.

Passing through resonances in the sense of an increasing frequency is not really dangerous. The increase in the beam cross section arising from the effects of the resonance weakens the space charge and hence the frequency rises, which accelerates passage through the resonance. Calculations show that the tolerances imposed on the magnetic field in respect of tranversing radial and mixed resonances may readily be upheld.

It is also possible, however, to pass through z resonances in the sense of a falling frequency. This extends the resonance interaction and may cause the beam to be drawn into resonance. In order to avoid dangerous passages through resonance it is essential to choose an n(r) law of variation such that

$$0.1 \leqslant \nu_z < 0.25. \qquad \text{(II.12)}$$

We then avoid the resonances $\nu_z = 1/2;\ 1/3;\ 1/4$.

6. Admission of the Ions. At the end of the compression process the hydrogen source starts operating. When the relativistic electrons collide with hydrogen molecules, molecular ions are chiefly formed (this is the most probable process, the ionization cross section being $\sigma = 10^{-19}$ cm^2), and in subsequent collisions with electrons these dissociate into a hydrogen atom and a proton. Collisions between ions leading to ionization and charge exchange may be neglected. Estimates show this to be valid for $(N_i/N_e) \ll 10^{-1}$ [6].

The admission of hydrogen into the adhesor takes place along the z axis so as to ensure uniform filling of the ring with neutral molecules. With this form of admission, the time for filling the volume of the ring with hydrogen molecules is determined by the dimensions of the small cross section of the ring, being equal to $t_v \approx 1$–$0.1\ \mu$sec. This time is much smaller than $t_g \approx t_u$, the characteristic times of dissociation and ionization. For $N_e = 10^{13}$–10^{14}, $t_u \approx (50-5)\ \mu$sec; we may therefore neglect the ring-filling time and consider that the ring is filled with hydrogen instantaneously. For times greater than t_u, the accumulation of protons in the ring takes place in accordance with a linear law. If $N_i = 10^{-2} N_e$, then for a hydrogen pressure of $p = 10^{-6}$ mm Hg the ion-injection time is $t = 50$ μsec. In the case of a greater hydrogen pressure this process may be accelerated.

One further point should be noted. In the presence of ions the betatron-oscillation frequencies will be slightly higher. It may well be that the corresponding shift in the radial frequency on injecting the ions will lead to a passage through the resonance $\nu_r = 1$. However, we must also remember the effect of the metal extraction tube (Section IV), which imparts a defocusing effect in the radial sense. The tube may prevent the frequency from passing through $\nu_r = 1$.

7. Other Possible Means of Creating an Electron Ring. The foregoing method of creating rings in a magnetic field that increases with time is not the only one possible. It is attractive to consider using an adhesor with a static magnetic field in order to create electron rings. It is simpler and cheaper to establish a static magnetic field than a varying one. Furthermore, in this case the number of accelerating cycles per unit time may be substantially increased, being limited solely by the potentialities of the injector.

Several methods of creating rings in a static magnetic field have been proposed [7-9]. The simplest of these [7] is as follows: an electron beam is injected at a certain angle in the weak fringing field of a solenoid. In a magnetic field increasing along the axis of the solenoid, the initial longitudinal momentum of the electrons P_z is transformed into a transverse momentum P_θ, and as a result of this a compressed electron ring is formed in the strong-field region in the center of the solenoid.

However, on using this method, the final transverse dimensions of the ring depend very substantially on the angular and energy spreads of the injected beam. It is accordingly quite impossible to achieve a ring of the desired parameters in this manner, and we shall pay no further attention to it.

There are two further methods by means of which an electron beam of the required parameters might (at least in principle) be obtained.

The first of these [8] lies in the fact that an electron ring formed by injection perpendicularly to the axis of the magnetic field is first accelerated in a magnetic field which diminishes along the z direction. This creates a field configuration satisfying the 2:1 condition:

$$\frac{\partial B_z}{\partial z} = \frac{1}{2} \cdot \frac{\partial \bar{B}_z}{\partial z}, \tag{II.13}$$

where $\bar{B}_z = \frac{2}{r^2} \int_0^r \xi B_z \, d\xi$ is the mean field in a circle of radius r. On satisfying condition (II.13) the radius of the ring remains constant. Then the accelerated ring is retarded to a complete standstill in a magnetic field adiabatically increasing along z. If at each z the magnetic field is almost uniform with respect to radius, we have $R \sim 1/Bz^{-/2}$. As a result of the asymmetry of this process, the ring finds itself in a stronger magnetic field, and acquires dimensions smaller than at the point of injection.

In view of the axial symmetry of the system, the azimuthal component of the generalized momentum should be conserved, this being written as

$$\tilde{M} = -R^2 \left(B_z - \frac{\bar{B}_z}{2}\right). \tag{II.14}$$

In order to create the required $\bar{B}_z$ in the accelerating section of constant radius, internal coils are required. In order to preserve the wholeness of the ring, $\bar{B}_z$ should be varied adiabatically, and the internal coils must therefore not break off sharply. At the end of the adhesor, the condition $\bar{B}_z = B_z$ should be observed in order to enable the ring to be extracted into the accelerating section. (Satisfaction of the condition $\bar{B}_z = B_z$ is not entirely obligatory if the ring is subsequently accelerated in a falling magnetic field.) Using Eq. (II.14) and the law of conservation of momentum we then have:

$$\left.\begin{aligned} \bar{B}_{z_0} &= 2B_{z_0} - B_{z_K} \frac{1}{s^2}; \\ B_{z_K} &= s \cdot B_{z_0}, \end{aligned}\right\} \tag{II.15}$$

where $s = R_0/R_k$ is the compression factor of the adhesor (the indices 0 and K refer to the beginning and end of the adhesor, respectively).

Maintenance of a small r dimension of the ring is ensured by the external magnetic field. In the z direction there is no focusing by the external field, since in the falling field there is no specially distinguished particle around which betatron oscillations are executed. (It was erroneously stated in one case [8] that the z dimension might also be kept intact by the external field.) Hence, the z dimension has to be preserved by some other means. In principle, focusing by a metal tube may be employed for this purpose (Sec. IV), by making use of the fact that the image forces have a focusing effect in the z direction. The tube profile then has to reproduce the varying radius of the ring. The question of focusing in this case demands special consideration. The injection of the ions in such an adhesor takes place at the final stage of compression, when the longitudinal velocity of the ring is low.

In order to extract the beam it is sufficient to make the field at the end of the adhesor a little smaller than the limiting value of B_{zk}, so that the ring may retain a slight velocity in the axial direction.

The second method of producing an electron beam of the required parameters [9] depends on the use of specially shaped internal and external solenoids; these create a magnetic-field configuration such that on a certain curve $r = r(z)$ $B_r \equiv 0$ [r(z) diminishes monotonically]. Then the equilibrium electrons injected on the curve r(z) in a field increasing along z will experience neither acceleration nor retardation.

The nonequilibrium electrons will execute oscillations around the curve r(z), the frequencies being $\nu_{1,2}^2 = 1$ and 0. Since one of the frequencies is zero, the ring will spread. In order to avoid this, the static magnetic field is augmented by a magnetic pit, depression, or well traveling along the z direction; this ensures focusing of the beam and directs it into the strong-field region. The traveling magnetic well is created by means of a spiral slowing-down system, a current pulse being fed into this. Since the frequency $\nu^2 = 0$ corresponds to a position of neutral equilibrium, only a small traveling field is required in order to ensure focusing.

The foregoing methods of creating electron rings in a static magnetic field have a common failing. The energy of the electrons is constant in a static field, and since, in order to compensate the Coulomb repulsion in the final state, it is essential to have a fairly large value of $\gamma_{\perp k}$, the electrons must be injected with an energy $\gamma_{\perp k}$, which makes the injector expensive and more complicated. Substantial difficulties associated with maintaining a small z dimension of the ring also arise. Furthermore, the creation of complex field configurations by means of internal coils and solenoids is quite a difficult problem.

8. Extraction of the Beam from the Adhesor. After the formation of the electron-ion ring has been completed, it has to be drawn out along the axis of the adhesor for further acceleration. The extraction of the ring from the adhesors considered in Paragraph 7 is not a particularly difficult matter, since in this case there is no magnetic field barrier on passing into the accelerating system. It is therefore of greatest interest to consider the extraction of the beam from an adhesor with a magnetic field increasing in time.

The process of compressing the electron ring takes place in a potential well formed by current-carrying coils or turns symmetrically arranged with respect to the plane in which the electron ring lies. The currents in each pair of turns are identical. The resultant magnetic field increases on both sides of the symmetry axis of the system.

Thus, in order to extract the beam from the adhesor it is essential to overcome the barrier created by the increasing magnetic field. At the same time it is required to preserve the dimensions of the ring achieved by virtue of the compression and also to restrain the ions.

Several ways of extracting the beam from the compression chamber have been suggested [2, 10-12]. These may be divided into two groups. In the first group we have methods of extracting the ring by displacing the potential well together with the ring to a position outside the chamber [10]. In the second group we have extraction methods based on removing the magnetic barrier within the chamber and creating a magnetic field diminishing along the axial direction [2, 11, 12]. In one case [10] the displacement of the potential well is achieved by using additional coils which are switched on after completing the compression process. The magnetic field of these coils changes the original field distribution in the chamber so that the B_z field minimum at which the ring is situated moves slowly in a longitudinal direction in the desired sense. This method is convenient because it creates no disruption in the ring-focusing conditions ($n \approx$ const) and provides the necessary field gradients for the retention of the ions. However, the magnetic barrier is simply moved outside the chamber, not lowered. Furthermore, the longitudinal velocity of the electron-ion beam here equals the velocity of the potential well and is very low ($\beta_Z \approx 10^{-5}$–10^{-4}, where β_Z is the longitudinal velocity of the ring referred to c). For this reason the second group of methods is preferable, the symmetry of the magnetic field distribution being disturbed in these so as to exert a repulsive force on the ring in the desired direction.

Let us consider the general requirements on any extraction system that distorts the magnetic barrier.

The equation of motion along the z axis for a constant, uniform field* will be:

$$\frac{d}{dt} m\gamma \dot{z}_s = -\mu \frac{dB_z}{dz}. \qquad \text{(II.16)}$$

The index s signifies the central (equilibrium) particle of the ring on which no Coulomb forces are acting. The quantity μ is the magnetic moment of the particles, constituting an adiabatic invariant:

$$\mu = \frac{m\gamma\beta_{\theta_0}^2 c^2}{2B_{z_0}} = \text{inv.}$$

Since γ = const, Eq. (II.16) may be written

$$\ddot{z}_s = -\frac{\beta_{\theta_0}^2 c^2}{2B_{z_0}} \cdot \frac{dB_z}{dz}. \qquad \text{(II.17)}$$

For the initial conditions $t = 0$; $z = 0$; $\dot{z} = 0$; $B_Z = B_{Z_0}$ the first integral of Eq. (II.17) will be

$$\beta_z = \beta_{\theta_0} \sqrt{1 - \frac{B_z}{B_{z_0}}}. \qquad \text{(II.18)}$$

We thus see that in order to extract the ring from the adhesor a magnetic field falling in the z direction must be created.

Let us estimate the permissible field gradients. In order to ensure that the ions should not be detached from the electron ring it is essential to prevent the acceleration of the electrons from exceeding the accelerations experienced by the ions. For an ion situated at the edge of the electron ring we may, according to Eq. (II.8), write down the following equation of motion:

$$\ddot{z}_i = \frac{2e^2 N_e}{\pi R^2 (b+g) M\gamma_z^3}, \qquad \text{(II.19)}$$

where $\gamma_Z^2 = (1/1-\beta_Z^2)$. Equating the left-hand sides of relations (II.17) and (II.19), we obtain the limiting magnetic-field gradient

$$\left|\frac{dB_z}{dz}\right|_{\lim} = \frac{4e^2 N_e B_{z_0}}{\pi R^2 (g+b) M\gamma_z^3 \beta_{\theta_0}^2 c^2}. \qquad \text{(II.20)}$$

The gradient of the magnetic field falling along the z direction (created in order to extract the ring from the compression chamber) must never exceed the limiting value specified by (II.20).

Focusing of the electron ring during compression is achieved by means of a weak barrel-shaped focusing magnetic field with a specified value of n. For the deviation of the particles from the equilibrium particle we have the linearized oscillation equations (II.4). The focusing conditions are ν_r^2 and $\nu_Z^2 > 0$.

When the symmetry of the field distribution in the adhesor starts breaking down and a field diminishing in the z direction is adiabatically created, n falls and may pass through zero. Hence, starting from a certain instant of time, the focusing conditions will cease to be satisfied, and focusing will now have to be achieved by external methods (for example, by virtue of the image forces in a metal tube).

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*More details are given in Sec. III, Paragraph 2.

All the methods of extracting the ring by creating a falling magnetic field are restricted by the foregoing common requirements as to field gradients and focusing conditions.

In two cases [2, 11] it was proposed that, in order to extract the ring from the adhesor by lifting the magnetic barrier, two open turns should be placed close to the maximum of the principal field and short-circuited at a specific instant of time. This instant would occur during the rise in the field created by the main turns. The induced field, directed in opposition to the field of the principal turns, would lift part of the magnetic barrier, and, since the current in the main turns would continue rising, at a specific instant of time the conditions required for the extraction of the ring (namely, the creation of a magnetic field falling along the z axis) would be created. This instant of time would be the instant at which the rise in current ceased in the main turns.

In order to smooth the field distribution, additional coils should be placed on the line of motion of the ring, the number of these coils and the currents passing through them being determined by the permissible field gradients.

The change in the magnetic field distribution in the adhesor amounts to a gradual reduction in the depth of the potential well and a simultaneous displacement of the latter, and subsequently to the creation of a diminishing magnetic field. While the depth of the potential well remains fairly large, it secures the focusing of the ring; as it diminishes, however, other means of focusing have to be used, unless one relies on the attainment of a self-focusing ring.

In another case [12] it was proposed that, in order to eliminate the magnetic barrier, the current in one of the main coils should be reduced after the end of the compression process. At the same time the current would remain constant in the symmetrical coil, or even increased. The symmetry of the field distribution would thus be disrupted, the field distorted, and ultimately a magnetic field falling in the direction of extraction of the ring would be created. This method assumes the possibility of creating pulsed currents greater than are required for compression at the specified ultimate dimensions of the ring.

The requirements imposed upon the gradients are satisfied by using a solenoid. The problem of focusing remains the same as in the previous method.

Thus, as a result of one means of distorting the shape of the magnetic field or another, the electron-ion ring falls into a region of diminishing magnetic field, in which the azimuthal velocity of the electrons is converted into longitudinal velocity and the ions in the ring are accelerated. The longitudinal velocity at the exit from the adhesor may be $\beta_i \approx (0.1–0.2)$.

III. Acceleration of an Electron-Ion Ring

1. Preliminary Comments. The problem of accelerating an electron ring charged with ions has a number of special features. The ring constitutes a compact formation with a large charge, and the intrinsic current created by this charge (of the order of tens of kiloamperes) loads the accelerating system severely.

The existence of rotational motion of the electrons leads to a substantial effective "weighting" of the ring. Hence, its acceleration to relativistic velocities takes place much more slowly than the acceleration of a simple group of electrons with the same charge. During acceleration the ring is polarized, and the ions are accelerated by their intrinsic Coulomb forces. These forces are determined by the parameters of the ring and are limited. Hence, the forces accelerating the ring must be equally limited.

These considerations taken together with the technical feasibilities and economic requirements of the accelerator determine the choice of structure for the accelerating system.

The principles to be followed in making this structure may clearly be the same as in the case of an ordinary charged-particle linear accelerator; however, in addition to this, the special characteristics of the problem in hand, involving the acceleration of a group of electrons in the form of a ring, admits a different kind of solution, for example, the use of a falling magnetic field and the combination of such a field with a system of resonators.

We shall now give some further consideration to the determination of the permissible accelerating fields and the use of a falling magnetic field together with a system of resonators for acceleration purposes (as we shall shortly show, this arrangement has great advantages over ordinary methods of acceleration), and finally, we shall discuss the important question of the power supply of the system.

2. Interaction between the Electron and Ion Components of the Ring, and the Permissible Accelerating Fields. The forces acting on the ions by virtue of the accelerated electron ring are determined from the following considerations. If the longitudinal dimensions of the electron ring remain constant in a system of coordinates accompanying the ring under the action of any particular external forces, then on accelerating the ring in a constant longitudinal electric field $\mathscr{E}$ stable acceleration of the ions by a constant force may readily be achieved [2]. The laws of longitudinal motion of the central electron and the central ion then take the following form:

$$m_{\perp}\gamma_{\parallel}^{3}\ddot{z}=e\left(\mathscr{E}-\frac{N_i}{N_e}\mathscr{E}_{\text{к}}\right); \qquad \text{(III.1)}$$

$$M\gamma_{\parallel}^{3}\ddot{z}=e\left(\mathscr{E}_{\text{к}}-\mathscr{E}\right) \qquad \text{(III.2)}$$

where

$$\mathscr{E}_{\text{к}}=\frac{2eN_e}{\pi r_0^2\left(b_c+g\right)}\Delta; \qquad \text{(III.3)}$$

$m_{\perp} = m\gamma_{\perp}$ is the "weighted" mass of the electron $\gamma_{\parallel} = (1-\beta_z^2)^{-1/2}$; $\gamma_{\perp} = (1-\gamma_{\parallel}^2\beta_{\theta}^2)^{-1/2}$; β_z is the longitudinal velocity of the electron (along the z axis) referred to the velocity of light, r_0 is the ring radius, b_c is the longitudinal semidimension of the ring cross section in an accompanying system of coordinates, referred to r_0, Δ is the distance between the central particles referred to b_c ($\Delta \ll 1$), which characterizes the degree of polarization.

We see from Eqs. (III.1) and (III.2) that in this case the acceleration of the ring in the accompanying system is constant.

The condition that the laws of motion of the central electron and ion should coincide determines the permissible electric accelerating field:

$$\mathscr{E}_g=\mathscr{E}_{\text{к}}\frac{m_{\perp}}{M}\cdot\frac{1+\dfrac{M}{m_{\perp}}\cdot\dfrac{N_i}{N_e}}{1+\dfrac{m_{\perp}}{M}}, \qquad \text{(III.4)}$$

The force accelerating the central electron is then equal to

$$e\mathscr{E}_{\text{к}}\frac{m_{\perp}}{M}\cdot\frac{1-\dfrac{N_i}{N_e}}{1+\dfrac{m_{\perp}}{M}},$$

while the force accelerating the central ion is $M/m_{\perp}$ times greater. We thus see that a considerable loading of the electron ring with ions (provided always that $N_i/N_e \ll 1$) has little effect on the efficiency of this method of acceleration.

For the parameters indicated in Paragraph 3 of Sec. II, if $\Delta = 0.5$, $\mathscr{E}_g$ varies between 25 kV/cm ($N_e = 10^{13}$) to 80 kV/cm ($N_e = 10^{14}$).

3. Acceleration of a Ring in a Falling Magnetic Field. Let us now consider the acceleration of a ring in a falling magnetic field (longitudinal and axially symmetric) [2]. In this case the energy accumulated in the rotational motion of the ring is transformed into translational motion. The equation of motion of the electron in such a field takes the form

$$\ddot{r}=r\dot{\theta}\left(\dot{\theta}+\frac{eB_z}{m\gamma c}\right); \qquad \text{(III.5)}$$

$$\frac{d}{dt}\left(m\gamma r^2\dot{\theta}+\frac{e}{c}\int_0^r B_z\,\xi d\xi\right)=0; \qquad \text{(III.6)}$$

$$\ddot{z} = -\frac{eB_z}{m\gamma c} r\dot{\theta}\frac{B_r}{B_z}. \tag{III.7}$$

On satisfying the following conditions:

$$\left|\gamma_{\parallel}\frac{r}{B_z}\cdot\frac{\partial B_z}{\partial z}\right| = \varepsilon \ll 1; \quad \left|\gamma_{\parallel}^2\frac{r^2}{B_z}\cdot\frac{\partial^2 B_z}{\partial z^2}\right| \leqslant \varepsilon^2, \tag{III.8}$$

which correspond to the drift approximation, the equations of motion in the linear approximation with respect to ε admit solutions of the form:

$$\dot{\theta} = -\frac{eB_z}{m\gamma c}; \tag{III.9}$$

$$r = r_0\sqrt{\frac{B_{z_0} - \overline{B}_{z_0}/2}{B_z - \overline{B}_z/2}}; \tag{III.10}$$

$$\ddot{z} = r^2\dot{\theta}^2\frac{1}{2B_z}\cdot\frac{\partial\overline{B}_z}{\partial z}. \tag{III.11}$$

In the case of an almost uniform field we have

$$r = r_0\sqrt{\frac{B_{z_0}}{B_z}}; \quad P_\theta = P_{\theta_0}\sqrt{\frac{B_z}{B_{z_0}}} \qquad (P_\theta = m\gamma r\dot{\theta}). \tag{III.12}$$

On satisfying the betatron condition

$$B_z = \frac{\overline{B}_z}{2} + \text{const}; \tag{III.13}$$

$$r = r_0; \quad P_\theta = P_{\theta_0}\frac{B_z}{B_{z_0}}. \tag{III.14}$$

If we now allow for the fact that the longitudinal motion of the ring should satisfy the condition of constant acceleration in the accompanying coordinate system, it is not hard to obtain the following expressions for the law of variation of the longitudinal magnetic field, which are valid if

$$\frac{N_i}{N_e} \ll 1 \text{ and } \frac{N_i\beta_{\theta_i}}{N_e\beta_{\theta_e}} \ll 1.$$

In the case of an almost uniform field

$$B_z = B_{z_0}\frac{1}{\gamma^2\beta_{\theta_0}^2}\left\{\frac{\gamma_{\perp 0}^2}{\left[1 + \frac{e\mathscr{E}_0(z - z_0)}{mc^2\gamma}\right]^2} - 1\right\}$$
$$\approx B_{z_0}\left[1 - \frac{2}{\gamma_{\parallel 0}^2\beta_{\theta_0}^2}\cdot\frac{e\mathscr{E}_0(z - z_0)}{mc^2\gamma}\right], \tag{III.15}$$

the latter if $\frac{e\mathscr{E}_0(z - z_0)}{mc^2\gamma} \ll 1$.

Under the 2:1 condition we have

$$B_z = B_{z_0}\sqrt{\frac{1}{\gamma^2\beta_{\theta_0}^2}\left\{\frac{\gamma_{\perp 0}^2}{\left[1 + \frac{e\mathscr{E}_0(z - z_0)}{mc^2\gamma}\right]^2} - 1\right\}}$$

$$\approx B_{z_0}\left[1-\frac{1}{\gamma_{\parallel_0}^2\,\beta_{\theta_0}^2}\cdot\frac{e\mathscr{E}_0(z-z_0)}{mc^2\gamma}\right]. \qquad \text{(III.16)}$$

Here the parameter $\mathscr{E}_0$ is chosen from the condition that the ions should be contained, and is equal to $\mathscr{E}_g$ determined from Eq. (III.4) with $\gamma_\perp = \gamma_{\perp 0}$, i.e., equal to the initial value. Equations (III.15) and (III.16) are only valid in a region in which the field is not changing too rapidly (not more than a factor of 2–3). The motion of the equilibrium particles in such fields constitutes a spiral. The angle of the spiral

$$\alpha=\operatorname{arctg}\frac{\beta_\theta}{\beta_z}\approx\frac{1}{\gamma_\parallel} \quad \text{for} \quad 1\ll\gamma_\parallel\ll\gamma. \qquad \text{(III.17)}$$

The fact that for fairly large $\gamma_\parallel$ the angle of the spiral is small may clearly be used in order to establish a central solenoid ensuring the satisfaction of the 2 :1 condition.

The motion of the deflected particles relative to the equilibrium orbit is approximately described by the usual equation

$$\ddot{\rho}+\dot{\theta}^2\rho=0, \qquad \text{(III.18)}$$

whence we readily see that the amplitude of the free radial oscillations varies in accordance with the law

$$a_r=a_{r_0}\sqrt{\frac{B_{z_0}}{B_z}}. \qquad \text{(III.19)}$$

It is clear that, if an initially accelerated ring falls into a growing magnetic field, it will start being retarded, and the energy of the forward motion will be converted into rotational energy.

4. Combined Accelerating System. Let us now consider a system using the earlier-mentioned properties of a falling and rising magnetic field in conjunction with a system of accelerating, appropriately phased resonators [2, 13]. This system has the following structure. In the region between the resonators the longitudinal field falls linearly and the ring is accelerated in it as a result of the energy of the rotational motion. Inside the resonator we find a rising longitudinal magnetic field with a configuration such that the energy communicated to the ring in the resonator is mainly converted into rotational motion, and only a part of it, corresponding to the permissible acceleration, passes into forward motion. The longitudinal magnetic field at the exit from the resonators is the same, and hence so is the azimuthal momentum. In the resonators the adiabatic parameter

$$\varepsilon_p=\left|\gamma_\parallel\frac{r}{B_z}\cdot\frac{\partial B_z}{\partial z}\right|\approx\left|\frac{\mathscr{E}_A}{B_z}\right|\ll 1, \qquad \text{(III.20)}$$

where $\mathscr{E}_A$ is the amplitude of the field in the resonator.

In this system the longitudinal magnetic field should satisfy the condition

$$\mathscr{E}_g=-\frac{1}{2}\,r_0\,\beta_{\theta_0}\,\gamma_{\parallel_0}\,\gamma_\parallel\,\frac{\partial B_z}{\partial z}+\mathscr{E}_A f\cos\Omega_r t, \qquad \text{(III.21)}$$

where f is a function reflecting the configuration of the z component of the electric field in the resonator, Ω_r is the frequency of the resonator field. It follows that

$$r_0\left[B_z(z)-B_z(z_0)\right]=\frac{2}{\gamma_\parallel}\left[-\mathscr{E}_g\cdot z-\mathscr{E}_A\int_{z_0}^{z} f\cos\Omega_r t\,d\xi\right] \qquad \text{(III.22)}$$

for

$$B_z(z_2)=B_z(z_0). \qquad \text{(III.23)}$$

Here z_0 corresponds to the beginning of the gap and z_2 to the beginning of the next gap, i.e., the exit from the resonator.

In the linear approximation the longitudinal motion in such a system will be described by Eq. (III.1), with $\mathscr{E} = \mathscr{E}_g$ determined by Eq. (III.21), and the radial motion will be described by

$$\rho'' + \frac{1}{\gamma_{\|}^2 \beta_z^2}\rho = \frac{1}{\gamma_{\|}^2 \beta_z^2}\left[\frac{1}{\gamma_{\|}}\left(\varepsilon_0 z - \varepsilon_p \int_{z_0}^{z} f \cos\Omega_r t d\xi\right)\right.$$

$$\left. + \frac{\gamma_{\|} r_0}{2}\varepsilon_p\left(\beta_z \varkappa \bar{f}\sin\Omega_r t - r_0 \frac{\partial \bar{f}}{\partial t}\cos\Omega_r t\right)\right], \qquad \text{(III.24)}$$

where $\varkappa = (2\pi r_0/\lambda)$; λ is the wavelength of the accelerating field, $\bar{f} = \frac{2}{r_0^2}\int_0^{r_0} f r dr$; $\varepsilon_0 = \frac{\mathscr{E}_g}{B_{z_0}}$; $\varepsilon_p = \frac{\mathscr{E}_A}{B_{z_0}}$; the prime denotes differentiation with respect to θ. The solution of this equation may be expressed in the form of the sum of a particular solution of the equation with the right-hand side satisfying the condition

$$\rho(z_0) = \rho(z_2) \text{ and } \rho'(z_0) = \rho'(z_2), \qquad \text{(III.25)}$$

which describes the orbit, and the general solution of the homogeneous equation describing free oscillations around this orbit.

The conditions of adiabatic motion are only not satisfied at the entrance and exit of the resonator.

Let us express f in the following form: $f = \sigma(z-z_1) + \sigma(z_2-z)-1$, and hence, $\partial f/\partial z = \delta(z-z_1)-\delta(z-z_2)$, where z_1 corresponds to the entrance into the resonator. Then it is not difficult to obtain a solution describing the orbit. This takes a fairly cumbersome form; however, its main relationships may be expressed in the following way:

$$\rho_1 = \frac{r_0}{\gamma_{\|}}\varepsilon_p \frac{\dfrac{z_n}{2r_0\gamma_{\|}\beta_z}}{\sin\dfrac{z_n}{2r_0\gamma_{\|}\beta_z}} F, \qquad \text{(III.26)}$$

where $F \sim 1$ is a function depending on z and the parameters of the accelerating system; $z_n = z_2 - z_0$ is the period of the system in question. We see from this equation that the accelerating structure should satisfy the requirement $\sin(z_n/2r_0\gamma_{\|}\beta_z) \neq 0$, i.e., that the period of the structure should not accommodate a whole number of free oscillations. For

$$\frac{z_n}{2r_0\gamma_{\|}\beta_z} \ll 1; \quad \varkappa\frac{z_n}{z_p}\operatorname{tg}\varphi \ll 1; \quad \varkappa^2\frac{z_n}{z_p} \ll 1, \qquad \text{(III.27)}$$

where $z_p = z_2 - z_1$ is the gap in the resonator, and φ is the phase of the field in the resonator at the instant at which the ring passes through its center, the orbit is proportional to the square bracket on the right-hand side of Eq. (III.24) with a coefficient 0.5. The amplitude of the oscillations of the orbit attenuates as $1/\gamma_{\|}$, while the amplitude of the free oscillations around this orbit will increase as $\sqrt{\gamma_{\|}}$.

The period of the oscillations of the ions in the intrinsic system, determined by the parameters of the ring, is $T_c = 6 \cdot 10^{-10}$ sec for $N_e = 10^{14}$. In the laboratory system $T = T_c\gamma_{\|}$. In the initial part of the accelerating system, T is of the order of the time of flight of the ring through the resonator, and hence, the condition (III.21) must be satisfied in this case to a high degree of accuracy.

For $\gamma_{\|} \sim 5$–10, T is of the order of the time required to pass through the whole period of the system, and here the requirements need only be imposed upon the integral condition (III.22)–(III.23). For $\gamma_{\|} \geq 50$–100, T is much greater than the time required to pass through the period of the system. In this case, clearly, modulation of the leading field is not required; discrete acceleration by a field $\mathscr{E}_A$ is equivalent to continuous acceleration by the mean field $\mathscr{E}_D$. The noise generation of oscillations as a result of the discrete nature of the system is clearly slight.

A system with a modulated field enables us to shorten the initial part of the accelerator considerably (for $\gamma_{\|} \leq 100$); the possibility of creating a larger field in the resonators in this case improves the energy supply of the beam and eases adjustment.

In the final section of the accelerator, acceleration in a falling magnetic field with a constant or increasing value of r_0 may be employed (Paragraph 2 of this Section).

5. Questions of Energy Supply in the Acceleration of the Ring and Radiation in the Accelerating Structure. Let us now consider the general question of the collection of energy by a relativistic grouping (ring) with a high charge. The fact that the intrinsic current loads the accelerating system severely means, in electrodynamic language, that the energy which has to be communicated to the ring per unit path constitutes a substantial proportion of the energy of the external field, stored in a region from which it can be accepted by the ring. In addition to this, the energy radiated by the ring on passing through the accelerating structure may be comparable with the energy acquired by the ring.

Let us first make an estimate of the power losses J associated with the intrinsic radiation of the electron ring during acceleration, expressed as a ratio of the energy acquired per unit time dE/dt. This ratio [14] is proportional to the square of the charge to mass ratio and is also proportional to the change in the energy per unit path (here the ring is regarded as a single charge with a common mass):

$$\frac{J}{\frac{dE}{dt}} = \frac{2}{3}\left(\frac{e}{m\gamma_{\perp}}\right)^2 \frac{eN_e \mathscr{E}_g}{c^4}. \tag{III.28}$$

Even for $N_e = 10^{14}$, $\mathscr{E}_d = 100$ kV/cm and $\gamma_{\perp} = 50$, the quantity (III.28) is of the order of 10^{-3}, i.e., the intrinsic radiation during acceleration does not constitute a decisive factor in the present problem.

In order to obtain fields capable of accelerating the ring to relativistic velocities, it is fundamentally essential to introduce spatial inhomogeneities into the accelerating tract.* Hence, in collecting energy in the accelerating tract a relativistic ring containing a high charge loses some of its energy in the coherent emission of external charges, which are excited by its electric field on passing along these spatial inhomogeneities, i.e., it loses energy in transitional and Cerenkov radiation.

The intensity of this radiation for technically permissible fields and numbers of particles in the ring may greatly exceed the losses given by Eq. (III.28) [13].

It was suggested earlier [13] that this radiation constituted a considerable proportion of the load on the accelerating system and determined the whole possibility of energy being taken from the external accelerating field by the accelerated ring. Thus, for example, in the case of acceleration in a periodic structure, substantial interaction with the accelerating wave is only possible if the phase velocity of the wave coincides with the velocity of the ring. However, this means that at the same time the condition for the development of Cerenkov radiation, excited by the charge of the moving ring at this frequency, will also be satisfied.

It was also shown earlier [15] that, when a ring passed into a cylindrical resonator in which an external (accelerating) field was excited, the "range of propagation" of the fields excited by the ring at the instant of emerging from the resonator was determined by the following inequalities†:

$$0 \leqslant r \leqslant r_0 + \frac{h}{\beta_z}; \quad 0 \leqslant z \leqslant h, \tag{III.29}$$

where h is the longitudinal dimension of the resonator, r_0 is the radius of the ring, while an unperturbed external field remains in the rest of the volume. Thus, in the course of acceleration only the store of energy of the external field existing in this "region of interaction" acquires a vital significance, not the total store of energy in the whole volume of the resonator.

It is convenient to follow the interaction of the ring with the accelerating field by way of the following example [15, 16]. In a specific interval of time, we consider a closed physical system, consisting of a

*A waveguide with an inner dielectric coating used for slowing the accelerating wave, in particular, also constitutes a spatially inhomogeneous system.

†It is assumed that β_z varies little during the flight of the ring through the gap in the resonator. This condition is naturally satisfied if $\beta_z \approx 1$.

transverse electromagnetic wave excited in the closed volume of a cylindrical resonator, an electron ring traveling through the resonator, and the field excited by its charge both as a result of the acceleration and as a result of the spatial inhomogeneity of the system. In order to describe this system we use the Hamiltonian method [17] and allow for the initial conditions governing the field oscillators and the motion of the charge.

From the condition of the conservation of the total Hamiltonian of the system (using the Coulomb calibration for the potentials)

$$\mathcal{H}=\mathcal{H}_1+\mathcal{H}_2-\mathcal{H}_3=\text{const}, \tag{III.30}$$

where

$$\mathcal{H}_1=\sqrt{M_0^2 c^4+c^2\left\{\mathbf{P}-\frac{1}{c}\int_V \rho\,[\mathbf{r}-\mathbf{Q}(t)]\,\mathbf{A}(\mathbf{r},t)\,dV\right\}^2} \tag{III.31}$$

is the Hamiltonian corresponding to the charge considered as a whole [18],

$$\mathcal{H}_2=\frac{1}{2}\sum_\lambda (p_\lambda^2+\omega_\lambda^2 q_\lambda^2) \tag{III.32}$$

is the Hamiltonian of the transverse field (λ is the complete set of indices determining the eigenfunctions of the resonator $A_\lambda(\mathbf{r}, t)$); $\mathcal{H}_3$ corresponds to the static Coulomb interaction, M_0 = mN is the total rest mass, **Q** is the radius vector of the center of gravity, **P** is the canonical momentum, $\rho\,[\mathbf{r}-\mathbf{Q}(t)]$ is the charge density in the ring. The quantities q_λ and **Q** are determined from the equations of motion:

$$q_\lambda+\omega_\lambda^2 q_\lambda=\frac{1}{c}\int_V \rho\mathbf{Q}\mathbf{A}_\lambda\,dV; \tag{III.33}$$

$$\frac{d\mathbf{P}}{dt}=-\frac{\partial\mathcal{H}}{\partial\mathbf{Q}};\quad \frac{d\mathbf{Q}}{dt}=\frac{\partial\mathcal{H}}{\partial\mathbf{P}}. \tag{III.34}$$

We see from Eq. (III.33) that the motion of the charge perturbs all the oscillators of the resonator, i.e., the collection of energy by the charge from the transverse field of the resonator, in accordance with Eqs. (III.30)–(III.34), can only take place in the presence of radiation and through radiation. As the ring moves, there is a redistribution of the energy between the various components of $\mathcal{H}$. Furthermore, the amplitude of the oscillations, and hence, the energy of the earlier-excited harmonic oscillator, changes in accordance with Eq. (III.33), and furthermore the remaining oscillators are excited. The static field of the charges induced on the inner walls of the resonator and entering into $\mathcal{H}_3$ creates an additional nonuniformity in the motion of the ring, in accordance with Eq. (III.34), although ultimately, in view of its potential nature, it does not make any contribution to the redistribution of the energy.

The change in the energy of the ring during its flight is, according to (III.30), equal to

$$\Delta\mathcal{H}_1=-\Delta\mathcal{H}_2, \tag{III.35}$$

where on allowing for the foregoing discussion $\Delta\mathcal{H}_2$ may be written as

$$\Delta\mathcal{H}_2=2\,(\mathbf{A}_{1_0}\,\delta\mathbf{A}_1)+\delta\mathbf{A}_1^2+\sum_{\lambda\neq 1}\delta\mathbf{A}_\lambda^2. \tag{III.36}$$

Here we have used the notation $A_\lambda = iP_\lambda + j\omega_\lambda q_\lambda$ (i and j are unit vectors); A_{1_0} is the initial excitation of the oscillator of the first harmonic; δA_λ is the perturbation determined by the equations of motion (III.33) and (III.34). The third term on the right-hand side of (III.36) determines the radiation in all the harmonics excluding the first. The second term determines the radiation in the first harmonic, while the first term determines the collection of energy by the charge from the initially excited oscillator.

The first term in Eq. (III.36) may conveniently be written as

$$2\,(\mathbf{A}_{1_0}\,\delta\mathbf{A}_1)=2\sqrt{\mathbf{A}_{1_0}^2\,\delta\mathbf{A}_1^2}\cos\varphi, \tag{III.37}$$

where q is the phase shift between the initial and radiated fields, i.e., it is proportional to the square root of the product of the energy of the initial excitation and the energy of the radiation in the harmonic itself.

Equation (III.36) may be expressed in the form

$$\Delta\mathcal{H}_2 = Aq + Bq^2, \tag{III.38}$$

where g is the total charge of the ring, A and B are coefficients, which, in the case of the specific idealization implied by distinguishing the system as being a closed one, depend on the following parameters: A on the initial external field and the geometry of the system, B on the geometry of the system alone.

The extent to which the system is a closed one in the case of the arguments here presented may be roughly estimated if, for example, we compare the charge arising as a result of the external field on the wall of the resonator with the charge induced in the wall by the ring. If the time of flight of the ring is short in comparison with the period of the external field [we consider that the radius of the resonator $R \gg r_0 + (h/\beta)$, see Eq. (3.29)], the density of the external charge on the end wall of the resonator may be expressed in terms of the amplitude of the external field by the formula $\sigma = \mathcal{E}_A/4\pi$. Hence, the external charge concentrated in the region of interaction equals

$$q_c \approx \left(r_0 + \frac{h}{\beta}\right)^2 \frac{\mathcal{E}_A}{2}.$$

Naturally a necessary condition for the system to be a closed one is that the inequality $q_c \geq q$ should be satisfied. This is equivalent to saying that the energy of the external field stored in the region of interaction is greater than $qh\mathcal{E}_A$.

Equation (III.38) shows that we are concerned with the theory of perturbations, and if the charge in the ring is very great then we are dealing with the most difficult aspect of perturbation theory in which the energy of the perturbation is comparable with the initial energy of the system.

Thus, in approaching the problem it is primarily essential to estimate this perturbation energy, namely, the term $Bq^2 = W_b$. This estimation is usually carried out by calculating the energy of the radiation induced by the ring for a specified law of motion of the latter. Since the transverse components of the electrostatic field of the ring in the laboratory system of coordinates increase as v approaches c, a second important characteristic determining the energy W_b is the relativistic factor γ. The chief and most difficult problem is that of obtaining the $W_b = W_b(\gamma)$ relationship. The difficulty in solving this problem lies in the necessity of considering the large frequency range within which the radiation is excited. With increasing γ, higher and higher frequencies have to be taken into account. This may readily be seen, for example, if we make use of a Fourier expansion of the field components of the moving ring (for simplicity we shall consider that $\mathbf{v} = \mathbf{v}_0 = \text{const}$), i.e., the field exciting the radiation. The Fourier components are proportional to

$$\left(\frac{\sin\frac{l\omega}{c\beta}}{\frac{l\omega}{c\beta}}\right)^2 \exp\left[-\frac{\omega}{c\beta\gamma}(r-r_0)\right], \tag{III.39}$$

where l and r_0 are, respectively, the longitudinal and transverse characteristic dimensions of the ring, $(r-r_0)$ is the target parameter (distance from the trajectory to the obstacle). The quantity (III.39) constitutes a frequency "cut-off factor," and its effect diminishes with increasing γ. It may well be said that all the frequencies up to $\omega_{max} \approx c\gamma(r-r_0)$ contribute to W_b.

The factor $\left(\sin\frac{\omega l}{c\beta} \Big/ \frac{\omega l}{c\beta}\right)^2$ appearing in the expression (III.39) allows for the incoherence of the exciting effect of the field of the ring for wavelengths smaller than its longitudinal dimension l. It should nevertheless be noted that with increasing γ the value of l becomes smaller: $l = l_c/\gamma$, where l_c is the longitudinal dimension of the ring in its own system.

In a fair number of papers (a list is given in the review articles [19, 20]) the ring-induced radiation has been calculated for various cases of spatial inhomogeneity.* The problems thus solved as yet present no very clear picture as to the dependence of the excited radiation on γ, since in many cases they constitute upper asymptotic estimates or numerical calculations of particular cases.

At the present time, at least in the case of the motion of a ring under the condition $v = v_0$ = const, we may clearly divide the whole problem into two parts: the flight of the ring past a "single obstacle," and the motion of the ring in a periodic structure.

For the first part of the problem there are two characteristic aspects: the flight of the ring through an aperture in an ideally conducting screen [21], and its flight through a single resonator [15, 16, 20, 22-29].

In the problem relating to the flight of a charge q at a constant velocity v_0 through a circular aperture of radius a in an infinite, ideally conducting screen, we have four parameters: q, a, v_0, and the velocity of light c. From these quantities we can set up only one combination having the dimensions of energy, q^2/a. Hence, the total energy losses may be written in the form

$$\Delta W = \frac{q^2}{a} f\left(\frac{v_0}{c}\right). \tag{III.40}$$

It was shown earlier [21] that f increased in proportion to γ. Analogous results are obtained in the problem relating to the radiation of the charge on flying past an ideally conducting wedge [30], and in that of a charge into or out of a semiinfinite, ideally conducting tube [31]. The results of a calculation carried out for the case of an infinite filament flying past an ideally conducting cylinder [32] show that the linear rise in ΔW with γ is not a consequence of the fact that the obstacles have a sharp edge. The relation giving the energy loss when a charge flies past any single obstacle apparently takes the form (III.40) with a linear dependence of f on γ, some characteristic dimension (e.g., target or aiming parameter) playing the part of a. The only difference lies in the numerical coefficients involved.

The question as to the radiation of a charge passing through a single cylindrical resonator has been considered by a number of authors [15, 16, 20, 22-29], starting with Kotov and Kolpakov [22]. It is very natural that this should be so, since resonators constitute vital elements of accelerating installations. Owing to the complexity involved in considering the true conditions when the resonator contains entrance and exit apertures or feeding waveguides, instead of a resonator certain authors have considered a closed cylindrical volume, in which the charge (either an assembly of charges or a current-carrying ring) penetrates through a wall. Kotov and Kolpakov [22] expanded the fields in terms of the eigenfunctions of the resonator, taking account of the zero initial values for the field oscillators, and obtained expressions for the energy of the radiation left in the resonator by the ring after escaping through the second wall. For a thin charged ring this expression takes the form:

$$\Delta W = \frac{8\pi^2 q^2}{R_r} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \left(\frac{2}{1+\delta_{m_0}}\right) \left[\frac{\sin\left(\frac{\pi l}{\beta h}\sqrt{\left(\frac{\nu_n h}{\pi R_r}\right)^2 + m^2}\right)}{\frac{\pi l}{\beta h}\sqrt{\left(\frac{\nu_n h}{\pi R_r}\right)^2 + m^2}}\right]^2$$

$$\times \left[\frac{r_2 J_1\left(\nu_n \frac{r_2}{R_r}\right) - r_1 J_1\left(\nu_n \frac{r_1}{R_r}\right)}{\frac{\nu_n}{R_r}\left(r_2^2 - r_1^2\right)}\right]^2 \frac{1}{\frac{\pi \nu_n}{2} J_1^2(\nu_n)}$$

$$\times \frac{\left(\frac{\nu_n h}{\pi R_r}\right)^3}{\left[\sqrt{\left(\frac{\nu_n h}{\pi R_r}\right)^2 + m^2} + \beta m\right]^2} \cdot \frac{\sin^2\left[\frac{\pi}{2\beta}\left(\sqrt{\left(\frac{\nu_n h}{\pi R_r}\right)^2 + m^2} - \beta m\right)\right]}{\left[\frac{\pi}{2\beta}\left(\sqrt{\left(\frac{\nu_n h}{\pi R_r}\right)^2 + m^2} - \beta m\right)\right]^2}, \tag{III.41}$$

*In what follows we shall not consider the radiation excited by the azimuthal current in the ring, since the retarding action of this radiation differs in no fundamental manner from that of the radiation induced by the charge. Only different modes are excited.

where R_r is the radius of the resonator, h is its length, r_2 and r_1 are the outer and inner radii of the annular group of electrons (the ring), l is its thickness along the z axis, ν_n are the roots of the equation $J_0(\xi)=0$; δ_{mk} is the Kronecker delta, and $\beta = v/c$, where c is the velocity of the ring along the z axis.

For the limiting cases of an infinitely thin ring and a point charge, the expression for ΔW is logarithmically divergent. For other cases, such an infinitely thin disc or any other two- or three-dimensional charge distribution, the double sum is convergent, although it is certainly very difficult to calculate. However, a complete calculation is not really necessary, since in the limiting case, in which apertures occur in the resonator, according to Eq. (III.39) the very high frequencies make only a small contribution. Kotov and Kolpakov [22] suggested allowing for the entrance and exit apertures phenomenologically, by truncating the spectrum of the radiated waves, assuming that (for purposes of calculation) it was reasonable to confine attention to the terms corresponding to waves having a spatial inhomogeneity of the fields of the order of the diameter of the apertures. On this assumption, the formula obtained for ΔW (in the case of a point charge) was

$$\Delta W = \frac{q^2 h}{2a^2}, \qquad \text{(III.42)}$$

where a is the radius of the aperture.

In the ultrarelativistic case this formula requires refinement, since the contribution of the high harmonics to Eq. (III.41) becomes greater for large γ.

In another paper [26] the complete spectrum in (III.41) was truncated in respect of the radial harmonics giving a field inhomogeneity in the radial consideration, while all the longitudinal harmonics were taken into account. According to (III.39), however, this type of truncation neglects a number of waves which are in fact excited, but instead of remaining within the resonator pass out through the outer apertures, so that for large γ the result is incomplete.

In order to discover the maximum possible value of ΔW for very large γ with this type of approach, asymptotic estimates of the total sum in (III.41) were subsequently attempted [27]. The resultant upper limit for a group in the form of an infinitesimally thin disc of radius r_0 takes the form

$$\Delta W < \frac{8q^2 h}{r_0^2} \qquad \text{(III.43)}$$

and is independent of the value of γ. In order to deduce the asymptotic behavior of ΔW it is extremely important that the factor $\sin^2\left[\frac{\pi}{2\beta}\left(\sqrt{\left(\frac{\nu_n h}{\pi R_r}\right)^2 + m^2} - \beta m\right)\right]$ in Eq. (III.41) should be properly taken into account, since for a considerable range of n and m values (expanding with increasing γ) this is much smaller than unity. If due allowance for this fact is not made, and if it is assumed (as has frequently been done) that the factor in question is approximately equal to unity, then ΔW is apparently proportional to γ, and the estimate is far too high. Numerical calculations of the sum (III.41) were made in [28]. In order to allow for the coherence factor in (III.39), rings of different longitudinal dimensions were considered. The dependence of the sum (III.41) on γ was also considered with due allowance for the Lorentz contraction of the longitudinal dimension of the ring, and it was found that over a certain range of γ ($\gamma < 200$) values the rise in ΔW was approximately proportional to $\gamma^{1/2}$. It is nevertheless essential to note that the calculated values of ΔW corresponding to $\gamma = 200$ are very much smaller than the upper estimate (III.43). Hence, on further increasing γ the dependence of ΔW on γ should vanish.

The foregoing approach, with or without phenomenological allowance for the apertures, lies a long way from the true situation, since in this case only the radiation excited during the flight and left in the resonator is taken into account. Actually a certain proportion of the energy is lost by the ring as it enters and leaves. Certain authors accordingly considered another model. In some cases [24, 25] the losses were estimated as the amount of energy of the radiation diffracted within the resonator cavity during the entrance of the charge through the feeding waveguide. The resultant formula, which for an infinitesimally thin ring of radius r_0 takes the form

$$\Delta W \approx \frac{0.44q^2 h^{1/2}}{a(a-r_0)^{1/2}} \gamma^{1/2}, \qquad \text{(III.44)}$$

and for a point charge

$$\Delta W \approx 0.6 \frac{q^2 h^{1/2}}{a^{3/2}} \gamma^{1/2}, \qquad \text{(III.45)}$$

indicates that the energy rises as $\gamma^{1/2}$.

An analogous relationship with a numerical factor roughly twice as large was also obtained by Sessler [20], who analyzed the same model by partly matching the fields in the resonator cavity and the fields in the waveguide.

Numerical calculations of the energy lost by a point charge in a resonator with semiinfinite entrance and exit waveguides were carried out for $\gamma \leq 30$ in [29]. The relationship so derived may be expressed in the following way for $\gamma > 10$ and $h/R_r = 1$:

$$[\Delta W \approx \frac{4q^2}{\pi a}(0.5 + 0.09\gamma). \qquad \text{(III.46)}$$

For $10 < \gamma < 30$ there are no serious quantitative differences between (III.45) and (III.46).

It is not entirely clear from the foregoing discussion whether any specific conclusion may be drawn regarding the variations in losses with increasing γ.

In connection with problems regarding the radiative energy losses of groups of electrons in the case of open and closed resonators, the question arises as to how the energy losses are distributed along the path of the electron group. The energy lost by a charge on passing through any particular structure may be expressed in the form of the work done by the retarding force of the radiation over a specific path. When the charge passes through a single resonator with apertures or with feeding waveguides, the total energy loss may be written in the form

$$W = q \int_{-\infty}^{\infty} \mathscr{E}_T \, dz. \qquad \text{(III.47)}$$

Assuming that $\mathscr{E}_T$ has only one main maximum, this expression may be written in the form

$$W = q\mathscr{E}_{T\max} L, \qquad \text{(III.48)}$$

where $\mathscr{E}_{T\max}$ is the maximum value of $\mathscr{E}_T$. Since, from the point of view of the acceleration of the charge in a specific accelerating system, we are interested in the retarding force averaged over a section of path comparable with the period of the system (or with the gap in the resonator), it is extremely important to know what determines the rise in the losses with increasing γ: is it due to a rise in $\mathscr{E}_{T\max}$, a rise in L, or to a rise in both of these, and, if the latter, then in what proportion? It is obvious that, if the rise in the losses with increasing γ is determined mainly by the increases in the path length L within which even the slightest retarding force occurs, then this relationship will not constitute any fundamental obstacle to the creation of an accelerating system.

We may further note that, as a result of the relativistic flattening of the intrinsic field of the group of electrons in the direction of motion (in the laboratory coordinate system), the maximum perturbation of the external charges occurs at the actual instant at which the group flies past an inhomogeneity, and hence the work done by the retarding force depends mainly on how long the group moves in phase with the excited retarding wave.

A good example (and one yielding an exact solution) for tracing the dependence of the retarding force on the position of the group in the course of its motion is that concerned with the excitation of an ideally conducting semiplane by a charged filament with a linear charge density $\varkappa$, flying past it at a distance a [19]. For simplicity, it is convenient to take the trajectory of the filament as being perpendicular to the semiplane. Using the expressions of [19] for the fields and integrating over the whole spectrum of frequencies, we obtain the following exact formula for the retarding force in relation to the position of the filament [33]:

$$F_T = -\frac{\varkappa^2}{2\gamma a\left[1+\left(\frac{y}{\gamma a}\right)^2\right]^2}\left\{\frac{1-\beta\frac{y}{a}}{\frac{y}{a}}\left(\frac{1}{\sqrt{1+\left(\frac{y}{a}\right)^2}}-1\right)\right.$$

$$+\frac{(2-\beta^2)\frac{y}{a}}{\sqrt{1+\left(\frac{y}{a}\right)^2}}-\frac{y}{a}\left(\frac{(2+\beta^2)\beta\frac{y}{a}}{\sqrt{1+\left(\frac{y}{a}\right)^2}}+3\right)+\left(\frac{y}{\gamma a}\right)^2$$

$$\left.\times\left(\frac{\frac{y}{a}}{\sqrt{1+\left(\frac{y}{a}\right)^2}}+\beta\right)-\left(\frac{y}{\gamma a}\right)^2\left(\frac{y}{a}\right)\left[\frac{\beta\frac{y}{a}}{\sqrt{1+\left(\frac{y}{a}\right)^2}}+(2-\beta^2)\right]\right\}. \qquad \text{(III.49)}$$

In Eq. (III.49) it is considered that the filament is moving at a velocity v = const from y = $-\infty$ toward y = $+\infty$. It is easy to show from the foregoing equation that, in the ultrarelativistic case, starting from the point $y \approx 0$ (y = 0 is the coordinate of the half-plane), the retarding force increases almost linearly, reaches a maximum (not depending on the value of γ) at approximately $y = 0.7\, a\gamma$, and then falls hyperbolically. Thus, the retarding field arising at the instant of passing the half-plane, which has the form of a pulse with a length of the order of $a/\gamma c$, is first propagated in such a way to overtake the source of excitation, after which dephasing gradually takes place, owing to the slight difference in velocities, and the field outstrips the filament. This retardation process occupies a time $t \approx a\gamma/c$. It is reasonable to suppose that this picture will remain qualitatively the same in any other case in which a single obstacle is excited, for example, in that of a resonator with feeding waveguides, and so on.

If a second obstacle is encountered at a certain distance L_1 after the first, then if the inequality $L_1 \ll 0.7\, a\gamma$ is satisfied it would appear reasonable to treat both obstacles as a single one, and in practice the foregoing picture of the retardation process will change very little. However, in the computing respect, the question as to the excitation of a complex system is made particularly difficult by the necessity of allowing for the screening of one element by another. For $L_1 \gg 0.7\, a\gamma$ the retarding effect of the second obstacle will in practice be independent of that of the first.

We have already pointed out that, for our own purpose of accelerating a ring to ultrarelativistic velocities, the accelerating system should comprise a series of accelerating elements, i.e., it should have a periodic or quasiperiodic structure. Hence, an extremely important problem is that of determining the radiative losses in such structures.

It was shown a long time ago [34] that the energy losses in such cases were of a resonance nature. However, up to the present time there has been no convincing success in elucidating the asymptotic dependence of the losses on the parameter γ.

So far as we know, the only exact solution which has been obtained relates to the excitation of an infinite "comb" of half-planes by a "charged filament" [35]. However, even in this case the analysis of the solution presents substantial difficulties. The asymptotic formula for the losses associated with one period of the structure [35] takes the form ($\gamma \gg 1$)

$$W = \frac{2\varkappa^2 D}{a}\cdot\frac{\beta}{\gamma}, \qquad \text{(III.50)}$$

where D is the period of the structure, a is the target (aiming) parameter, $\varkappa$ is the charge per unit length of the filament. However, this formula only allows for the contribution of the high frequencies $ka \geq \gamma$. Numerical calculations were carried out elsewhere for this type of structure [35, 20]. The calculations show that the losses diminish with increasing γ.

Calculations based on the exact formulas in the nonrelativistic and slightly relativistic cases [35] agree closely with those based on (III.50). In one case [29] the relativistic problem ($5 \leq \gamma \leq 200$) yielded a fall in losses varying as $\gamma^{-1/2}$.

The calculation carried out by Sessler [20] also shows a close agreement between the total losses per structure period calculated by integrating the retarding force created by the secondary excited field and the estimated amount of energy radiated by this field in the gap between the corresponding two planes of the structure.

It should be noted that a comb constitutes an open structure, and thus some of the excited waves pass out of the structure into space, and the main contribution to the force retarding the group of electrons arises from the delayed surface waves propagating at a phase velocity equal to the velocity of the particles.

An analogous picture of the retardation of a group by a surface wave is obtained for other open structures. Thus, an approximate solution of the integral equations determining the currents flowing in various elements of the structure [36] yielded asymptotic estimates of the losses for fairly large γ in a system comprising an infinite number of plane screens with apertures of radius a in the center, and also in a system comprising an infinite series of drift tubes. For the first structure, the total loss of energy by the group of particles at a frequency ω on passing through a distance equal to one period D is given by the equation

$$W_{\omega}^{\mathrm{I}}=\frac{2qk}{c\beta\gamma}\,\mathrm{Re}\left\{\int_{a}^{\infty} j_{r\omega}^{0}(r)\,K_1\left(\frac{kr}{\beta\gamma}\right) r\,dr\right\}, \tag{III.51}$$

and for the second structure by

$$W_{\omega}^{\mathrm{II}}=-\frac{2qak}{c\beta^{2}\gamma^{2}}\,K_0\left(\frac{ka}{\beta\gamma}\right)\mathrm{Im}\left\{\int_{0}^{d} j_{z\omega}^{0}(\xi)\,e^{-i\frac{\omega}{c\beta}\xi}\,d\xi\right\}, \tag{III.52}$$

where d is the length of the tube and a is its radius.

Equations (III.51) and (III.52) are exact, and may be used to obtain the corresponding numerical values of W_{ω}^{I} and W_{ω}^{II} if we know the values of the Fourier components of the currents $j_{r\omega}^{0}(r)$ and $j_{z\omega}^{0}(\xi)$ flowing in any arbitrarily-chosen element of the structure. For $\gamma \gg 1$ estimates of the currents may be obtained by the stationary-phase method, and the corresponding formulas for the losses, integrated over the frequency range within the limits of $ka \geq 1$, are as follows:

$$W^{\mathrm{I}}=\frac{5}{24\pi}\cdot\frac{q^{2}D}{a^{2}}; \tag{III.53}$$

$$W^{\mathrm{II}}\approx A\frac{q^{2}d}{a^{2}\gamma}, \tag{III.54}$$

where A is a certain numerical factor (not derived in [36]).

If the source of excitation is an infinitesimally thin ring of radius r_0, then instead of (III.53) we have

$$W^{\mathrm{I}}\approx 10^{-2}\,\frac{q^{2}D}{a(a-r_0)}. \tag{III.55}$$

The same computing method gives the following expression for the loss of energy by a charged filament in a comb

$$W=\frac{\varkappa^{2}D}{16a}. \tag{III.56}$$

For $(a/D) < 1$ the latter formula in no way contradicts the earlier numerical results [35, 20], since the quantitative estimates based on Eq. (III.56) lie below these figures.* Equation (III.56) evidently corresponds to a more strongly relativistic situation, and the difference between this and Eq. (III.50) may be explained by the fact that (III.56) allows for the range $\gamma \geq ka \geq 1$, which was neglected in (III.50).

*We note, however, that the results of the earlier paper [20] imply an approximately quadratic dependence on the ratio a/D (for $0.5 \leq (a/D) \leq 2$).

By basing our considerations on the foregoing discussion as to the retardation of a charge by a single obstacle, we may (at any rate for open structures) qualitatively represent the retardation of charge in a structure as follows. If the period of the structure $D \gg \gamma a$ [here a is the target parameter, or in the case of a ring $(a-r_0)$], the retarding effect of each structural element will manifest itself independently, and the losses per structural period will be proportional to γ. When we increase γ far enough to satisfy the opposite relationship $(\gamma a \gg D)$, the interaction between the structural elements during radiation becomes very substantial, and the curve relating the losses to γ develops a plateau. Of course any real accelerating system is closed.

The qualitative picture of the losses (the dependence of the losses on γ and the fact that the charge is retarded by the accompanying wave) in closed structures is plainly the same as in the open structures just considered. Only quantitative changes are to be expected, although these are of course important for designing any specific accelerating system.

However, any calculation of the energy losses associated with the motion of a charge through closed systems (such as diaphragmed waveguides or a system of resonators linked by drift tubes) is extremely difficult, and can certainly only be carried out numerically on an electronic computer, occupying a very considerable amount of machine time.

Nevertheless, for a structure consisting of narrow resonators connected by tubes, an analytical formula was obtained in [37] with due allowance for the low-frequency part of the losses experienced by a point charge ($ka \leq 1$, a is the radius of the apertures):

$$W = \frac{q^2}{2a}\left(\frac{d}{a}\right)\left(\frac{d}{D}\right), \tag{III.57}$$

where d is the length of a resonator, D is the period of the structure $(d/a < 1)$; this formula may be regarded as a reasonable approximation for the slightly-relativistic case.

Numerical computer calculations [38] were employed in order to analyze the more general case of a system of coupled resonators excited by a ring. Allowance was made for a larger range of wavelengths than in the earlier case [37] ($ka \leq 30$), and the results were extended to the region of $\gamma \leq 50$. The resultant numerical values of the losses were almost independent of γ. It should nevertheless be noted that the calculation made no allowance for a certain range of high frequencies which might well make a finite contribution, since, according to (III.39), the truncating parameter $k(a-r_0)/\gamma$ (r_0 = radius of the ring) is much smaller than unity in the case in question, and its effect is not particularly obvious. This may very well explain why the results obtained [38] vary little with the distance of the ring from the walls of the waveguide.

A comparison between the results obtained for a closed system and the earlier results obtained for an open system of the comb type [20] shows that the losses are several times greater in the closed case. Thus, for example, the ratio of the energy lost per structural period by unit length of ring in a heavily diaphragmed waveguide to the square of the linear charge density of the ring equals two in [38], while in the case of a comb [20] the same ratio equals 0.5. In both cases the ratio of the target parameter (for a ring this is $a-r_0$) to the structural period equals 0.5.

All the foregoing arguments lead to the following conclusions: firstly, the energy lost by a group of electrons (e.g., a ring) as radiation in each period of the structure does not, at any rate, actually increase with increasing relativistic factor γ. Secondly, apart from this qualitative type of conclusion, we cannot at the present time specify any particular general law for determining the extent of these losses, so that careful loss calculations must be carried out on a computer for each specific case, although this will demand a great deal of machine time.

IV. Focusing of an Electron Ring

1. General Comments. In a collective accelerator the ions are captured and contained by the potential well of an electron group, which is itself accelerated by external fields. It is therefore quite clear how important it is to keep the electron density in the group or ring sufficiently high to ensure the required depth of the potential well. The efficiency of the collective method of acceleration largely depends on the solution of this problem.

At the present time the most promising arrangement is that in which the group of electrons constitutes a ring of toroidal shape, formed by relativistically rotating electrons. The repulsion between the electrons in such a ring associated with the Coulomb charge is considerably weakened by the magnetic attraction. In principle no external focusing is really needed if the self-focusing condition $N_i = N_e/\gamma_\perp^2$ is satisfied [2]; however, focusing is certainly necessary until the condition of self-focusing has been established, and even after this condition has been satisfied it is very desirable, since the ring in the potential well is stable with respect to a large number of types of perturbation. Calculations (Sec. V) show that in certain cases a potential well eliminates hydrodynamical instabilities. Subsequently, we shall only consider focusing by external fields and image forces.

For a ring of toroidal shape the requirement that the density should be preserved is equivalent to the necessity of keeping the cross-sectional dimensions small. The major radius of the ring and the radial dimension of the cross section may be kept constant fairly efficiently by means of a magnetic field. We shall therefore devote our principal attention to focusing in the axial direction. Let us make a brief analysis of possible methods of focusing. The simplest method of focusing would appear to be that of using the traveling-wave automatic-phasing effect. However, this method is not very effective for accelerating rings, since it requires the use of powerful generators, the creation of a retarding system, and also a change in the velocity of the wave during the acceleration process. Furthermore, since the gradients of the focusing field in the intrinsic coordinate system fall off as $1/\gamma_\parallel^2$, for a focusing-field amplitude of $\mathscr{E}_0 \approx 100$ kV/cm and $\lambda \leq 10$ cm focusing is only possible up to $\gamma_\parallel = 4$.

Different methods of focusing may find a practical use in different versions of the collective method of acceleration (the acceleration of heavy ions or the acceleration of protons to extremely high energies). We shall consider focusing by means of the azimuthal component of the magnetic field (H_φ focusing), focusing based on the use of the high-frequency Miller potential well involving oppositely directed waves, and focusing based on the use of image forces [39].

2. H_φ Focusing. The mechanism of H_φ focusing may readily be understood by considering a straight filament in a longitudinal magnetic field. Under the influence of the intrinsic repulsive forces the particles in the filament acquire velocities perpendicular to the direction of the magnetic field; the latter turns the particles, not allowing them to leave the filament. In the case of a ring the azimuthal component of the magnetic field (H_φ), created, for example, by a current-carrying conductor with the current directed along the z axis, may provide this longitudinal field.

The focusing conditions for a ring moving along the axis are given by two inequalities:

$$\left.\begin{aligned} \omega_{H_\varphi}^2 &> \Omega_\Lambda^2/\gamma_\perp^2\,\gamma_\parallel^2, \\ \Omega_\Lambda^2/\gamma_\parallel^2 &> \omega_{H_z}^2 - 2\omega_{H_z}\omega_{H_\varphi}\gamma_\parallel, \end{aligned}\right\} \qquad \text{(IV.1)}$$

where Ω_Λ is the Langmuir frequency of the ring, $\omega_{H_\varphi} = eH_\varphi/m\gamma_\perp c$; $\omega_{H_z} = eH_z/m\gamma_\perp c$, Ω_Λ, and $\gamma_\perp$ are taken in the intrinsic coordinate system of the ring.

An analysis of Eq. (IV.1) shows that, on increasing the relativistic factor $\gamma_\parallel$ with a constant field H_z and H_φ, the major radius of the ring becomes greater. This limits the length of the acceleration path of the ring for the type of focusing in question.

The limitation may be removed if as $\gamma_\parallel$ is increased the value of the H_φ field component (in the laboratory coordinate system) is correspondingly lowered.

3. Focusing Based on Oppositely Directed Waves. There is also the so-called Miller mechanism of focusing, which is capable of taking place in a system of two waves moving in opposite directions. It is easy to show that in this case a unique value of the particle velocity may be defined, i.e., we may find an equilibrium particle around which all the other particles execute stable oscillations. A system of this kind has a number of advantages. Let us suppose that in the laboratory coordinate system we have an external "corrugated" magnetic field with a phase $\psi_0 = k_0'z'$, while a wave with a phase $\psi = k'z' - \omega't'$ travels to meet the ring. Let us transform to a system in which the frequencies of these waves coincide; then

$$\left.\begin{aligned} &\omega = \gamma_\parallel(\omega' - vk'); \; k = \gamma_\parallel(k' - v'\omega); \\ &\omega_0 = \gamma_\parallel v k_0'; \; k_0 = \gamma_\parallel k_0' \end{aligned}\right\} \qquad \text{(IV.2)}$$

and $\omega = \omega_0$; hence

$$c\beta_z = \frac{\omega'}{k' + k_0'} < 1. \qquad \text{(IV.3)}$$

We accordingly see that such a system requires no retardation; the value of $\omega'/k' + k_0'$ may be varied as acceleration proceeds by using k_0', which is very convenient. If we take $k' \gg k_0'$, $\omega' \approx k'c'$, then it is easy to find $\dot{\gamma}_\parallel \approx \sqrt{\frac{k'}{2k_0'}}$; hence any error in k_0' affects $\gamma_\parallel$, while the error in the velocity (or the condition of exact synchronism between the potential well and the particle) equals $\delta\beta_z \approx 1/\gamma_\parallel^2 \cdot \delta\gamma_\parallel/\gamma_\parallel$, i.e., it diminishes with increasing $\gamma_\parallel$. This demonstrates the usefulness of the method in question for focusing at ultrarelativistic velocities.

Let us write down the equation of the z oscillations in the intrinsic system (as an example, we consider a wave of the H_{01} type):

$$\ddot{z} + \frac{e\beta_0}{m\gamma_\perp}\left(\frac{k}{k'} h_1 \cos\psi - \gamma_\parallel h_0 \cos\psi_0\right) = 0, \qquad \text{(IV.4)}$$

where $h_1 = H_1 J_1(kr_0)$ is the amplitude of the traveling wave; $h_0 = H_0 I_0(kr_0)$ is the amplitude of the corrugated field.

We assume that the time required for changes in the external parameters is large compared with the period of the averaged oscillations. This equation may be solved by the Bogolyubov method. We find a well with the following characteristics:

$$\omega^2 \gg \frac{e^2 h_1 h_0}{m^2 c^2 \gamma_\perp^2} > \frac{2}{\gamma_\perp^3} \cdot \frac{c}{a_0^2} \cdot \frac{e^2 N_e}{mc^2 2\pi r_0}; \ \omega' \approx k', \ k' \gg k_0'. \qquad \text{(IV.5)}$$

Here a_0 is the small dimension of the ring (cross-sectional radius). The left-hand equation is the condition for the applicability of the method; the right-hand equation constitutes the condition for the compensation of the forces of Coulomb repulsion between the electrons.

The use of this method will be quite feasible if we can obtain powerful sources of radiation in the short-wave range (in order to hold a ring with $N_e = 10^{13}$, we require a traveling wave with an amplitude of about 1 kOe).

4. Focusing by Image Forces. We shall pay rather more attention to the details of this method, since it constitutes the most practical method of focusing in all known installations of the kind in question.

The motion of the electron ring during acceleration takes place under conditions of close screening: in all systems capable of practical use the ring is surrounded by a metal tube or a screen of more complicated configuration.

The interaction of a charged ring with a screen is of a "coherent" nature, i.e., the force acting on each particle is proportional to the number of particles within the ring. This emphasizes the necessity of allowing for screening when considering the motion of the particles in the ring and the behavior of the ring as a whole. One particularly interesting fact is that the screening may be used in order to focus the beam. Let us consider a straight charged filament formed by a flux of electrons and screened by a conducting metal plane. If the distance from the center of the filament to the plane is much greater than its small dimension, then the image field may be replaced by the field of an infinitesimally thin filament formed by charged particles of the opposite sign.

An elementary construction of the forces of interaction between the "image" filament and the extreme particles of the real one gives the focusing force. The value of this force would be much smaller than the forces of Coulomb repulsion between the particles if there were no longitudinal motion of the particles. As a result of this motion, the intrinsic repulsive forces in the beam are weakened by a factor of $\gamma_\perp^2$ and there is a clear possibility of compensating the Coulomb repulsion by the image forces. A filament moving as a whole in a direction perpendicular to the direction of motion of the particles within it may be considered as a "straight" model for a ring traveling in a screening tube. In this case, however, the electric field of the filament becomes time dependent, and this leads to the appearance of a magnetic screening field, which weakens the focusing image forces by a factor of $\gamma_\perp^2$ and makes them smaller than the intrinsic defocusing forces. The use of the image forces for focusing is nevertheless very attractive in view of the fact that focusing by this method requires no supplementary power sources and is independent of the velocity of the ring up to fairly large values of $\gamma_\|$. This is why attempts have been made at finding systems which will retain the focusing effect of the electric screening while reducing the defocusing effect of the magnetic screening.

If we consider a ring screened by a cylinder, we find that the geometrical curvature itself leads to the desired result. In the intrinsic fields, the curvature may be neglected only if $1/\gamma_0^2 \gg (a_0^2/r_0^2)\ln(8r_0/a_0)$, whereas in the induced fields the corresponding condition is $1/\gamma_\perp^2 \gg (a-r_0/r_0)\ln(r_0/a-r_0)$; we may thus find ourselves able to create conditions under which the intrinsic field may be regarded as "straight" (weakened by a factor of $\gamma_\perp^2$) while the induced field is "curved" (and thus not weakened at all). Here a is the radius of the screening tube. The force gradient of present interest takes the following form in this case:

$$\frac{1}{\omega_0^2 m\gamma_\perp}\cdot\frac{\partial F_z}{\partial z}=-\frac{4e^2}{mc^2}\cdot\frac{1}{\gamma_\perp}\cdot\frac{N_e}{2\pi r_0}\int_0^\infty dt\cdot t^2\left[I_0^2(t\xi)\frac{K_0(t)}{I_0(t)}\right.$$

$$\left.-\beta^2 I_1^2(t\xi)\frac{K_1(t)}{I_1(t)}\right]=-\frac{4e^2}{mc^2}\cdot\frac{1}{\gamma_\perp}\cdot\frac{N_e}{2\pi r_0}T_z, \qquad \text{(IV.6)}$$

where I_n and K_n are modified Bessel functions. In the range $0.8 < \xi = r_0/a < 0.95$, $0.4 < T_z < 0.8$. The force gradient of the Coulomb repulsion is $\approx (4e^2/mc^2)\cdot(1/\gamma_\perp)\cdot(N_e/2\pi r_0)\cdot 1/[b_c(g+b_c)]\cdot(1/\gamma_\perp^2)$. For $N_e = 10^{13}$, $\gamma_\perp = 30$, $g = 0.02$, $r_0 = 5$ cm, and $\xi = 0.8$ may be achieved with a focusing parameter $b_c \approx 0.2$, while the ratio of the square of the frequency of the betatron oscillations in the z direction to the square of the frequency of rotation of a particle in the ring will then be of the order of $3\cdot 10^{-3}$. It thus follows that, in order to be able to use this type of focusing in practice, it is essential to increase its efficiency.

It is clear that the focusing efficiency increases as the defocusing effect of the screening magnetic field diminishes. Of the various possible ways of reducing the magnetic screening we select the following [40].

We may suppose that, if the metal tube is coated on the inside with a thin layer of dielectric with a fairly high dielectric constant, then the reflection of the electric field will occur at the radius of the dielectric, while the reflection of the magnetic field will occur at the radius of the metal. Since the layer of dielectric is situated closer to the ring than the metal, the contribution of the electric field to the force acting on the particles in the ring will be greater than that of the magnetic field.

This proposition appears very interesting; however, a system incorporating a dielectric may lead to serious losses by way of Cerenkov radiation, and this constitutes its weak point.

Dolbilov [41] proposed a system very effectively reducing the effects of the magnetic screening. This system proposes using a metal cylinder cut into strips along the generators ("squirrel cage"). The ring as a whole moves coaxially with the cylinder; hence, this system does not form a retarding configuration, and no Cerenkov radiation is produced.

A screen of the "squirrel-cage" type gives the following expression for the gradient of the axial force component:

$$\left.\begin{aligned}&\frac{1}{\omega_0^2 m\gamma_\perp}\cdot\frac{\partial F_z}{\partial z}=-\frac{4e^2}{mc^2}\cdot\frac{1}{\gamma_\perp}\cdot\frac{N_e}{2\pi r_0}\left[\widetilde{T}_0^z+\sum_{n\neq 0}^{\infty}\widetilde{T}_n^z\, e^{i\,kn\varphi}\right];\\&\widetilde{T}_0^z=\xi^3\int_0^\infty dt\cdot t^2\left[I_0^2(t\xi)\frac{K_0(t)}{I_0(t)}(x_0^{э}-1)\right]\\&-\beta^2\xi^3\int_0^\infty dt\cdot t^2 I_1^2(t\xi)\cdot\frac{K_1(t)}{I_1(t)}x_0^{M}\equiv T_{0.e}^z+T_{0.m}^z\end{aligned}\right\} \qquad \text{(IV.7)}$$

Here k is the number of cuts in the screen.

TABLE 1

ξ	$T^z_{0.e}$	$T^z_{0.m}$	$\tilde{T}^z_0$
0,8	1,84	−0,16	1,68
0,6	0,28	−0,009	0,27

The coefficients T^z_n are determined by the corresponding integrals of $x_n^{e.m}$. Analogous expressions are obtained for the force gradient in the radial direction. The determination of $x_n^{e.m}$ involves the substitution of the boundary conditions at the screen. These conditions lead to the following functional equations:

$$\left.\begin{aligned}&\sum_n e^{i\,nk\varphi}\,x_n^{e.m}=0 \quad \text{for} \quad \frac{\pi q^{e.m}}{l}<|\varphi|<\pi;\\ &\sum_{n\neq 0} e^{i\,nk\varphi}\,|n|\,(1-\varepsilon_n^{e.m})\,x_n^{e.m}\\ &=-\varkappa^{e.m}(1-x_0^{e.m}) \quad \text{for} \quad |\varphi|<\frac{\pi q^{e.m}}{l};\\ &\varepsilon_n^{e}=1-\frac{1}{2\,|n|\,kI_s(t)\,K_s(t)}; \quad \varepsilon_n^{m}=1+\frac{2tI_s'(t)\,K_s'(t)}{|n|\,k};\\ &\varkappa^{e}=\frac{l}{4\pi aK_0(t)\,I_0(t)}; \quad \varkappa^{m}=-2t^2I_1(t)\,K_1(t),\end{aligned}\right\} \qquad \text{(IV.8)}$$

where q^e is the width of a strip, q^m is the width of the slit, $l = q^e + q^m$; $s = nk$. This system may be solved by means of an electronic computer.

The possibility of finding a solution to a high accuracy depends on having a small parameter, $\varepsilon_n^{e.m}$, which possesses the following property: $\varepsilon_n^{e.m}\to 0$ as $|n|\to\infty$.

In order to realize this focusing method experimentally we must choose optimum values for ξ, k, and $\zeta = \pi q^e/l$. Let us take $\xi = 0.8$. It is clear that with increasing distance between the wall and the ring the focusing force diminishes.

Table 1 shows how the quantities $T^z_{0.e}$, $T^z_{0.m}$, and T^z_0 vary with ξ for k = 30 and $\zeta = \pi/2$.

On approaching the screen, instability of the ring as a whole may develop; this increases the demands made upon the accuracy with which the ring is set relative to the tube axis. If the ring originally deviated from the axis, then on satisfying the inequality

$$\frac{8e^2}{mc^2}\cdot\frac{1}{\gamma_\perp}\cdot\frac{N_e}{2\pi r_0}\cdot\frac{1}{\beta^2}\cdot\xi^2\Phi(\xi)\leqslant 1$$

it will not be drawn to the wall, but will execute an angular drift around the tube axis. The function $\Phi(\xi)$ is nonlinear. For $\xi = 0.5$, $\Phi(\xi)\approx 1$, for $\xi = 0.8$, $\Phi(\xi) = 4.5$, and for $\xi = 0.95$, $\Phi(\xi)\approx 140$. However, even on satisfying the foregoing inequality, the deviation of the center of the ring from the axis of the tube should never exceed distances of the order of the smaller radius, in order to prevent the latter from "swelling" as a result of the drift of the ring.

It is convenient to make the lengths of the gaps and strips equal ($\zeta = \pi/2$). For $\zeta = 0$ the "squirrel cage" transforms into a continuous cylinder and the focusing force is simply determined by the difference between the electric and magnetic forces associated with the curvature of the system. The value of $\zeta = \pi$ corresponds to the ring in free space. Calculations show that the focusing force depends very little on ζ when ζ is close to $\pi/2$. The number of divisions is determined by the dependence of the amplitudes of the harmonics of the focusing force on this number. Estimates indicate that

$$T^z_{(n+1)e}/T^z_{ne}\sim(\xi)^k.$$

Numerical calculations confirm this estimate (see Table 2, which represents the values of T^z_{ne} and T^z_{nm} for different n with k = 30, $\xi = 0.8$, and $\zeta = \pi/2$).

With increasing number of divisions the focusing force becomes greater and the defocusing force smaller. Table 3 gives the values of $T^z_{0.e}$, $T^z_{0.m}$, and T^z_0 for various k and $\xi = 0.8$; $\zeta = \pi/2$.

TABLE 2

n	T^z_{ne}	T^z_{nm}	n	T^z_{ne}	T^z_{nm}
0	1,844	—0,1657	2	$0{,}36\cdot10^{-6}$	$-0{,}39\cdot10^{-5}$
1	$0{,}53\cdot10^{-3}$	$-0{,}30\cdot10^{-2}$	3	$0{,}49\cdot10^{-9}$	$-0{,}76\cdot10^{-8}$

TABLE 3

k	$T^z_{0.e}$	$T^z_{0.m}$	$\tilde{T}^z_0$
5	1,25	—0,38	0,87
10	1,61	—0,36	1,25
30	1,84	—0,16	1,68

Thus, the "squirrel cage" constitutes an anisotropic screen which holds back the axial electric field and passes the normal magnetic field. The improvement in the anisotropy with increasing number of divisions may be understood in the following manner. For $\zeta = \pi/2$ and a small number of divisions, the distance between the strips is fairly large, and some of the axial electric field penetrates between them and passes outside. On increasing k, the number of lines of force characterizing the electric field increases, the field being defined by the charges on the strips (charges at the edges); this means that the field inside the system increases.

The defocusing magnetic force in F_z is associated with the component H_r, for which the condition $H_r = 0$ should be satisfied in the metal strip. This means that the H_r component of the magnetic field passes freely into the gap between the strips, curving considerably around them. The field associated with the curvature of the magnetic lines of force around the middle of the strip in general remains inside the system (if k = 0, the whole magnetic field is reflected). On increasing k and keeping ζ constant, the part played by the edges of the strips increases more and more, and so does the associated protrusion of the magnetic field.

For these reasons the parameters chosen for the focusing system of the Joint Institute for Nuclear Research model of a collective linear ion accelerator are as follows: $\xi = 0.8$; $\zeta = \pi/2$; and k = 30. This gives $T^z_0 = 1.68$ and $T^r_0 = 1.33$ and ensures the same dimensions of the ring on acceleration as occur in the adhesor at the end of the compression process.

The use of image-force focusing is clearly limited in the case of large $\gamma_\|$, for which the effect of the real conductivity of the chamber walls on the motion of the ring becomes appreciable. An estimate gives a maximum value of $\gamma_\| \sim 40$.

The foregoing focusing methods are of course not exhaustive. Many laboratories are even now looking for different methods. Among these investigations the complicated and independent problem of obtaining a stabilized ring (Budker) occupies a leading place.

V. Stability of an Electron-Ion Ring

In order to achieve the collective method of acceleration it is essential to hold the parameters of the electron-ion ring within a specified range of values. The characteristic times required for these parameters to undergo changes as a result of various perturbations should be much greater than the acceleration time. It is practically impossible to determine this characteristic time; hence, in analyzing the stability we are compelled to take various model representations and study those cases which are deemed to be the most dangerous.

In considering the foregoing means of creating and accelerating an electron ring, it is natural to divide the problem of stability into two. The first of these is the problem of the stability of the electron ring (without ions) in a magnetic field with weak focusing; the second is a study of the stability of an electron-ion ring (charged plasma).

In this section we shall set out the results (by no means completely) of an analysis of these problems based on earlier review articles [42, 43].

In considering the instabilities of a one-component system (the electron ring), we classify the instabilities arising as, respectively, single-particle and coherent. By "single-particle" we mean instabilities associated with the motion of an individual particle in an external field and the field of all the other particles, which is regarded as being given. Single-particle instabilities are associated with a frequency shift of the betatron oscillations, arising from space charge and resonance perturbations of the external field. We have already given a description of these instabilities. Here we shall consider certain "coherent" instabilities. These instabilities are characterized by a change in both the motion of the particles and the intrinsic fields during the development of the instability. The method of studying such instabilities theoretically lies in solving the Vlasov system of self-consistent equations.

Coherent instabilities of the electron ring are divided into longitudinal (associated with the azimuthal motion of the particles) and transverse. We distinguish three types of longitudinal instability:

1) instability of the negative-mass type (NMI) [44, 45];

2) radiative instability RI (induced cyclotron radiation) [46, 47];

3) resistive longitudinal instability [48].

The main physical characteristic of the first of these instabilities is associated with the specific dependence of the particle rotation frequency in a magnetic field on the energy. If the rotation frequency falls with increasing particle energy, as occurs, for example, in a weakly focusing magnetic field, then the particle moves in the opposite direction to the force under the action of the azimuthal forces, just as if its mass were negative. In association with this, there may be a self-bunching of the particles and an increase in density fluctuations. The mean velocity of the beam remains constant, and the instability is not accompanied by the emission of propagating electromagnetic waves.

An analysis of the negative-mass type of instability in the collective linear method of acceleration is especially important at the initial stage of ring formation, when the ring is close to the cylindrical walls of the chamber. This kind of geometry differs greatly from that traditionally employed in ordinary accelerators, and, since the increment of the negative-mass instability depends on the geometrical factor, special calculations allowing for the effects of specific screening are essential. Calculations of the increments were carried out for a monoenergetic, infinitely thin cylindrical electron beam (an E layer) rotating with relativistic velocity in a constant magnetic field. The beam is surrounded by an infinitely conducting cylindrical screen situated coaxially with respect to the beam [49, 50]. The following expression was obtained for the negative-mass-instability increment (gain) [49]:

$$\mathrm{Im}\,\omega = \Omega \frac{n}{\gamma_\perp} \sqrt{\frac{2(a-r_0)}{r_0}}\,. \tag{V.1}$$

Here

$$\Omega = \sqrt{\frac{2\pi r^* \sigma_0 r_0 \omega_0^2}{\gamma_\perp e}}\,;$$

σ_0 is the surface electron density, a is the radius of the cylindrical screen; n is the number of the harmonic of the azimuthal perturbation of the electron beam; ω_0 is the electron rotation frequency. In calculating the increment of the negative-mass instability it was assumed that the following inequality held:

$$\frac{a-r_0}{r_0} \ll \frac{1}{\gamma_\perp^2}\,, \tag{V.2}$$

this occurring when the beam is immediately next to the screen and $\gamma_\perp$ is not too high. We see from Eq. (V.1) that the geometrical factor $\Lambda = \sqrt{\frac{2(a-r_0)}{r_0}}$ may be very small owing to the influence of the screen. It was shown elsewhere [50] that on satisfying the condition opposite to (V.2) no negative-mass instabilities arose at all. For beams involving a finite energy spread, on satisfying (V.2) the negative-mass increment falls in relation to (V.1), and an instability threshold appears [51]. The threshold spread may be found by using existing results [44, 45] and the geometrical factor indicated above.

In another paper [51] the negative-mass increment was also calculated for a very thin ring (the ring dimension l along the generator of the cylindrical screen being considered much smaller than the distance from the ring to the screen). In contrast to the case of the E layer, for this kind of ring (without allowing

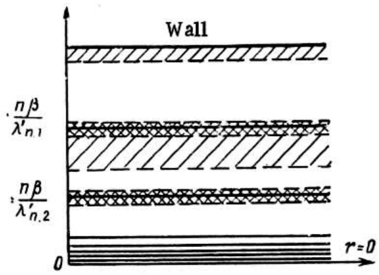

Fig. 1. Region of negative-mass and radiation instability. Values of r/b are shown along the vertical axis.

for the energy spread) no instability cutoff occurs in the relativistic beam. If we take equal values of $2\pi r_0\sigma_0$ and N/l, the increment calculated for the ring is a factor of $\sqrt{\frac{l}{a-r_0}}$ smaller than (V.1). Thus, quite a small energy spread is needed in order to suppress negative-mass instability in a ring lying close to the wall. This conclusion is qualitatively supported by experiment [52]. An experiment with a beam close to the screen shows that the initial azimuthal density fluctuations are smoothed out after a few rotations of the particles.

Subsequent consideration of negative-mass instability when the ring moves away from the screen involves computing difficulties. The regions in which this instability appears occur at ring-screen distances such that the harmonic of the mean particle rotation frequency approximately coinciding with the frequency of the perturbation wave differs fairly sharply from the arbitrary eigenfrequency of the chamber corresponding to this harmonic. In this case the condition for the radiation of transverse waves is not satisfied, and we find ourselves concerned with negative-mass instabilities in pure form [51]. It is convenient to describe the situation in terms of beam and chamber impedance, so as to identify the longitudinal instabilities [50, 51, 53]; thus, the regions in which negative-mass instability appears correspond to a capacitive impedance.

These regions are shown shaded in Fig. 1 for the harmonic with number n. The unshaded bands correspond to regions of longitudinal instability of the ring (inductive impedance).

One method of stabilizing negative-mass instability [50] is in fact based on increasing the inductive part of the impedance by means of inductive screens [50]. We also note the possibility of stabilizing this effect in the nonlinear stage of development. If the negative-mass increment is small in comparison with the energy spread in the beam, and the nonlinear interaction of the harmonics of the electromagnetic perturbation field may be neglected, we may use the quasilinear theory in order to analyze the development of negative-mass instability [54]. We find that the energy spread in this case increases with time and reaches a threshold value at which an instability cut-off occurs.

The cross-hatched regions (Fig. 1) represent the region of radiation instability (active impedance). Here we find resonance between the oscillations in the beam and the intrinsic modes of the chamber, and also electromagnetic radiation. The radiation instability was considered for the collective linear method of acceleration in earlier papers [55-60]. We may consider two cases:

1. The radiation practically occurs in only one mode of the chamber oscillations, when the beam is fairly close to the wall, and the time required for the development of the instability is much smaller than the time for the signal to pass from the ring to the wall.

2. In the condition opposite to the foregoing, the resonances merge into a continuous band and radiation occurs as in free space.

In the first case, for a monoenergetic beam, we have the increment [58]

$$\operatorname{Im}\omega=\left(\frac{cr^{*}N\omega_0\sqrt{n\omega_0}}{\gamma_{\perp}a^2}f(n)\right)^{2/5} \qquad (V.3)$$

and the real part of the frequency

$$\operatorname{Re}\omega=n\omega_0+\operatorname{Im}\omega, \qquad (V.4)$$

where $f(n)$ is a function increasing as $n^{4/3}$ up to $n\sim\gamma_{\perp}^3$ and then falling exponentially. We note that for real beams of finite thickness the results presented here are only valid for the first few harmonics, since it was assumed that the length of the perturbation wave was much greater than the thickness of the beam. For a large spread the increments diminish substantially; however, the instability does not have any threshold.

In the second case, the radiation-instability increment for a monoenergetic beam is smaller than (V.3), and furthermore a threshold exists [60]. The threshold number of particles is determined from the following inequality:

$$\frac{r^* N}{2\pi r_0 \gamma_\perp} < n^{2/3} \left(\frac{\Delta\omega_0}{\omega_0}\right)^2. \qquad (V.5)$$

Here $\Delta\omega_0$ is the scatter in the particle rotation frequencies. On making allowance for the nonlinear interaction of the harmonics in the development of radiation instability in a monoenergetic beam a long way from the screen we find that the instability passes very rapidly into the nonlinear stage with increasing energy spread [60].

All the results which we have just been mentioning were obtained on the assumption of infinite conductivity in the walls of the chamber. The effect of a finite Q in a chamber of the same geometry was discussed earlier [59]. The question as to the transverse instability of the electron beam was discussed by a number of authors [43, 61, 62].

It was shown in [61] that the existence of an infinitely conducting cylindrical screen eliminated the transverse instabilities of the E layer. The condition imposed on the $\Delta\nu_{r,z}$ spread in the betatron frequencies, leading to the suppression of the resistive transverse instability [63] in the case in which the side walls of the chamber exert a substantial influence, takes the form

$$\Delta\nu_{r,z} > \frac{r^* r_0 N}{2\pi\nu_{r,z} a_0^2 \gamma_\perp^3} + \frac{r^* r_0 N}{\pi\nu_{r,z}\gamma_\perp (\Delta h)^2}\left[\frac{r_0 c}{8\pi\sigma(n-\Delta\nu)_{r,z}(\Delta h)^2}\right]^{1/2}, \qquad (V.6)$$

where a_0 is the small dimension of the ring, Δh is the distance from the wall to the middle plane of the ring, and σ is the wall conductivity.

The case in which the electron ring lies close to the cylindrical surface of the chamber was considered in [62]. The maximum increment in the resistive transverse instability for a monoenergetic beam is

$$\operatorname{Im}\omega = \frac{r^* N}{2\pi r_0 \gamma_\perp}\left(\frac{r_0}{2a}\right)^2 \frac{c}{a}\left[\frac{\omega_0}{2\pi\sigma(1-\nu_{r,z})}\right]^{1/2}. \qquad (V.7)$$

For beams with an energy spread there is an instability threshold associated with Landau damping. The spread with respect to the longitudinal energy w required to suppress the instability has to satisfy the inequality

$$\Delta w > \frac{1}{2}\cdot\frac{\nu}{\gamma_\perp}\left(\frac{c}{a}\right)^2 \frac{1}{\frac{d}{dw}\left[\omega_0^2(w)(1-\nu_{r,z}(w))\right]\big|_{w=w(\omega_0)}}. \qquad (V.8)$$

Now let us turn to instabilities in the electron-ion ring. Here we confine attention to beam instability [64], the stability of the ring with respect to transverse bending [65], and the stability of a slightly inhomogeneous cylindrical layer [66].

Analysis of the beam instability is carried out by using the model of a cylindrical quasicentral plasma filament (pinch) through which a compensated beam of charged particles is moving. In this model no allowance is made for the intrinsic fields of the ring in the stationary state, nor for its curvature. The density of the beam is regarded as being much smaller than the density of the plasma, which in the case of a ring corresponds to the consideration of instability in a coordinate system linked to the electrons.

There are two mechanisms of exciting waves by a beam traveling through the plasma: the Vavilov-Cerenkov effect, and the excitation of waves due to the anisotropy of the particle velocity-distribution function. For three-dimensional waves with a wavelength much smaller than the radius of the filament, the increments in beam instability are the same as in an unlimited medium. In the cases of a relativistic monoenergetic beam of low density currently concerning us, the increment in the instability associated with Cerenkov radiation was given earlier [67]. It should be noted that in the case of a ring this instability may not arise at all if the condition $\omega_0\gamma\ /\Omega_e$ is satisfied, where Ω_e is the Langmuir frequency of the electrons. The excitation of an axially symmetric surface wave of the same type in a cylindrical beam is greatly impeded; the corresponding increments are exponentially small [68]. An aperiodic instability of the beam was studied elsewhere [68–70]. In the hydrodynamic approximation, using the linear theory, the instability increment for three-dimensional waves in the laboratory system of coordinates equals

$$\mathrm{Im}\,\omega = \Omega_i\,\gamma_\perp, \tag{V.9}$$

where Ω_i is the Langmuir frequency of the ions.

The increment of surface waves is only half as much as (V.9). The quasilinear theory of aperiodic instability shows that in the nonlinear stage the instability breaks off, the relative energy losses of the beam being of the order of $mn_i/Mn_e \ll 1$.

The stability of an electron–ion ring with a partly compensated charge and external focusing with respect to transverse bending was studied in [71, 72]. In these papers the model of two charged cylindrical filaments with a constant density over the cross section was taken, and the instability of the motion of the centers of mass of the filaments during the formation of "snakes" (characteristic configurations) was examined.

In the case currently concerning us, in which the number of ions is small and the condition $\eta = m\gamma/M \cdot n_0/n_i \gg 1$ [71], for the continuous spectrum of wave vectors k there is a region of instability

$$kR = \sqrt{n_{\text{eff}}} + \frac{\omega_i}{\omega_0} + s\sqrt{\frac{\omega_i^3}{\sqrt{n_{\text{eff}}}\,\omega_0^3\,\eta}}, \tag{V.10}$$

where $\omega_i = \frac{c}{a_0}\sqrt{\frac{2m}{M}\nu_e}$; n_{eff} is the effective magnetic-field fall-off index (allowing for the intrinsic field of the ring), while s varies over the range $-1 \le s \le 1$. The instability increment is

$$\mathrm{Im}\,\omega = \frac{\omega_i}{2}\sqrt{\frac{1-s^2}{\sqrt{n_{\text{eff}}}\,\eta}}. \tag{V.11}$$

In a ring the spectrum of wave vectors is discrete ($k = l/r_0$). Here l is an arbitrary whole number; hence, there will be no instability if no wave vector falls in the range (V.10). The conditions of ring stability take the form

$$2\sqrt{\frac{\omega_i^3}{\omega_0^3}} < 1 - \left\{\sqrt{n_{\text{eff}}} + \frac{\omega_i}{\omega_0} - \sqrt{\frac{\omega_i^3}{\sqrt{n_{\text{eff}}}\,\eta\omega_0^3}}\right\} \tag{V.12}$$

or

$$1 > \sqrt{n_{\text{eff}}} + \frac{\omega_i}{\omega_0} + \sqrt{\frac{\omega_i^3}{\sqrt{n_{\text{eff}}}\,\eta\omega_0^3}}, \tag{V.13}$$

where the symbol $\{A\}$ denotes the fractional part of the number A. Consideration of the Landau damping and radiative friction [71] showed that these effects did not lead to any stabilization of the instability.

Low-frequency potential oscillations in an electron–ion ring were considered for the case of a cylindrical layer of relativistic particles rotating in a uniform magnetic field in [66]. Certain simplifying assumptions were made: the frequency of the oscillations was considered to be much smaller than the cyclotron frequency of the particles, the phase velocity of the waves much smaller than the velocity of light, the momentum distribution of the particles along the magnetic field nonrelativistic, and the longitudinal wavelength much smaller than the radius of the layer. In this case, the methods of geometrical optics suffice to find the spectrum of oscillations for the layer, and the increment may be obtained from perturbation theory. The oscillations of the mode with $|n| = 1$ corresponding to ionic sound in a homogeneous plasma are unstable, the growth increment of the oscillations being

$$\mathrm{Im}\,\omega = \sqrt{\pi}\cdot\frac{n_e}{n_i}\,\frac{(\mathrm{Re}\,\omega)^2}{c\,|k_3|\,u_e}\cdot\frac{\int_0^\infty dsf\,\sqrt{1+u_{pe}^2\,s}\,\left|\frac{d\Phi}{ds}\right|^2}{\int_0^\infty dsfs\left|\frac{d\Phi}{ds}\right|^2}, \tag{V.14}$$

where k_3 is the wave number in the direction of the magnetic field, the function f describes the particle distribution across the magnetic field, u_{pe}^2 and u_e^2 are, respectively, expressed in terms of the mean squares of the transverse and longitudinal momenta of the electron, and Φ is the potential (in the zero approximation) with respect to a small parameter, namely, the ratio of the increment to the oscillation frequency ω. This kind of kinetic instability corresponds to drift instability in a weakly inhomogeneous nonrelativistic plasma. It was later shown [66] that, for a group of particles limited in the magnetic-field direction, the length of the group being much smaller than the ratio of the thermal velocity of the particles to the wave frequency, this instability could not develop at all. An analysis of the instabilities carried out for specific dimensions of the annular groupings (rings) used in experiments on the collective method of acceleration shows that no serious changes in the ring parameters take place during the period of acceleration.

CONCLUSION

We have thus shown that it is perfectly possible to employ the collective method in order to create charged-particle accelerators. We have considered only one particular system of acceleration, and only from the point of view of creating very-high-energy accelerators. This does not mean, however, that the collective method of acceleration is limited to this. The principle may clearly be used in order to create accelerators of very different types, starting with accelerators capable of being used for the heaviest ions and ending with accelerators yielding extremely high energies. Estimates show that accelerators with beams moving in opposite directions may also be based on the collective principle.

In order to illustrate the prospects of the method under consideration, we here present some general estimates relating to the use of a relativistically stabilized beam in a collective accelerator. For a strong electron beam compensated with ions in such a way that $N_i = N_e/\gamma_{\perp}^2$, magnetic compression gives rise to a dimunition in the beam cross section as a result of the conversion of some of the energy of the transverse motion of the particles into radiation. This reduction in cross section continue until the scattering of electrons by ions starts to play an appreciable part. A stationary (steady) state then sets in. The cross-sectional dimensions of the ring then amount to 10^{-3}–10^{-4} cm. In such a ring the maximum field strength acting on an ion may reach 10^9 V/cm. This means that if all the potentialities of the annular grouping of particles (the ring) are exploited it should be possible to make accelerators with dimensions of only 1 m for each 100 GeV.

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